Successes of SO(10) GUT Models

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KITP Talk
20 February 2003

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Introduction

Many models in literature attempt to explain only neutrino masses and mixings.

More ambitious attempts construct SUSY GUT models to understand both lepton and quark sectors.

My talk will be restricted mainly to $SO(10)$ GUT models with 3 families in 4 dimensions.

We'll see that several models are still quite successful in understanding the data.

Specific details will be given for one paticular model.

Some considerations will also be given to extensions to 5 dimensions.

SO(10) Model Structure

Essential Ingredients:

- 3 families of 16 LH q's and ℓ's $\rightarrow \mathbf{16}_i, \ i = 1, 2, 3$
- Higgs fields in $\mathbf{45}_H$, $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ are needed to break $SO(10) \rightarrow$ SM.
- 2 Higgs doublets $\rightarrow \mathbf{10}_H \supset 5 + \bar{5}$ of $SU(5)$ or $\rightarrow \mathbf{10}_H \supset (6,1,1) + (1,2,2)$ of $SU(4) \times SU(2) \times SU(2)$.
- Doublet-triplet splitting can be achieved via Dimopoulos-Wilczek mechanism, if $\langle \mathbf{45}_H \rangle$ points in $B - L$ direction.
- With only one $\mathbf{10}_H$ effecting the electroweak breaking, $\tan\beta \equiv v_u/v_d \sim 55$.

Additional Higgs fields may be desirable:

- $\mathbf{16}'_H$, $\overline{\mathbf{16}}'_H$ can help to stabilize doublet-triplet splitting.
- If $\langle \bar{5}(\mathbf{16}'_H) \rangle \neq 0$, then $H_d \sim \bar{5}(\mathbf{10}_H)\cos\gamma + \bar{5}(\mathbf{16}'_H)\sin\gamma$ and $\tan\beta \sim 1 - 55$ is possible.
- $\mathbf{126}_H$ and $\overline{\mathbf{126}}_H$ also possibilities.

Additional Matter Fields may be desirable:

- $\mathbf{16}$, $\overline{\mathbf{16}}$ pairs may get supermassive and can be integrated out in Froggatt-Nielsen type diagrams for the mass matrix elements.

Horizontal Flavor Symmetries

While $SO(10)$ relates q's and ℓ's of one family, it is necessary to invoke some horizontal flavor symmetry to avoid the bad $SU(5)$ relations: $m_d = m_e$, $m_s = m_\mu$. This can be done with 4 different levels of model building:

- Level 1: Simply impose a certain texture such as a modified Fritzsch form for the mass matrices.
- Level 2: Introduce an effective λ expansion for each mass matrix. The prefactors typically are not precisely determined, however.
- Level 3: Assign effective operators for each matrix element possibly with some flavor symmetry imposed.
- Level 4: Introduce a horizontal flavor symmetry which assigns flavor charges to every Higgs and matter superfield. Higgs and Yukawa superpotentials are constructed in terms of renormalizable (and possibly some non-renormalizable) terms which obey that flavor symmetry. Matrix elements then follow from Froggatt-Nielsen diagrams.

General Observations

- $SO(10)$ models differ by their choice of Higgs structure, horizontal flavor symmetry and flavor charge assignments (if any).
- Desirable Georgi-Jarlskog relations, $m_s = m_\mu/3, \; m_d = 3m_e$, can be obtained if $\langle \mathbf{45}_H \rangle$ points in the $B-L$ direction, or if $\langle \bar{5}(\mathbf{126}_H) \rangle$ is involved.
- Presence of $\langle \bar{5}(\mathbf{16}_H) \rangle$ and a flavor symmetry will typically lead to lopsided down quark and charged lepton mass matrices, D and L. This is useful to explain small V_{cb} and large $U_{\mu 3}$ but leads to enhanced $\tau \to \mu\gamma$ decay.
- Most early models were easily able to accommodate SMA solar solution, while some could accommodate LOW or QVO solution as well.
- To obtain the LMA solution, some fine tuning is generally required.
  - Typically models which require special features of the Dirac and right-handed Majorana mass matrices, N and M_R, to get maximal atmospheric mixing have trouble getting LMA.
  - Easier if M_R can be independently adjusted to get LMA, while N and L conspire to give maximal atmospheric mixing.
  - Several recent papers attempt to achieve the bi-large neutrino mixings from a doubly lopsided nature of the charged lepton mass matrix. The mixings and mass spectrum issues are then separated, but it is somewhat difficult to achieve diagonal Dirac and Majorana mass matrices for the flavor symmmetries imposed.

Example of One Predictive Model Leading to Maximal Atmospheric and LMA Solutions

Albright, Barr

Model is based on a Level 4 $U(1) \times Z_2 \times Z_2$ flavor symmetry from which the Higgs and Yukawa superpotentials can be constructed.

The structure of the Higgs sector with that flavor symmetry solves the doublet-triplet splitting problem. Barr, Raby

- Higgs Superfields

  $\mathbf{45_H}$,
  $\mathbf{16_H}, \; \overline{\mathbf{16}}_\mathbf{H}, \; \mathbf{16'_H}, \; \overline{\mathbf{16'}}_\mathbf{H}$,
  $\mathbf{10_H}, \; \mathbf{10'_H}, \; \ldots$,
  $\mathbf{1_H}$'s,

  where
  $\langle \mathbf{45_H} \rangle_{B-L}, \; \langle \mathbf{1}(\mathbf{16_H}) \rangle, \; \langle \mathbf{1}(\overline{\mathbf{16}}_\mathbf{H}) \rangle$ break $SO(10) \to$ SM;

  $v_u = \langle 5(\mathbf{10_H}) \rangle, \; v_d = \langle \bar{5}(\mathbf{10_H}) \rangle \cos\gamma + \langle \bar{5}(\mathbf{16'_H}) \rangle \sin\gamma$
  break EW symmetry.

- Matter Superfields

  $\mathbf{16_1}, \; \mathbf{16_2}, \; \mathbf{16_3}; \; \mathbf{16}, \; \overline{\mathbf{16}}, \; \mathbf{16'}, \; \overline{\mathbf{16'}}$
  $\mathbf{10_1}, \; \mathbf{10_2}$
  $\mathbf{1'}$s, ...

  where all but the $\mathbf{16_i}$, $i = 1, 2, 3$ get superheavy and are integrated out.

The mass matrices then follow from Froggatt-Nielsen diagrams involving vertex terms appearing in the superpotentials.

- Dirac Mass Matrices

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \quad D = \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D,$$

$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \quad L = \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D,$$

where

$$\begin{aligned} M_U &\simeq 113 \text{ GeV}, & M_D &\simeq 1 \text{ GeV}, \\ \sigma &= 1.78, & \epsilon &= 0.145, \\ \delta &= 0.0086, & \delta' &= 0.0079, \\ \phi &= 54^o, & \eta &= 8 \times 10^{-6} \end{aligned}$$

are input parameters defined at the GUT scale to fit the low scale observables after evolution downward from Λ_G.

- Above textures were obtained by imposing the Georgi-Jarlskog relations at Λ_G:

  $$m_s^0 \simeq m_\mu^0/3, \quad m_d^0 \simeq 3m_e^0$$

  and Yukawa coupling unification with $\tan\beta \sim 5$.

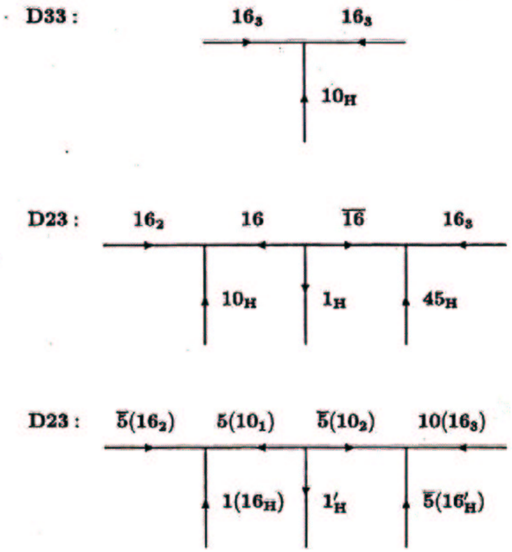
- Froggatt-Nielsen diagrams illustrate various contributions and features:

  "1" obtained from $\mathbf{16_3} \cdot \mathbf{16_3} \cdot \mathbf{10_H}$ vertices.

  "ϵ" obtained from $\langle \mathbf{45_H} \rangle_{B-L}$ suppression.

  "σ" obtained from the $\mathbf{16_2} \cdot \mathbf{16_H} \cdot \mathbf{16'_H} \cdot \mathbf{16_3}$ effective operator which contributes only to D and L in a lop-sided fashion.

Froggart-Nielsen diagrams can be drawn for the Dirac matrix elements with the left-handed conjugate fields on the left and the left-handed fields on the right.



- All 9 quark and charged lepton masses plus the 3 CKM angles and 1 phase are well-fitted with the 8 input parameters (after evolution from the GUT scale):

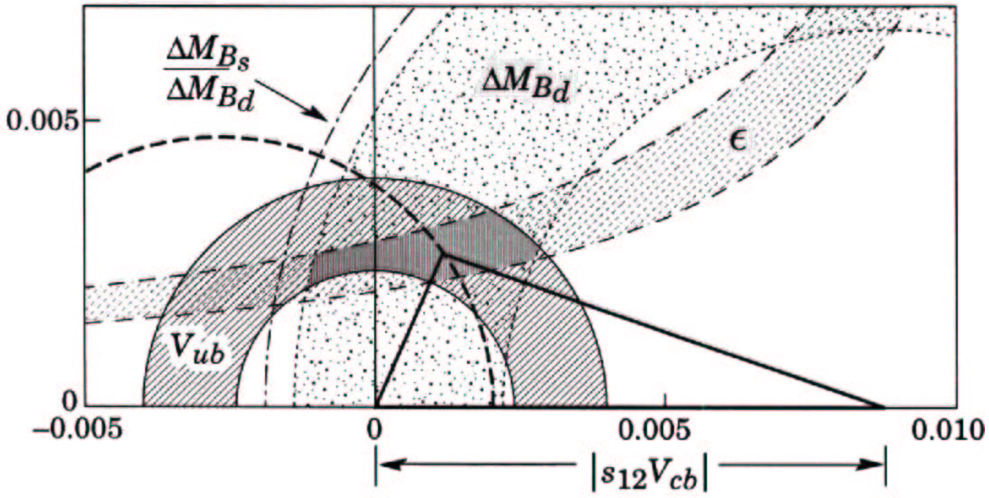
$$\begin{aligned} m_t(m_t) &= 165\ \text{GeV}, & m_\tau &= 1.777\ \text{GeV} \\ m_u(1\ \text{GeV}) &= 4.5\ \text{MeV}, & m_\mu &= 105.7\ \text{MeV} \\ V_{us} &= 0.220, & m_e &= 0.511\ \text{MeV} \\ V_{cb} &= 0.0395, & \delta_{CP} &= 64^\circ \end{aligned}$$

which lead to the following predictions:

$$\begin{aligned} m_b(m_b) &= 4.25\ \text{GeV}, & m_c(m_c) &= 1.23\ \text{GeV} \\ m_s(1\ \text{GeV}) &= 148\ \text{MeV}, & m_d(1\ \text{MeV}) &= 7.9\ \text{MeV} \\ |V_{ub}/V_{cb}| &= 0.080, & \sin 2\beta &= 0.64. \end{aligned}$$

- $U^\dagger U$, $D^\dagger D$, and $N^\dagger N$ are diagonalized with small LH rotations, while $L^\dagger L$ is diagonalized by a large LH rotation.

  This accounts for the fact that $V_{cb} = (U_U^\dagger U_D)_{cb}$ is small while $U_{\mu 3} = (U_L^\dagger U_\nu)_{\mu 3}$ is large for any reasonable M_R.



The unitarity triangle for $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ is displayed along with the experimental constraints on $V_{ud}V_{ub}^*$, which is the upper vertex in the triangle. The constraints following from $|V_{ub}|$, B-mixing and ϵ extractions from experimental data are shown in the lightly shaded regions. The experimentally allowed region is indicated by the darkly shaded overlap. The model predicts that $V_{ud}V_{ub}^*$ will lie on the dashed circle. The particular point on this circle used to draw the triangle shown is obtained from the CP-violating input phase, $\delta_{CP} = 64^\circ$.

- Right-Handed Majorana Matrix

The type of $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ solar neutrino mixing is determined by the texture of M_R, since the solar and atmospheric mixings are essentially decoupled in this model.
The LMA solution requires a nearly hierarchical texture which can be understood with Froggatt-Nielsen diagrams:

$$M_R = \begin{pmatrix} b^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R$$

The pre-SNO allowed LMA region could be covered with

$1.0 \lesssim a \lesssim 2.5, \quad 1.8 \lesssim b \lesssim 5.2$ Albright, Geer

With the recent SNO results, nearly half of the allowed parameter reigion is eliminated, as

$$\tan^2\theta_{12} \gtrsim 0.30, \ \sin^2 2\theta_{12} \gtrsim 0.71 \Rightarrow a \lesssim 1.8$$

- As an interesting special case, we note that with

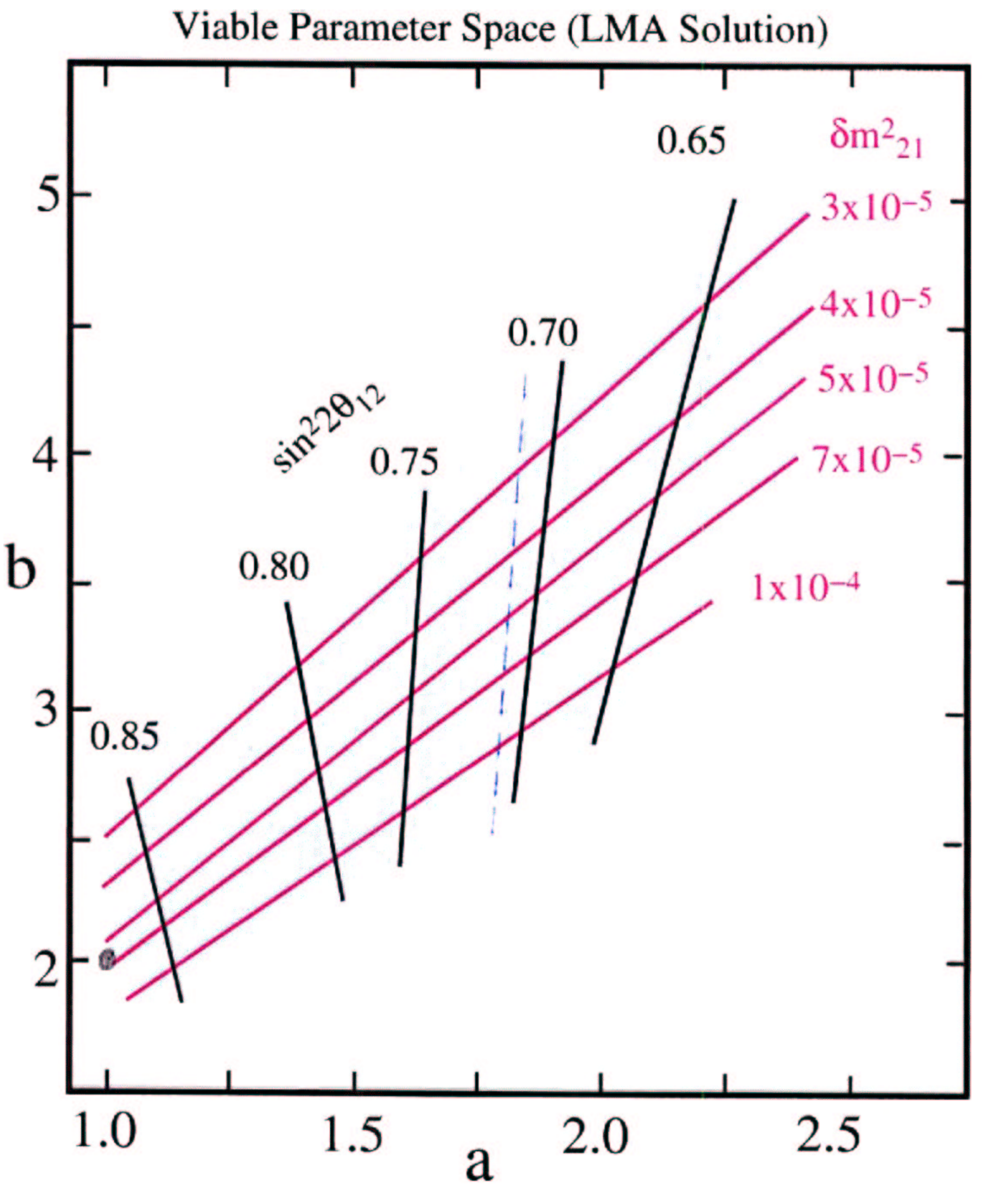
$$a = 1, \ b = 2 \text{ and } \Lambda_R = 2.72 \times 10^{14} \text{ GeV}$$

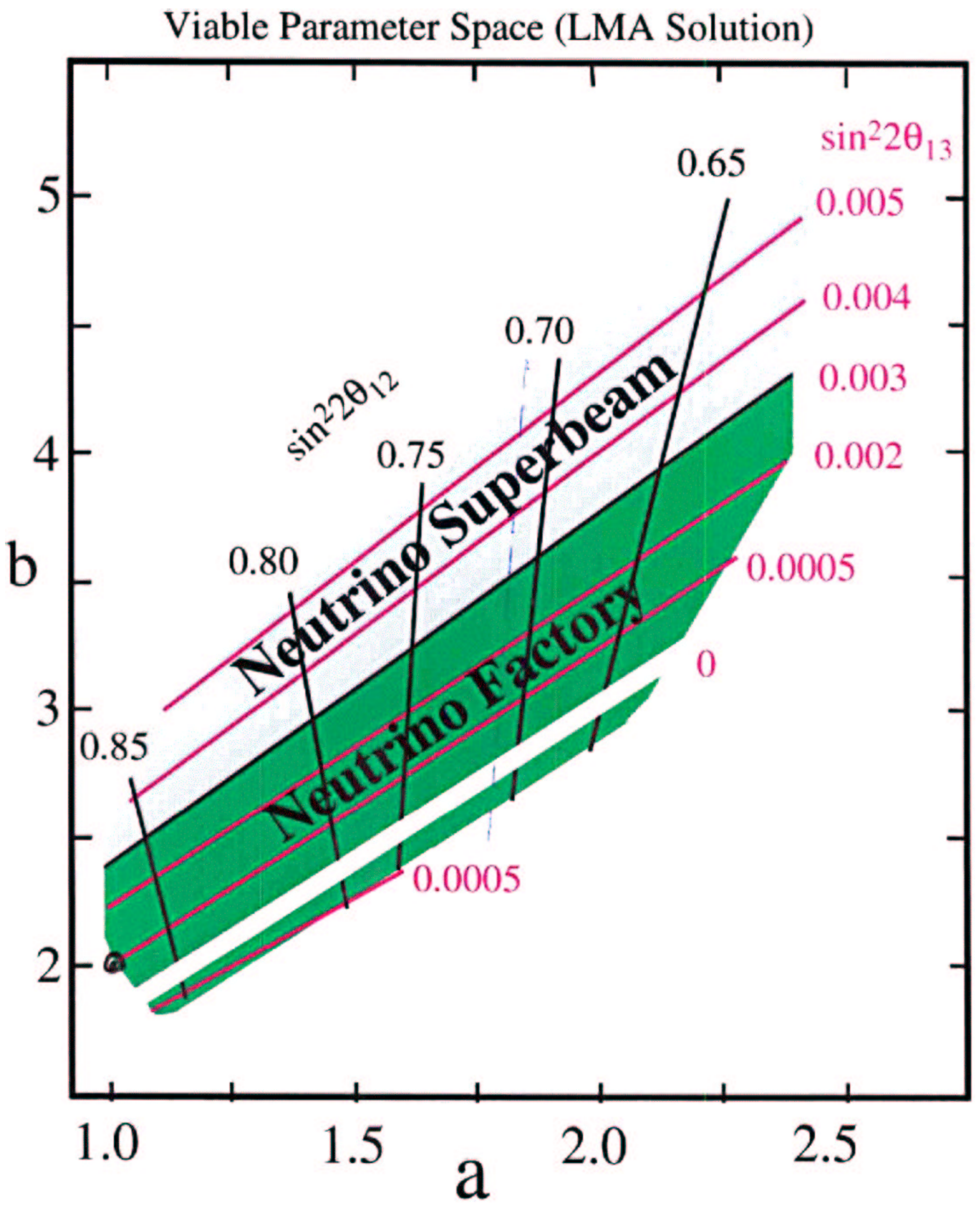
the seesaw mechanism leads to

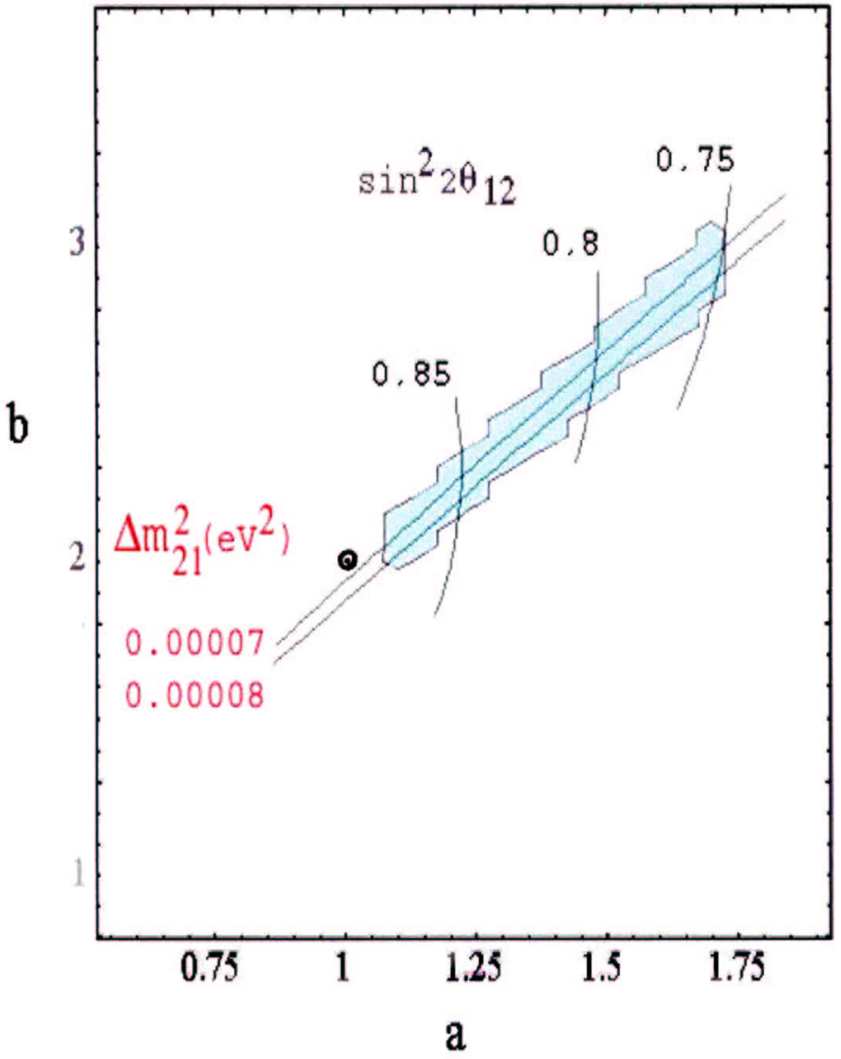
$$M_\nu = \begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} M_U^2/\Lambda_R$$

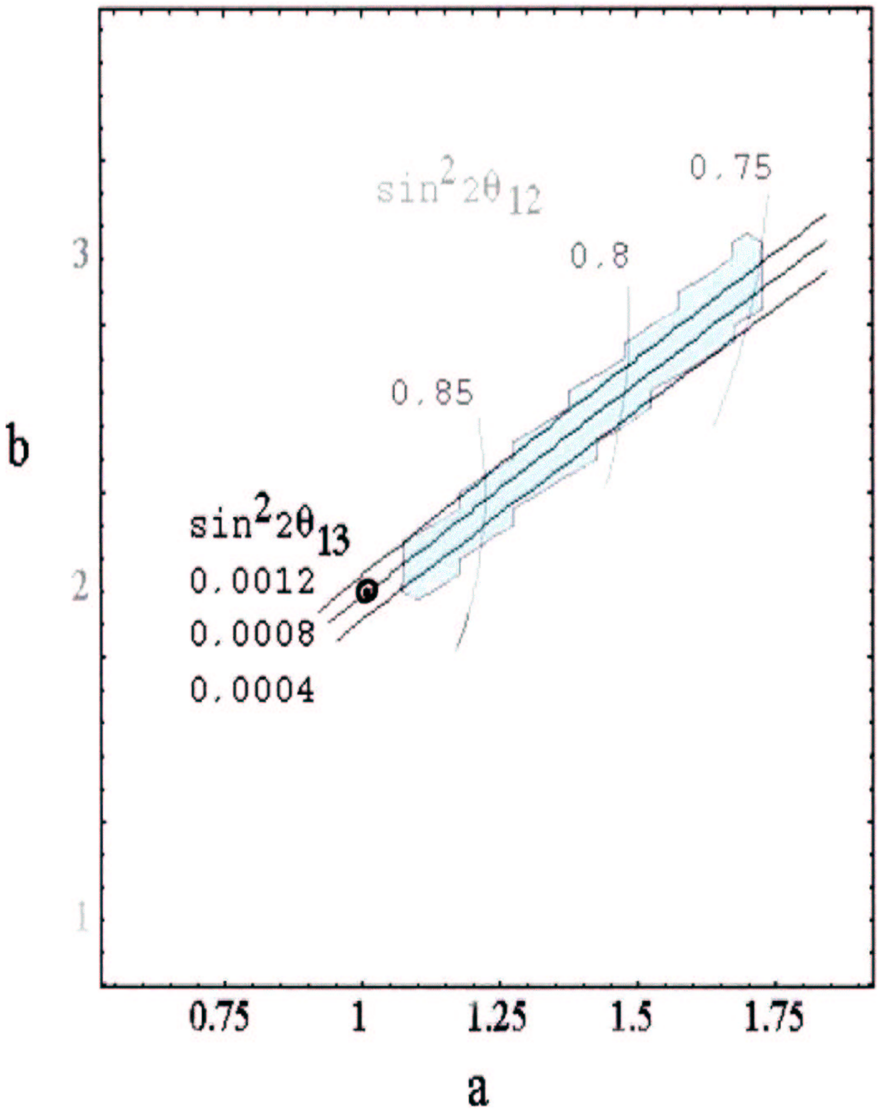
$M_1 = 3.2 \times 10^8, \ M_2 = 3.6 \times 10^8, \ M_3 = 2.8 \times 10^{14}$ GeV,
$m_1 = 4.9$ mev, $m_2 = 8.7$ mev, $m_3 = 51$ mev,
$\Delta m^2_{32} = 2.5 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{atm} = 0.994$,
$\Delta m^2_{21} = 5.1 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{sol} = 0.88$, $\tan^2\theta_{sol} = 0.49$,
$U_{e3} = -0.014$, $\sin^2 2\theta_{reac} = 0.0008$

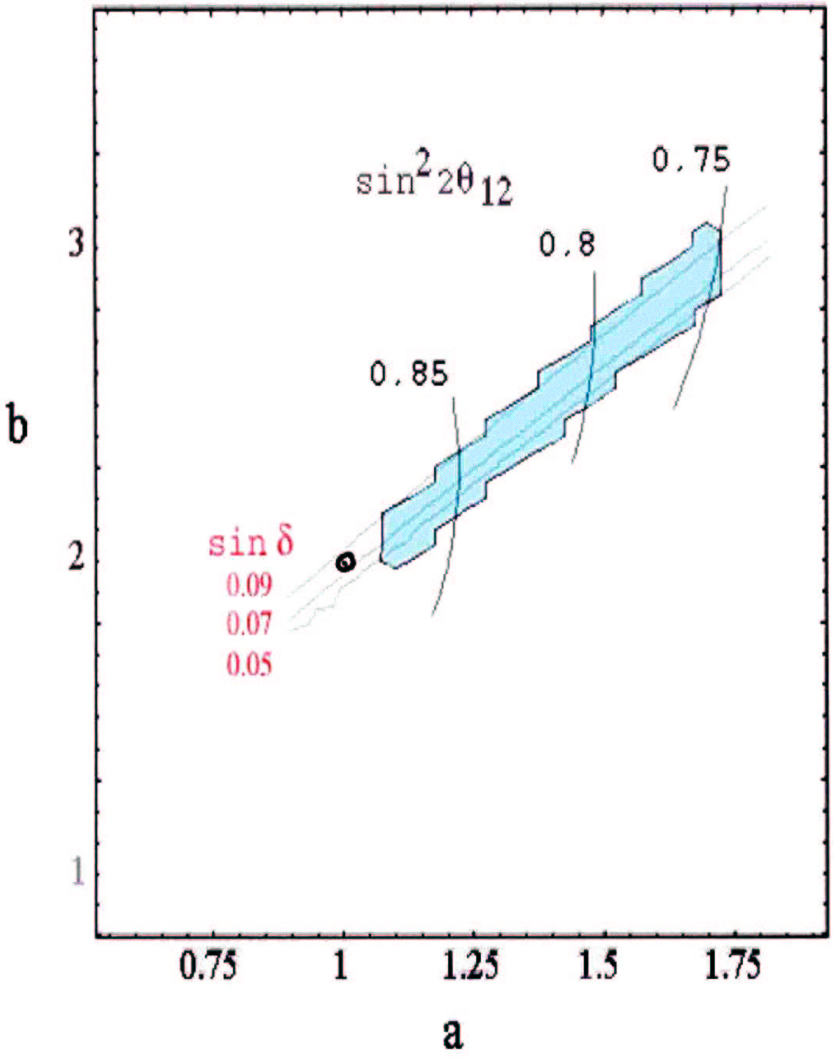
Whereas one might have expected an inverted hierarchy with M_1 and M_2 so close and much smaller than M_3, the resultant form of M_ν leads to a normal hierarchy.

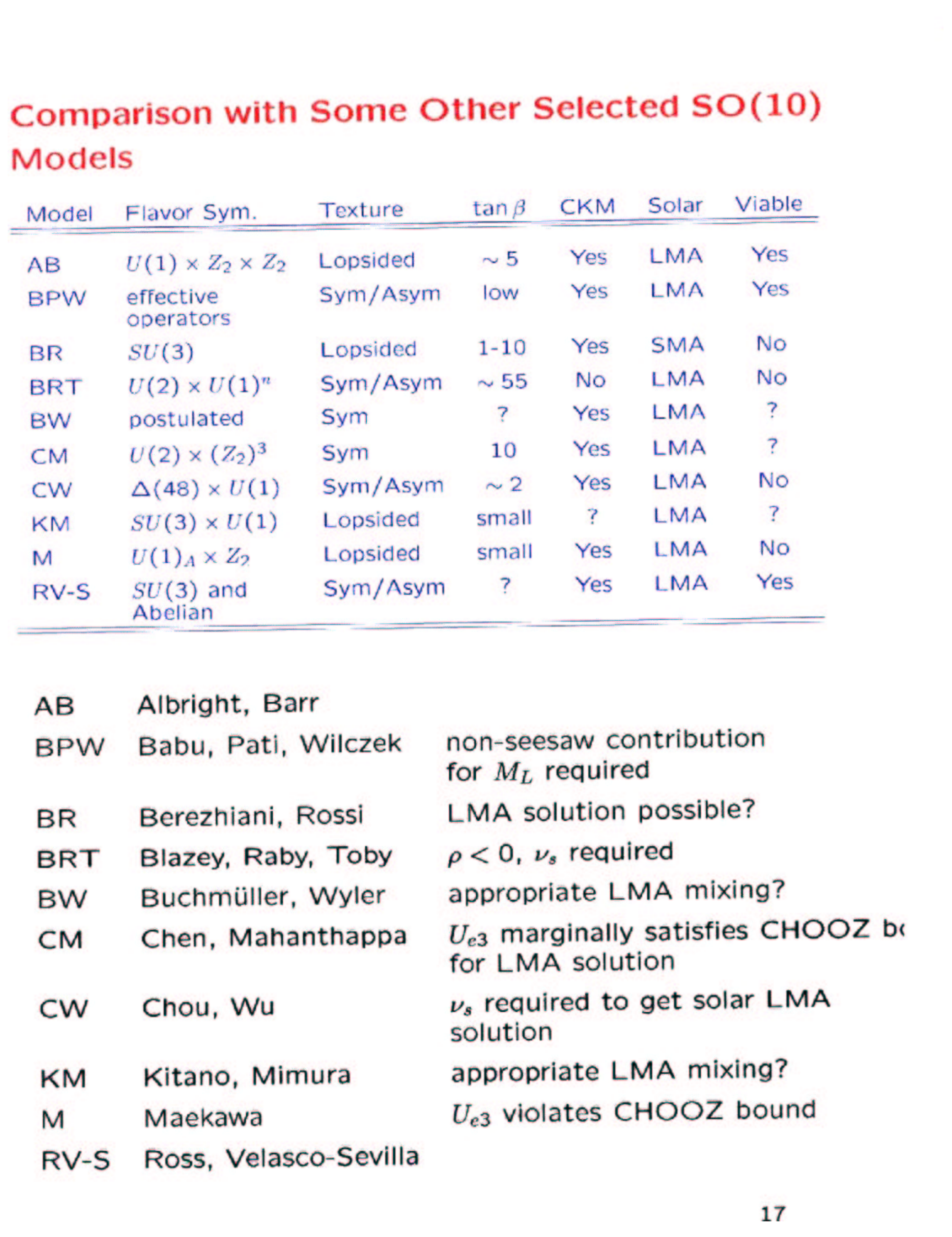



Viable Parameter Space (LMA Solution)
$\sin^2 2\theta_{13}$
0.005
0.004
0.003
0.002
0.0005
0
0.0005
$\sin^2 2\theta_{12}$
0.65
0.70
0.75
0.80
0.85
Neutrino Superbeam
Neutrino Factory
b
5
4
3
2
a
1.0
1.5
2.0
2.5


$\sin^2 2\theta_{12}$
0.75
0.8
0.85
$\Delta m^2_{21}(\mathrm{eV}^2)$
0.00007
0.00008
b
3
2
1
a
0.75
1
1.25
1.5
1.75


$\sin^2 2\theta_{12}$
0.75
0.8
0.85
$\sin^2 2\theta_{13}$
0.0012
0.0008
0.0004
b
3
2
1
0.75
1
1.25
1.5
1.75
a

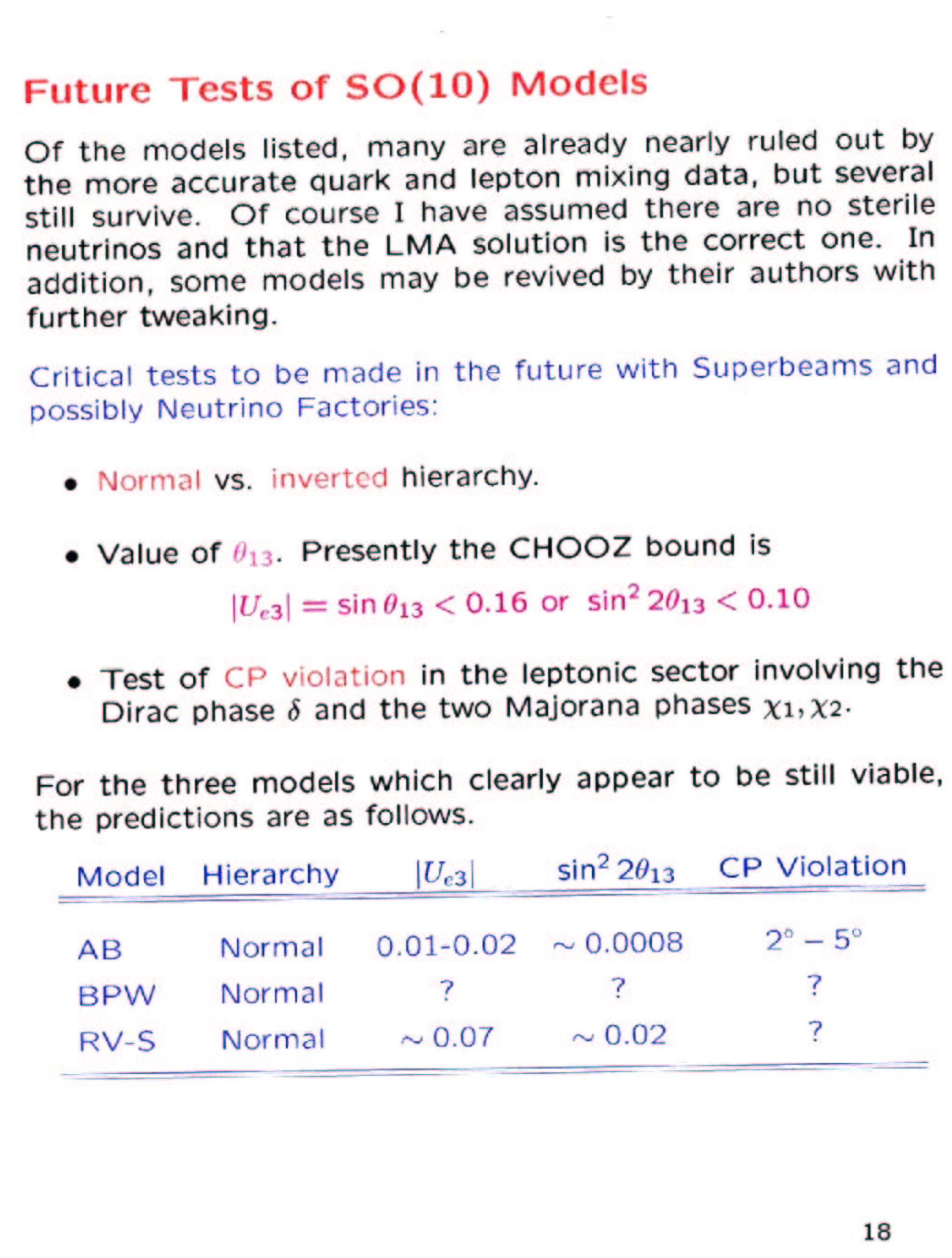

$\sin^2 2\theta_{12}$
0.75
0.8
0.85
$\sin\delta$
0.09
0.07
0.05
b
3
2
1
0.75
1
1.25
1.5
1.75
a



Comparison with Some Other Selected SO(10) Models

Model	Flavor Sym.	Texture	$\tan\beta$	CKM	Solar	Viable
AB	$U(1)\times Z_2\times Z_2$	Lopsided	~ 5	Yes	LMA	Yes
BPW	effective operators	Sym/Asym	low	Yes	LMA	Yes
BR	$SU(3)$	Lopsided	1-10	Yes	SMA	No
BRT	$U(2)\times U(1)^n$	Sym/Asym	~ 55	No	LMA	No
BW	postulated	Sym	?	Yes	LMA	?
CM	$U(2)\times (Z_2)^3$	Sym	10	Yes	LMA	?
CW	$\Delta(48)\times U(1)$	Sym/Asym	~ 2	Yes	LMA	No
KM	$SU(3)\times U(1)$	Lopsided	small	?	LMA	?
M	$U(1)_A\times Z_2$	Lopsided	small	Yes	LMA	No
RV-S	$SU(3)$ and Abelian	Sym/Asym	?	Yes	LMA	Yes

AB	Albright, Barr	
BPW	Babu, Pati, Wilczek	non-seesaw contribution for M_L required
BR	Berezhiani, Rossi	LMA solution possible?
BRT	Blazey, Raby, Toby	$\rho < 0$, ν_s required
BW	Buchmüller, Wyler	appropriate LMA mixing?
CM	Chen, Mahanthappa	U_{e3} marginally satisfies CHOOZ b for LMA solution
CW	Chou, Wu	ν_s required to get solar LMA solution
KM	Kitano, Mimura	appropriate LMA mixing?
M	Maekawa	U_{e3} violates CHOOZ bound
RV-S	Ross, Velasco-Sevilla	



Future Tests of SO(10) Models

Of the models listed, many are already nearly ruled out by the more accurate quark and lepton mixing data, but several still survive. Of course I have assumed there are no sterile neutrinos and that the LMA solution is the correct one. In addition, some models may be revived by their authors with further tweaking.

Critical tests to be made in the future with Superbeams and possibly Neutrino Factories:

- Normal vs. inverted hierarchy.
- Value of θ_{13}. Presently the CHOOZ bound is
$$|U_{e3}| = \sin\theta_{13} < 0.16 \text{ or } \sin^2 2\theta_{13} < 0.10$$
- Test of CP violation in the leptonic sector involving the Dirac phase δ and the two Majorana phases χ_1, χ_2.

For the three models which clearly appear to be still viable, the predictions are as follows.

Model	Hierarchy	$\lvert U_{e3}\rvert$	$\sin^2 2\theta_{13}$	CP Violation
AB	Normal	0.01-0.02	~ 0.0008	$2^\circ - 5^\circ$
BPW	Normal	?	?	?
RV-S	Normal	~ 0.07	~ 0.02	?

Extension of the Models to Extra Dimensions

There are several advantages to lifting the 4D SO(10) models to 5 dimensions:

- gauge coupling unification can be improved with

  $\alpha_s(M_Z) \simeq 0.125 \rightarrow 0.118,$ Hall, Nomura

- doublet-triplet splitting achieved without Dimopoulos-Wilczek mechanism,
- dim-5 operator proton decay via colored higgsino exchange can be avoided.

- Procedure
  - Compactify the 5th dimension on an $S^1/(Z_2 \times Z_2)$ orbifold where

    $$S_1: \quad y = y + 2n\pi R, \quad y' = y + \pi R/2,$$

    Z_2 maps $y \leftrightarrow -y$, Z_2' maps $y' \leftrightarrow -y'$, so fundamental region is restricted to $-\pi R/2 \leq y \leq 0$, with physical brane at O $(y = 0)$, hidden brane at O' $(y = -\pi R/2)$.
  - Generic bulk fields $\phi(x^\mu, y)$ of definite (P, P') parity can be expanded in a Fourier series in y with 4D space-time coefficient functions.

    ϕ^{-+}, ϕ^{--} bulk fields vanish on visible brane,

    ϕ^{+-}, ϕ^{--} bulk fields vanish on hidden brane

  - Assume all $SO(10)$ $\mathbf{45}_g$ gauge fields and the $\mathbf{10}_H$ and $\mathbf{45}_H$ Higgs fields live in the bulk, while all other Higgs and matter fields live on the physical brane.
  - Assume the orbifold compactification breaks $N = 1$ 5D SUSY to $N = 1$ 4D SUSY under the action of Z_2, while $SO(10)$ is broken to $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$ under action of Z_2'. In 4D all bulk fields become massive except for the K-K zero modes of the $\phi^{++}(x^\mu, y)$ fields. Dermisek, Mafi
  - Parities are assigned so that this occurs:

$$\begin{aligned}\mathbf{45}_H = \ & \Phi^{++}_{(15,1,1)} + \Phi^{++}_{(1,3,1)} + \Phi^{++}_{(1,1,3)} + \Phi^{+-}_{(6,2,2)} \\ & + \Phi^{c--}_{(15,1,1)} + \Phi^{c--}_{(1,3,1)} + \Phi^{c--}_{(1,1,3)} + \Phi^{c-+}_{(6,2,2)} \\ \mathbf{10}_H = \ & \Phi^{++}_{(1,2,2)} + \Phi^{+-}_{(6,1,1)} + \Phi^{c--}_{(1,2,2)} + \Phi^{c-+}_{(6,1,1)}\end{aligned}$$

- Findings
  - Superpotential on the hidden brane where the gauge symmetry is G_{PS} leads to $\langle \mathbf{45}_H \rangle$ pointing in the $B - L$ direction, which corresponds to one of the generators of G_{PS}.
  - On the visible brane, massless zero mode of $\Phi^{++}_{(1,2,2)}$ contains two Higgs doublets, while the colored Higgs triplets in $\Phi^{+-}_{(6,1,1)}$ are made superheavy by orbifold compactification.
  - Dim-5 proton decay operators are absent, since color-triplet Higgs do not have superheavy mass terms connecting them, so no exchange of colored higgsinos mediate proton decay.
  - Higgs superpotential is simplified, while Yukawa superpotential and mass matrices derived from it remain essentially unaltered. Albright, Barr

Summary

- A number of $SO(10)$ SUSY GUT models have been proposed in the literature. Some have been or are on the verge of being eliminated, while some still survive and are able to explain all the known quark and lepton mass and mixing data.

- Long baseline experiments which can determine whether the neutrino mass hierarchy is normal or inverted appear to have a direct bearing on the survival of $SO(10)$ vs conserved-lepton-number-type models.

- The observed value of $\sin^2 2\theta_{13}$ will further narrow down the list of viable models. Some predict that θ_{13} lies just below the CHOOZ bound and will be observable with off-axis beams and/or Superbeams. Others favor such low values of θ_{13} that a Neutrino Factory may be required to determine its value.

- The issue of proton decay via dim-5 operators is potentially a serious one with 4D models, if proton decay is not detected shortly. On the other hand, by formulating an $SO(10)$ model in 5 dimensions, one can eliminate the dim-5 operator contributions entirely. The dim-6 operators involving colored Higgs exchange will still be present, but the proton decay lifetime is then expected to be in the 10^{35-36} year range.