Introduction

Present

- Atmospheric Neutrino

$$\theta_{23} \sim \frac{\pi}{4}, \quad |\Delta m^2_{31}| \sim 3 \times 10^{-3} \mathrm{eV}^2$$

- Solar Neutrino

$$\theta_{12} \lesssim \frac{\pi}{4}, \quad \Delta m^2_{21} \sim 6 \times 10^{-5} \mathrm{eV}^2$$

- CHOOZ

$$\theta_{13} \leq 0.2$$

↓ Neutrino Beam with High Intensity

Future

- Super-beam or Neutrino Factory

$$\theta_{13}, \quad \mathrm{sgn}(\Delta m^2_{31}), \quad \delta$$

Remark

1. Long Baseline Experiments will be planned.
2. Matter effects would be important.

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New Formula for Three Neutrino CP Violation

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Reference

1. H. Yokomakura, K. Kimura and A. T.
   Phys. Lett. B544 (2002) 286
2. K. Kimura, A. T. and H. Yokomakura
   Phys. Rev. D66 (2002) 073005
3. K. Kimura, A. T. and H. Yokomakura
   Phys. Lett. B537 (2002) 86
4. H. Yokomakura, K. Kimura, and A. T.
   Phys. Lett. B496 (2000) 175

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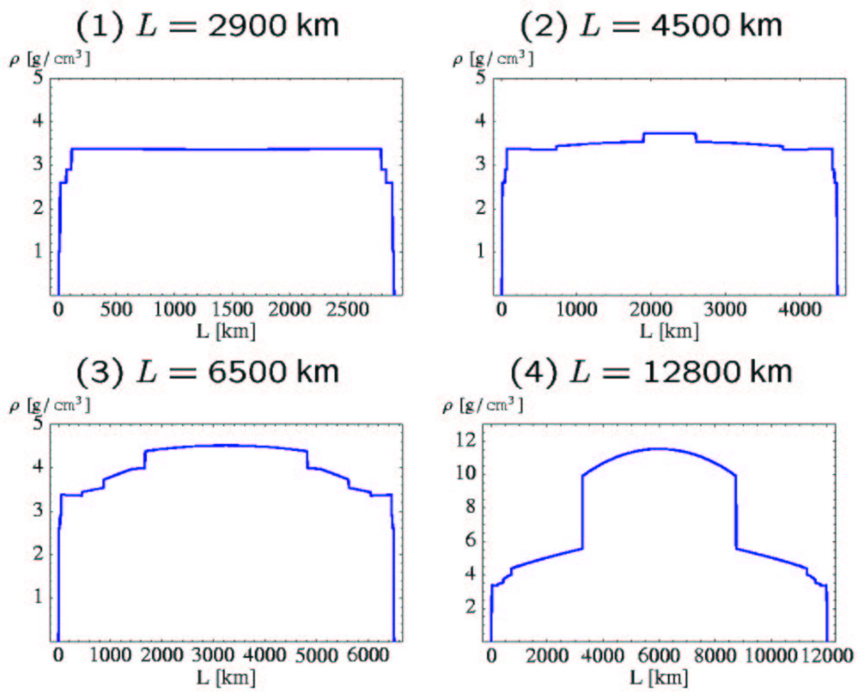
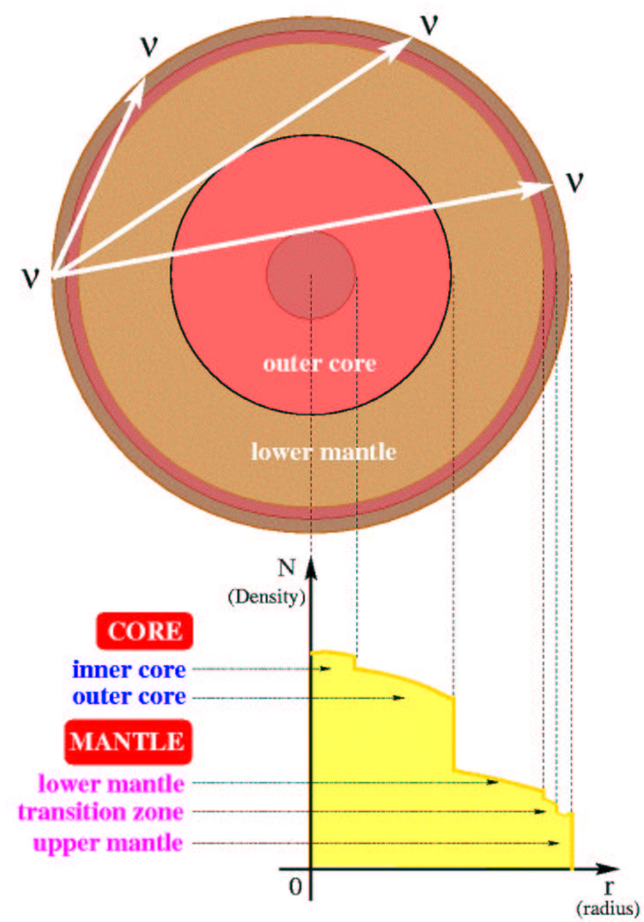
Density Profile along the Neutrino Path



Remark

1. Matter density is almost constant in $L < 5000$ km
2. Matter profile effect is important in $L > 5000$ km

Preliminary Reference Earth Model



Effective Mixing Angles and CP Phase

Effective Mixing Angles and CP Phase

$$\sin^2\tilde\theta_{13} = \frac{\lambda_3^2-\alpha\lambda_3+\beta}{\tilde\Delta_{13}\tilde\Delta_{23}}$$
$$\sin^2\tilde\theta_{12} = \frac{-(\lambda_2^2-\alpha\lambda_2+\beta)\tilde\Delta_{31}}{(\lambda_1^2-\alpha\lambda_1+\beta)\tilde\Delta_{32}-(\lambda_2^2-\alpha\lambda_2+\beta)\tilde\Delta_{31}}$$
$$\sin^2\tilde\theta_{23} = \frac{G^2s_{23}^2+F^2c_{23}^2+2GFs_{23}c_{23}\cos\delta}{G^2+F^2}$$
$$e^{-i\tilde\delta} = \frac{(G^2e^{-i\delta}-F^2e^{i\delta})s_{23}c_{23}+GF(c_{23}^2-s_{23}^2)}{\sqrt{(G^2s_{23}^2+F^2c_{23}^2+2GFs_{23}c_{23}\cos\delta)(G^2c_{23}^2+F^2s_{23}^2-2GFs_{23}c_{23}\cos\delta)}}$$

where

$$\alpha = m_3^2c_{13}^2+m_2^2(c_{13}^2c_{12}^2+s_{13}^2)+m_1^2(c_{13}^2s_{12}^2+s_{13}^2)$$
$$\beta = m_3^2c_{13}^2(m_2^2c_{13}^2+m_1^2s_{12}^2)+m_2^2m_1^2s_{13}^2$$
$$G = [\Delta_{31}(\lambda_3-m_1^2-\Delta_{21})-\Delta_{21}(\lambda_3-m_1^2-\Delta_{31})s_{12}^2]c_{13}s_{13}$$
$$F = (\lambda_3-m_1^2-\Delta_{31})\Delta_{21}c_{12}s_{12}c_{13}$$

H.W.Zaglauer and K.H.Schwarzer, Z.Phys.C40,273 (1988)

Remark

1. $\tilde\theta_{12}$ and $\tilde\theta_{13}$ do not depend on the CP phase δ.
2. $\tilde\theta_{23}$ and $\tilde\delta$ are especially complicated.

$$\Rightarrow \quad \tilde J = \tilde s_{12}\tilde c_{12}\tilde s_{23}\tilde c_{23}\tilde s_{13}\tilde c_{13}^2\sin\tilde\delta = ?$$

7

Effective Masses

Standard Parametrization

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12}-s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Effective Masses

$$\lambda_1 = \frac{1}{3}s-\frac{1}{3}\sqrt{s^2-3t}\left[u+\sqrt{3(1-u^2)}\right]$$
$$\lambda_2 = \frac{1}{3}s-\frac{1}{3}\sqrt{s^2-3t}\left[u-\sqrt{3(1-u^2)}\right]$$
$$\lambda_3 = \frac{1}{3}s+\frac{2}{3}u\sqrt{s^2-3t}$$

where

$$s = \Delta_{21}+\Delta_{31}+a$$
$$t = \Delta_{21}\Delta_{31}+a[\Delta_{21}(1-s_{12}^2c_{13}^2)+\Delta_{31}(1-s_{13}^2)]$$
$$u = \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{2s^3-9st+27a\Delta_{21}\Delta_{31}c_{12}^2c_{13}^2}{2(s^2-3t)^{3/2}}\right)\right]$$

V. Barger et. al., Phys. Rev. D22,2718 (1980)

Remark Effective Masses

1. do not depend on 2-3 mixing and CPphase
2. depend on both 1-2 mixing and 1-3 mixing

6

Oscillation Probability in Constant Matter

$\nu_e \to \nu_\mu$ transition

$$P(\nu_e \to \nu_\mu) = \left| < \nu_\mu | e^{-iHt} | \nu_e > \right|^2$$

$$U^\dagger H U = \frac{1}{2E}\text{diag}(0, \Delta_{21}, \Delta_{31})$$

$$\Delta_{ij} = m_i^2 - m_j^2$$

$U_{\alpha i}$: MNS matrix

$$P(\nu_e \to \nu_\mu) = \underbrace{-4 \sum_{(ij)}^{\text{cyclic}} \text{Re} J_{e\mu}^{ij} \sin^2 \Delta'_{ij}}_{\text{CP even}} \underbrace{-2 \sum_{(ij)}^{\text{cyclic}} J \sin 2\Delta'_{ij}}_{\text{CP odd}}$$

$$J_{e\mu}^{ij} \equiv U_{ei}U_{\mu i}^*(U_{ej}U_{\mu j}^*)^* \quad J \equiv \text{Im } J_{e\mu}^{12} \quad \Delta'_{ij} \equiv \frac{L}{4E}\Delta_{ij}$$

vacuum $\Rightarrow$ matter

$$H \Rightarrow \tilde{H} = H + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \quad \tilde{U}^\dagger \tilde{H} \tilde{U} = \frac{1}{2E}\text{diag}(\lambda_1, \lambda_2, \lambda_3)$$

Probability in matter is obtained by the replacements

$$\begin{cases} U_{\alpha i} \to \tilde{U}_{\alpha i} \\ \Delta_{ij} \to \tilde{\Delta}_{ij} \equiv \lambda_i - \lambda_j \end{cases}$$

5

CP Dependence of Probability

⋆ Standard Parametrization

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\begin{aligned} p &= \Delta_{21}U_{e2}U_{\mu 2}^* + \Delta_{31}U_{e3}U_{\mu 3}^* \\ &= p_1 e^{-i\delta} + p_2, \\ q &= p(\Delta_{21} + \Delta_{31} + a) + \Delta_{21}\Delta_{31}U_{e1}U_{\mu 1}^* \\ &= q_1 e^{-i\delta} + q_2 \end{aligned}$$

Therefore

$$\begin{aligned} \tilde{U}_{ei}\tilde{U}_{\mu i}^* &= \frac{-p(\lambda_j + \lambda_k) + q}{\tilde{\Delta}_{ji}\tilde{\Delta}_{ki}} \\ &= X_1 e^{-i\delta} + X_2 \end{aligned}$$

$$P(\nu_e \to \nu_\mu) = \Big| \sum_i \tilde{U}_{\mu i}\tilde{U}_{ei}^* \text{diag}\left(e^{i\lambda_1 L'}, e^{i\lambda_2 L'}, e^{i\lambda_3 L'}\right)_{ii} \Big|^2$$

Finally we find

$$P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C$$

11

Exact Formula for Oscillation Probabilities

$$\tilde{H}^n = (H + \text{diag}(a,0,0))^n$$

$$\tilde{H} = \tilde{U}\text{diag}(\lambda_1,\lambda_2,\lambda_3)\tilde{U}^\dagger/(2E),$$
$$H = U^\dagger\text{diag}(0,\Delta_{21},\Delta_{31})U/(2E)$$

$$\tilde{U}\text{diag}(\lambda_1^n,\lambda_2^n,\lambda_3^n)\tilde{U}^\dagger = \{U^\dagger\text{diag}(0,\Delta_{21},\Delta_{31})U + \text{diag}(a,0,0)\}^n$$

n = 0 (Unitarity)

$$\tilde{U}_{e1}\tilde{U}^*_{\mu 1} + \tilde{U}_{e2}\tilde{U}^*_{\mu 2} + \tilde{U}_{e3}\tilde{U}^*_{\mu 3} = 0$$

n = 1

$$\lambda_1\tilde{U}_{e1}\tilde{U}^*_{\mu 1} + \lambda_2\tilde{U}_{e2}\tilde{U}^*_{\mu 2} + \lambda_3\tilde{U}_{e3}\tilde{U}^*_{\mu 3} = p$$

n = 2

$$\lambda_1^2\tilde{U}_{e1}\tilde{U}^*_{\mu 1} + \lambda_2^2\tilde{U}_{e2}\tilde{U}^*_{\mu 2} + \lambda_3^2\tilde{U}_{e3}\tilde{U}^*_{\mu 3} = q$$

where

$$p = \Delta_{21}U_{e2}U^*_{\mu 2} + \Delta_{31}U_{e3}U^*_{\mu 3},$$
$$q = p(\Delta_{21} + \Delta_{31} + a) + \Delta_{21}\Delta_{31}U_{e1}U^*_{\mu 1}$$

Finally

$$\tilde{U}_{ei}\tilde{U}^*_{\mu i} = \frac{-p(\lambda_j + \lambda_k) + q}{\tilde{\Delta}_{ji}\tilde{\Delta}_{ki}}$$

Our New Method

Probability

$$P(\nu_e \to \nu_\mu) = |S_{\mu e}|^2$$

Amplitude

$$S_{\mu e} = \exp\left(i\tilde{H}L\right)_{\mu e} = \sum_{i=1,2,3}\tilde{U}_{\mu i}\tilde{U}^*_{ei}\text{diag}\left(e^{i\lambda_1 L'}, e^{i\lambda_2 L'}, e^{i\lambda_3 L'}\right)_{ii}$$

where we use

$$\tilde{U}^\dagger\tilde{H}\tilde{U} = \frac{1}{2E}\text{diag}(\lambda_1,\lambda_2,\lambda_3), \quad L' = \frac{1}{2E}L$$

$\tilde{U}_{\alpha i}$ complicate	$\Rightarrow$	$\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i}$ simple (?)

Key Points

1. We need only the product of $\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i}$ and do not need the calculating of single $\tilde{U}_{\alpha i}$.
2. We expect the expression of the product of $\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i}$ is simple rather than the single $\tilde{U}_{\alpha i}$.

Naumov-Harrison-Scott identity

- Naumov, Int.J.Mod.Phys. D1(1992)379
- Harrison and Scott, Phys. Lett. B476(2000)349

$$\tilde{H} = H + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \Rightarrow \tilde{H}_{\alpha\beta} = H_{\alpha\beta}$$

$$\mathrm{Im}(\tilde{H}_{e\mu}\tilde{H}_{\mu\tau}\tilde{H}_{\tau e}) = \mathrm{Im}(H_{e\mu}H_{\mu\tau}H_{\tau e})$$

↓ $\tilde{H} = \tilde{U}\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)\tilde{U}^\dagger$

$$\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}\tilde{J} = \Delta_{12}\Delta_{23}\Delta_{31}J$$

$$J = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2\sin\delta$$

Matter Effects $\Rightarrow \Delta_{ij}$
CP Effects $\Rightarrow \sin\delta$

Remark

1. Both matter effects and CP effects are simple in $\tilde{J}$.
2. How about Probability or Amplitude have simple matter effect and CP dependence?

Summary

We have calculated $P(\nu_e \to \nu_\mu)$ in constant matter.

1. We have derived an exact and simple formula.

$$P(\nu_e \to \nu_\mu) = |\sum_i \tilde{U}_{\mu i}\tilde{U}^*_{ei}\mathrm{diag}\left(e^{i\lambda_1 L'}, e^{i\lambda_2 L'}, e^{i\lambda_3 L'}\right)_{ii}|^2$$

$$\tilde{U}_{ei}\tilde{U}^*_{\mu i} = \frac{-p(\lambda_j + \lambda_k) + q}{\tilde{\Delta}_{ji}\tilde{\Delta}_{ki}}$$

2. We have found the simple CP dependence.

$$P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C$$