U.S. Department of Energy's
Office of Science

DOUBLE BETA DECAY

Conference on Neutrinos: Data, Cosmos, and Planck Scale

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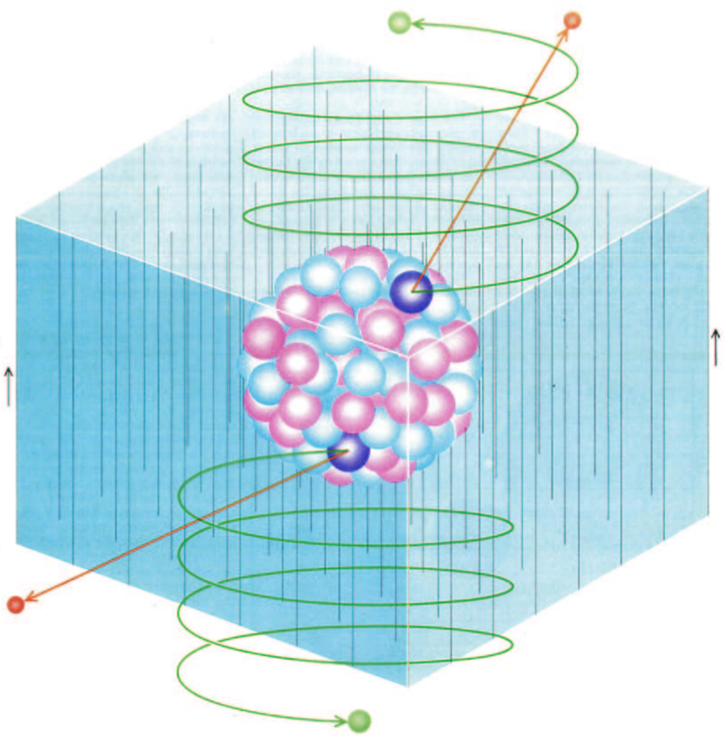
Double Beta Decay

1935	Maria Goeppert-Mayer – meta-stability of even-even isotopes
1937	E. Majorana – symmetric theory of electron, neutrino
1937	G. Racah – $\nu \equiv \nu^c$? and exchange of real neutrinos
	– Ray Davis 1st exist in 1955
1939	W. Furry – $\nu \equiv \nu^c$? and exchange of virtual neutrinos
	– Lifetime ~ 10^6 shorter than 2-neutrino decay
1949, 1950	Inghram and Reynolds – 1st geochemical experiment
1968	Kirsten, Schaeffer, Norton, Stoenner – $^{130}Te \rightarrow {}^{130}Xe$ $\tau \sim 2 \times 10^{21}$ yrs
1987, 1988	Elliott, Hahn, and Moe – 1st direct observation of double beta decay

weakly with matter—so weakly, in fact, that it was not observed until 1956 when Clyde L. Cowan, Jr., and Frederick Reines of the Los Alamos Science Laboratory captured a few neutrinos emanating from a nuclear reactor.

According to the Standard Model, the neutrino accompanying a negative beta ray is the distinct antiparticle of the one accompanying a positive beta ray (just as the positron is the distinct antiparticle of the electron). Theories that go beyond the Standard Model and assign a mass to the neutrino, however, predict that the particle emitted with a negative beta ray should be the same as the one emitted with a positive beta ray. In other words, the neutrino would be its own antiparticle. How can we tell whether these predictions are right?

Double-beta decay is the ideal process in which to seek an answer to this question. If the neutrino has mass and is its own antiparticle, then the neutrino emitted in the first stage of the process might be reabsorbed in the



NUCLEAR SIGNATURE of double-beta decay emanates from the nucleus of an atom of selenium consisting of 48 neutrons (*blue and purple*) and 34 protons (*red*). Two of the neutrons (*purple*) decay simultaneously into two protons and in the process generate two beta rays (electrons) (*green*) and two antineutrinos (*orange*). An external magnetic field (*gray*) causes the paths of the ejected electrons to spiral. The double spiral is an observable signal of the double-beta event. The resulting atom has two more protons and two fewer neutrons than the original state; it has been transformed from selenium to krypton.

didates, would produce another noble gas, xenon. Minerals that contain selenium or tellurium should therefore accumulate krypton or xenon over time. To be sure, the amount of the gas produced in the billion-year history of a mineral would be small—less than one part in 100 million, if Mayer's estimates were essentially correct.

In 1949 Michael G. Inghram and John H. Reynolds, both of the University of Chicago, pioneered a technique for examining fossil gases trapped within ancient selenium and tellurium ores. They released the gases into a mass spectrometer to determine their composition. In 1968, after some refinement of this geochemical method, Till Kirsten, now at the Max Planck Institute in Heidelburg, the late Oliver A. Schaeffer of the State University of New York at Stony Brook and Elinor F. Norton and Raymond W. Stoenner, both of the Brookhaven National Laboratory, found a definitive excess of xenon 130 in 1.3-billion-year-old tellurium ore. This result provided the earliest undisputed evidence that double-beta decay actually occurs.

From the age of the ore and the fraction of tellurium that had decayed to xenon, the half-life of double-beta decay was determined for tellurium 130. There were two important problems with these geochemical experiments. For one, processes other than double-beta decay might have created small amounts of xenon. For another, small amounts of the gas might have been lost from the ore through slow diffusion processes or sudden catastrophic events that heated the ore. The investigators were able to argue that these problems were not serious, but doubts persisted.

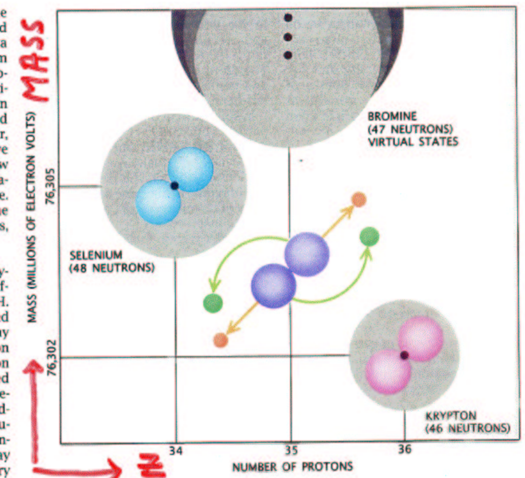
In 1939, four years after Mayer's calculations of theoretical half-lives were published, Wendell H. Furry of Harvard University suggested the possibility that double-beta decay could take place without the emission of neutrinos. Although conservation of energy and momentum required the emission of a neutrino in single-beta decay, there was no corresponding requirement for neutrinos in double-beta decay. Energy and momentum could be conserved in a decay releasing two electrons only. Furry recognized that if neutrinos were Majorana particles—identical to their antiparticles—then no-neutrino double-beta decay could compete with Mayer's two-neutrino double-beta decay. Furry estimated that no-neutrino double-beta decay should occur a million times more frequently than the two-neutrino mode. Nevertheless, the half-life was still on the order of 100 billion years; double-beta decay would still be rare enough to account for the apparent stability of even-even nuclei.

It seemed that the large difference between the predicted lifetimes of the two decay modes might make it possible to determine whether no-neutrino decay actually was taking place. Edward L. Fireman of Princeton University took up the challenge in 1948. He obtained two samples of tin, one artificially enriched in the double-beta-decay candidate tin 124 and the other depleted in that isotope. He placed each sample between a pair of Geiger-Müller tubes (Geiger counters) so each tube could receive one of the two electrons from double-beta decay; consequently, the tubes would fire simultaneously whenever a double-beta decay occurred. He found that simultaneous firing of the tubes took place significantly more often with the enriched sample than with the depleted one. From the data he calculated a half-life much closer to Furry's value than to Mayer's. He concluded that he had observed the no-neutrino mode.

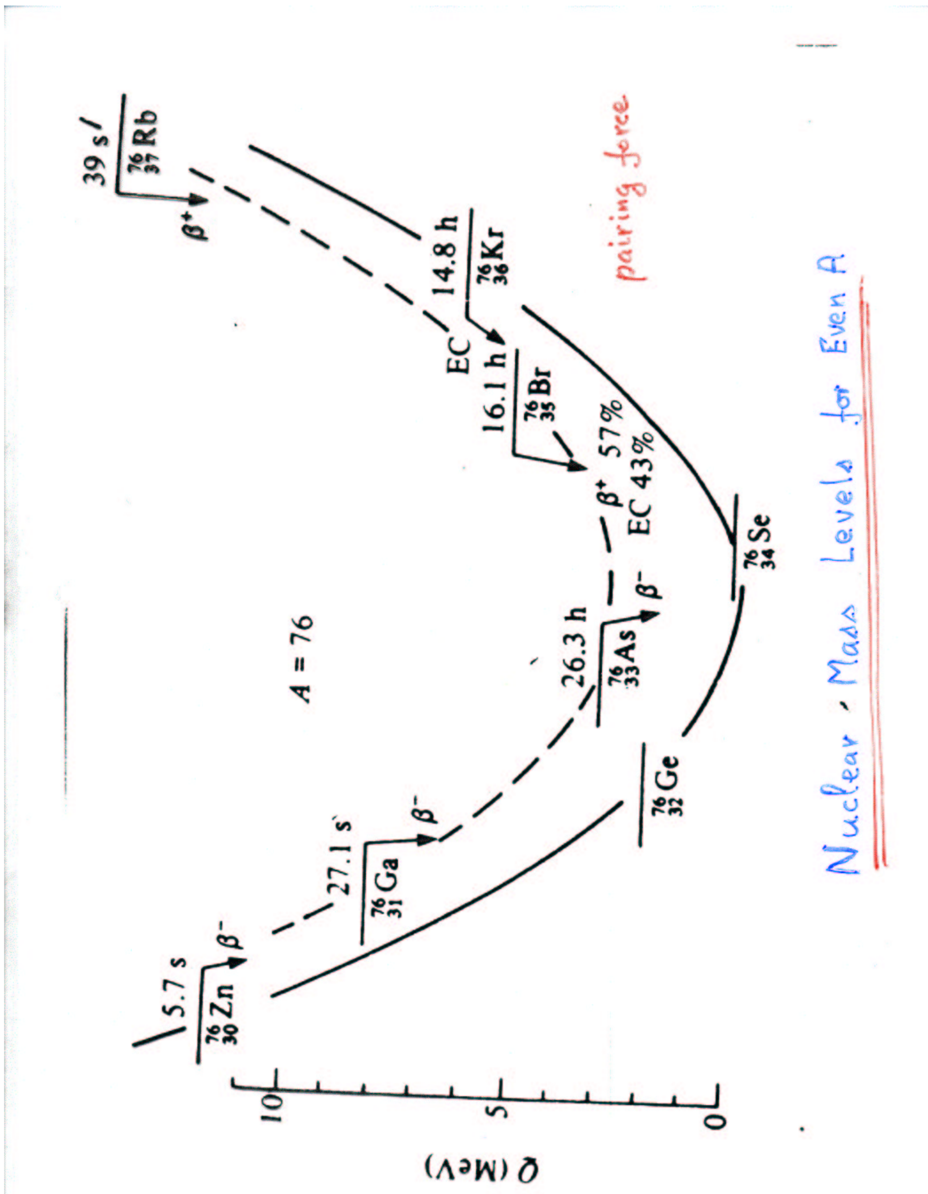
The excitement following this result was short-lived, however. Experiments performed a few years later, including one by Fireman himself, were unable to confirm that the result was actually caused by double-beta decay. Fireman finally conceded that his original results were probably distorted by a small trace of a radioactive impurity in the enriched sample of tin 124.

Until recently all attempts at direct detection of double-beta decay were frustrated by the same problem that Fireman had encountered. Traces of radioactive elements in quantities as small as one part per billion easily masked significant events in double-beta-decay sources, because the half-life of the decay—even by optimistic estimates—was at least a billion times longer than that of common radioactive decays.

The stumbling blocks nature placed



DOUBLE-BETA DECAY from selenium 82 to krypton 82 occurs by way of bromine 82. As in all double-beta decays, the parent isotope is heavier than the final product, but both are lighter (less energetic) than the intermediate state. Single-beta decay of selenium to bromine is energetically forbidden, but double-beta decay to krypton, through a "virtual" intermediate state in bromine, is allowed by the uncertainty principle. The final state of krypton appears after two neutrons become two protons, with the emission of two electrons and usually (perhaps always) two antineutrinos.



Kinematical Features of Decay Modes.

1) Two-neutrino decay has four-body phase space :

$$\left(1/\tau^{2\nu}_{1/2}\right) \propto Q^{10-11} \qquad Q \approx 2\text{-}3\ \text{MeV}$$

2) No-neutrino decay has two-body phase space plus integral over virtual neutrino. Crudely speaking :

$$\left(1/\tau^{0\nu}_{1/2}\right) \propto \langle E_\nu \rangle^5 Q^5 m^2_{\beta\beta}$$

mean energy of virtual neutrino

Branching Ratio for 0ν increases as Q decreases. $U^{238} \to Pu^{238}$ Cowan, Turkevich Economou

$$\text{Ratio } R = \left(\tau^{2\nu}_{1/2} \Big/ \tau^{0\nu}_{1/2}\right) \propto \left(\frac{\langle E_\nu \rangle}{Q}\right)^5 m^2_{\beta\beta}$$

For $\langle E_\nu \rangle \approx 50$ MeV, $Q \approx 3$ MeV, $R \approx 10^6 m^2_{\beta\beta}$

$\therefore$ Were all other factors equal, No-ν decay would be much faster than 2ν-decay;

$\therefore$ No-ν very sensitive to small $\Delta L \neq 0$ parameters

Dirac versus Majorana Neutrino

- **Dirac Neutrino** : $\nu^c \not\equiv \nu$

  Analogues:
  - $\overline{K^0} \not\equiv K^0$ by virtue of strangeness, hypercharge
  - $\bar{n} \neq n$ by virtue of baryon number, magnetic moment.

- **Majorana Neutrino** : $\nu^c \equiv \nu$

  Analogues:
  - $(\pi^0)^c \equiv \pi^0$ zero charge, hypercharge..
  - $(\gamma)^c \equiv -\gamma$ zero charge, ...etc

∴ Distinction meaningless in Standard E-W model by virtue of helicity and zero mass: ν_L and $\bar{\nu}_R$ cannot "communicate."

Formal Analysis :

ν_L , ν_R and $(\nu_L)^c \equiv (\bar{\nu})_R$

$\psi_\nu^c \equiv C\,\overline{\psi_\nu}^T$ $\qquad (\nu_R)^c \equiv (\bar{\nu})_L$

General Mass Term :

$$= m_D\,(\overline{\psi_\nu}\,\psi_\nu) + m_m\,(\overline{\psi_\nu}\,\psi_\nu^c)$$

+ Hermitian Conjugate

Eigenvalues : $\frac{1}{2}(m_D \pm m_m)$

Eigenvectors : $\frac{1}{2}(\psi_\nu \pm \psi_\nu^c) = \Psi_\pm$

Majorana Neutrinos :

$$\Psi_\pm^c = \pm\,\Psi_\pm \equiv \lambda\,\Psi_\pm$$

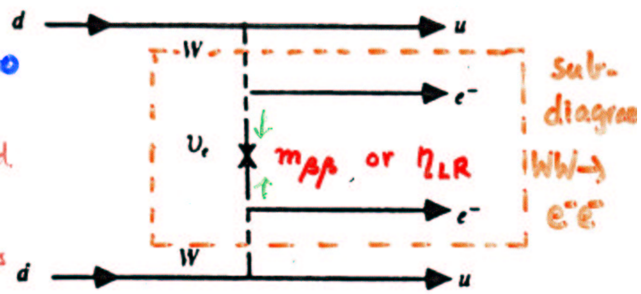
$\psi_{Dirac} = \Psi^+ + \Psi^-$

$\equiv$ superposition of two Majorana's with opposite λ

N.B
$[CPT]|\nu_L\rangle \equiv e^{i\phi}|\nu_R\rangle$
$|\nu_R\rangle$ = Lorentz B. $|\nu_L\rangle$
B. Kayser
$m_\nu \neq 0$

No-neutrino ββ:

- non-standard model
- Majorana Mass

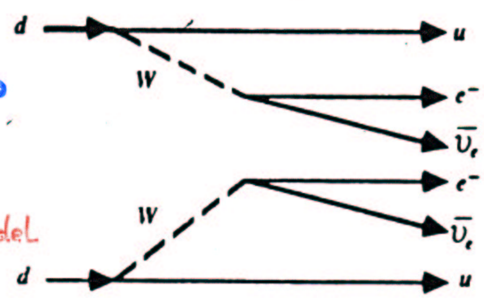


(b)

- $m_{\beta\beta} = \sum_i m_i' \lambda_i U_{ei}^2$
- $\eta_{LR} = \sum \lambda_i U_{ei} V_{ei} \frac{\not{q}}{(q^2+m_i^2)}$

Two-Neutrino ββ:

- Allowed by standard model



(a)

Effective Beta Decay Interaction:

Arising from some Gauge Theory beyond standard model:

$$H_{eff} = \frac{G}{\sqrt{2}} \left\{ J_L^{Had.} j_L^{lep.} + \eta J_L^{Had} j_R^{lep.} + \eta' J_R^{Had} j_L^{lep.} + \eta'' J_R^{Had.} j_R^{lep.} \right\}$$

$$j_L^{lep.} \equiv (\bar{e} \gamma_\lambda \nu_{eL})$$

$$j_R^{lep} \equiv (\bar{e} \gamma_\lambda \nu_{eR})$$

η, η', η'' all expected to be small! Vary from gauge model to gauge model.

Effective Mass for ββ:

$$m_{\beta\beta} = \sum_i \lambda_i m_i U_{ei}^2$$

Effective RHC parameter:

$$\langle \eta \rangle = \sum_i \lambda_i U_{ei} V_{ei} \frac{\cancel{q}\ \langle q \rangle}{(q^2+m_i^2)}$$

Leptonic Part of Amplitude :

permutes electrons

$$\propto \frac{1}{\sqrt{2}}\left(1 - P(e_i, e_j)\right) \langle L_\lambda L_\mu \rangle$$

$$\langle L_\lambda L_\mu \rangle \equiv \left(\bar{e}_i^- \gamma_\lambda (\nu_L + \eta \nu_R)\right)\left((\nu_L + \eta\nu_R)^T \gamma_\mu^T \bar{e}_j^{-T}\right)$$

Majorana neutrino propagator :

$$\left(\frac{(i\not{q} + m)}{q^2 + m^2}\right)$$

links LL , RR

links LR , RL

Majorana Mass amplitude $\sim \left(\frac{\lambda\, m}{q^2 + m^2}\right)$

RHC amplitude $\sim \left(\eta\, \frac{\lambda (i\not{q})}{q^2 + m^2}\right)$

$$e^- e^- \to W_a W_b$$

diverges quadratically unless

also $\sum m_i \lambda_i U_{ei}^2 = 0$ $\qquad \sum \lambda_i U_{ei} V_{ei} = 0$

- Assume this condition satisfied .

$$\langle \eta \rangle \approx \sum \lambda_i U_{ei} V_{ei} \frac{\not{q}}{q^2}\left\{1 - \frac{m_i^2}{q^2}\right\}$$

$$\approx \sum \lambda_i U_{ei} V_{ei} \left(-\frac{m_i^2}{q^2}\right)\frac{\not{q}}{q^2}$$

- Implies : (i) $m_i \neq 0$ for RHC

(ii) Lower bound on m_{max}

$$m_{max}^2 \geq \langle \eta \rangle (q^2)^2 / \langle q \rangle^2$$

$$\approx 10^{-8} \langle q^2 \rangle$$

Details depend on nuclear physics.

Kayser , Petcov , Rosen

- If $0\nu\beta\beta$ is seen with $\langle m_{\beta\beta} \rangle \approx 1$ eV in ^{76}Ge .

$$m_{max} \gtrsim 1\,\text{eV} \left[\frac{10^{24}}{\tau(^{76}Ge)}\right]^{\frac{1}{2}} \qquad \textbf{KPR}$$

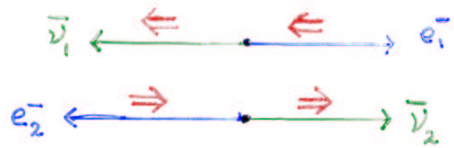
(8)

Angular Correlation for Decay Modes

Two - Neutrino Decay :

Nuclear Transition $J^P = 0^+ \to 0^+$

$\therefore$ Leptons arrange themselves as :

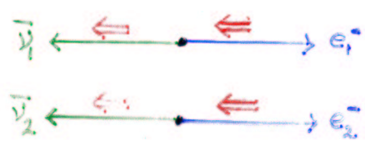


Conserves linear momentum and J_z without nuclear recoil.

$\therefore$ for ground state $\to$ ground state transition

$$A(e_1, e_2) \propto (1 - \lambda \cos\theta_{12}) \qquad \lambda = \frac{v_1 v_2}{c^2}$$

Nuclear Transition $J^P = 0^+ \to 2^+$ (excited state) 500 keV



allowed

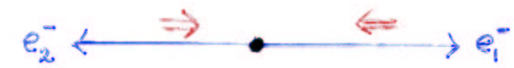
$$A'(e_1, e_2) \propto (1 + \tfrac{1}{3}\lambda \cos^2\theta_{12})$$

(9)

No - Neutrino Decay : ν - Mass Mechanism

Nuclear Transition $J^P = 0^+ \to 0^+$

g.s. $\to$ g.s.



Electrons have $\boxed{\text{same}}$ helicity. $\therefore$ can conserve momentum & J_z without nuclear recoil.

$$A(e_1, e_2) \propto (1 - \lambda \cos\theta)$$

Nuclear Transition $J^P = 0^+ \to 2^+$

g.s. $\to$ excited state (500 keV)

Cannot pick up enough angular momentum from electron spins alone : need orbital angular momentum $\to$ $\therefore$ forbidden

Majoron Decay $J^P = 0^+ \to 0^+$

Again electrons have same helicity and must come out back-to-back to conserve J_z. Thus Majoron cannot ~~attain~~ have its max momentum & tends to be soft.

Theoretical Bounds ~~of~~ on Lepton-Violating Mass Parameters :

(1) $\langle m_{\beta\beta} \rangle = \sum m_i \lambda_i U_{ei}^2 \leq \sum m_{max} \lambda_i U_{ei}^2$

$\leq m_{max} \sum U_{ei}^2 \leq m_{max}$

$\therefore$ Mass of heaviest "light" neutrino

$\geq$ effective mass measured in no-neutrino $\beta\beta$ decay

$\boxed{\text{or}}$

$m_{\text{"light"}} \ll \langle q \rangle \approx 10\text{-}100\,\text{MeV}$

At least one "light" neutrino heavier than $0\nu\beta\beta$ effective mass.

(2) RHC Bound from High Energy Behaviour of $W_a^- W_b^- \to e^- e^-$

Subdiagram of $0\nu\beta\beta$:

Neutrino Flavor Eigenstates as Superpositions of Mass Eigenstates

Take mass eigenstates to be Majorana neutrinos such that

$$(\nu_i)^c = \lambda_i \nu_i$$

λ_i is a phase factor, can be taken as (± 1). Mass $= m_i$.

$$\nu_{eL} = \sum_i U_{ei} \nu_{iL}, \quad \nu_{eR} = \sum_i V_{ei} \nu_{iR}$$

Similarly for ν_μ , ν_τ .

Effective Beta Decay Interaction

$$H_{eff} = \frac{G}{\sqrt{2}} \left\{ J_L^{Had} j_L^{Lepton} + \eta J_L^{Had} j_R^{Lepton} + \eta' J_R^{Had} j_L^{Lept} + \eta'' J_R^{H} j_R^{Lept} \right\}$$

Klapdor $\tau_{1/2}({}^{76}Ge)_{\beta\beta 0\nu} \approx 1.6\times10^{25}$ yrs

$m_{\beta\beta} \approx 400\ \text{meV}$

"Best Value"

KPR $m_{max} \geq m_{\beta\beta}$

3-ν model:

$m_3 = 400\ \text{meV}$, $m_2 = 400-\delta_2$, $m_1 = m_2 - \delta_1$

$\Delta m^2_{ATMOS} \approx 3\times10^3\ \text{meV}^2 \Rightarrow \delta_2 \approx 4\ \text{meV}$

$\Delta m^2_{LMA} \approx 30\ \text{meV}^2 \Rightarrow \delta_1 \approx 4\times10^{-2}\ \text{meV}$

What about LSND?

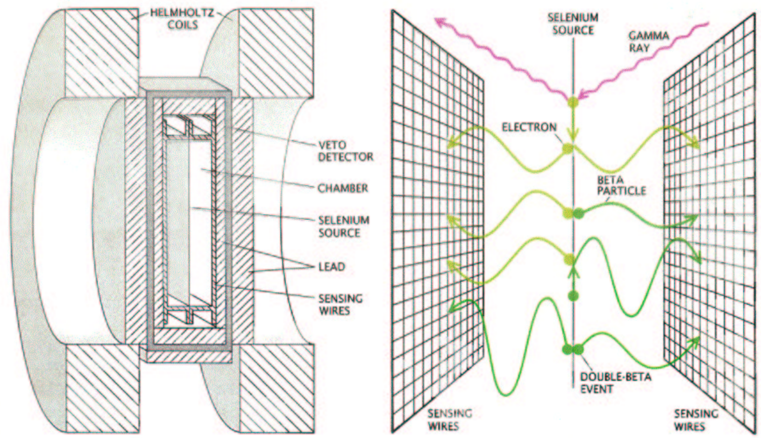
ν_4 —

$\Delta m^2_{LSND} \approx 10^6\ \text{meV}^2$

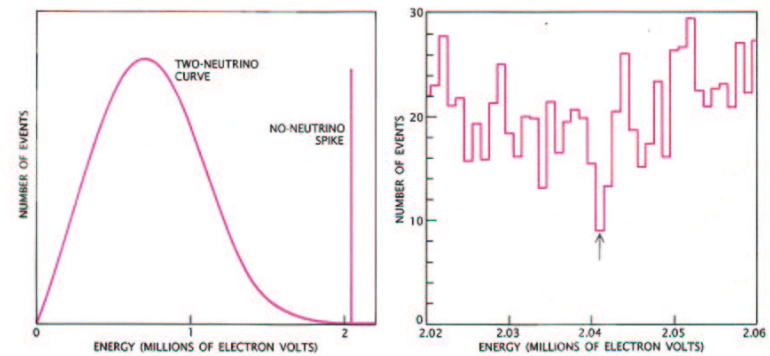
$\nu_{3,2,1}$ ≡

$\Rightarrow m_4 \approx 1{,}100\ \text{meV}$

$\approx 1.1\ \text{eV}$



TIME PROJECTION CHAMBER (*left*) provided the first direct evidence for double-beta decay by tracking the emitted electrons. A sample of selenium 82 is supported in the central plane of the detector. Around the sample is a chamber filled with helium. A lead casing shields the chamber from outside radioactivity, and a "veto" detector warns of incoming cosmic rays. A Helmholtz coil generates a magnetic field, which causes beta rays emitted in the chamber to follow helical paths. As the beta particles move through the helium, they ionize it. An applied electric field causes the resulting free electrons to drift into sensing wires, which register their arrival time and position. The pattern of free electrons is analyzed to recreate the helical paths of the beta rays. The size and pitch of a helix yield the beta-ray energy. The double-beta-decay signature (*bottom event at right*) can be mimicked by the rare background events shown above the signature. Such imposters are usually revealed within a few hours when a daughter nucleus resulting from the event decays at the same spot.



ENERGY SPECTRUM of the electrons associated with germanium 76 decay is expected to include a broad curve for the two-neutrino mode of double-beta decay and a spike for the no-neutrino mode (*left*). The most sensitive measurements to date (*right*) have not revealed the spike, which is predicted to appear at 2.041 million electron volts (*arrow*). The bumpiness in the spectrum is largely the result of statistical fluctuations in the background. If the neutrinoless double-beta contribution is assumed to be less than the size of the statistical fluctuations, the half-life of germanium 76 for the no-neutrino mode must be greater than 2.3×10^{24} years. David O. Caldwell and his co-workers accumulated the data over several years.

Heavy Right-Handed Neutrino:

For mass-induced $0\nu\beta\beta$, appropriate part in matrix element is:

$$P = \left(\frac{m_\nu \langle q \rangle}{\langle q^2 \rangle + m_\nu^2} \right)$$

"Light" neutrino, $m_\nu \ll \langle q \rangle$:

$$P \approx \frac{m_\nu}{\langle q \rangle}$$

"Heavy" neutrino, $M_\nu \gg \langle q \rangle$

$$P \approx \frac{\langle q \rangle}{M_\nu}$$

"See-Saw":

$$m_\nu M_\nu \approx \langle q \rangle^2 \left(\frac{M_{W_L}}{M_{W_R}} \right)^4$$

$$(1\,\mathrm{eV})(10^2\,\mathrm{GeV}) \approx (30\,\mathrm{MeV})^2 \, (10^{-4})$$

$$\approx (m_e)^2$$

(5)

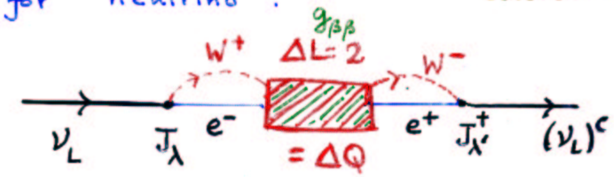
Note (i). Gauge theory condition comes from requirement of good high energy behaviour of:

$$e^- e^- \xrightarrow[\text{exchange}]{\text{neutrino}} W_a^- W_b^-$$

in theories with no doubly charged gauge bosons.

Note (ii). Even if some new interaction is responsible for $0\nu-\beta\beta$ decay, its existence implies Majorana mass term for neutrino.

Schechter + Valle



On dimensional grounds: expected to be small

$$\delta m_{\text{Majorana}} \approx g_{\beta\beta} \cdot G_F^2 \, E^5$$

Take $E \approx 100$ MeV → mean separation of nucleons $\approx 1\text{-}2\,f$ in nuclei.
→ probably optimistic

$$\delta m_{\text{Majorana}} \approx g_{\beta\beta} \cdot 10^{-6}\,\mathrm{eV} \approx 10^{-9}\,\mathrm{eV}$$

$$\delta(m_{K_L} - K_S) \approx 10^{-5}\,\mathrm{eV}$$

{ Suppression of ov rel. phase sp.

$$e^- e^- \to W_a W_b$$

diverges quadratically unless

$$\sum \lambda_i U_{ei} V_{ei} = 0$$

- Assume this holds true.

$$\langle \eta \rangle \approx \sum \lambda_i U_{ei} V_{ei} \frac{\not{q} \langle q \rangle}{q^2} \left\{ 1 - \frac{m_i^2}{q^2} \right\}$$

$$\approx \sum \lambda_i U_{ei} V_{ei} \left(-\frac{m_i^2}{q^2} \right)$$

Implies $m_i \neq m_j \neq 0$ for RHC

Bounds on m_i from nuclear physics.

From ^{76}Ge

$$m_{max} \gtrsim 1\,\text{eV} \left[\frac{10^{24}}{\tau(^{76}Ge)_{0\nu}} \right]$$

Nuclear MATRIX Elements

Second-order in β-decay Hamiltonian:

$$\mathcal{M} = \sum_{m, \ell_m} \frac{\langle f, \ell_f | H_\beta | m, \ell_m \rangle \langle m, \ell_m | H_\beta | i \rangle}{(E_m - E_i)}$$

$$(E_m - E_i) = \underbrace{W_{mi}}_{\substack{\text{energy difference between nuclear states} \\ |m\rangle \text{ and } |i\rangle.}} + \underbrace{E_{\nu_m} + E_{e_m}}_{\substack{\text{Energy of intermediate} \\ \text{lepton state } |\ell_m\rangle}}$$

$$|\ell_f\rangle \equiv \begin{cases} |e_1 e_2 ; \nu_1 \nu_2\rangle & 2\nu\beta\beta \\ |e_1, e_2\rangle & 0\nu\beta\beta \end{cases}$$

- For 2νββ replace $E_m - E_i$ by an average value $\langle E_m - E_i \rangle = \langle W_{mi} \rangle + \frac{1}{2} Q + m_e c^2$; can use closure over $|m\rangle$, but not necessary in modern calculations.
- For 0νββ, integrate over intermediate neutrino energy using closure over $|m\rangle$. Finite because of ⟨separation⟩ of neutrons.

Brief Survey of Existing Experiments

Heidelberg - Moscow Enriched ^{76}Ge:

Hint of peak at end-point going away:

$\tau_{1/2} > 2.9 \times 10^{24}$ yrs ; $\langle m_{\beta\beta} \rangle < 0.9$ eV

IGEX enriched ^{76}Ge, 19.6 kg source

most bkgd cosmogenic $^{68}(Ge)$ $(\tau_{1/2} \approx 288d)$

0.3 counts/keV/kg/yr (H-M ≈ 0.23)

CalTech ^{126}Xe TPC, being upgraded

$\tau_{1/2} > 3.7 \times 10^{23}$ yr ; $\langle m_{\beta\beta} \rangle < 2.8$ eV

(F. Boehm)

Milano - G.S. ^{130}Te, being expanded ($1 \to 4$ xtals)

Hint of peak at end pt (13 ± 6 cts, 2σ)

$\tau_{1/2} \gtrsim 1.4 \times 10^{22}$ yrs $\langle m_{\beta\beta} \rangle < 3.4 - 6.7$ eV

Enriched in 130 and 128

UCSB - LBL ^{150}NdF cryogenic detector

100 gms enriched ^{150}Nd : reach H-M limit in few weeks

UCSB - LBL ^{150}NdF cryogenic bolometer.

100 gms enriched ^{150}Nd, 70 times more sensitive than ^{76}Ge. Phase space $30 \times$ ^{76}Ge (higher Q, higher Z)

Better energy resolution than ^{76}Ge.

Raghavan idea: put Xe in liquid of Borexino detector:

Low bkgd, very clean, energy resolution good enough for $m_{\beta\beta} \approx 0.1$ eV.

PRL 72, 1411 (1994)

Mike Moe: • looking for 2ν in enriched ^{48}Ca. $\tau \approx 10^{19}$ yrs – could go to 10^{20} yrs

• 0ν in ^{126}Xe, eliminate bkgd in large source, see daughter.

NEMO ^{100}Mo Detector, under construction

10 kg ^{100}Mo cylinder 2 m high, 2.8 diameter

tracking chamber; scintillator wires

2000 PMT's.

Should get to 10^{25} yrs, $\langle m_{\beta\beta}\rangle < 0.3$ eV

Double beta decays experiments are moving from the bench-top to the large-scale, large collaboration efforts.

Definitely a much-needed development if we are to have any chance of discovering $0\nu\beta\beta$ (if it is there).

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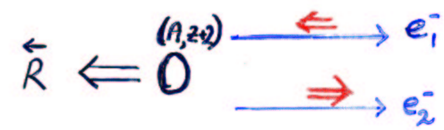
(10)

No-Neutrino Decay: L-R Interference Mechanism.

Cross-Term between Left- and Right-handed currents. ∴ Electrons have $\boxed{\text{opposite}}$ helicities.

Nuclear Transition: $J^P \Rightarrow 0^+ \to 0^+$
g.s. → g.s.

Cannot conserve momentum and J_z simultaneously unless there is a nuclear recoil:



$$\therefore \quad A(e_1, e_2) \propto (1 + \lambda \cos\theta_{12})$$

¿ ARE TPC & SANDWICH DETECTORS BIASSED AGAINST THIS TYPE OF CORRELATION ?

Nuclear Transition $J^P = 0^+ \to 2^+$ (g.s. → excited st. at 500 keV)

Electron Spins, plus Recoil + P-wave provide enough momenta for $\Delta J = 2$ transition.

∴ This Transition is $\boxed{\text{allowed}}$ in L-R Mechanism

∴ $\boxed{0^+ \to 2^+ \text{ no-}\nu \text{ decay} \Rightarrow \text{L-R Mechanism}}$

Nuclear Operators

For $0^+ \to 0^+$ nuclear $\beta\beta$ transitions, the principal operators are:

$$H_\beta^F = \sum_{\text{nucleons}} \tau_k^+ \equiv T^+ \qquad \text{Fermi, isospin raising}$$

$$H_\beta^{GT} = \sum_{\text{nucleons}} \tau_k^+ \vec{\sigma}_k \qquad \text{Gamow-Teller}$$

- For $2\nu\beta\beta$, $\mathcal{M}^F$ vanishes on grounds of isospin and we are left with:

$$\mathcal{M}^{GT}_{2\nu} = \frac{1}{\langle E_m - E_i \rangle} \sum_m \langle f | \sum_k \tau_k^+ \vec{\sigma}_k | m \rangle \cdot \langle m | \sum_{k'} \tau_{k'}^+ \vec{\sigma}_{k'} | i \rangle$$

modern calculations can correct for isospin breaking.

- For $0\nu\beta\beta$, both types of operator can contribute. Closure over intermediate nuclear state $|m\rangle$ and integration over virtual neutrino energy yields:

No-Neutrino Decay

- Closure over intermediate nuclear states.
- Integrate over virtual neutrino energies.

$$\mathcal{M}_F^{0\nu} = \langle f | \sum_{k,l} \frac{\tau_k^+ \tau_l^+}{r_{kl}} I(r_{kl}) | i \rangle$$

$$\mathcal{M}_{GT}^{0\nu} = \langle f | \sum_{k,l} \frac{\tau_k^+ \tau_l^+}{r_{kl}} \vec{\sigma}_k \cdot \vec{\sigma}_l \, I(r_{kl}) | i \rangle$$

$I(r_{kl})$ involves $\int$al over virtual neutrino energy: Haxton Representation for $m_\nu \neq 0$

$$\frac{1}{r} I(r) = \frac{1}{r} I(r)\Big|_{m_\nu = 0} + \sum \frac{\pi m_\nu}{2r} \frac{(e^{-m_\nu r} - 1)}{m_\nu + a \langle E_{ni} \rangle}$$

(For RHC this gives proportionality between $M^{0\nu}$ and $M^{2\nu}$.)

$$\frac{1}{r} I(r)\Big|_{m_\nu = 0} = \int_0^\infty \frac{\sin qr}{q + \langle W_{mi} \rangle} dq$$

Decay Rates

For $2\nu\beta\beta$ the rate for $0^+ \to 0^+$ decay is:

$$[T_{1/2,2\nu}(0^+ \to 0^+)]^{-1} = G_{2\nu}(E_0, Z)\, |m^{GT}_{2\nu}|^2$$

Phase space and Coulomb wavefunction factor.
Polynomial up to E_0^{10}, or E_0^{11} power.
Tabulated in many review articles.

For $0\nu\beta\beta$ the $0^+ \to 0^+$ decay rate is:

$$[T_{1/2,0\nu}(0^+ \to 0^+)]^{-1} = G_{0\nu}(E_0, Z)\, \left|m^{GT}_{0\nu} - \frac{g_V^2}{g_A^2} m^F_{0\nu}\right|^2 \times f(\langle m_{\beta\beta}\rangle, \eta_{LR})$$

Phase space and Coulomb; polynomial to E_0^5

Lepton non-conserving factor of form

$$f(\langle m_{\beta\beta}\rangle, \eta) = C_1 \frac{\langle m_{\beta\beta}\rangle^2}{m_e^2} + C_2 \frac{\langle m_{\beta\beta}\rangle}{m_e}\eta + C_3 \eta^2$$

where coefficients are known for specific cases.

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Nuclear Matrix Elements

Two - Neutrino Decay

Typical Matrix Element of form

$$m = \sum_m \frac{\langle f | H_\beta | m \rangle \langle m | H_\beta | i \rangle}{(E_m - E_i)}$$

$$(E_m - E_i) = W_{mi} + E_{\nu_i} + E_{e_i}$$

Energy difference between g. s. of parent nucleus and state m of intermediate nucleus

energy of neutrinos, electrons

- Replace energy denominator by an average.
  $\langle E_m - E_i \rangle = \langle W_{mi} \rangle + \frac{1}{2} Q + m_e c^2$
- Can (but not necessary) use closure over $|m\rangle\langle m|$

For $0^+ \to 0^+$ transitions, the operators H_β are:

(isospin raising operator) $H_\beta = \sum_{\text{nucleons}} \tau_k^+ = T^+$ Fermi

([illegible]) $H_\beta = \sum_{\text{nucleons}} \tau_k^+ \vec{\sigma}_k$ Gamow-Teller

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Fermi matrix element

$$\mathcal{M}_F = \frac{1}{\langle E_m - E_i \rangle} \sum_m \langle f | T^+ | m \rangle \langle m | T^+ | i \rangle$$

involves isospin raising operator T^+ which increases T_3 by 1 unit, but leaves T unchanged. Therefore $|m\rangle$ and $|f\rangle$ must have same isospin as $|i\rangle$ for matrix element to be non-zero.

Fermi operator wants to take g.s. of (A, Z) to double isobaric analogue state of $(A, Z+2)$!

$$\therefore \boxed{\mathcal{M}_F \, (g.s \to g.s.) = 0}$$

What about Gamow-Teller Matrix element?

$$\mathcal{M}_{GT} = \frac{1}{\langle E_m - E_i \rangle} \sum_m \langle f | \sum_k \tau_k^+ \vec{\sigma}_k | m \rangle \cdot \langle m | \sum_{k'} \tau_{k'}^+ \vec{\sigma}_{k'} | i \rangle$$

Should there be a good symmetry scheme with $\sum \tau^+ \vec{\sigma}$ as part of the set of generators, then $\mathcal{M}_{GT}$ will vanish when $|i\rangle$ and $|f\rangle$ belong to different representations.

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Take matrix element and insert complete set of states :

$$\sum_m |\langle m | \tau^+ \vec{\sigma} | 0^+ \rangle|^2 - \sum_n |\langle n | \tau^- \vec{\sigma} | 0^+ \rangle|^2 = 3(N-Z)$$

Giant Gamow Teller resonance + low-lying 1^+ states do not exhaust sum rule.

(accounts for 60%)

$\therefore$ take axial vector coupling constant

$\boxed{g_A = 1}$ in heavier nuclei instead of 1.26

Haxton, ...

$\therefore$ Must turn to more complicated schemes to evaluate $\mathcal{M}_{GT}$ which gives major contribution to lifetime:

$$\boxed{1/\tau_{1/2}^{2\nu}(0^+ \to 0^+) = |\mathcal{M}_{GT}|^2 \, G_{GT}(A, Z)}$$

matrix element

phase space, Coulomb factor

Calculation of Nuclear Matrix Elements

by : nuclear shell model; and

quasi-particle random phase approximation (QRPA).

QRPA has been used by most recent authors – gives a good representation of nuclear forces.

$2\nu\beta\beta$	Theory* and	Expt.+
^{76}Ge	1.3×10^{21} yrs	$(9\pm1)\times10^{20}$ yrs
^{82}Se	1.2×10^{20} yrs	$(1.08^{+0.26}_{-0.06})\times10^{20}$ yrs
^{100}Mo	6×10^{18} yrs	$(1.15^{+0.30}_{-0.20})\times10^{19}$ yrs
^{128}Te	5.5×10^{23} yrs	$(7.7\pm0.4)\times10^{24}$ yrs
^{130}Te	2.2×10^{20} yrs	$(2.7\pm0.1)\times10^{21}$ yrs

* Engel, Vogl, Zirnbauer. + selected