NEUTRINO MASSES AND GUTS:

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KITP ν-CONFERENCE

03/03 - 07/03.

1. INTRODUCTION
2. SEESAW MECHANISM
   TYPE I & II
3. UNDERSTANDING LARGE MIXINGS USING TYPE II SEESAW
4. INVERTED HIERARCHY, $L_e - L_\mu - L_\tau$ AND HORIZONTAL SYM.
5. TESTS: U_{e3}, $\beta\beta_{0\nu}$, Δm^2_A

1

- SOLAR + ATMOSPH.

$\Rightarrow m_{\nu_i} \neq 0$;

$\Rightarrow$ THEY MIX $(\theta_{ij} \neq 0)$

- SMALL Δm^2 FROM OSC. DATA + TRITIUM DECAY RESULT (≤ 2.2 eV)

$\Rightarrow$ ALL MASSES, $m_{\nu_i} \lesssim 2.2\,\text{eV}$

$\Rightarrow$ COSMOLOGY (2dF, WMAP)

$\Rightarrow \sum_i m_{\nu_i} \leq .69\,\text{eV}$

HOW MANY ν'S ?

DEPENDS ON WHETHER LSND IS CORRECT OR NOT ?

A) WITHOUT LSND:

ν_e, ν_μ, ν_τ SUFFICIENT TO EXPLAIN ALL DATA.

B) INCLUDE LSND

$\nu_e, \nu_\mu, \nu_\tau + \nu_s$

- NO EVIDENCE YET FROM SOLAR DATA
- NEW PRESSURE FROM WMAP (MURAYAMA, PIERCE '03; GIUNTI '03)
- WAIT FOR MINI-BOONE

• 3 NEUTRINOS

• ASSUME ν = MAJORANA
(OR $\nu = e^{i\alpha} C \bar{\nu}^T$).
4-COMP.

• $\mathcal{L}_{mass} = \nu_L^T C^{-1} M_\nu \nu_L + \bar{\ell}_L M_\ell \ell_R + h.c.$

DIAGONALIZE:
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_\nu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}; \ \ell \ldots$$

MIXING MATRIX: $U_{PMNS} = U_{\ell,L}^{\dagger} U_\nu \quad (U_{\alpha i})$ $\alpha = e, \mu, \tau$; $i = 1, 2, 3$

MIXING PATTERN

SOLAR: PRESENT FAVORITE

$\nu_e\,\nu_\mu$ MIXING (U_{e2}) LARGE!!

$.55 \leq \sin^2 2\theta_{12} \leq .95 \quad 3\sigma$.

KAMLAND: $\sin^2 2\theta_{12} \simeq .86 - 1$
(+ ELIMINATES ALL NON-OSC. SOLUTIONS TO $\nu_\odot$-PROB)

ATMOSPH.: $\sin^2 2\theta_{23} \simeq .8 - 1$

REACTOR EXPT. CHOOZ, PALOVERDE
$U_{e3} < .15$ FOR $\Delta m^2 \gtrsim 10^{-3}\ eV^2$ (θ_{13})

$$\Rightarrow U_{MNSP} = \begin{pmatrix} c & -s & \epsilon \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

?MAXIMAL (BI-LARGE)

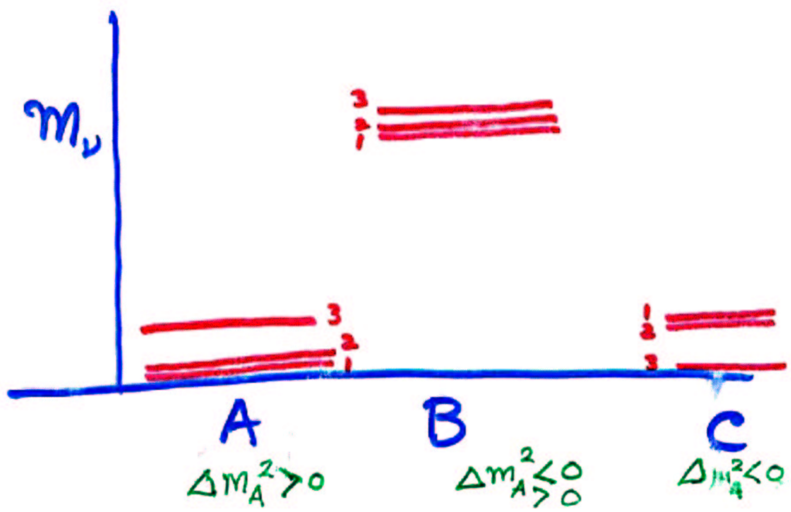
MASSES

THREE PATTERNS:

A) $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$ (HIERARCHICAL)

B) $m_{\nu_1} \simeq m_{\nu_2} \simeq m_{\nu_3}$ (DEGENERATE)

C) $m_{\nu_1} \simeq m_{\nu_2} \gg m_{\nu_3}$ (INVERTED)



WE STILL NEED TO KNOW:

(i) $\nu = \bar{\nu}$?

$\beta\beta_{0\nu}$

(ii) $U_{e3} = ?$

LBL (NUMI-OFFAXIS, JHF, ...)

(iii) MASS PATTERN:

- $\Delta m_A^2 \geq 0$ OR ≤ 0

  LBL

- $m_1 \ll m_2 \ll m_3$ OR DEG.

  $\beta\beta_{0\nu}$; KATRIN, COSMOLOGY

TOWARDS A FUNDAMENTAL THEORY

HOW TO UNDERSTAND

(i) $m_\nu \ll m_{e,u,d}$;

(ii) BI-LARGE MIXING ;

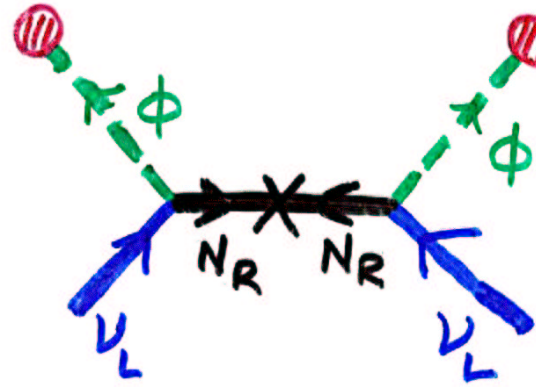
WHY θ_{12}, θ_{23} LARGE BUT $\theta_{13} \ll 1$?

(iii) MASS PATTERN

e.g. WHY $\frac{\Delta m^2_\odot}{\Delta m^2_A} \ll 1$?

(i) WHY $m_\nu \ll m_{e,u,d}$?

$\Rightarrow$ INTRODUCE ν_R AND GIVE IT A LARGE MAJORANA MASS. (NEW SCALE)



$$m_\nu \simeq M^T_{\nu D} \frac{1}{M_R} M_{\nu D} \ll m_{\ell,q},$$

(FOR $M_R \gg v_{wk}$)

SEESAW MECHANISM (TYPE I) : $M_R \ll M_{P\ell}$

GELL-MANN, RAMOND, SLANSKY; YANAGIDA; R.N.M., SENJANOVIĆ; GLASHOW '79

$\sqrt{\Delta m^2_A} \simeq .05\ eV \Rightarrow$ AT LEAST ONE $m_{\nu_i} \geq .05\ eV$

$\Rightarrow \quad m_{\nu_3^D} \leq m_t \Rightarrow M_{N_R} \lesssim 10^{15}$ GeV

WHY $M_{N_R} \ll M_{Pl}$?

- NEW LOCAL SYM. AT M_{N_R}
  e.g. B-L , $SU(2)_H$
- ELIMINATE $M_{P\ell}$
  e.g. EXTRA DIM. ($M_{N_R} \simeq M_*$)

A LOCAL B-L EXAMPLE:

SM + N_R → MINIMAL ANOMALY FREE EW SYM.

$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \subset SO(10)$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{pmatrix} N_R \\ e_R \end{pmatrix}$$

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}$$

$$\Rightarrow \Delta I_{3R} = -\frac{1}{2}\Delta(B-L)$$

MAJORANA ν

m_ν AND SPONTANEOUS BREAKING OF PARITY:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\begin{array}{cc} \nu_L & N_R \\ \end{array}$$
$$\begin{pmatrix} 0 & 0 \\ 0 & f v_R \end{pmatrix}$$

$$\Big\downarrow \; v_R \text{ (NEW SCALE)}$$

$$SU(2)_L \times U(1)_Y$$

$$\begin{array}{cc} \nu_L & N_R \\ \end{array}$$
$$\begin{pmatrix} f v_L & m_{\nu D} \\ m_{\nu D} & f v_R \end{pmatrix}$$

$$\Big\downarrow \; v_{wk}$$

$$U(1)_{em}$$

$$m_{\nu_D} \simeq h v_{wk} \ll f v_R$$

$$m_\nu = f \frac{v_{wk}^2}{v_R} - M_{\nu D}^T \frac{f^{-1}}{v_R} M_{\nu D}$$

(TYPE II)

PARITY SYM. $\Rightarrow$ BOTH TERMS.

TYPE I AND TYPE II SEESAW FORMULAE:

MODELS WITHOUT PARITY SYM.

TYPE I: $M_\nu = - M_{\nu D} M_R^{-1} M_{\nu D}^T$

MODELS WITH PARITY SYM: (LR, SO(10), E_6, ... $SU(3)^3$)

$\downarrow$

TYPE II

$$M_\nu = f v_L - M_{\nu D} \frac{1}{f v_R} M_{\nu D}^T$$

Mohap-- - 09

$\boxed{m_\nu \text{ PROBES THE SCALE OF}}$

B-L OR $SU(2)_H$

ATMOS. $m_{\nu_\tau} \sim 10^{-1}$ eV

(IF $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$)

$$M_{BL,H} \simeq 10^{12} - 10^{15} \text{ GeV}$$
$$\simeq M_{N_R}$$

FITS VERY WELL WITH:

(i) BARYOGENESIS VIA LEPTO GENESIS, WHICH REQUIRES HIGH MASS RH N_R'S;

(ii) UNIFICATION OF GAUGE COUPLING

⇒ FITS INTO SO(10) GUT.

UNDERSTANDING LARGE ν- MIXINGS WITH "SEESAW":

$$\mathcal{M}_\nu = f\lambda \frac{v_{wk}^2}{V_{BL}} - M_{\nu D} M_{N_R}^{-1} M_{\nu D}^T$$

$M_{N_R} \approx f V_{BL}$

OF PARAMETERS ≥ 12

OF OBSERVABLES = 6 (WITHOUT CP)

⇒ NEED EXTRA SYM. AT HIGH SCALE SO(10), FAMILY, DISCR....

(i) SECOND TERM DOMINATES

(ii) FIRST TERM DOMINATES

(iii) BOTH IMPORTANT

LARGE MIXING FROM SO(10) WITH TYPE II SEESAW

- ALL SM FERMIONS $+ \nu_R \subset \{16\}$
- $G_{STD} \xrightarrow{V_{B-L}} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$

$$\downarrow$$

$$SU(2)_L \times SU(2)_R \times SU(4)_{PS}$$

$$M_U \downarrow$$

$$SO(10)$$

- $M_{W,Z} \ll V_{B-L} \simeq M_{N_R} \lesssim M_U$
- SEESAW SCALE LINKED TO GRAND UNIFICATION.
- SO(10) CAN MAKE THE MODEL PREDICTIVE

TWO CLASSES OF SO(10) MODELS DEPENDING ON HOW M_{N_R} ARISES:

$\boxed{\text{(i)}\ \Delta(B-L) = 1}$

$$16_H \Rightarrow M_{N_R} \Leftarrow \frac{(16_F 16_H)^2}{M_{Pl}}, \frac{(16_F \overline{16}_H)^2}{M_{Pl}}$$

ALBRIGHT, BARR; BABU, PATI, WILCZEK, BLAZEK, RABY, TOBE, ⋮

!! NEED EXTRA SYMMETRIES BEYOND SO(10) TO HAVE STABLE DARK MATTER !!

$\boxed{\text{(ii)}\ \Delta(B-L) = 2}$

$$126_H: \quad M_{N_R} \Leftarrow \underbrace{16_F 16_F \overline{126}_H}_{f \nu^c \nu^c \Delta^c}$$

$\Rightarrow$ R-PARITY AUTOMATIC - STABLE DARK MATTER!!

GOOD NEWS AND BAD NEWS FROM SO(10) — × —

$SO(10) \supset SU(4)_{PS}$

$\Rightarrow$ LINKS $M_{\nu D}$ TO M_u

$\Rightarrow \boxed{\text{GOOD}}$ CAN REDUCE PARAMETERS

$\Rightarrow m_{u,d,e} \ll m_{c,s,\mu} \ll m_{t,b,\tau}$

$\Rightarrow m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$

$\Rightarrow$ EXPLAINS WHY $\Delta m^2_\odot \ll \Delta m^2_A$

$\boxed{\text{BAD}}$ $\theta_{q,ij} \approx \theta_{\nu,ij}$

DESIRED FORM OF $\mathcal{M}_\nu$ FOR THE CASE OF NORMAL HIERARCHY i.e. $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$:

$$\mathcal{M}_\nu = \begin{pmatrix} \epsilon^{n'} & \epsilon^n & b\epsilon \\ \epsilon^n & 1+a\epsilon & 1 \\ b\epsilon & 1 & 1+\epsilon \end{pmatrix} \sqrt{\Delta m^2_A}$$

$$a, b \sim 1$$

$$n, n' \geq 1$$

CAN SUCH A FORM NATURALLY IN SO(10) ?

A MINIMAL SO(10) EXAMPLE

BABU, R.N.M. '92

MINIMAL HIGGS: $\{10\}$ $\{\overline{126}\}$

(WITH STABLE DARK MATTER)

$$h\,\psi\psi\{10\} + f\,\psi\psi\{\overline{126}\}$$

$$\Rightarrow M_u = h\langle 10\rangle_u + f\langle\overline{126}\rangle_u$$

$$M_d = h\langle 10\rangle_d + f\langle\overline{126}\rangle_d$$

$$M_\ell = h\langle 10\rangle_d - 3f\langle\overline{126}\rangle_d$$

$$M_{\nu D} = h\langle 10\rangle_u - 3f\langle\overline{126}\rangle_u$$

$$M_\nu = f\langle 126\rangle_{LL} - M_{\nu D}(f v_R)^{-1} M_{\nu D}^T$$

BOTH $M_{\nu D}$ AND M_{N_R} $(= f v_R)$ FLAVOR STRUCTURE PREDICTED!

SMALL $\theta_\odot$: LAVOURA '93; Lee, R.N.M. '95; BRAHMACHARI, R.N.M. '98; ODA, TAKASUGI, TANAKA, YOSHIMURA '99

CP PHASE: FUKUYAMA, KOIDE, MATSUDA, NISHIURA '02; FUKUYAMA, OKADA '02

$\Rightarrow$ BILARGE

IN PARTICULAR, FOR TYPE II SEESAW ⇒ SUMRULE

$$\Rightarrow \quad M_{\nu_{LL}} \simeq c(M_d - M_\ell) + O\left(\frac{m_D^2}{M_R}\right)$$

SMALL

2-3 SECTOR:

$$M_d = \begin{pmatrix} \varepsilon & \varepsilon' \\ \varepsilon' & 1 \end{pmatrix} m_b$$

ε's $\ll 1$

$$M_\ell = \begin{pmatrix} \varepsilon''' & \varepsilon'' \\ \varepsilon'' & 1 \end{pmatrix} m_\tau$$

AT GUT SCALE, $m_b \simeq m_\tau$

$$\Rightarrow \quad M_d - M_\ell = \begin{pmatrix} \varepsilon & \tilde{\varepsilon} \\ \tilde{\varepsilon} & \sim 0 \end{pmatrix} \approx M_\nu$$

⇒ LARGE MIXING ANGLE NATURALLY!

$\theta_{12} = ?$

BAJC, SENJANOVIĆ, VISSANI '02

EXTENSION TO 3 GEN.

CASE:

GOH, NG, R.N.M. '03

DETAILED ANALYSIS

⇒ POSSIBLE TO GET LARGE $\theta_\odot$ FOR A RANGE OF ALLOWED QUARK MASSES.

PROCEDURE: ELIMINATE h, f

$$\boxed{k\tilde{M}_\ell = r\tilde{M}_d + \tilde{M}_u} \qquad \tilde{M} = \frac{M}{m_3}$$

INPUT $\quad m_q, V_{CKM} \Rightarrow 8$ PARAM

OUTPUT $\quad m_\ell, \theta^\ell_{ij}, k, r$

$\quad 3 + 3 + 1 + 1 = 8$

$$\Rightarrow \quad \boxed{M_\nu \simeq a(M_d - M_\ell)}$$

GUT SCALE VALUES:

BARGER, BERGER, CHIANG; C. DAS, PARIDA hep-ph/0010004

FUSAOKA, KOIDE;

input observable	$tan\beta = 10$
m_u (MeV)	$0.7238^{+0.1365}_{-0.1467}$
m_c(MeV)	$210.3273^{+19.0036}_{-21.2264}$
m_t(GeV)	$82.4333^{+30.2676}_{-14.7686}$
m_d (MeV)	$1.5036^{+0.4235}_{-0.2304}$
m_s (MeV)	$29.9454^{+4.3001}_{-4.5444}$
m_b (GeV)	$1.0636^{+0.1414}_{-0.0865}$
m_e (MeV)	0.3585
m_μ(MeV)	$75.6715^{+0.0578}_{-0.0501}$
m_τ (GeV)	$1.2922^{+0.0013}_{-0.0012}$

INPUT

$$V_{CKM} = \begin{pmatrix} 0.974836 & 0.222899 & -0.00319129 \\ -0.222638 & 0.974217 & 0.0365224 \\ 0.0112498 & -0.0348928 & 0.999328 \end{pmatrix}$$

FITS TO $m_{e,\mu;\tau} \Rightarrow r, k$

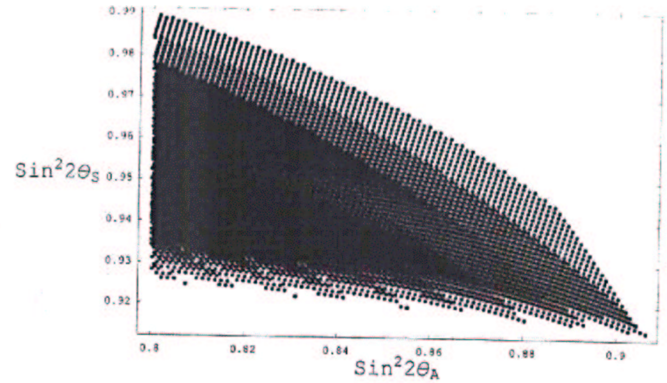
$m_{s,d,c} < 0$

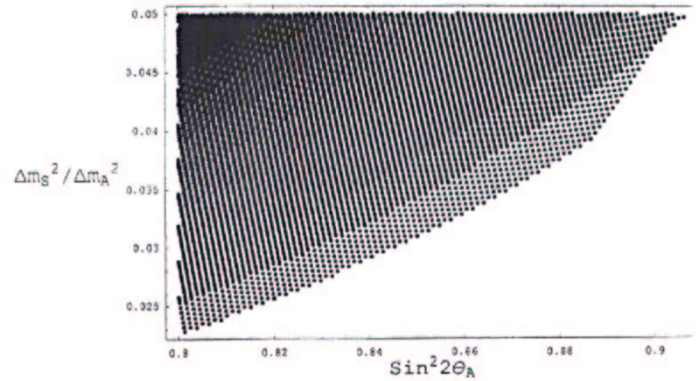
$-.78 \leq r \leq -.74$

$.23 \leq k \leq .26$

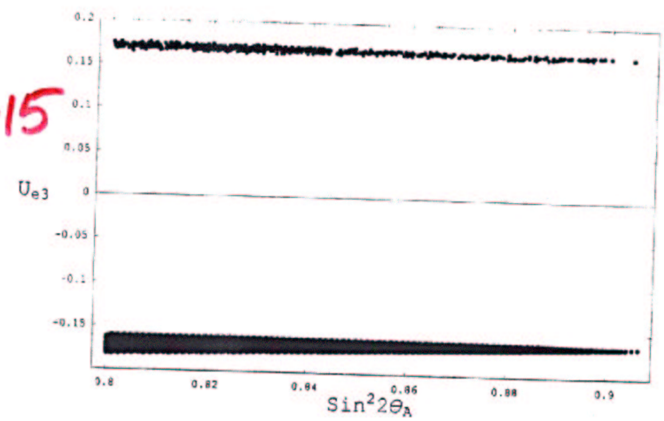
$$\mathcal{M}_\nu \simeq \begin{pmatrix} 10^{-3} & 10^{-3} & \sim 10^{-3} \\ 10^{-3} & \sim .2 & .3 \\ 10^{-3} & \sim .3 & \sim .3 \end{pmatrix} a$$

$\tan\beta = 10$





$U_{e3} \simeq .15$



TESTABLE MODEL

(i) $U_{e3} \sim .15$

(ii) $\sin^2 2\theta_A \sim .8-.9$

(iii) $\sin^2 2\theta_{12} \gtrsim .9$

(iv) $A_{\beta\beta_{0\nu}}$ NEGLIGIBLE

LARGE MIXING FOR QUASI-DEG. ν'S:

- Q–L UNIFICATION (SO(10)…)

  ⇒ AT SEESAW SCALE $\theta_{quark} \approx \theta_{lepton}$

- TYPE II SEESAW FOR QUASI-DEGENERACY

$$\mathcal{M}_\nu = f v_L - \frac{1}{v_{BL}} M_{\nu D} f^{-1} M_{\nu D}^T \quad (\text{AT } \mu = v_{BL})$$

- RADIATIVE MAGNIFICATION OF SMALL MIXINGS:

$$\mathcal{M}_\nu(M_Z) = \begin{pmatrix} 1+\delta_\mu & \\ & 1+\delta_\tau \end{pmatrix} \mathcal{M}_\nu(v_{BL}) \begin{pmatrix} 1+\delta_\mu & \\ & 1+\delta_\tau \end{pmatrix}$$

$$\delta_\ell = \frac{h_\ell^2}{16\pi^2} \ln \frac{v_{BL}}{M_Z}$$

2-GEN.

AT $\mu = V_R$, SEESAW SCALE

(BALAJI, DIGHE, PARIDA, R.N.M PRL, 2000.)

$$M_{\nu_R} = \begin{pmatrix} m_0 & \delta \\ \delta & m_0 + \delta' \end{pmatrix}; \quad \delta \ll \epsilon \ll m_0$$

$$\tan 2\theta = \frac{2\delta}{\delta'} \ll 1$$

(AS IN QUARK SECTOR)

EXTRAPOLATE TO WEAK SCALE:

$$M(\nu_{wk}) \simeq \begin{pmatrix} m_0 & \delta \\ \delta & m_0 + \delta' - \epsilon_\tau m_0 \end{pmatrix}$$

$(\epsilon_\mu \ll \epsilon_\tau)$

$$\Rightarrow \tan 2\bar{\theta} \simeq \frac{2\delta}{\delta' - \epsilon_\tau m_0}$$

IF $m_0 \epsilon_\tau \approx \delta'$, $\boxed{\bar{\Theta} \text{ MAXIMAL!!}}$

NOTE $\Delta m_A^2 \simeq 2 m_0 \delta' \simeq 2 m_0^2 \epsilon_\tau \simeq 2.5 \times 10^{-3}$ eV^2
FOR $m_0 \simeq .3$ eV $\Rightarrow \tan\beta = 30$.

ANOTHER WAY TO SEE RADIATIVE MAGNIFICATION:

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - c_{23}s_{13}s_{12} & c_{13}c_{23} \end{bmatrix} \quad (1)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij} (i, j = 1, 2, 3)$. U diagonalizes the mass matrix M in the flavor basis with $U^T M U = \text{diag}(m_1, m_2, m_3)$. The RGEs for the mass eigen values can be written as [13, 14]

$$\frac{dm_i}{dt} = -2F_\tau m_i U_{\tau i}^2 - m_i F_u, \; (i = 1, 2, 3). \quad (2)$$

CASAS, ESPINOSA, IBARRA, NAVARRO, 2000

For every $\sin\theta_{ij} = s_{ij}$, the corresponding RGEs are,

$$\frac{ds_{23}}{dt} = -F_\tau c_{23}{}^2 (-s_{12}U_{\tau 1}D_{31} + c_{12}U_{\tau 2}D_{32}), \quad (3)$$

$$\frac{ds_{13}}{dt} = -F_\tau c_{23}c_{13}{}^2 (c_{12}U_{\tau 1}D_{31} + s_{12}U_{\tau 2}D_{32}), \quad (4)$$

$$\frac{ds_{12}}{dt} = -F_\tau c_{12} (c_{23}s_{13}s_{12}U_{\tau 1}D_{31} - c_{23}s_{13}c_{12}U_{\tau 2}D_{32} + U_{\tau 1}U_{\tau 2}D_{21}). \quad (5)$$

$$F_\tau = -\frac{h_\tau^2}{16\pi^2 \cos^2\beta}; \quad D_{ij} = \frac{m_i + m_j}{m_i - m_j}$$

MIXING UNIFICATION AND LARGE $\theta_\odot$ AND θ_A

R.N.M., PARIDA, RAJASEKARAN '03

ASSUME AT $\mu = M_R$

$$\theta_{12} = \theta_{\text{CABIBBO}}\ ;\quad \theta_{23} = V_{cb}\ ;\quad \theta_{13} = V_{ub}$$

RAD. CORRECTIONS + QUASI-DEGENERACY

$\Rightarrow$ AT WEAK SCALE

$$\sin^2 2\theta_A \simeq .84 - .94$$

$$\sin^2 2\theta_\odot \simeq .78 - .86$$

$$U_{e3} \simeq .08$$

$$\boxed{m_{\beta\beta} \gtrsim .1\ \text{eV}}$$

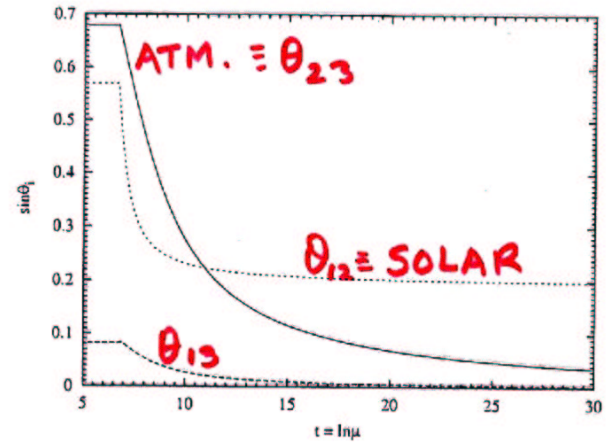


FIG. 1: Evolution of small quark-like mixings at the see-saw scale to bilarge neutrino mixings at low energies .The solid, dashed and dotted lines represent $\sin\theta_{23}$, $\sin\theta_{13}$, and $\sin\theta_{12}$, respectively, as defined in the text.

DATA :

HEIDELBERG-MOSCOW

KLAPDOR-K et. al. $\Rightarrow$ $m_\nu \approx .11 - .56\ eV$

WMAP + MIXING UNIF.

$\Rightarrow .1\ eV \leq m_i \leq .23\ eV$

TESTABLE IN $\beta\beta_{0\nu}$ EXPTS

e.g. GENIUS, EXO, MAJORANA, CUORE, MOON

TO 0.02 eV

$SU(2)_H$ SYMMETRY, 3X2 SEESAW AND LARGE MIXINGS

X

KUCHIMANCHI, R.N.M.
PRD66, 051301 (2002)
PLB, 552, 198 (2003)

$SU(2)_H$ ON 1st AND 2nd GEN:

e.g.

$$(L_e, L_\mu) \quad L_\tau$$
$$(e_R, \mu_R) \quad \tau_R$$

$\Rightarrow$ GLOBAL WITTEN ANOMALY

$\pi_4(SU(2)) = Z_2$

$\Rightarrow$ THERE MUST BE A $(N_{eR}, N_{\mu R})$ TO MAKE TH. ANOMALY FREE

$\Rightarrow$ 3 X 2 SEESAW

IMPLICATIONS FOR ν-MASS

$$SU(2)_H \supset U(1)_{e-\mu}$$

$$\Big\downarrow \text{SYM. BR.}$$

APPROX. $L_e - L_\mu - L_\tau$ SYM.

$$M_{\nu D} = \begin{pmatrix} a & 0 \\ 0 & a \\ 0 & b \end{pmatrix}; \quad M_{N_R} = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}$$

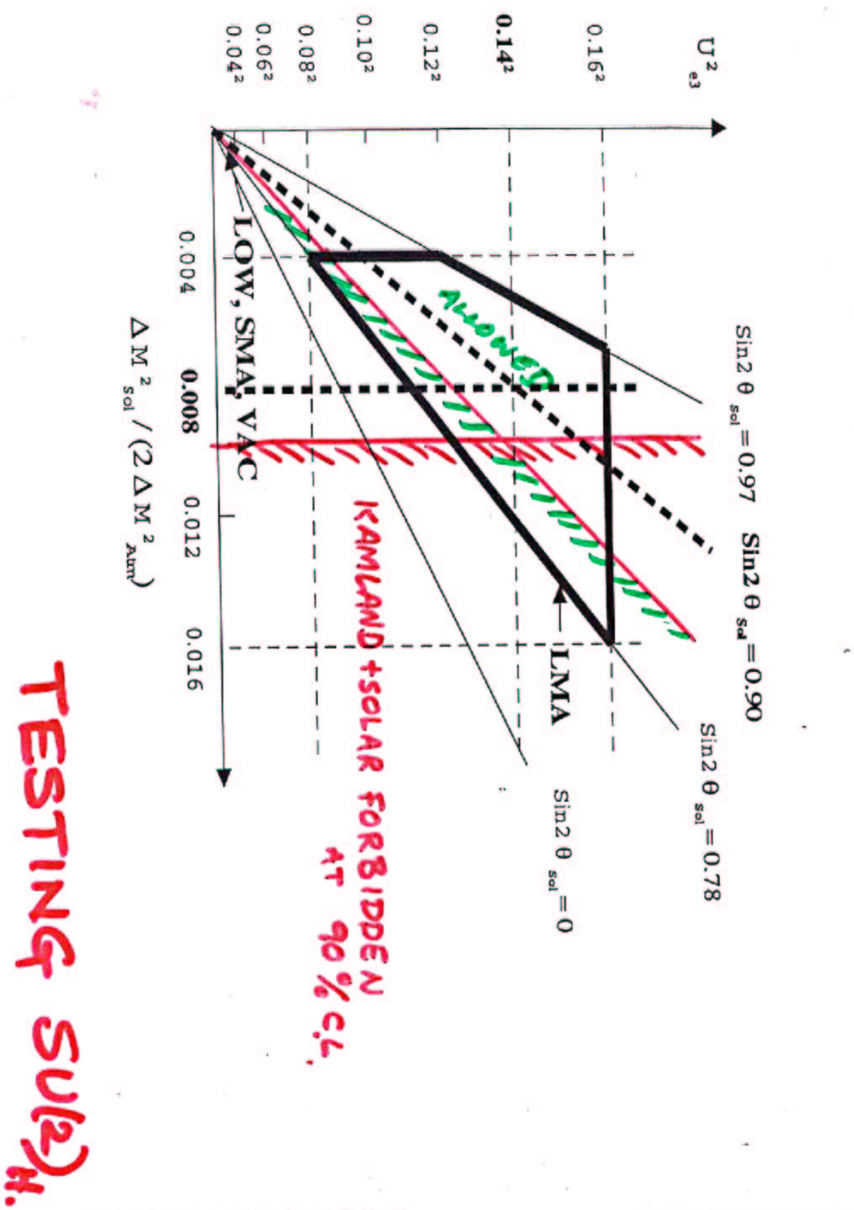
$$\Rightarrow \quad \mathcal{M}_\nu = \begin{pmatrix} 0 & m_1 & m_2 \\ m_1 & 0 & 0 \\ m_2 & 0 & 0 \end{pmatrix} + \text{small}$$

$\Rightarrow$ $\sin^2 2\theta_\odot = 1$; $\sin^2 2\theta_A$ LARGE

$U_{e3} = 0$; $\Delta m^2_\odot = 0$; $\Delta m^2_A = m_1^2 + m_2^2$;

$\mathcal{M}_\ell$ BREAKS $L_e - L_\mu - L_\tau$ SYM.
$\Rightarrow \Delta m^2_\odot, U_{e3} \neq 0$.

- INVERTED HIERARCHY
- $$U_{e3}^2 \cos 2\theta_\odot = \frac{\Delta m^2_\odot}{2\Delta m^2_A} + \text{SMALL CORR.}$$
- SMALLNESS OF $\Delta m^2_\odot / \Delta m^2_A$ AND U_{e3}^2 RELATED !!
- $\boxed{\text{MEASUREMENT OF } U_{e3} \text{ VERY IMPORTANT FOR TESTING } L_e - L_\mu - L_\tau \text{ AND } SU(2)_H}$



CONCLUSION

(i) SEESAW IS A STANDARD PARADIGM FOR $m_\nu \ll m_{u,d,l}$

⇒ • LOCAL B-L SYM; PERHAPS LEFT-RIGHT SYM. BEYOND SM

• FITS INTO GUT, SO(10) TYPE

• HEAVY N_R ⇒ FRAMEWORK FOR ORIGIN OF MATTER VIA LEPTOGEN.

(ii) UNDERSTANDING MIXINGS:

NO STD PARADIGM YET BUT SOME PROMISING DIRECTIONS:

A) INVERTED HIERARCHY

$\Rightarrow L_e - L_\mu - L_\tau \subset SU(2)_H$

3 × 2 SEESAW

"LARGE" U_{e3} - A TEST.

B) DEGENERATE MASSES

RADIATIVE MAGNIFICATION OF θ_{12} & θ_{23}. $m_{ee} \gtrsim .1\,eV$ ($\beta\beta_{0\nu}$)

C) NORMAL HIERARCHY: SO(10)