Neutrino Masses and Flavor-Changing Radiative Corrections

Ernest Ma
UC Riverside

SBITP (BKITP) → KITP

* Brief history of radiative contributions to m_ν
* The form of m_ν (2002 edition) and its underlying symmetry
* Three-fold degeneracy and FCRC
* Two-fold degeneracy and FCRC

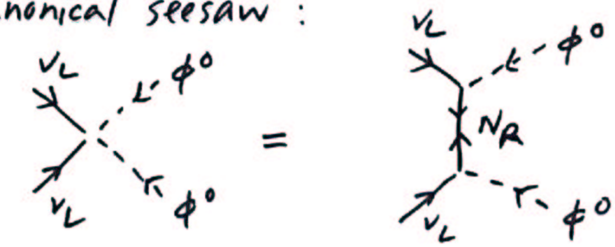
* Where does m_ν come from?

Weinberg (79):

$$\mathcal{L}_M = -\frac{f_{ij}}{2\Lambda}(\nu_i\phi^0 - \ell_i\phi^+)(\nu_j\phi^0 - \ell_j\phi^+) + H.c.$$

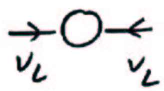
$$\Rightarrow (m_\nu)_{ij} = \frac{f_{ij}\langle\phi^0\rangle^2}{\Lambda}$$

Canonical seesaw:



is just 1 of 3 tree-level realizations. There are also 3 generic 1-loop realizations. [Ma (98)]

There are 2 kinds of radiative corrections.

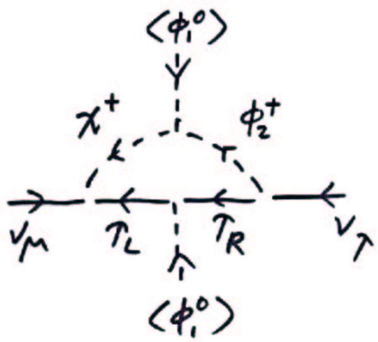
(I) 

, i.e. mass correction

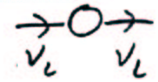
First example: Zee (80)

Add χ^+ and (ϕ_2^+, ϕ_2^0)

$\Rightarrow$



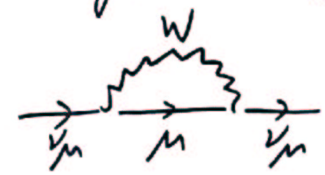
Result: $m_\nu \to m_\nu + \delta m_\nu$

(II) 

, i.e. wave-function correction

First example: Babu, Leung, Pantaleone (93)

Standard Model:



Most general such corrections:

$$R = \begin{pmatrix} r_{ee} & r_{e\mu} & r_{e\tau} \\ r_{e\mu}^* & r_{\mu\mu} & r_{\mu\tau} \\ r_{e\tau}^* & r_{\mu\tau}^* & r_{\tau\tau} \end{pmatrix}$$

Result:

$$m_\nu \to (1+R)\, m_\nu\, (1+R^T)$$

Atmospheric & Solar neutrino data

$$\Rightarrow \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

to a good first approximation

$\Rightarrow \mathcal{M}_\nu$ is a function of 4 real parameters (m_1, m_2, m_3, θ) and may be written in the form

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b+c & d & d \\ d & b & a+b \\ d & a+b & b \end{pmatrix},$$

then $\frac{d^2}{2c^2} = \frac{\tan^2\theta}{(1-\tan^2\theta)^2} = \left(\frac{\tan 2\theta}{2}\right)^2$ [Ma (02)]

$$m_{1,2} = a+2b+c \mp \sqrt{c^2+2d^2}, \quad m_3 = -a$$

Consider the most general $\mathcal{M}_\nu$, i.e.

$$\mathcal{M}_\nu = \begin{bmatrix} A & D & E \\ D & B & F \\ E & F & C \end{bmatrix}$$

and the discrete symmetry

$\nu_e \leftrightarrow \nu_e$, $\nu_\mu \leftrightarrow \nu_\tau$, i.e. $\mathcal{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$,

then $\mathcal{U} \mathcal{M}_\nu \mathcal{U}^T = \mathcal{M}_\nu$

$$\Rightarrow \mathcal{M}_\nu = \begin{bmatrix} A & D & D \\ D & B & F \\ D & F & B \end{bmatrix}$$

$= \mathcal{M}_\nu$ of p.5 if A, B, D, F are real.

<u>Examples</u>:

$A = B + F$ [Mohapatra, Nussinov (99)]
$A + D = B + F$ [Harrison, Scott (02)]
$A + B + F = 0$ [Joshipura (02)]

Consider $A = F$, $B = D = 0$, i.e.

$$\mathcal{M}_\nu = m_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

This pattern is the result of the symmetry A_4 [Ma, Rajasekaran (01)] for the complete theory, with

$$\left.\begin{array}{l} (\nu_i, l_i)_L \sim \underline{3}, \\ l_{1R} \sim \underline{1},\ l_{2R} \sim \underline{1}',\ l_{3R} \sim \underline{1}'' \end{array}\right\} \text{under } A_4$$

$$\Rightarrow \mathcal{M}_l = \underbrace{\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}}_{U_L} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad [\omega = e^{2\pi i/3}]$$

$$\Rightarrow \mathcal{M}_\nu = U_L^T \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_L = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

as desired.

What is A_4?

and why is it special?

Theaetetus (~ 390 BCE) $\Rightarrow$

only 5 perfect geometric solids

solid	faces	vertices	Plato	Group
Tetrahedron	4	4	fire	A_4
Octahedron	8	6	air	S_4
Icosahedron	20	12	water	A_5
Hexahedron	6	8	earth	S_4
Dodecahedron	12	20	?	A_5

A_4 = group of <u>even</u> permutations of 4 objects

<u>Character Table</u>

Class	n	h	$\chi^{(1)}$	$\chi^{(2)}$	$\chi^{(3)}$	$\chi^{(4)}$
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	1	1	-1

n = number of elements

h = order of each element

$\omega = e^{\frac{2\pi i}{3}}$

$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}$

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Add the most general radiative correction [Babu, Ma, Valle (02)]

i.e. $\mathcal{M}_\nu \to (1+R)\,\mathcal{M}_\nu\,(1+R^T)$

$$= m_0 \begin{pmatrix} 1+\delta_0+\delta+\delta^*+2\delta' & \delta'' & \delta''^* \\ \delta'' & \delta & 1+\delta_0+\frac{\delta+\delta^*}{2} \\ \delta''^* & 1+\delta_0+\frac{\delta+\delta^*}{2} & \delta^* \end{pmatrix}$$

where
$$\delta_0 \equiv r_{\mu\mu} + r_{\tau\tau} - r_{\mu\tau} - r^*_{\mu\tau}$$
$$\delta \equiv 2 r_{\mu\tau}$$
$$\delta' \equiv r_{ee} - \tfrac{1}{2} r_{\mu\mu} - \tfrac{1}{2} r_{\tau\tau} - \tfrac{1}{2} r_{\mu\tau} - \tfrac{1}{2} r^*_{\mu\tau}$$
$$\delta'' \equiv r^*_{e\mu} + r_{e\tau}$$

$\Rightarrow \mathcal{M}_\nu$ = that of p. 5 <u>automatically</u> for all δ's real, i.e. b, c, d are generated radiatively !

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SM (no FCRC) : $\delta'' = \delta = 0$

To get a realistic $\mathcal{M}_\nu$, we need new flavor-changing physics.

Example: Back to the Zee model, but do not add second Higgs doublet. Instead add [Ma (02)]

$$\eta_i^+ \sim \underline{3} \text{ under } A_4, \quad \text{breaks } A_4 \text{ softly}$$

with $\mathcal{L} = f\epsilon_{ijk}(\nu_i e_j - e_i \nu_j)\eta_k^+ + \overbrace{m_{ij}^2 \eta_i^+ \eta_j^-}^{\text{breaks } A_4 \text{ softly}}$

$$\Rightarrow \quad \delta = -\frac{f^2}{4\pi^2}\sum_{i=1}^{3} U_{\mu i}^* U_{\tau i} \ln m_i^2$$

$$\delta' = -\frac{f^2}{8\pi^2}\sum_{i=1}^{3}\left(\frac{1}{2}|U_{\mu i} - U_{\tau i}|^2 - |U_{ei}|^2\right)\ln m_i^2$$

$$\delta'' = -\frac{f^2}{8\pi^2}\sum_{i=1}^{3}\left(U_{\mu i}^* U_{ei} + U_{\tau i} U_{ei}^*\right)\ln m_i^2$$

$$(\Delta m^2)_{atm} \simeq 4\delta m_0^2, \quad U_{e3} \simeq \frac{i\,Im\,\delta''}{\sqrt{2}\,\delta},$$

$$\left[\frac{(\Delta m^2)_{sol}}{(\Delta m^2)_{atm}}\right]^2 \simeq \left[\frac{\delta'}{\delta} + |U_{e3}|^2\right]^2 + 2\left[\frac{Re\,\delta''}{\delta}\right]^2$$

$m_0 < 0.23$ eV [WMAP(03)]

$$\Rightarrow \delta \gtrsim \frac{2.5\times 10^{-3}}{4(0.23)^2} \simeq 1.2\times 10^{-2}$$

[compared to say QCD : $\alpha_s/2\pi \simeq 1.9\times 10^{-2}$]

$\Rightarrow$ m_0 cannot be much below 0.2 eV

$\Rightarrow$ near-future $0\nu\beta\beta$ decay experiments should measure m_0 !

Recall the condition $\mathcal{U} \mathcal{M}_\nu \mathcal{U}^T = \mathcal{M}_\nu$.

This implies $\mathcal{U}^n \mathcal{M}_\nu (\mathcal{U}^T)^n = \mathcal{M}_\nu$.

Unless $\mathcal{U}^n = 1$ for some $n = \bar{n}$, the only possible solution for $\mathcal{M}_\nu$ is $\mathcal{M}_\nu \propto 1$ (and $\mathcal{U}$ real).

Example:

$$\mathcal{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathcal{U}^2 = 1$$

* Suppose we want $\mathcal{U}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$,

now $\mathcal{U}^4 = 1$, but what is $\mathcal{U}$?

* Solution (i)

$$\mathcal{U}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} \end{pmatrix}$$

$$\Rightarrow \mathcal{M}_\nu = \begin{pmatrix} A & D & D \\ D & B & B \\ D & B & B \end{pmatrix} = \begin{pmatrix} 2b+c & d & d \\ d & b & b \\ d & b & b \end{pmatrix}$$

$$m_{1,2} = 2b + c \mp \sqrt{c^2 + 2d^2}, \quad m_3 = 0,$$

i.e. inverted hierarchy with

$$(\Delta m^2)_{atm} \simeq (2b+c)^2 \simeq 4b^2$$

$$(\Delta m^2)_{sol} \simeq 4(2b+c)\sqrt{c^2+2d^2} \simeq 8b\sqrt{c^2+2d^2}$$

$$m_{\nu_e}(0\nu\beta\beta) \simeq \sqrt{(\Delta m^2)_{atm}} \simeq 0.05\ \text{eV}$$

* Solution (ii)

$$\mathcal{U}_2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad \omega = e^{\frac{2\pi i}{3}}$$

$$\Rightarrow c = d, \text{ i.e. } \tan^2\theta = 2 - \sqrt{3} = 0.27.$$

$$\frac{(\Delta m^2)_{sol}}{(\Delta m^2)_{atm}} \simeq \frac{10^{-4}\,eV^2}{2.5\times 10^{-3}\,eV^2} = 0.04$$

$$\Rightarrow \frac{\sqrt{c^2+2d^2}}{b} \simeq 0.02$$

$\therefore$ c, d may come from FCRC

$$\text{Let } \mathcal{M}_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix},$$

then $(1+R)\mathcal{M}_\nu(1+R^T)$

$$= m_0 \begin{pmatrix} 1+2r_{ee} & \frac{3r_{e\mu}+r_{e\tau}}{2} & \frac{r_{e\mu}+3r_{e\tau}}{2} \\ \frac{3r_{e\mu}+r_{e\tau}}{2} & \frac{1}{2}+r_{\mu\mu}+r_{\mu\tau} & \frac{1+r_{\mu\mu}+r_{\tau\tau}+r_{\mu\tau}+r_{\mu\tau}^*}{2} \\ \frac{r_{e\mu}+3r_{e\tau}}{2} & \frac{1+r_{\mu\mu}+r_{\tau\tau}+r_{\mu\tau}+r_{\mu\tau}^*}{2} & \frac{1}{2}+r_{\tau\tau}+r_{\mu\tau}^* \end{pmatrix}$$

Example: $\mathcal{L} = f_e(\nu_e\mu - e\nu_\mu)\chi^+ + f_\tau(\nu_\tau\mu - \tau\nu_\mu)\chi^+ + H.c.$

i.e. $L_e + L_\tau - L_\mu$ symmetry, with

$r_{e\mu} = r_{\mu\tau} = 0$, $r_{\mu\mu} = r_{ee} + r_{\tau\tau}$, $r_{e\tau}^2 = r_{ee}r_{\tau\tau}$

$$\Rightarrow m_1 = m_0(1+r_{ee})$$
$$m_2 = m_0(1+2r_{ee}+2r_{\tau\tau})$$
$$m_3 = 0$$
$$\tan^2\theta = \frac{r_{ee}}{2r_{\tau\tau}} < 0.8$$
$$|U_{e3}| \simeq \sqrt{\frac{r_{ee}r_{\tau\tau}}{2}} < 0.005$$

Conclusion

* Symmetry + FCRC

$\Rightarrow$ realistic $\mathcal{M}_\nu$

(i) 3-fold degeneracy :

$m_0(0\nu\beta\beta)$ should not be much below 0.23 eV (WMAP bound)

(ii) 2-fold degeneracy :

inverted hierarchy with $m_0 \simeq 0.05$ eV, predicts WMAP $\Sigma m_\nu = 0.1$ eV

* FCRC $\Rightarrow$

likely to be observable !

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