Neutrino Oscillations and CP and/or T Violation

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Borislab

with H. Minakata and H. Nunokawa

Outline:

- Leptonic CP/T Violation
- The Anatomy of the Bi-Probability Plots
- Parameter Degeneracies:
  $\nu_\mu \to \nu_\mu$ and $\nu_\mu \to \nu_e$
- JHF/NuMI Complementarity
- Summary

Boris loves ···

- Physics especially B's and Nu's
- Chocolate !!!
- Puzzles:

e.g.

How many zeros appear at the end of One Million Factorial ???

(one quarter million minus a few)

This is base 10,

What about base 16 ???

3 active flavors

(but can be easily modified to accommodate 3+1)

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

The parameterization used for the unitary MNS matrix, U, is

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

where $c_{jk} \equiv \cos\theta_{jk}$ and $s_{jk} \equiv \sin\theta_{jk}$.

The primary element of interest here is

$|U_{e3}|^2$ or $\sin^2 2\theta_{13}$

and δ.

Leptonic CP and T Violation in Oscillations

CP

$$\nu_\mu \to \nu_e \quad \Longleftrightarrow \quad \bar{\nu}_\mu \to \bar{\nu}_e$$

T $\Updownarrow$ $\Updownarrow$ T

$$\nu_e \to \nu_\mu \quad \Longleftrightarrow \quad \bar{\nu}_e \to \bar{\nu}_\mu$$

CP

IN GENERAL (in vacuum):

CP Violation:

$$\alpha \neq \beta \qquad P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

T Violation:

$$\begin{array}{ll} \alpha \neq \beta & P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha) \\ \text{and} & P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq P(\bar{\nu}_\beta \to \bar{\nu}_\alpha) \end{array}$$

CPT Violation:

$$\text{any } \alpha,\beta \quad P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

Amplitude for $\nu_\mu \to \nu_e$

$$a_{\mu\to e} \equiv$$

$$\left[\; 2U_{\mu 3}U^*_{e3}e^{i\frac{\delta m^2_{31}L}{4E}} \; \sin\frac{\delta m^2_{31}L}{4E} \right.$$

$$+$$

$$\left. 2U_{\mu 2}U^*_{e2}e^{i\frac{\delta m^2_{21}L}{4E}} \; \sin\frac{\delta m^2_{21}L}{4E} \; \right]$$

Difference in

Relative Phases:

Changes Interference

$\Downarrow$

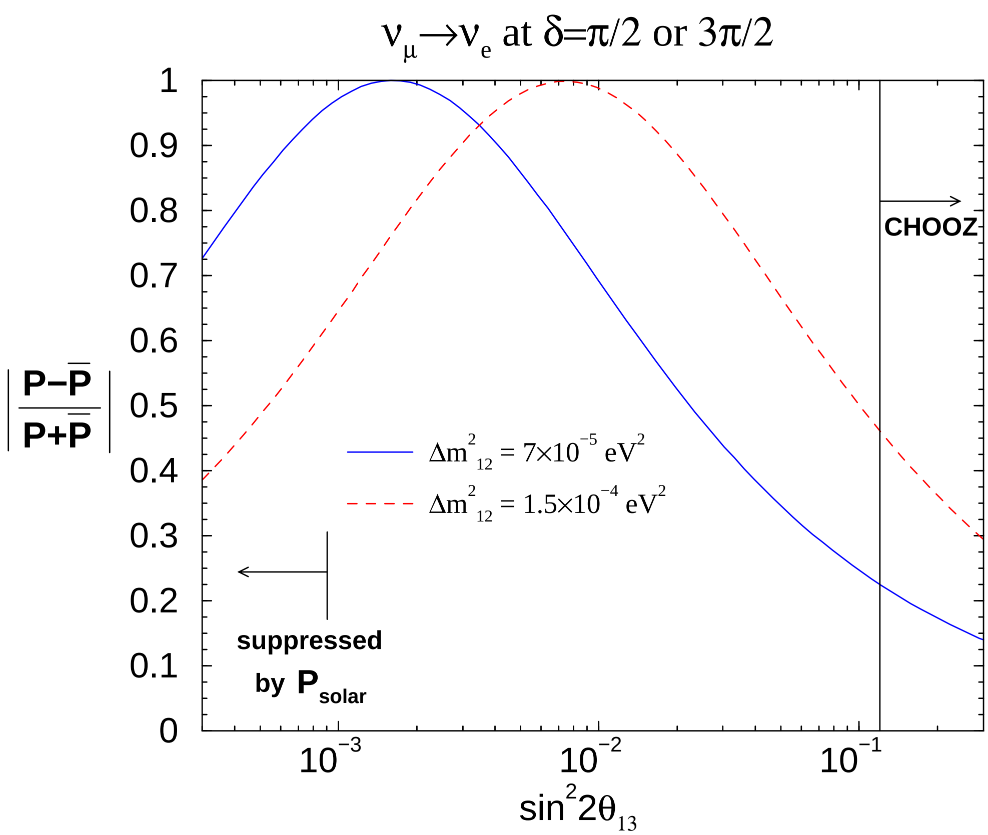
CP (or T) Violation

used $\sum U_{\mu i}U^*_{ei} = 0$

Why Everybody is Excited!

- Maximum Allowed Asymmetry ($\delta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$) for $\nu_\mu \to \nu_e$ at first Oscillation Maximum in vac:

- $P,\ \bar{P} = |a^{atm}_{\mu\to e} + a^{\odot}_{\mu\to e}|^2 \approx (\sin\theta_{23}\sin 2\theta_{13} \pm \sqrt{P_\odot})^2$
  - $|P - \bar{P}| \approx 4\sqrt{P_\odot}\sin\theta_{23}\ \sin 2\theta_{13}$
  - $P + \bar{P} \approx 2\sin^2\theta_{23}\ \sin^2 2\theta_{13} + 2P_\odot$



- Peak occurs at

$$\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2\theta_{23}}\left[\frac{\pi}{2}\ \frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$$

at OM $\sqrt{P_\odot} = \cos\theta_{13}\cos\theta_{23}\sin 2\theta_{12}\sin\left(\frac{\pi}{2}\ \frac{\delta m^2_{12}}{\delta m^2_{13}}\right)$

- For BK

Oscillation Highlights:

With three neutrinos we can access: two δm^2, three mixing angles, θ and one CP or T violating phase, δ.

(Majorana neutrinos have two more CP phases inaccessible in oscillations. These effect neutrinoless double beta decay.)

ATMOSPHERIC:

$|\delta m^2_{atm}| = 2.5 \times 10^{-3} eV^2$

$\sin^2 2\theta_{23} \approx 1.0 \qquad \theta_{23} \sim \frac{\pi}{4} = 45^\circ \quad |U_{\mu 3}|^2 \approx \frac{1}{2}$

SOLAR: LMA

$\delta m^2_\odot = +7 \times 10^{-5} eV^2$

$\sin^2 2\theta_{12} = 0.85 \qquad \theta_{12} \sim \frac{\pi}{6} = 30^\circ \quad |U_{e2}|^2 \approx \frac{1}{4}$

REACTOR: (Chooz)

$\sin^2 2\theta_{13} < 0.1 \qquad \theta_{13} < \frac{\pi}{20} = 9^\circ \quad |U_{e3}|^2 < 2.5\%$

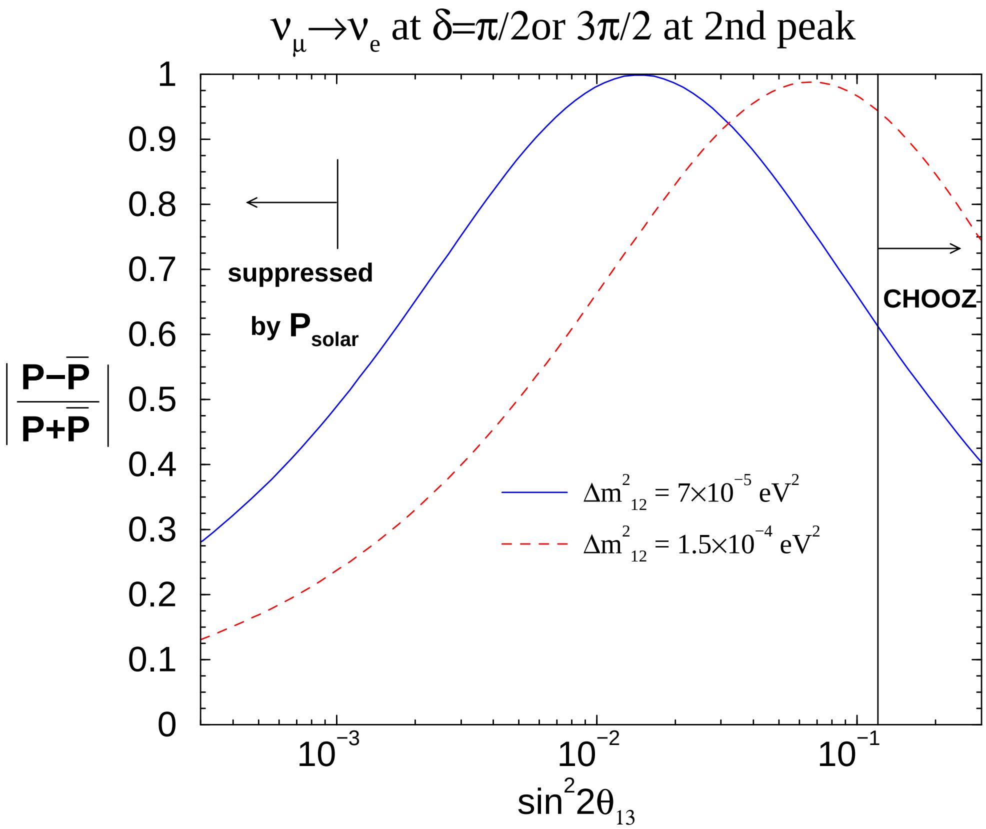
LBL ENERGIES AND BASELINES:

$$E_{OM}^{JHF} = 0.6 \text{ GeV} \left(\frac{L}{295 \text{ km}}\right) \left(\frac{\delta m^2_{atm}}{2.5\times 10^{-3} \text{ eV}^2}\right)$$

$$E_{OM}^{NuMI} = 1.5 \text{ GeV} \left(\frac{L}{732 \text{ km}}\right) \left(\frac{\delta m^2_{atm}}{2.5\times 10^{-3} \text{ eV}^2}\right)$$

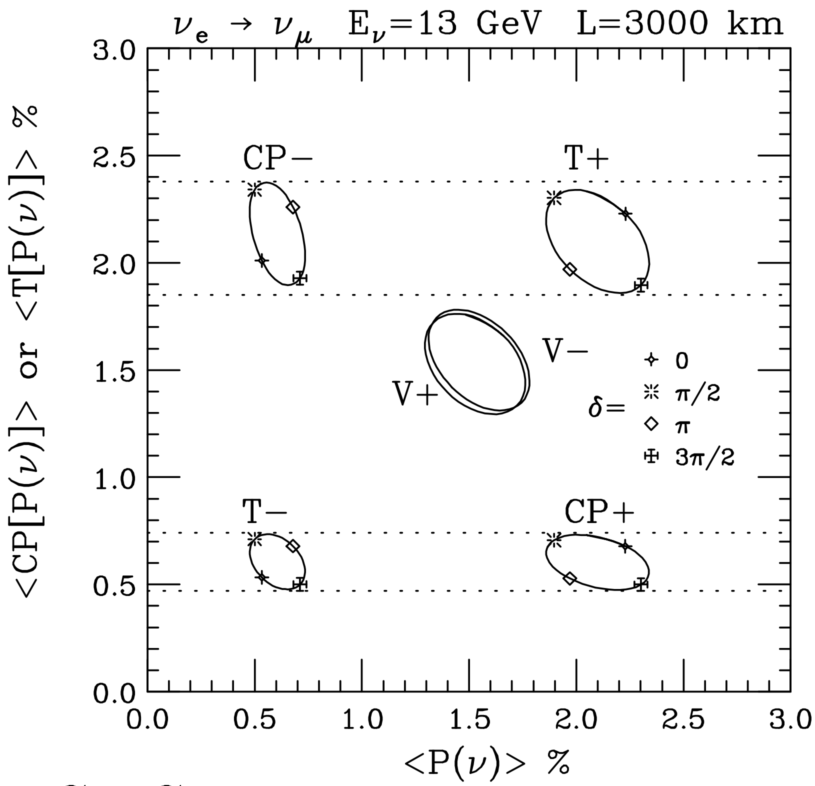
Energies ~30% higher are 0.8 and 2.0 GeV resp.

2nd Peak



- Peak occurs at $\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2 \theta_{23}} \left[\frac{3\pi}{2} \; \frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$

Anatomy of the Bi-Probability Plot:



- The CP-CP relation:

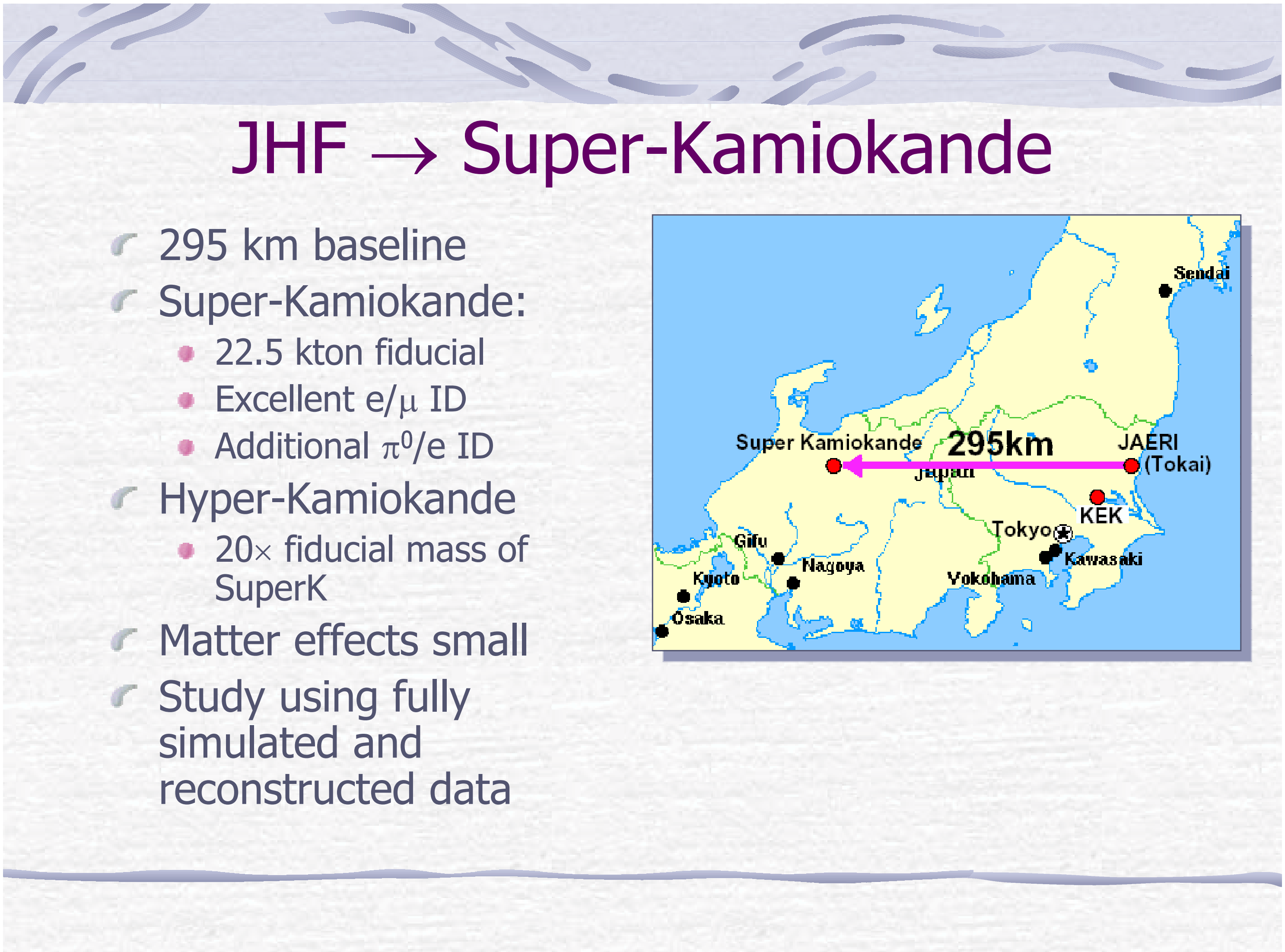
$$
\begin{aligned}
& P(\nu_e \to \nu_\mu; \Delta m^2_{31}, \Delta m^2_{21}, \delta, a) \\
= \quad & P(\bar{\nu}_e \to \bar{\nu}_\mu; -\Delta m^2_{31}, -\Delta m^2_{21}, \delta, a) \\
\approx \quad & P(\bar{\nu}_e \to \bar{\nu}_\mu; -\Delta m^2_{31}, +\Delta m^2_{21}, \pi + \delta, a)
\end{aligned}
$$

- The T-CP relation:

$$
\begin{aligned}
& P(\nu_\mu \to \nu_e; \Delta m^2_{31}, \Delta m^2_{21}, \delta, a) \\
= \quad & P(\bar{\nu}_e \to \bar{\nu}_\mu; -\Delta m^2_{31}, -\Delta m^2_{21}, -\delta, a) \\
\approx \quad & P(\bar{\nu}_e \to \bar{\nu}_\mu; -\Delta m^2_{31}, +\Delta m^2_{21}, \pi - \delta, a)
\end{aligned}
$$

- $\approx$ trade sign of δm^2_{12} for shift by π of δ:

$$(\cdots) + \delta m^2_{12}[(\cdots)\cos\delta + (\cdots)\sin\delta]$$



Requires New Beamline:

http://www-nu.kek.jp/jhfnu/

LOI: hep-ex/0106019

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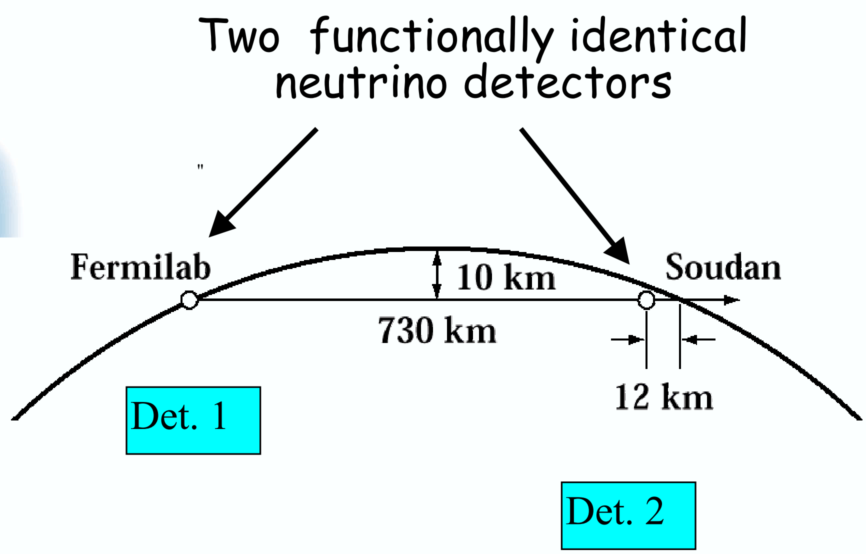
The NUMI Beamline



New Detector Required:

http://www-off-axis.fnal.gov/

LOI: hep-ex/0210005

Brookhaven to Homestake OR WIPP

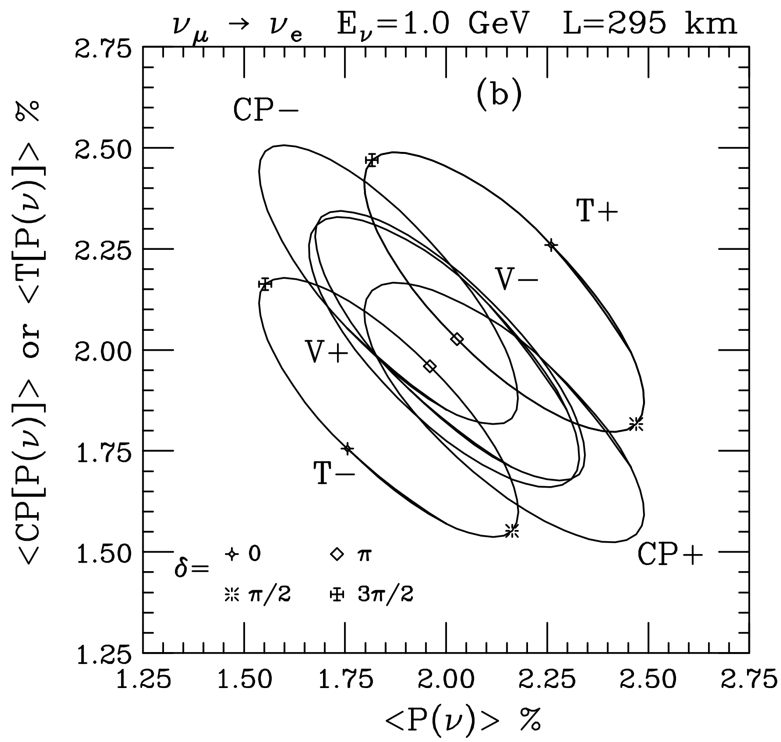
L= 2540 km or 2880 km

New Beamline, New Detector:

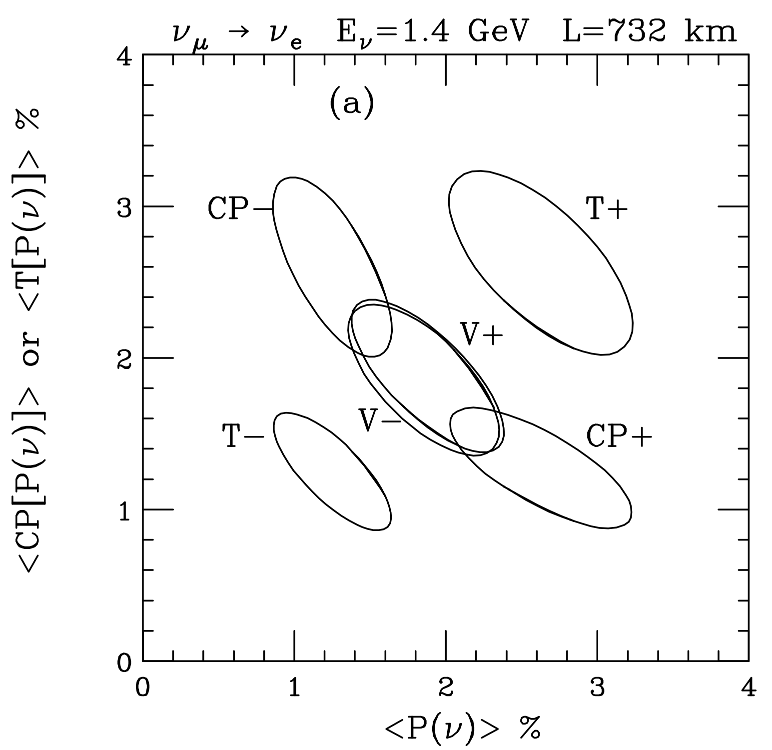
http://www.neutrino.bnl.gov/

LOI: hep-ex/0205040

- JHF to SuperK energy and distance:



- NuMI energy and distance:

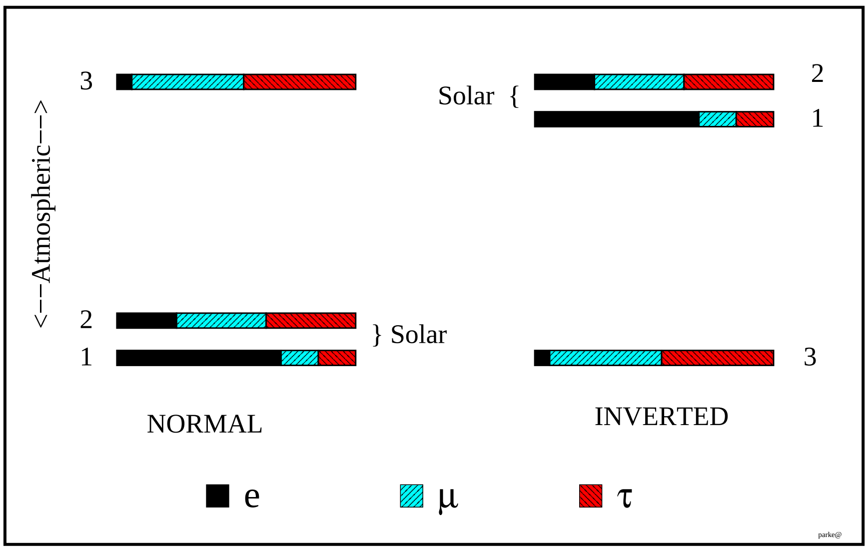


Parameter Degeneracies: $\nu_\mu \to \nu_\mu$

Precision Measurement of

$$|\delta m^2_{23}| \quad \text{and} \quad \sin^2 2\theta_{23}$$

- Mass Hierarchy and sign of δm^2_{23}



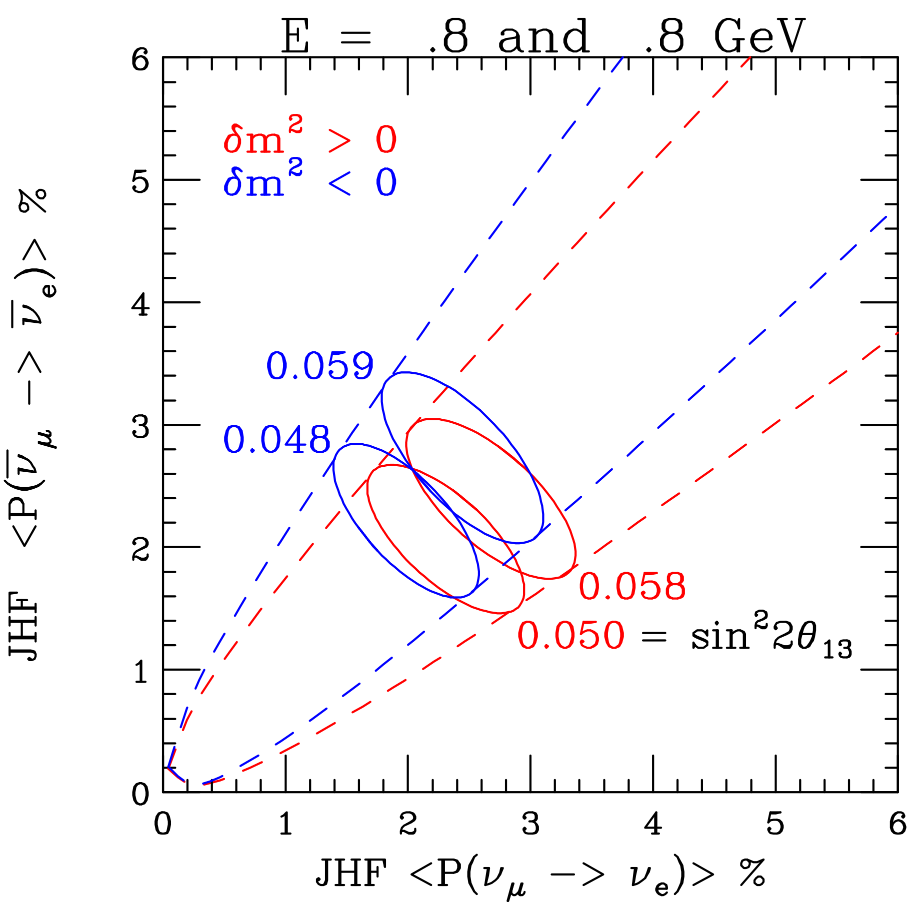
- $\sin^2 2\theta_{23} = 1 - \epsilon^2 \quad \Rightarrow$

$$2\sin^2\theta_{23} = 1 \mp \epsilon \qquad 2\cos^2\theta_{23} = 1 \pm \epsilon$$

$\sin^2 2\theta_{23}$	0.91	0.96	0.99	1.00	$\sin^2 2\theta_{23}$
ϵ	0.3	0.2	0.1	0.0	ϵ
$\sin^2\theta_{23}$	0.35	0.40	0.45	0.50	$\cos^2\theta_{23}$
$\cos^2\theta_{23}$	0.65	0.60	0.55	0.50	$\sin^2\theta_{23}$

$\nu_\mu \rightarrow \nu_e$ Degeneracy:

- Varying the CP or T violating phase, δ, with all other parameters fixed gives an ellipse.
- Scaling of axes by cross section, flux and detector size gives event rate - requires experimental expertise.

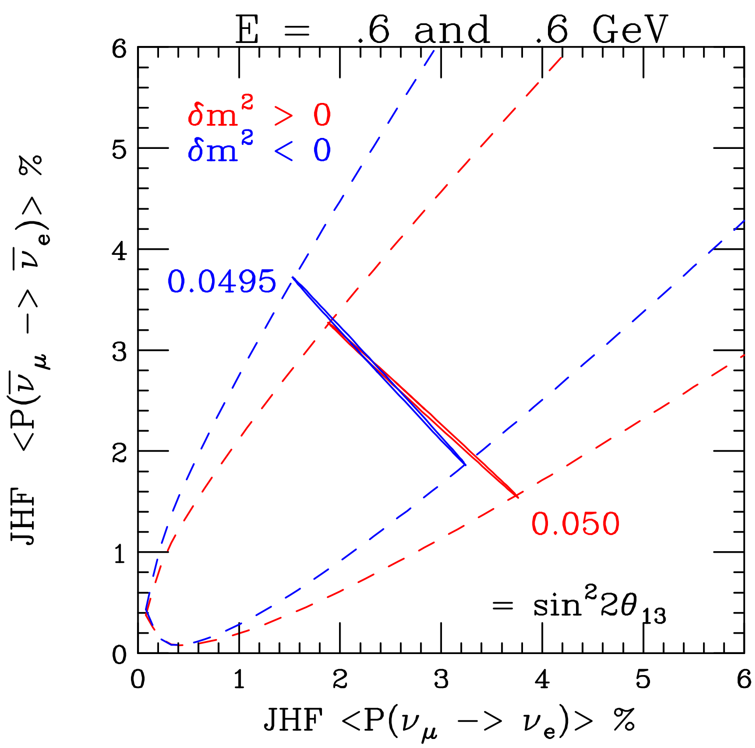


- Two Solutions (θ, δ) for each hierarchy if we know all other parameters.

Bi-Probability plots invented by Minakata+Nunokawa:

- At Oscillation Maximum (Kajita+Minakata+Nunokawa)

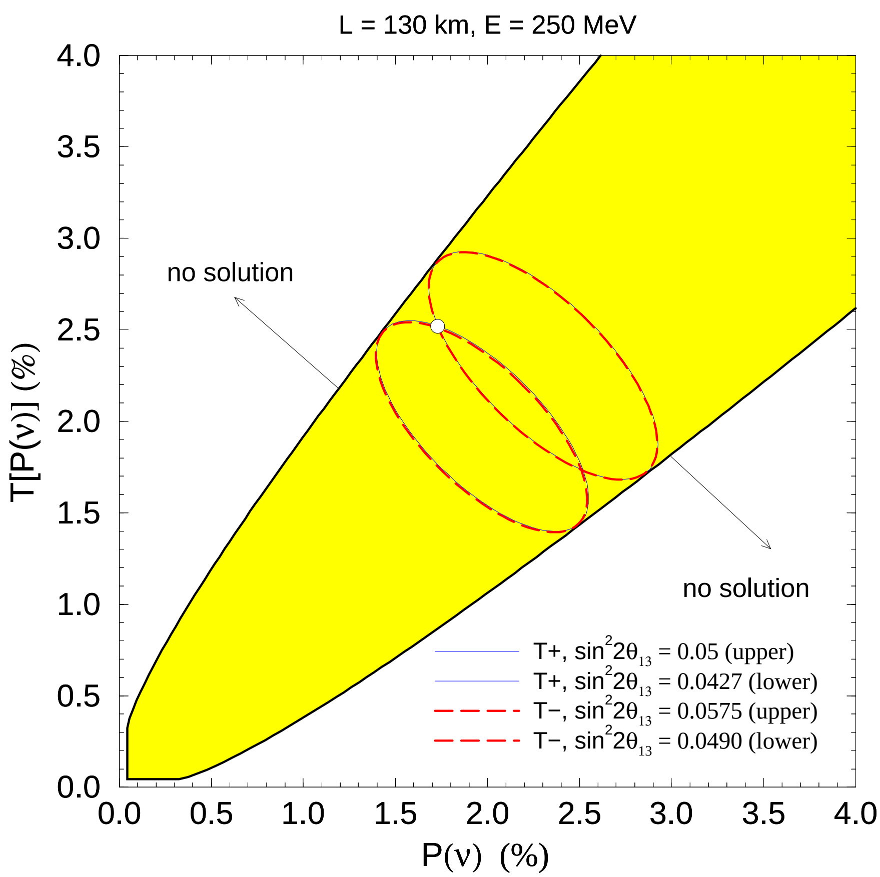
same θ different δ's - δ and $\pi - \delta$



- The different Hierarchies give almost same θ

Degeneracy for T-Violation:

- Cannot distinguish hierarchy!!!



Parameter Degeneracies: $\nu_\mu \rightarrow \nu_e$

Fogli+Lisi, Minakata+Nunokawa, Barger+Marfatia+Whisnant, Burguet-Castell+Gavela+Gome-Cadenas+Hernandez+Mena, Huber+Lindner+Winter, Minakata+Nunokawa+Parke

Using $\theta \equiv \sin\theta_{13}$ then

$$P(\nu_\mu \rightarrow \nu_e) = X_\pm \theta^2 + Y_\pm \theta \cos(\delta + \Delta_{13}/2) + P_\odot$$

$$\bar{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = X_\mp \theta^2 - Y_\mp \theta \cos(\delta - \Delta_{13}/2) + P_\odot$$

where (Burguet-Castell et al Nucl. Phys. B608, 301 (2001))

$$X_\pm = 4 s_{23}^2 \left(\frac{\Delta_{13}}{B_\mp}\right)^2 \sin^2\left(\frac{B_\mp}{2}\right),$$

$$Y_\pm = \pm 8 c_{12} s_{12} c_{23} s_{23} \left(\frac{\Delta_{12}}{aL}\right)\left(\frac{\Delta_{13}}{B_\mp}\right) \sin\left(\frac{aL}{2}\right) \sin\left(\frac{B_\mp}{2}\right)$$

$$P_\odot = c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{aL}\right)^2 \sin^2\left(\frac{aL}{2}\right); \; Y_\pm = \pm 2\sqrt{X_\pm P_\odot}$$

$$\Delta_{ij} \equiv \frac{|\Delta m_{ij}^2| L}{2E}, \quad B_\pm \equiv |\Delta_{13} \pm aL|, \quad a = \sqrt{2} G_F N_e.$$

- The X's are typically of order $1 \leftrightarrow 2$
- and the Y's of order $\pm\frac{1}{20}$
- Thus $\frac{Y^2}{X} \ll 0.01$ Complete the Square!!!

(for $\nu_e \rightarrow \nu_\tau$ interchange s_{23} and c_{23})

Solution to these equations, ignoring terms of $\mathcal{O}(\frac{Y^2}{X})$,

$$\begin{aligned} \theta &= \sqrt{\frac{P - P_\odot}{X_\pm}} - \frac{Y_\pm}{2X_\pm}\cos\left(\delta + \frac{\Delta_{13}}{2}\right) \\ &= \sqrt{\frac{\bar{P} - P_\odot}{X_\mp}} + \frac{Y_\mp}{2X_\mp}\cos\left(\delta - \frac{\Delta_{13}}{2}\right). \end{aligned}$$

The last equality gives us an equation of the form

$$(\cdots)\sin\delta + (\cdots)\cos\delta = (\cdots)$$

which can be used to solve for δ. It involves a $\pm\sqrt{(\cdots)}$ which gives us two solutions and setting this square root to zero gives us the boundary of the allowed region.

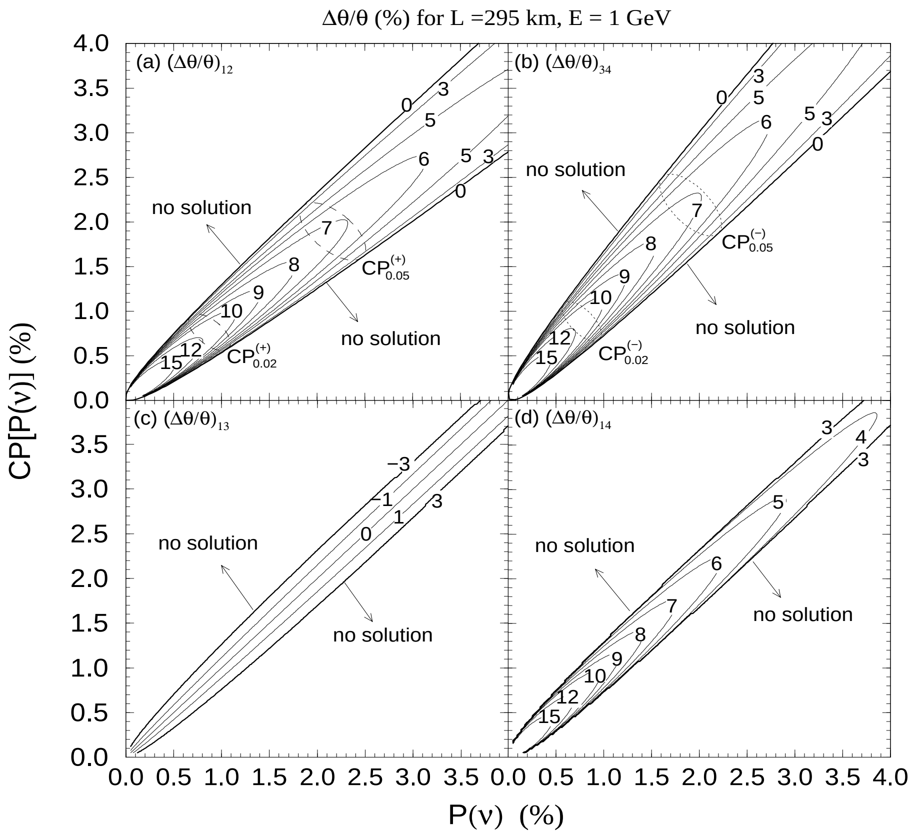
θ's are then obtained by substitution.

- So with 2 signs of δm^2
- and 2 possibilities for $\sin^2\theta_{23}$
- we have in general 8 solutions.

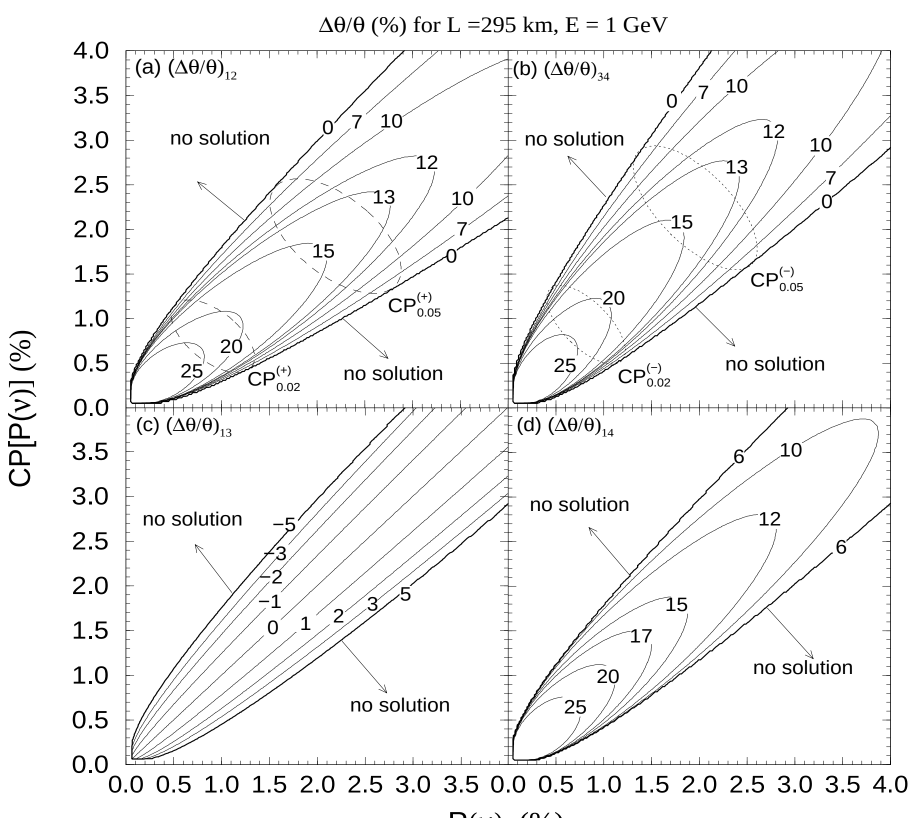
$\Rightarrow 2\frac{(\theta_i - \theta_j)}{(\theta_i + \theta_j)}$ which is to be compared with

the experimental resolution.

$$\frac{\Delta\theta}{\theta} = 2\frac{(\theta_i - \theta_j)}{(\theta_i + \theta_j)}.$$



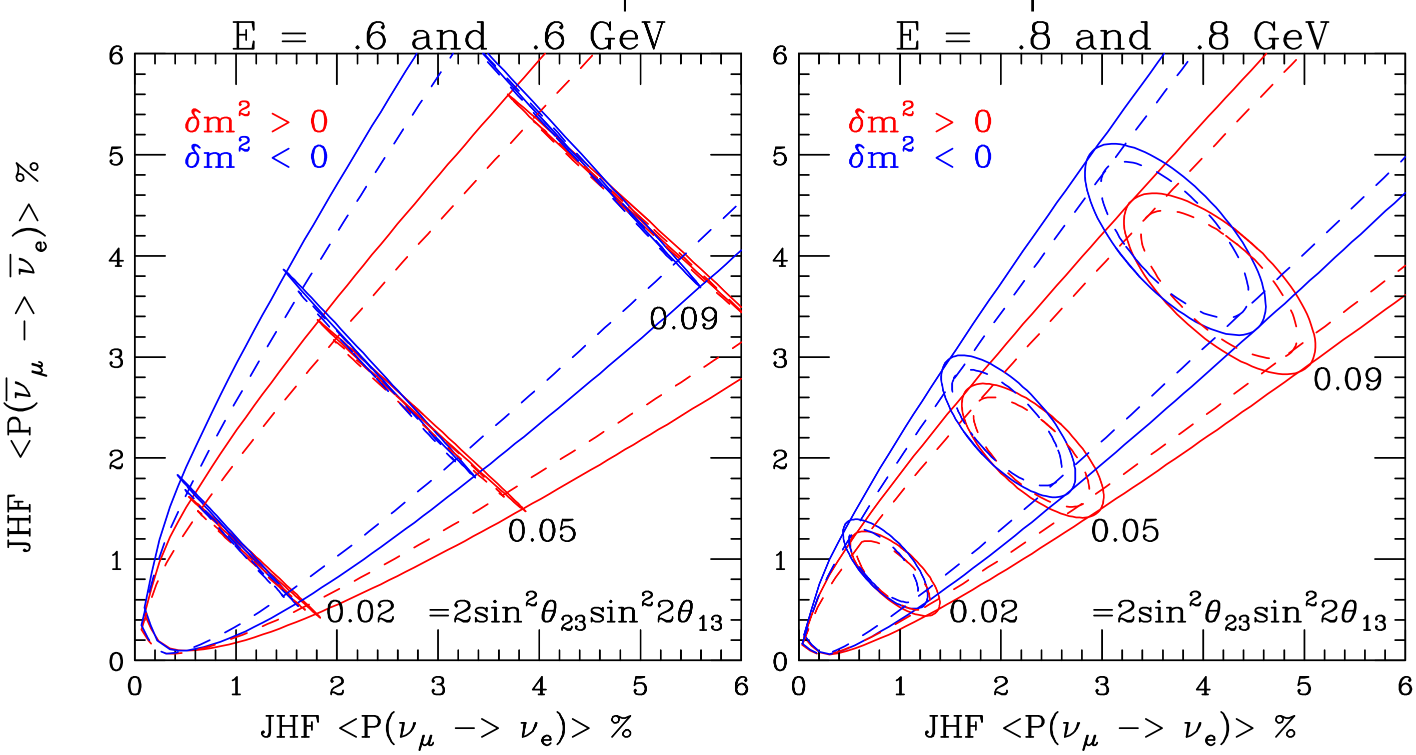
- $\delta m^2_{12} = 5 \times 10^{-5}$ (above) 1×10^{-4} (below) eV^2



$\sin^2 2\theta_{23}$ "Scaling":

- Using $\sin^2 2\theta_{23} = 0.96 = 4 * (0.4) * (0.6)$:

$2\sin^2\theta_{23}\sin^2 2\theta_{13}$	$\sin^2 2\theta_{13}^{(1)}$ $2\sin^2\theta_{23}^{(1)} = 0.8$	$\sin^2 2\theta_{13}^{(2)}$ $2\sin^2\theta_{23}^{(2)} = 1.2$
0.02	0.0250	0.0167
0.05	0.0625	0.0417
0.09	0.1125	0.075
	solid	dashes



If $\sin^2 2\theta_{23} \neq 1$ then two solution for θ_{13}

are related by $\theta_{13}^{(2)} \approx \frac{\sin\theta_{23}^{(1)}}{\sin\theta_{23}^{(2)}}\ \theta_{13}^{(1)}$

- Exact at Oscillation Maximum:

— small corrects at other energies.

A Solution:

- at Oscillation Maximum determine:

$$\sqrt{2}\sin\theta_{23}\sin\theta_{13}$$

$$\sqrt{2}\cos\theta_{23}\sin\delta$$

(no dependence on $\cos\delta$ at OM)

- At another E/L determine:

$$\sqrt{2}\cos\theta_{23}\cos\delta$$

Squaring the last two gives $\cos^2\theta_{23}$ remember we only have to distinguish between $>$ or $<$ 0.5

$$\Rightarrow \theta_{13},\ \theta_{23} \text{ and } \delta$$

Maybe two sets of solution since hierarchy may not be determined: See JHF v NuMI next.

POSSIBLE HELP FROM

- neutrinoless double β-decay
- Reactors: $\bar{\nu}_e \rightarrow \bar{\nu}_e$
- Nu Factory: $\nu_e \rightarrow \nu_\tau$

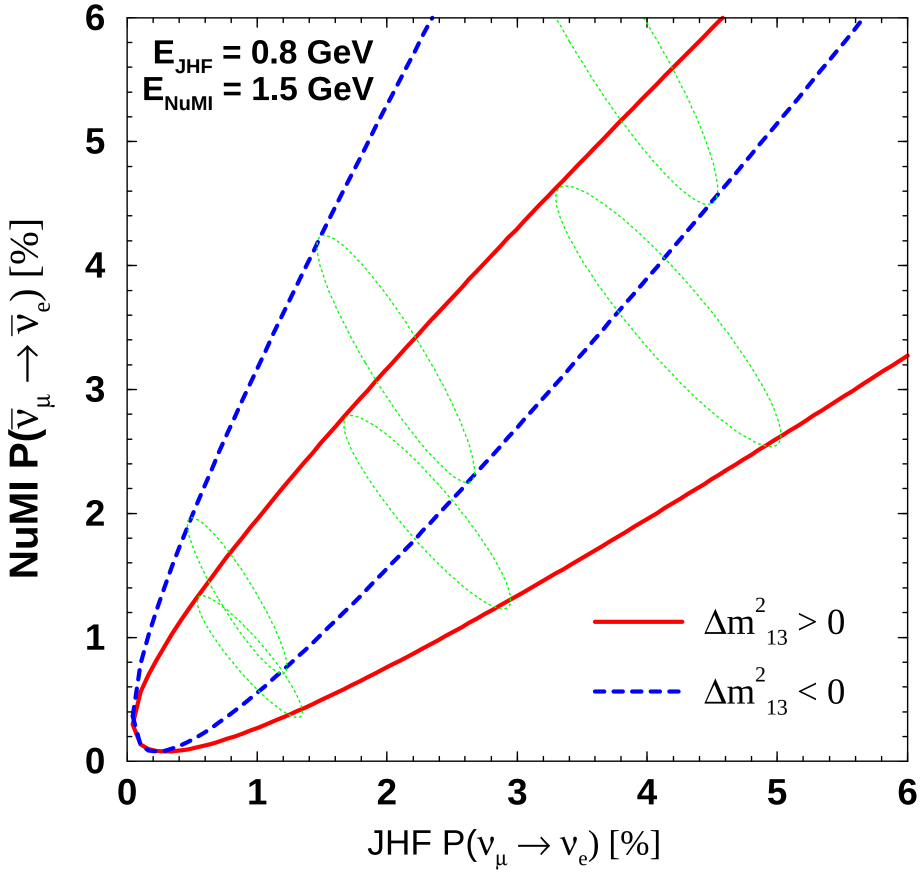
$\nu_\mu \rightarrow \nu_e$ JHF/NuMI Complementarity:

Barger+Marfatia+Whisnant hep-ph/0210428

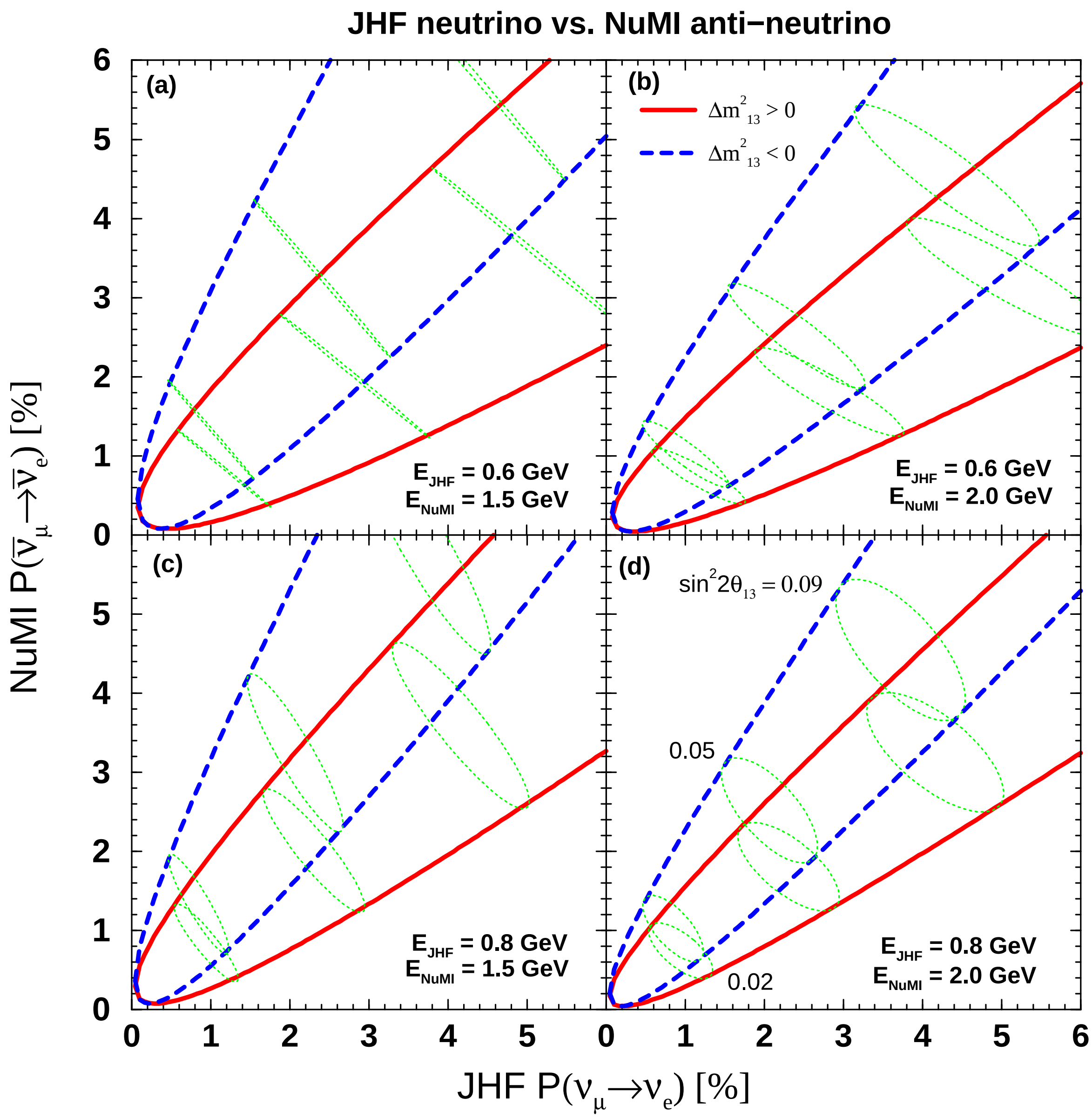
Huber+Lindner+Winter hep-ph/0211300

Minakata+Nunokawa+Parke hep-ph/0301210

- JHF Neutrinos - NuMI Anti-Neutrinos



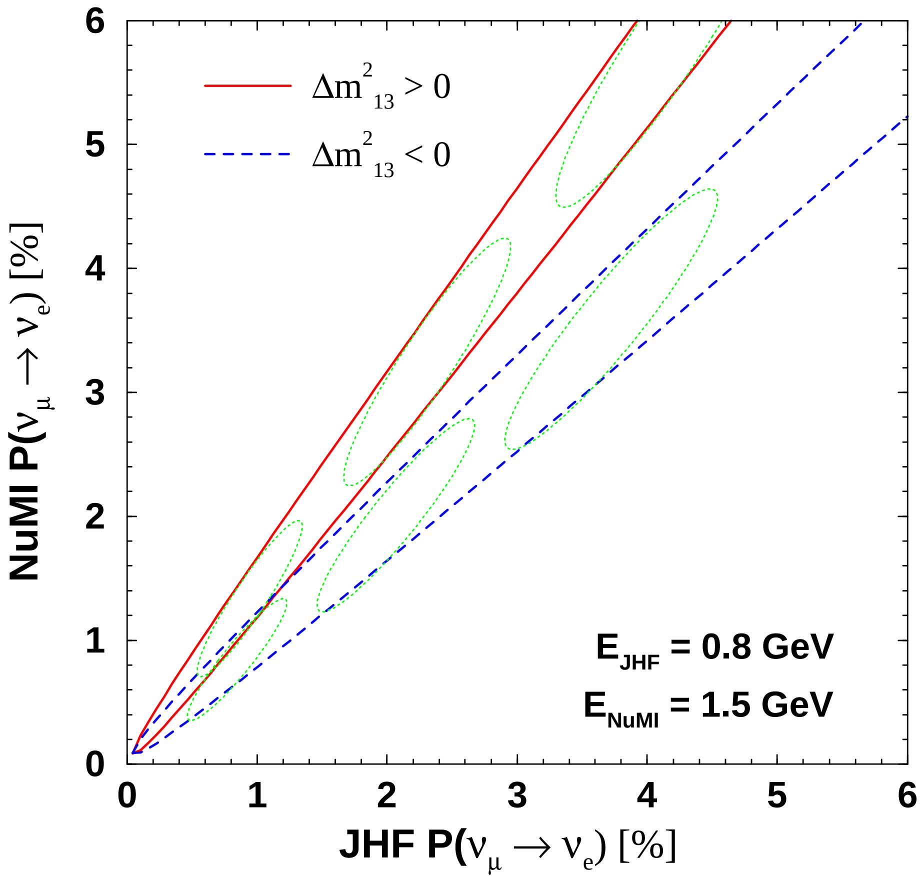
- Similar to JHF (NuMI) ν - JHF (NuMI) $\bar{\nu}$



- Width of the Cigars: $\left| \frac{Y_{\pm}^{N}}{X_{\pm}^{N}} - \frac{Y_{\mp}^{J}}{X_{\mp}^{J}} \right| \theta$

Since $Y_{\pm}$ have opposite sign NO cancellation.

• JHF Neutrinos - NuMI Neutrinos



• Separation of Hierarchies!!!

• Ratio of Slopes:

$$\frac{X_+^N}{X_-^N} / \frac{X_+^J}{X_-^J} \approx 1 + G(\Delta_{13}^N)(aL)|_N - G(\Delta_{13}^J)(aL)|_J$$

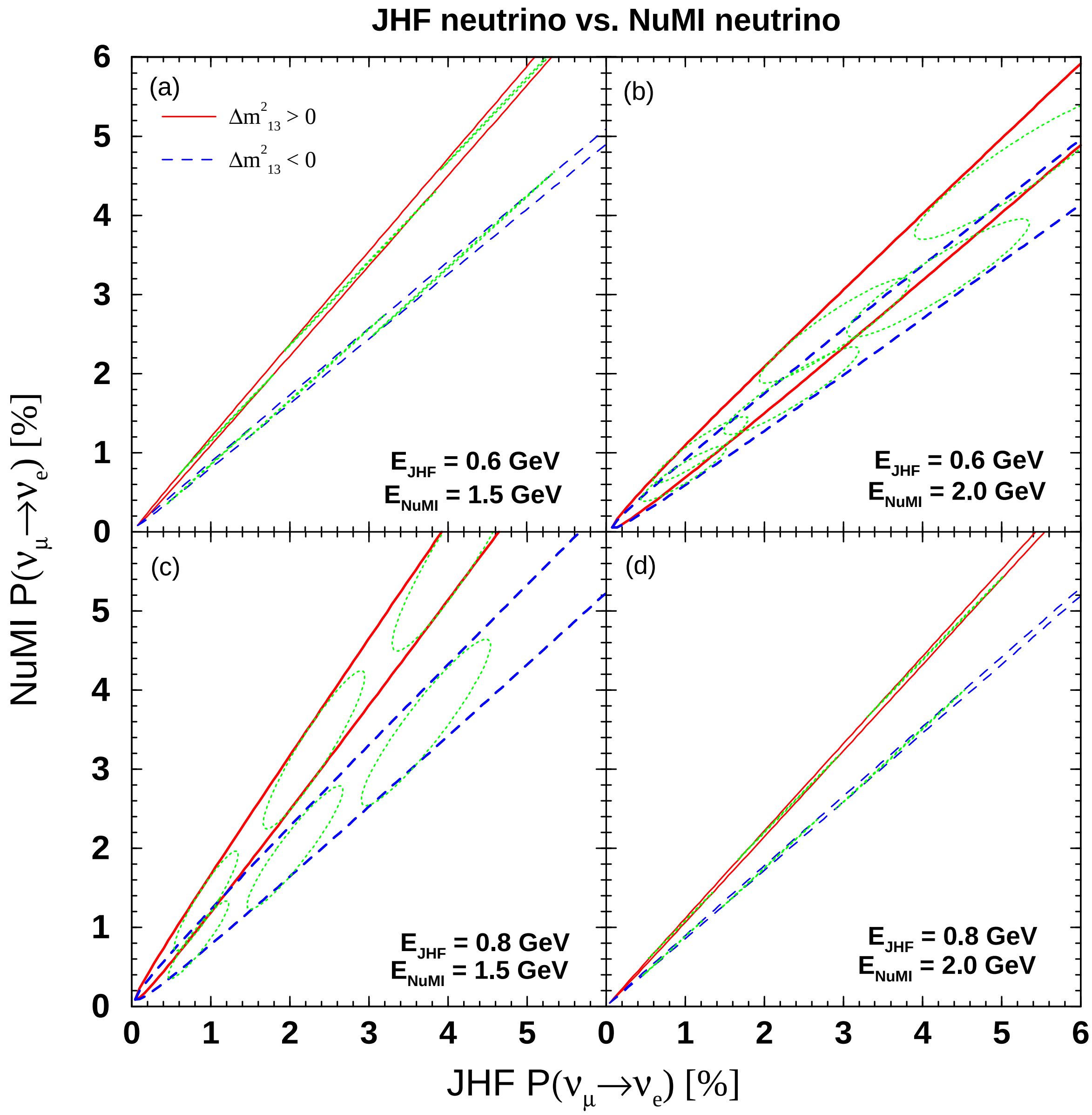
where $G(\Delta_{13}) \equiv 2\left[\frac{2}{\Delta_{13}} - \cot\left(\frac{\Delta_{13}}{2}\right)\right]$ is a monotonically increasing (decr) function of Δ_{13} (E).

E^J up: larger slope ratio, E^N up: smaller slope ratio.

• Width of Pencils $\left|\frac{Y_\pm^N}{X_\pm^N} - \frac{Y_\pm^J}{X_\pm^J}\right| \theta$

at same $\left(\frac{E}{L}\right)$ we have an identity $\frac{Y_\pm^N}{\sqrt{X_\pm^N}} = \frac{Y_\pm^J}{\sqrt{X_\pm^J}}$

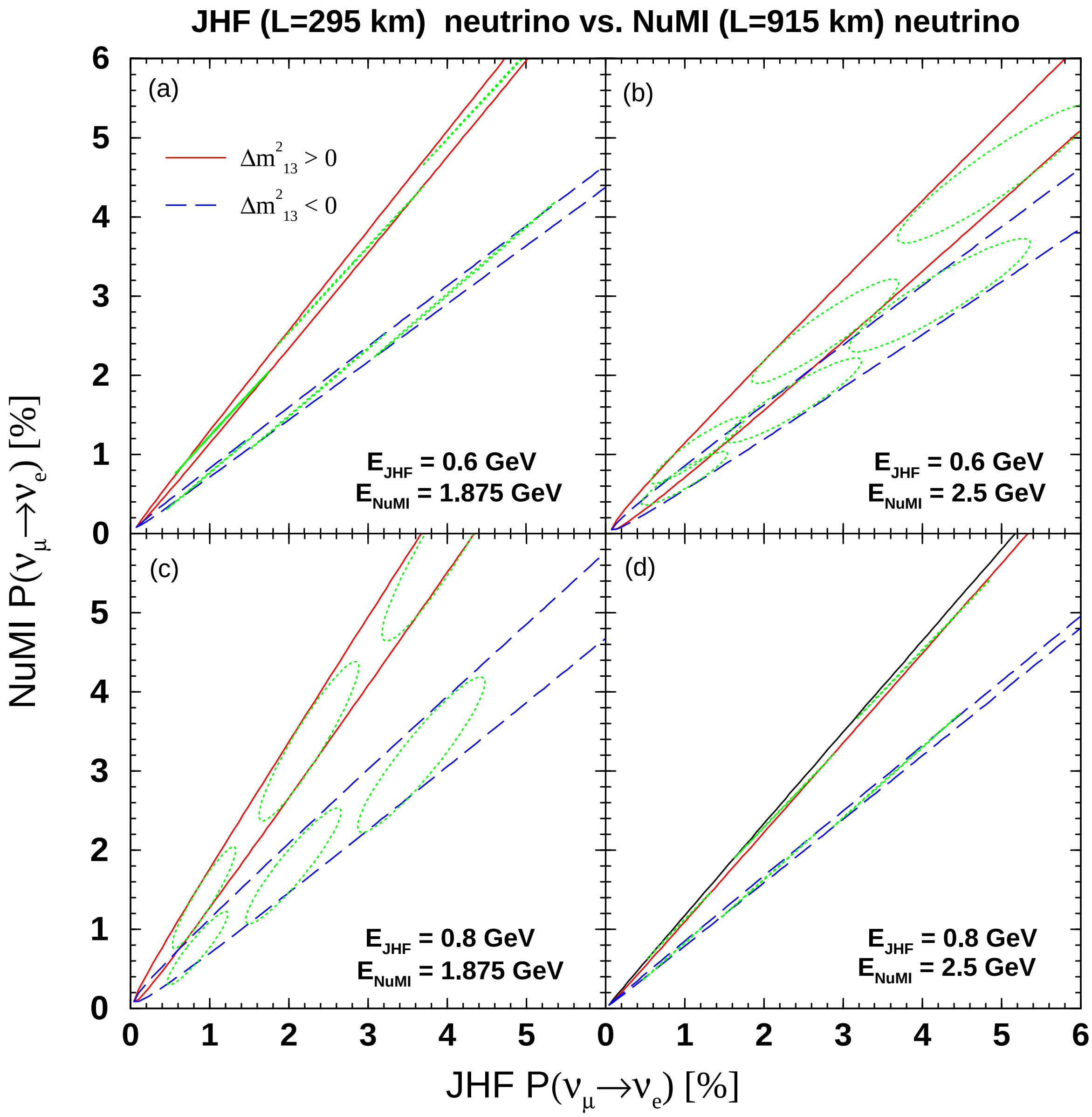
implies small width.



- Separation provided $\left(\frac{E}{L}\right)_{NuMI} \leq \left(\frac{E}{L}\right)_{JHF}$

- Best Separation at Oscillation Maximum

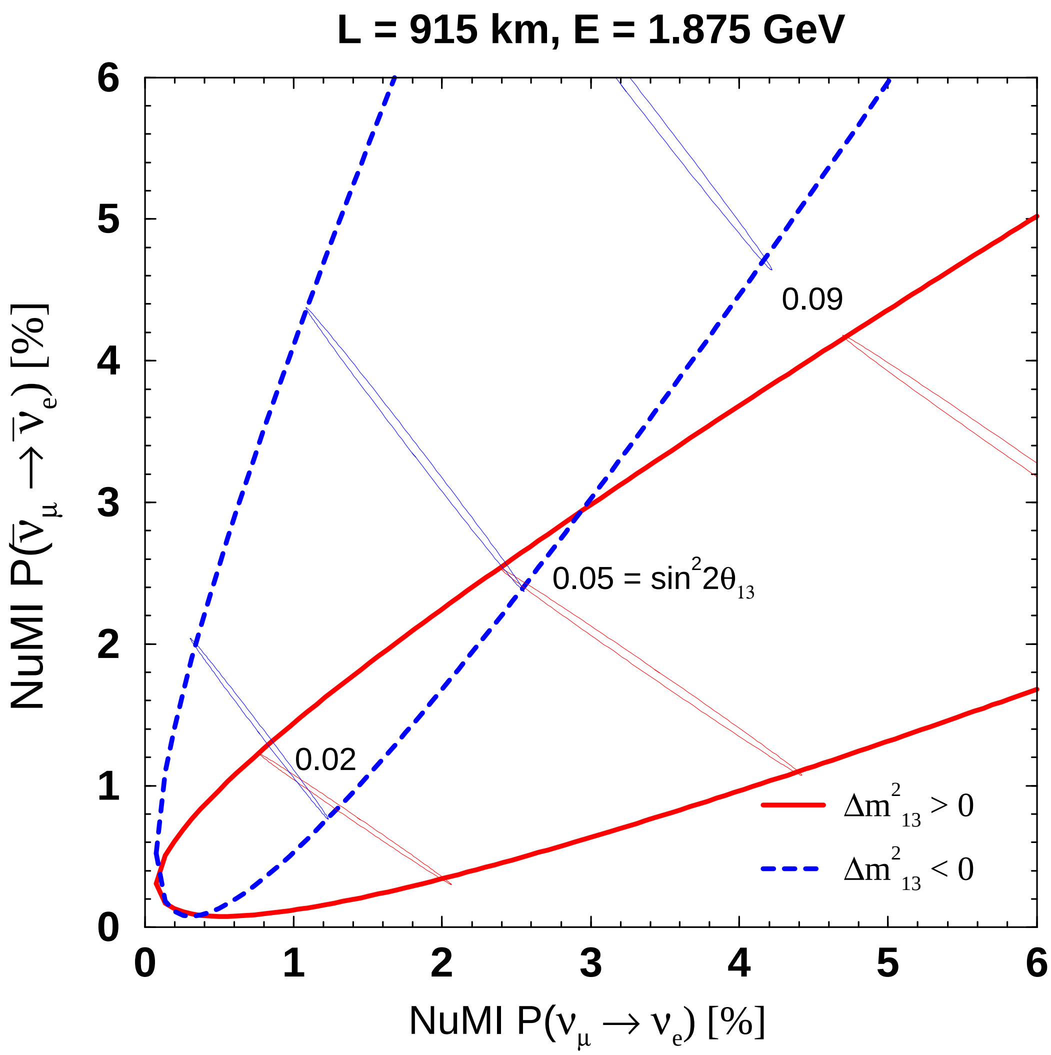
Longer Baseline for NuMI



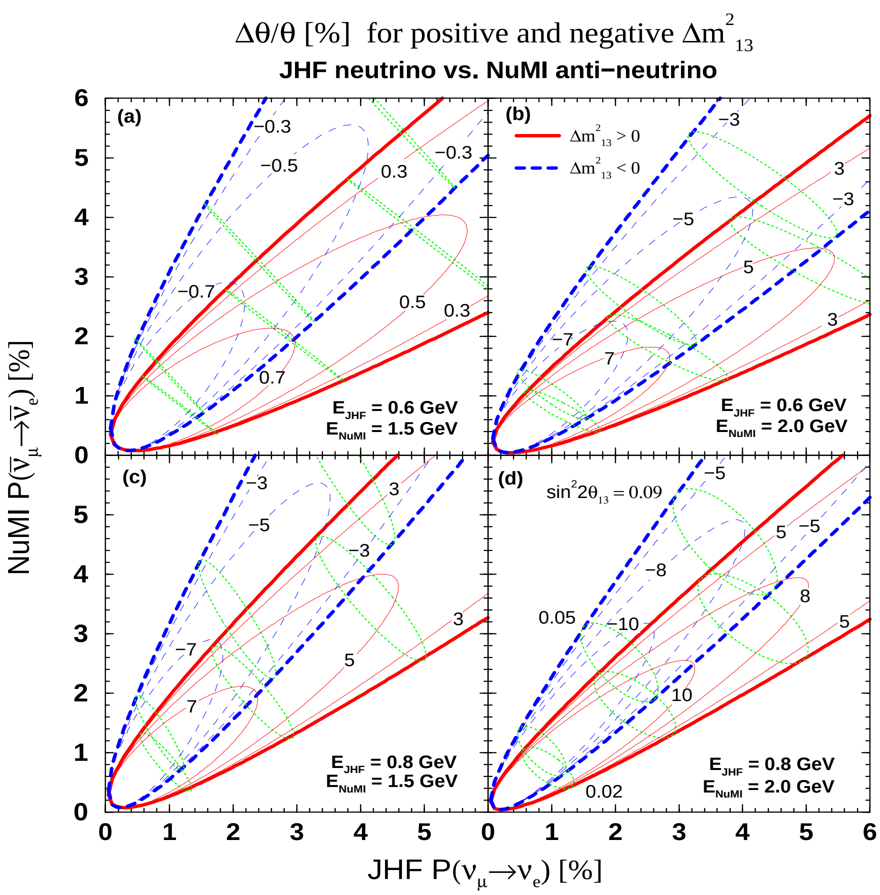
- Better separation at same E/L.

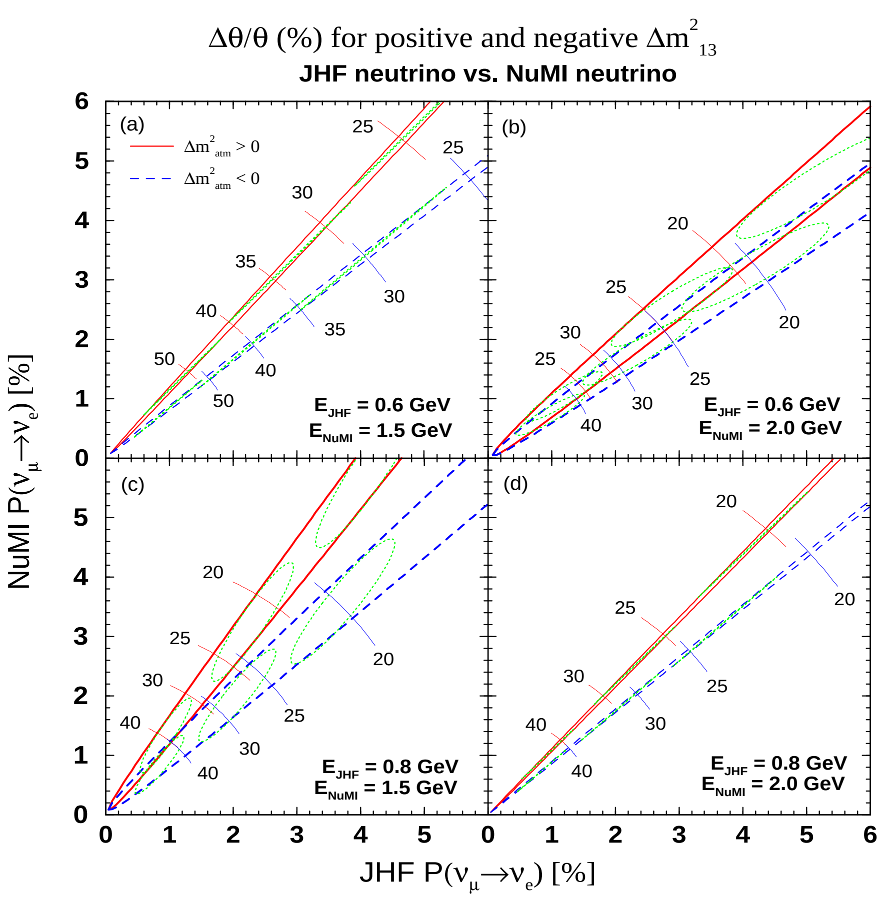
Higher E good, Larger L bad for statistics.

$\nu - \bar{\nu}$ at a Longer Baseline NuMI

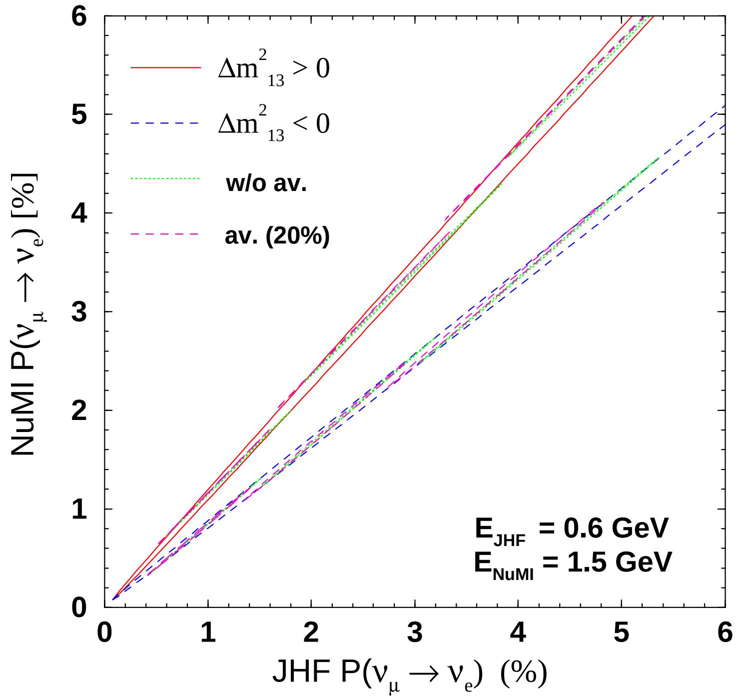


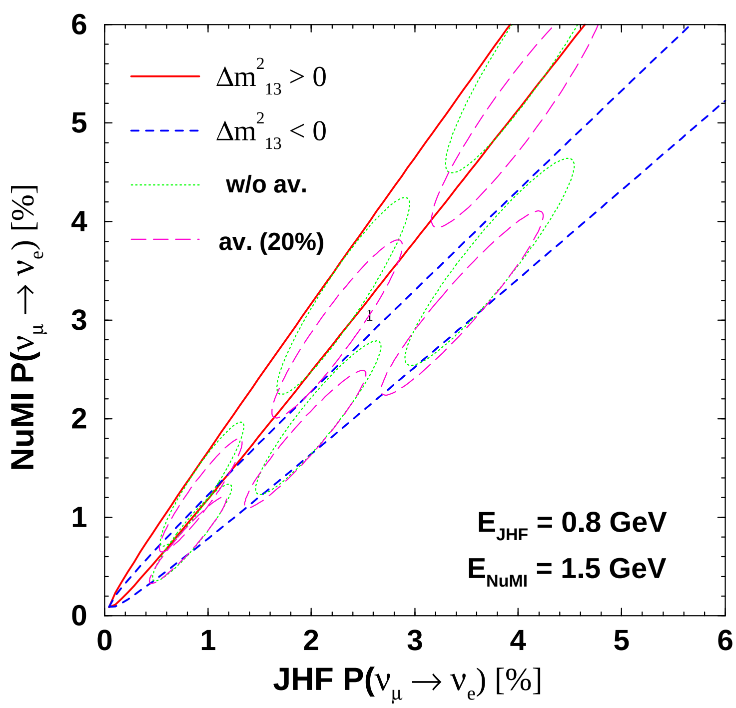
- Hierarchy separation good but NOT guaranteed.


Δθ/θ [%] for positive and negative Δm²₁₃
JHF neutrino vs. NuMI anti−neutrino
(a)
(b)
(c)
(d)
Δm²₁₃ > 0
Δm²₁₃ < 0
sin²2θ₁₃ = 0.09
E_JHF = 0.6 GeV
E_NuMI = 1.5 GeV
E_JHF = 0.6 GeV
E_NuMI = 2.0 GeV
E_JHF = 0.8 GeV
E_NuMI = 1.5 GeV
E_JHF = 0.8 GeV
E_NuMI = 2.0 GeV
NuMI P(ν̄_μ→ν̄_e) [%]
JHF P(ν_μ→ν_e) [%]


Δθ/θ (%) for positive and negative Δm²₁₃
JHF neutrino vs. NuMI neutrino
(a)
(b)
(c)
(d)
Δm²_atm > 0
Δm²_atm < 0
E_JHF = 0.6 GeV
E_NuMI = 1.5 GeV
E_JHF = 0.6 GeV
E_NuMI = 2.0 GeV
E_JHF = 0.8 GeV
E_NuMI = 1.5 GeV
E_JHF = 0.8 GeV
E_NuMI = 2.0 GeV
NuMI P(ν_μ→ν_e) [%]
JHF P(ν_μ→ν_e) [%]

Energy Averaging:





- Energy Averaging does NOT effect separation.

SUMMARY:

$P(\nu_\mu \to \nu_e)$ from $\delta m^2 \sim 3 \times 10^{-3} eV^2$
is a WONDERFUL opportunity.

- The 8 fold degeneracy issue can be solved with multiple measurements of $\nu_\mu \to \nu_e$:
e.g.
Neutrino and Anti-Neutrino at Osc. Max. with large matter effect PLUS
Neutrino at a higher E/L with smaller matter effect is SUFFICIENT, if chosen carefully.

- JHF-NuMI both neutrinos is good for distinguishing the mass hierarchy provided

$$\frac{E}{L}\Big|_{NuMI} \leq \frac{E}{L}\Big|_{JHF}.$$

- JHF-NuMI one neutrinos and one anti-neutrinos is good for determining θ_{13} and δ.
Similar to JHF-J$\bar{\text{H}}$F and NuMI-Nu$\bar{\text{M}}$I.

- The Community needs to exploit this Opportunity Coherently !!