TACHYON DYNAMICS IN OPEN STRING THEORY

1. REVIEW OF TACHYON CONDENSATION CONJECTURES IN
   a) SUPERSTRING THEORY
   b) BOSONIC STRING THEORY

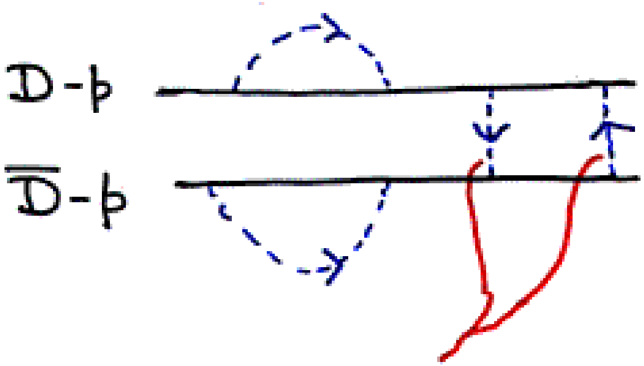
2. "DERIVATION" OF THESE CONJECTURES USING STRING FIELD THEORY

3a

OUR OBJECT OF STUDY:

A COINCIDENT Dp-$\bar{\text{D}}p$ PAIR IN IIA/IIB:

→ HAS FOUR KINDS OF OPEN STRING EXCITATIONS



EACH OF THESE OPEN STRINGS CONTAINS A STATE OF $m^2 < 0$

$m^2 = -\frac{1}{2\alpha'}$ mass

⇒ TACHYONIC MODE

Banks & Susskind
Green & Gutperle

⇒ CAN BE DESCRIBED BY 2 REAL OR 1 COMPLEX FIELD T WITH NEGATIVE (MASS)^2

⇒ POTENTIAL $V(T)$ HAS A MAXIMUM AT $T=0$

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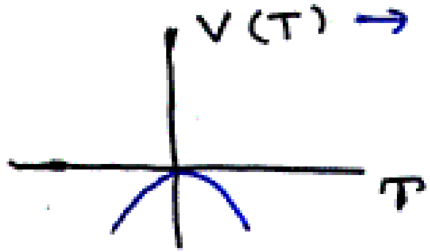
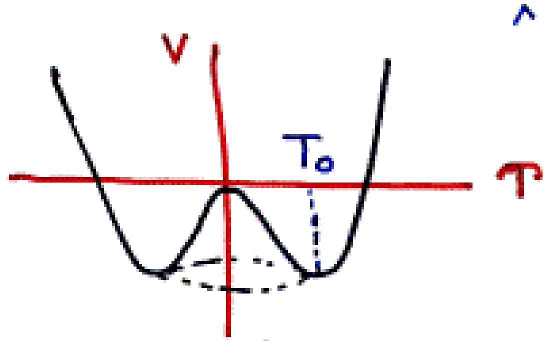


→ CLASSICAL TACHYON POTENTIAL

Q. DOES V(T) HAVE A STABLE MINIMUM, AND IF IT DOES, WHAT PHYSICAL CONFIGURATION DOES IT DESCRIBE?

CONJECTURES: A.S.

① V(T) DOES HAVE STABLE MINIMA.



$V(T_0)$ EXACTLY CANCELS THE TENSION OF Dp - $\overline{\text{Dp}}$ SYSTEM

$$\boxed{V(T_0) + \mathcal{T}_p + \overline{\mathcal{T}}_p = 0}$$

⇒ TOTAL ENERGY DENSITY = 0

② THE MINIMA DESCRIBE CLOSED STRING THEORY VACUUM WITHOUT ANY D-BRANE.

⇒ THERE ARE NO PERTURBATIVE OPEN STRING EXCITATIONS AROUND THIS MINIMUM

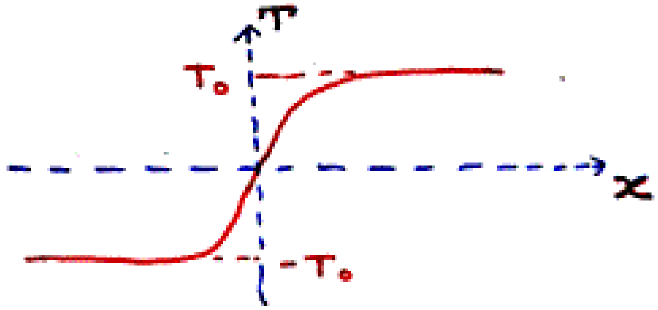
③ THERE ARE SOLITONIC EXCITATIONS AROUND THE MINIMUM

→ THESE DESCRIBE LOWER DIMENSIONAL D-BRANES

e.g. IN THE PRESENT EXAMPLE WE CAN CONSTRUCT A VORTEX SOLUTION INVOLVING T (& A $U(1)$ GAUGE FIELD LIVING ON $D\bar{D}$ SYSTEM)

THIS DESCRIBES A D-$(p-2)$ BRANE.

ANOTHER EXAMPLE: THE KINK



$\Rightarrow$ DESCRIBES THE NON-BPS D-(p-1) BRANE (UNSTABLE)

SIMILAR CONJECTURES EXIST FOR TACHYON CONDENSATION ON A SINGLE NON-BPS D-BRANE

WHY ARE THESE RESULTS OF INTEREST?

CONSIDER TYPE IIB STRING THEORY WITH A CERTAIN NUMBER OF $\underbrace{D9-\bar{D}9}_{}$ PAIRS

SPACE-FILLING BRANES

FULL DYAMICS OF THESE D9-$\bar{D}$9 BRANES IS DESCRIBED BY AN OPEN STRING FIELD THEORY (SFT)

→ A FIELD THEORY WITH ∞ # OF FIELDS

ACCORDING TO THE CONJECTURE THIS FIELD THEORY HAS A STABLE CLASSICAL GROUND STATE WHICH REPRESENTS TYPE IIB STRING THEORY WITHOUT ANY D-BRANE BACKGROUND.

IF WE EXPAND THE SFT ACTION AROUND THIS VACUUM THEN THE THEORY DOES NOT HAVE ANY PERTURBATIVE EXCITATIONS ALTHOUGH THERE ARE ∞ NO. OF FIELDS

HOWEVER THIS FIELD THEORY HAS CLASSICAL SOLITON SOLUTIONS WHICH DESCRIBE ALL THE D-BRANES OF TYPE IIB STRING THEORY

HOPE: WE CAN USE THIS OPEN STRING FIELD THEORY TO GIVE A NON-PERTURBATIVE DEFINITION OF TYPE IIB STRING THEORY

NOTE: OPEN STRING FIELD THEORIES ARE MUCH BETTER UNDERSTOOD THAN THEIR CLOSED STRING COUNTERPART.

IN ORDER FOR THIS PROGRAM TO BE SUCCESSFUL WE NEED TO SHOW THAT THIS STRING FIELD THEORY CONTAINS ALL KNOWN OBJECTS IN TYPE IIB STRING THEORY

(1) D-BRANES. ✓

(2) CLOSED STRINGS : ✓

Yi
Bergman, Hori, Yi
Gibbons, Hori, Yi
A.S.
⋮

THERE IS INDIRECT EVIDENCE THAT THEY EXIST IN THIS THEORY

(3) NS 5-BRANES ?

A SIMILAR PROGRAM CAN BE SET UP FOR TYPE IIA STRING THEORY

STARTING POINT: OPEN STRING FIELD THEORY FOR UNSTABLE D9-BRANES OF IIA STRING THEORY

Q. HOW DO WE PROVE / FIND EVIDENCE FOR THESE THREE CONJECTURES?

THERE ARE VARIOUS APPROACHES

1. CONFORMAL FIELD THEORY
2. RENORMALIZATION GROUP FLOW
3. STRING FIELD THEORY
4. BACKGROUND INDEPENDENT SFT
5. NON-COMMUTATIVE SOLITONS

OF ALL THE APPROACHES, STRING FIELD THEORY SEEMS TO BE MOST COMPLETE

→ HAS THE CAPABILITY OF ADDRESSING ALL THREE CONJECTURES.

WE SHALL FOCUS ON THIS APPROACH

WE SHALL STUDY THIS PROBLEM IN A SIMPLER CONTEXT.

→ BOSONIC STRING THEORY

REASON: CLASSICAL PROPERTIES OF D-BRANES IN BOSONIC STRING THEORY ARE VERY SIMILAR TO THAT OF BRANE-ANTIBRANE SYSTEM IN SUPERSTRING THEORY

HOPE: ONCE WE UNDERSTAND THE CLASSICAL DYNAMICS IN BOSONIC STRING THEORY D-BRANES WE SHOULD BE ABLE TO GENERALIZE THIS TO BRANE-ANTIBRANE SYSTEM IN SUPERSTRING THEORY

PROPERTIES OF D-BRANES IN BOSONIC STRING THEORY

(25+1)-DIMENSIONAL BOSONIC STRING THEORY HAS D-p-BRANES FOR ALL $p \leq 25$

IN THIS CASE EVEN A SINGLE D-p-BRANE HAS A TACHYONIC MODE

$\Rightarrow$ A REAL SCALAR FIELD T WITH NAGATIVE $(\text{MASS})^2$

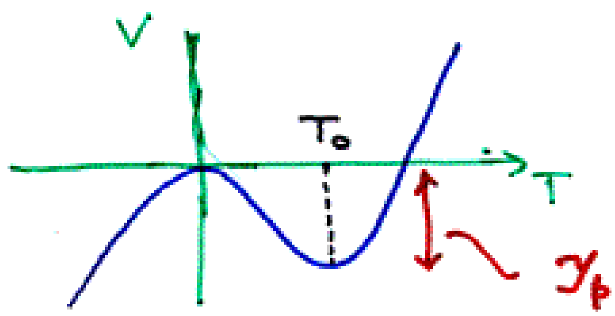
$\Rightarrow$ $V(T)$ HAS MAXIMUM AT $T=0$

Q. IS THERE A (LOCAL) MINIMUM OF THE TACHYON POTENTIAL $V(T)$?

CONJECTURES:

① THE TACHYON POTENTIAL HAS A (LOCAL) MINIMUM WHERE THE NEGATIVE CONTRIBUTION FROM THE POTENTIAL EXACTLY CANCELS THE D-p BRANE TENSION

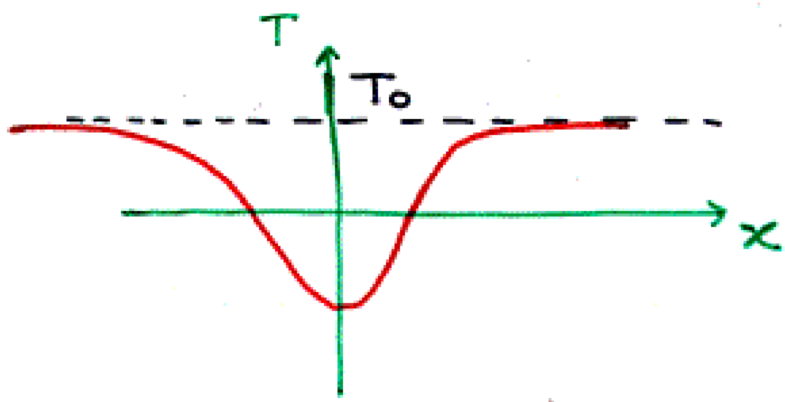
$$\boxed{V(T_0) + \mathcal{T}_p = 0}$$



② THE LOCAL MINIMUM T_0 OF $V(T)$ REPRESENTS CLOSED STRING VACUUM WITHOUT ANY D-BRANE.

$\Rightarrow$ THERE SHOULD NOT BE ANY OPEN STRING EXCITATION AROUND THIS VACUUM

3 THE THEORY CONTAINS CLASSICAL LUMP SOLUTIONS WHICH DESCRIBE LOWER DIMENSIONAL D-BRANES



NOTE: ALTHOUGH WE HAVE BEEN TALKING ABOUT TACHYON POTENTIAL / TACHYONIC LUMP SOLUTION, FULL DYNAMICS OF D-BRANES IS DESCRIBED BY A FIELD THEORY OF ∞ # OF COUPLED FIELDS

$\Rightarrow$ STRING FIELD THEORY

REVIEW OF SFT DESCRIBING A D-BRANE DYNAMICS:

Witten

STRING FIELD $|\Phi\rangle$: A STATE OF GHOST NO. 1 IN THE HILBERT SPACE $\mathcal{H}$ OF MATTER-GHOST CFT DESCRIBING FIRST QUANTIZED OPEN STRING THEORY

$$\boxed{|\Phi\rangle = \sum_r \psi_r \, |\phi_{1,r}\rangle}$$

ψ_r: DYNAMICAL VARIABLES OF SFT

$|\phi_{1,r}\rangle$: A BASIS OF STATES OF GHOST NO. 1

ACTION :
$$\boxed{S = -\frac{1}{g^2}\left\{ \frac{1}{2} \langle \Phi | Q_B | \Phi \rangle + \frac{1}{3} \langle \Phi | \Phi * \Phi \rangle \right\}}$$

$\langle \, | \, \rangle$: BPZ INNER PRODUCT

Q_B : BRST CHARGE

$|A * B\rangle$: WITTEN'S $*$- PRODUCT

$\hookrightarrow \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$

GAUGE INVARIANCE:

$$\boxed{\delta|\Phi\rangle = Q_B|\Lambda\rangle + |\Phi * \Lambda\rangle - |\Lambda * \Phi\rangle}$$

$|\Lambda\rangle$: AN ARBITRARY STATE OF GHOST NO. 0 $= \sum_r \lambda_r |\phi_{0,r}\rangle$

→ INFINITESIMAL GAUGE TRS. PARAMETER

PROOF OF GAUGE INVARIANCE NEEDS

$$Q_B^2 = 0$$

$$Q_B|A * B\rangle = |(Q_B A) * B\rangle + (-1)^{n_A}|A * Q_B B\rangle$$

GHOST NO. OF $|A\rangle$

$$\langle Q_B A|B\rangle = -(-1)^{n_A}\langle A|Q_B|B\rangle$$

$$\langle A|B\rangle = (-1)^{n_A n_B}\langle B|A\rangle$$

$$\langle A|B * C\rangle = \langle A * B|C\rangle$$

$$|A * (B * C)\rangle = |(A * B) * C\rangle$$

START WITH A D-25- BRANE

SUPPOSE $|\Phi_0\rangle$ IS THE CLASSICAL SOLUTION REPRESENTING THE TACHYON VACUUM

$$\boxed{Q_B|\Phi_0\rangle + |\Phi_0 * \Phi_0\rangle = 0}$$

→ EQS. OF MOTION

DEFINE $\boxed{|\Psi\rangle = |\Phi\rangle - |\Phi_0\rangle}$

$$\boxed{S = + S(\Phi_0) - \frac{1}{g^2}\underbrace{\left[\frac{1}{2}\langle\Psi|Q|\Psi\rangle + \frac{1}{3}\langle\Psi|\Psi * \Psi\rangle\right]}_{\tilde{S}(\Psi)}}$$

$$Q|A\rangle \equiv Q_B|A\rangle + |\Phi_0 * A\rangle - (-1)^{n_A}|A * \Phi_0\rangle$$

Q SATISFIES: Witten

$$\boxed{\begin{gathered} Q^2 = 0, \quad \langle QA|B\rangle = -(-1)^{n_A}\langle A|QB\rangle \\ Q|A * B\rangle = |(QA) * B\rangle + (-1)^{n_A}|A * QB\rangle \end{gathered}}$$

⇒ GAUGE INV. OF $\tilde{S}(\Psi)$:

$$\boxed{\delta|\Psi\rangle = Q|\Lambda\rangle + |\Psi * \Lambda\rangle - |\Lambda * \Psi\rangle}$$

CONJECTURE 1

$\Rightarrow \quad S(\Phi_0) = V^{(26)} \; \mathcal{T}_{25}$

$\hookrightarrow$ D-25-BRANE TENSION

VOLUME OF SPACE-TIME

CONJECTURE 3

$\Rightarrow$ EQS. OF MOTION HAVE CODIMENSION (25-p) LUMP SOLUTION $\Phi^{(p)}$ WITH

$$S(\Phi^{(p)}) - S(\Phi_0) = -V^{(p+1)} \; \mathcal{T}_p$$

D-p-BRANE WORLD-VOLUME

D-p-BRANE TENSION

$\Rightarrow$ DESCRIBES A D-p-BRANE SOLN.

CONJECTURE 2

$\Rightarrow$ Q HAS VANISHING COHOMOLOGY

ALL THREE CONJECTURES HAVE BEEN TESTED NUMERICALLY USING LEVEL TRUNCATION SCHEME.

Kostelecky & Samuel

BASIC IDEA: IN $\sum_r \psi_r |\phi_{1,r}\rangle$ RESTRICT THE EXPANSION TO THOSE $|\phi_{1,r}\rangle$ FOR WHICH

LEVEL $(1+h_r) \leq L \longrightarrow$ SOME FIXED NO.

CONFORMAL WEIGHT OF $\phi_{1,r}$

FOR FINITE L, WE HAVE A FINITE NO. OF FIELDS.

$\Rightarrow$ CAN SOLVE SFT EQS. OF MOTION, COMPUTE $S(\Phi_{cl})$ AND COMPUTE COHOMOLOGY OF Q NUMERICALLY

CONJECTURE 1 :

A.S. & Zwiebach
Moeller & Taylor
Hata, Shinohara
Harvey & Kraus

CONJECTURE 2 :

de Mello Koch, Jevicki,
Mihailescu, Tatar
Moeller, A.S., Zwiebach
de Mello Koch, Rodrigues
Moeller
Mukhopadhyay, A.S.
Ellwood & Taylor

CONJECTURE 3 :

Feng, He, Moeller
Taylor; A.S., Zwiebach

SIMILAR ANALYSIS HAS BEEN CARRIED OUT FOR SUPERSTRING THEORY

GETTING ANALYTICAL RESULTS SEEMS DIFFICULT IN THIS APPROACH

THERE IS HOWEVER AN INDIRECT APPROACH IN WHICH ANALYTICAL STUDY SEEMS POSSIBLE.

Rastelli, A.S., Zwiebach

WHAT IS THE FORM OF Q?

→ NOT POSSIBLE TO FIND DIRECTLY UNLESS WE KNOW $|\Phi_0\rangle$?

AT PRESENT $|\Phi_0\rangle$ IS KNOWN ONLY NUMERICALLY

OUR APPROACH : TRY TO GUESS THE FORM OF Q.

REQUIREMENTS ON Q :

(a) Q MUST SATISFY IDENTITIES E.

(b) Q MUST HAVE VANISHING COHOMOLOGY

→ ABSENCE OF PHYSICAL OPEN STRING STATES

→ IMPLEMENTS CONJECTURE (2).

(c) THE NEW ACTION MUST HAVE CLASSICAL SOLUTIONS REPRESENTING THE ORIGINAL D-BRANE AND OTHER D-BRANES

→ IMPLEMENTS CONJECTURES (1) AND (3)

(d) Q SHOULD BE "UNIVERSAL"

STARTING WITH ~~SOME~~ ANY D-BRANE WE SHOULD END UP WITH THE SAME $\tilde{S}(\Psi)$ AFTER TACHYON CONDENSATION TO VACUUM VALUE

→ GUESS: Q IS MADE PURELY OF GHOST FIELDS

→ DOES NOT INVOLVE ANY OPERATOR FROM MATTER CFT

IS THERE A Q SATISFYING ALL OF THESE CONDITIONS?

IT IS EASY TO FIND Q SATISFYING CONDITIONS (a), (b) & (d).

gauge inv. trivial Cohomology universality

EXAMPLE:

$$\boxed{Q = \sum_n a_n \left(c_n + (-1)^n c_{-n} \right)}$$

ARBITRARY CONSTANTS

Horowitz, Morrow-Jones, Martin, Woodard

NON-TRIVIAL TASK: CONDITION (c)

→ CONSTRUCTION OF CLASSICAL SOLUTIONS REPRESENTING VARIOUS D-BRANES

NOTE: MANY (BUT NOT ALL) DIFFERENT CHOICES OF a_n ARE RELATED BY FIELD REDEFN.

WE SHALL PROCEED WITHOUT COMMITTING OURSELVES TO A SPECIFIC FORM OF Q.

EQS. OF MOTION:

$$\boxed{Q|\Psi\rangle + |\Psi * \Psi\rangle = 0}$$

FACTORIZATION ANSATZ FOR SOLN.

Taylor

$$\boxed{|\Psi\rangle = |\Psi_g\rangle \otimes |\Psi_m\rangle}$$

ghost matter

$\Rightarrow$ EQ. OF MOTION SPLITS:

$$\boxed{Q|\Psi_g\rangle + |\Psi_g * \Psi_g\rangle = 0}$$

$$\boxed{|\Psi_m\rangle = |\Psi_m * \Psi_m\rangle}$$

$$S_{cl} = -\frac{1}{6g^2}\langle\Psi|Q|\Psi\rangle$$

$$= -\frac{1}{6g^2}\underbrace{\langle\Psi_g|Q|\Psi_g\rangle}_{\equiv K}\langle\Psi_m|\Psi_m\rangle$$

$\equiv K \rightarrow$ CAN'T BE CALCULATED WITHOUT KNOWING Q

UNIVERSALITY ANSATZ FOR SOLUTION

$|\Psi_g\rangle$ IS THE SAME FOR DIFFERENT SOLUTIONS REPRESENTING DIFFERENT D-BRANES, WHEREAS $|\Psi_m\rangle$ DIFFERS.

$\Rightarrow$ IF $|\Psi_m\rangle$ AND $|\Psi_m'\rangle$ DESCRIBE TWO DIFFERENT D-BRANE SOLUTIONS, THEN THE RATIO OF THEIR ENERGY IS:

$$\boxed{\langle \Psi_m | \Psi_m \rangle / \langle \Psi_m' | \Psi_m' \rangle}$$

NOTE: K DROPS OUT

WE SHALL NOW LOOK FOR SOLUTIONS OF

$$\boxed{|\Psi_m\rangle = |\Psi_m * \Psi_m\rangle}$$

USING TECHNIQUES DEVELOPED BY KOSTELECKY AND POTTING.

(hep-th/0008252)

$*$: A MAP FROM $\mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$

$$\underset{1}{\mathcal{H}} \otimes \underset{2}{\mathcal{H}} \to \underset{3}{\mathcal{H}}$$

$$|A * B\rangle_3 = {}_1\langle A| \otimes {}_2\langle B| V^{(3)}\rangle_{123}$$

Gross-Jevicki
Samuel

$$\boxed{|V^{(3)}\rangle_{123} = \int d^{26}p_{(1)}\, d^{26}p_{(2)}\, d^{26}p_{(3)} \times \delta^{(26)}(p_{(1)} + p_{(2)} + p_{(3)})\, \exp(-E)\, |0,p\rangle_{123}}$$

$$|0, p\rangle_{123} \equiv |p_{(1)}\rangle_1 \otimes |p_{(2)}\rangle_2 \otimes |p_{(3)}\rangle_3$$

FOCK VACUA CARRYING MOMENTUM $p_{(i)}$

$$\boxed{E \equiv \frac{1}{2} \sum_{\substack{r,s=1 \\ m,n \geq 1}}^{3} \eta_{\mu\nu}\, a_m^{(r)\mu\dagger}\, V_{mn}^{rs}\, a_n^{(s)\nu\dagger} + \sum_{\substack{r,s=1 \\ n \geq 1}}^{3} \eta_{\mu\nu}\, p_{(r)}^{\mu}\, V_{0n}^{rs}\, a_n^{(s)\nu\dagger} + \frac{1}{2} \sum_{r=1}^{3} \eta_{\mu\nu}\, p_{(r)}^{\mu}\, V_{00}^{rr}\, p_{(r)}^{\nu}}$$

$a_m^{(r)\mu\dagger}$: CREATION OPERATORS ASSOCIATED WITH X^μ IN $\mathcal{H}_{(r)}$.

V_{mn}^{rs} : CALCULATED BY Gross-Jevicki; Samuel

NORMALIZATION CONVENTION:

$$[a^{(r)\mu}_m, a^{(s)\nu\dagger}_n] = \eta^{\mu\nu}\,\delta_{rs}\,\delta_{mn}$$

$$\langle p | p' \rangle = \delta^{(26)}(p+p')$$

$$\delta^{(26)}(0) = V^{(26)}/(2\pi)^{26}$$

D-25-BRANE WORLD-VOLUME

$\alpha' = 1$

ANSATZ FOR THE SOLUTION REPRESENTING A D-25-BRANE:

$$|\Psi_m\rangle = \mathcal{N}^{26} \exp[-\tfrac{1}{2}\eta_{\mu\nu}\sum_{m,n\geq 1} S_{mn}\, a^{\mu\dagger}_m a^{\nu\dagger}_n]\,|0\rangle$$

CARRIES ZERO MOMENTUM

$\Rightarrow$ HAS 26-DIMENSIONAL TRANSLATION INVARIANCE AS REQUIRED.

NOTE: THE ANSATZ IS A PRODUCT OF 26 FACTORS $\Rightarrow$ ONE FOR EACH X^μ

SUBSTITUTE THIS ANSATZ INTO THE EQ. $|\Psi_m * \Psi_m\rangle = |\Psi_m\rangle$:

RESULT :

$$\boxed{\begin{aligned} S &= V^{11} + (V^{12} \;\; V^{21})(1-\Sigma\mathcal{V})^{-1}\Sigma \begin{pmatrix} V^{21} \\ V^{12} \end{pmatrix} \\ \mathcal{N} &= (\det(1-\Sigma\mathcal{V}))^{1/2} \end{aligned}}$$

V^{rs}, S : MATRICES WITH COMPONENTS V^{rs}_{mn}, S_{mn} FOR

$$1 \le m, n < \infty$$

$$\boxed{\Sigma \equiv \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}, \qquad \mathcal{V} \equiv \begin{pmatrix} V^{11} & V^{12} \\ V^{21} & V^{22} \end{pmatrix}}$$

SOLUTION : Kostelecky & Potting

$S = CT,$	$C_{mn} \equiv (-1)^m \delta_{mn}$
$T = (2X)^{-1}\left(1 + X - \sqrt{(1+3X)(1-X)}\right),$	$X \equiv CV^{11}$
$\mathcal{N} = \{\det(1-X)\ \det(1+T)\}^{1/2}$	

RECALL:

$$\boxed{S_{cl} = K\,\langle\psi_m|\psi_m\rangle}$$

$$K \to -\frac{1}{6g^2}\langle\psi_g|Q|\psi_g\rangle \to \text{UNKNOWN}$$

SUBSTITUTE $|\psi_m\rangle$:

$$\Rightarrow \boxed{S_{cl} = K.\,\frac{V^{(26)}}{(2\pi)^{26}}\left\{\det(1-X)^{3/4}\det(1+3X)^{1/4}\right\}^{26}}$$

$$\Downarrow$$

FROM $\langle 0|0\rangle = \delta^{(26)}(0)$

$\Rightarrow$ D-25-BRANE TENSION:

$$\boxed{\begin{aligned}\mathcal{T}_{25} &= -S_{cl}/V^{(26)}\\ &= -K.\,\frac{1}{(2\pi)^{26}}\left\{\det(1-X)^{3/4}\det(1+3X)^{1/4}\right\}^{26}\end{aligned}}$$

NOTE: $X = \underline{CV''}$ IS A KNOWN $\infty \times \infty$ MATRIX BUT K IS UNKNOWN

$\Rightarrow \mathcal{T}_{25}$ CANNOT BE CALCULATED

D-(25-k)-BRANE SOLUTION:

(FOLLOWING SUGGESTION BY KOSTELECKY & POTTING)

GENERAL GUIDELINES:

① NOTE THAT $|\Psi_m\rangle$ FOR D-25-BRANE IS A PRODUCT OF 26 FACTORS, ONE FOR EACH X^μ.

② D-(25-k)-BRANE SOLUTION IS ALSO TAKEN TO BE A PRODUCT OF 26 FACTORS

③ FOR DIRECTIONS $\parallel$ THE D-(25-k) BRANE WE KEEP THE FACTOR TO BE IDENTICAL TO THE ONE APPEARING IN D-25-BRANE SOLN.

④ FOR EACH DIRECTION $\perp$ THE D-(25-k)-BRANE WE CHOOSE A DIFFERENT FACTOR

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$x^{\bar\mu}$: DIRECTIONS $\parallel$ D-(25-k)-BRANE
$(0 \le \bar\mu \le 25-k)$

x^{α} : DIRECTIONS $\perp$ D-(25-k)-BRANE
$(26-k \le \alpha \le 25)$

DEFINE OSCILLATORS:

*Gross-Jevicki

$$\boxed{\begin{aligned} a_0^{\alpha} &= \frac{1}{2}\sqrt{b}\,\hat{p}^{\alpha} - \frac{i}{\sqrt{b}}\,\hat{x}^{\alpha} \\ a_0^{\alpha\dagger} &= \frac{1}{2}\sqrt{b}\,\hat{p}^{\alpha} + \frac{i}{\sqrt{b}}\,\hat{x}^{\alpha} \end{aligned}}$$

b : ARBITRARY CONSTANT

$\hat{x}^{\alpha}, \hat{p}^{\alpha}$: ZERO MODES OF X^{α}, P^{α}

$$\boxed{[a_0^{\alpha}, a_0^{\beta\dagger}] = \delta^{\alpha\beta}}$$

DEFINE $|\Omega_b\rangle$ SUCH THAT:

$$\boxed{a_0^{\alpha}\,|\Omega_b\rangle = 0 \quad \forall\ \alpha}$$

$\prod_{\alpha} (a_0^{\alpha\dagger})^{n_\alpha} |\Omega_b\rangle$: HARMONIC OSCILLATOR BASIS OF STATES

CONVERSION FROM MOMENTUM BASIS TO HARMONIC OSCILLATOR BASIS:

$$\boxed{|\{p^\alpha\}\rangle = \left(\frac{2\pi}{b}\right)^{-k/4} \exp\left[-\frac{b}{4} + \sqrt{b}\, a_0^{\alpha\dagger} p^\alpha - \frac{1}{2} a_0^{\alpha\dagger} a_0^{\alpha\dagger}\right] |\Omega_b\rangle}$$

NOW REEXPRESS THE 3-STRING VERTEX $|V^{(3)}\rangle_{123}$ IN THIS BASIS:

$$\Rightarrow |V^{(3)}\rangle_{123} = \int d^{26-k} p_{(1)}\, d^{26-k} p_{(2)}\, d^{26-k} p_{(3)}\, \delta^{(26-k)}(p_{(1)} + p_{(2)} + p_{(3)})$$

$$\times \mathrm{EXP}\left[-\frac{1}{2} \sum_{r,s} \sum_{m,n\geq 1} \eta_{\bar{\mu}\bar{\nu}}\, a_m^{(r)\bar{\mu}\dagger} V_{mn}^{rs} a_n^{(s)\bar{\nu}\dagger} - \eta_{\bar{\mu}\bar{\nu}} \sum_{r,s} \sum_{n\geq 1} p_{(r)}^{\bar{\mu}} V_{0n}^{rs} a_n^{(s)\bar{\nu}} - \frac{1}{2} \sum_r \eta_{\bar{\mu}\bar{\nu}}\, p_{(r)}^{\bar{\mu}} V_{00}^{rr} p_{(r)}^{\bar{\nu}}\right] |0, p\rangle$$

$$\otimes |V_\perp^{(3)}\rangle_{123}$$

$$\boxed{|V_\perp^{(3)}\rangle_{123} = \left\{\frac{\sqrt{3}}{(2\pi b^3)^{1/4}}\left(V_{00}^{**}+\frac{b}{2}\right)\right\}^{-k} \times \mathrm{EXP}\left[-\frac{1}{2}\sum_{r,s}\sum_{m,n\geq 0} a_m^{(r)\alpha\dagger} V'^{rs}_{mn}\, a_n^{(s)\alpha\dagger}\right]|\Omega_b\rangle_{123}}$$

$$|\Omega_b\rangle_{123} \equiv |\Omega_b\rangle_1 \otimes |\Omega_b\rangle_2 \otimes |\Omega_b\rangle_3$$

V'^{rs}_{mn} : KNOWN IN TERMS OF

V^{rs}_{mn} AND b (Gross-Jevicki)

ANSATZ FOR D-(25-k)-BRANE SOL^N.

$$\boxed{|\Psi'_m\rangle = \mathcal{N}^{26-k}\exp\left[-\frac{1}{2}\sum_{m,n\geq 1} S_{mn}\, a_m^{\bar\mu\dagger}\, a_n^{\bar\nu\dagger}\,\eta_{\bar\mu\bar\nu}\right]|0\rangle \times (\mathcal{N}')^k \exp\left[-\frac{1}{2}\sum_{m,n\geq 0} S'_{mn}\, a_m^{\alpha\dagger}\, a_n^{\alpha\dagger}\right]|\Omega_b\rangle}$$

SUBSTITUTE INTO:

$$|\Psi'_m\rangle_3 = |\Psi'_m * \Psi'_m\rangle_3$$

$$= {}_1\langle\Psi'_m| \otimes {}_2\langle\Psi'_m|\, V^{(3)}\rangle_{123}$$

A SOLUTION:

$\mathcal{N}$, S_{mn} : SAME AS FOR D-25-BRANE

$S' = C'T'$, $\quad T' \equiv (2X')^{-1}\left(1 + X' - \sqrt{(1+3X')(1-X')}\right)$

$C'_{mn} \equiv (-1)^m \delta_{mn}$, $\quad X' \equiv C' V'^{11}$

$$\mathcal{N}' = \left\{ \frac{\sqrt{3}}{(2\pi b^3)^{1/4}} \left(V^{11}_{00} + \frac{b}{2} \right) \right\} \left\{ \det(1 - X')(1 + T') \right\}^{\frac{1}{2}}$$

NOTE: FOR PRIMED VARIABLES

$m, n \geq 0$

$$S_{cl} = K \langle \Psi'_m | \Psi'_m \rangle$$

$$= K \frac{V^{(26-k)}}{(2\pi)^{26-k}} \left\{ \det(1-X)^{3/4} \det(1+3X)^{1/4} \right\}^{(26-k)}$$

$$\times \left\{ \frac{3}{\sqrt{2\pi}\, b^3} \left(V^{11}_{00} + \frac{b}{2} \right)^2 \right\}^k$$

$$\times \left\{ \det(1 - X')^{3/4} \det(1 + 3X')^{1/4} \right\}^k$$

$\equiv -V^{(26-k)}\, \mathcal{T}_{25-k} \rightarrow$ TENSION OF D-(25-k)-BRANE

$$\mathcal{T}_{25-k} = -\frac{K}{(2\pi)^{26-k}} \left\{ \det(1-X)^{3/4} \det(1+3X)^{1/4} \right\}^{26-k}$$

$$\times \left\{ \frac{3}{\sqrt{2\pi b^3}} \left(V^{11}_{00} + \frac{b}{2} \right)^2 \right\}^k \left\{ \det(1-X')^{\frac{3}{4}} \det(1+3X')^{\frac{1}{4}} \right\}^k$$

$\Rightarrow$

$$\frac{\mathcal{T}_{24-k}}{2\pi\, \mathcal{T}_{25-k}} = \frac{3}{\sqrt{2\pi b^3}} \left(V^{11}_{00} + \frac{b}{2} \right)^2$$

$$\times \quad \frac{\left\{ \det(1-X')^{3/4} \det(1+3X')^{1/4} \right\}}{\left\{ \det(1-X)^{3/4} \det(1+3X)^{1/4} \right\}}$$

NOTE: K HAS DROPPED OUT

$X = CV''$, $X' = CV''$: KNOWN $\infty \times \infty$ MATRICES

WE EVALUATE THE DETERMINANTS BY RESTRICTING X, X' BELOW CERTAIN LEVEL L.

RESULT FOR $\mathcal{I}_{24-k} / (2\pi\, \mathcal{I}_{25-k})$

(EXPECTED ANSWER = 1)

L	b = .5	b = 2	b = 8
50	.94968	.96518	1.02101
200	.95906	.97172	1.01802
800	.96539	.97606	1.01558
3200	.97007	.97927	1.01361
∞	1.00042	1.00063	.99939

↓ OBTAINED USING AN INTERPOLTING FUNCTION:

$$\boxed{f(L) = a_0 + \frac{a_1}{\log L} + \frac{a_2}{(\log L)^2} + \frac{a_3}{(\log L)^3}}$$

(USED MORE DATA POINTS AND A LEAST SQUARE FIT)

RESULTS → STRONG EVIDENCE THAT OUR SOLUTIONS REALLY DESCRIBE D-BRANES

⇒ INITIAL ANSALZ FOR $\widetilde{S}(\psi)$ IS CORRECT.

NOTES:

① WE HAVE A 1-PARAMETER FAMILY OF SOLUTIONS LABELLED BY b

(IN FACT WE COULD CHOOSE DIFFERENT b'S FOR DIFFRENT DIRECTIONS $\perp$ THE D-(25-k)-BRANE

$\Rightarrow$ MULTI-PARAMETER FAMILY OF SOL$^{\underline{N}}$)

STRONG EVIDENCE (NOT PROOF):

THESE DIFFERENT SOLUTIONS ARE RELATED BY GAUGE TRANSF.

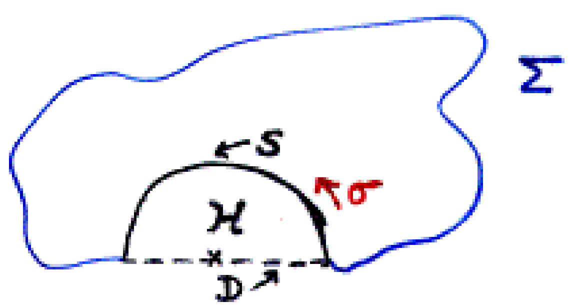
SINCE $b \rightarrow$ WIDTH OF THE SOLUTION $\perp$ THE D-BRANE

$\Rightarrow$ WIDTH OF THE SOLUTION IS A GAUGE ARTIFACT

GEOMETRICAL DESCRIPTION OF THE D-25-BRANE SOLUTION: $|\Psi_m\rangle$

CONSIDER A ~~R~~ DISK Σ, WITH A PUNCTURE AT THE BOUDARY

CHOOSE SOME "LOCAL COORDINATE" ξ AT THE PUNCTURE SUCH THAT $\xi = 0$ IS THE PUNCTURE AND UPPER HALF UNIT DISK H IS PART OF Σ



PUT THE CFT ON $\Sigma - H$ WITH STANDARD OPEN STRING B.C. ON $(\partial\Sigma - D)$

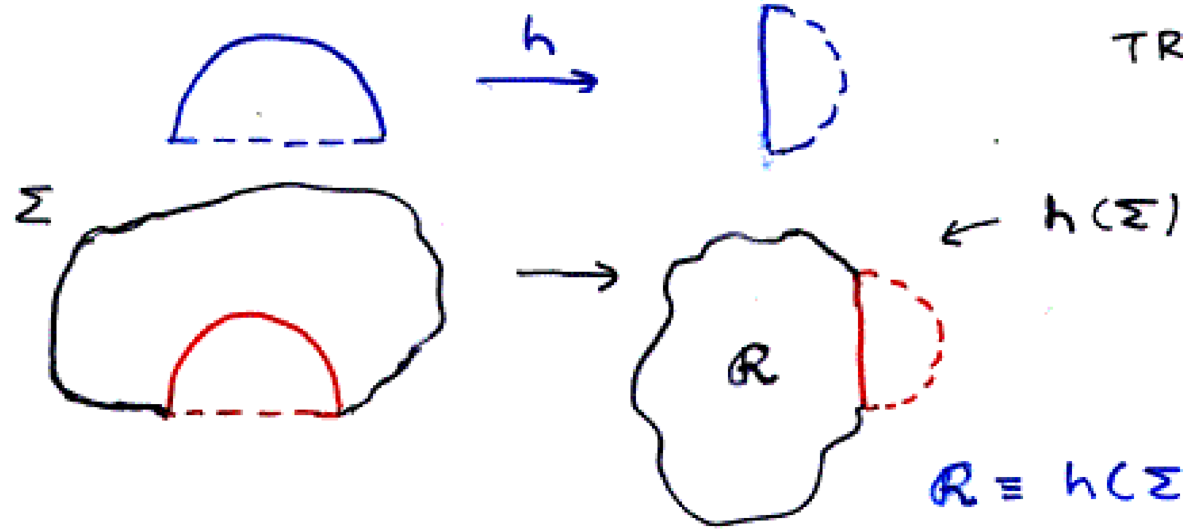
$\Rightarrow$ A FUNCTIONAL OF FIELDS ON S

$\Rightarrow$ THE SURFACE STATE $\langle\Sigma|$

$$\langle\Sigma|\phi\rangle = \langle\phi(0)\rangle_\Sigma \quad \forall\ |\phi\rangle$$

DEFINE $h(\xi) = \frac{1+i\xi}{1-i\xi}$

$$\langle \Sigma | \phi \rangle = \langle \underbrace{h \circ \phi(0)}_{\text{CONFORMAL TRS.}} \rangle_{h(\Sigma)}$$

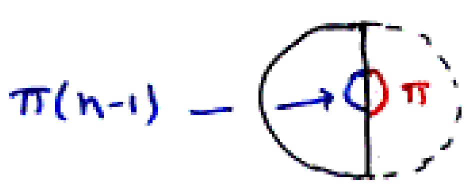


$R \equiv h(\Sigma - H)$

DEFINE THE STATE $\langle n|$ AS THE SURFACE STATE ASSOCIATED WITH A SPECIFIC R.

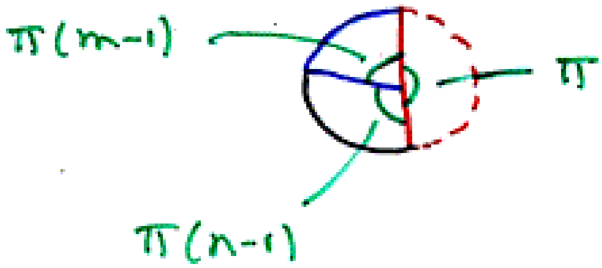
$R + h(H)$: A CONE SUBTENDING AN ANGLE (πn) AT THE ORIGIN.

Rastelli & Zwiebach



IT IS EASY TO CALCULATE

$$\langle m| * |n|$$



$\to \langle m+n-1|$

$$\langle m| * \langle n| = \langle m+n-1|$$

NOTE $\langle 1| * \langle 1| = \langle 1|$ $\to$ IDENTITY OF $*$ PROD.

$$\langle\infty| * \langle\infty| = \langle\infty| \qquad \langle\infty| \equiv \langle\Xi|$$

$\to$ $\langle\Psi_m|$: THE D-25-BRANE SOLUTION "SLIVER" $\langle\Xi_m|$

HOW DO WE KNOW THAT $\langle\Psi_m|$ IS THE SAME AS $\langle\Xi_m|$ CONSTRUCTED ALGEBRAICALLY?

$$\langle \Xi | \phi \rangle = \lim_{n\to\infty} \langle h \circ \phi(0) \rangle_{\hat{D}_n}$$

$\hat{D}_n$: A CONE SUBTENDING AN ANGLE $n\pi$ AT THE ORIGIN

$$h(\xi) = \frac{1+i\xi}{1-i\xi}$$

DEFINE :

$$F_n(\omega) = \frac{n}{2} \tan\left\{\frac{1}{in} \ln \omega\right\}$$

$$\hat{D}_n \xrightarrow{F_n} \text{U.H.P.}$$

$$\langle \Xi | \phi \rangle = \lim_{n\to\infty} \langle F_n \circ h \circ \phi(0) \rangle_{UHP}$$

$$= \langle f \circ \phi(0) \rangle_{UHP}$$

$$f(\xi) = \lim_{n\to\infty} F_n \circ h(\xi) = \tan^{-1}\xi$$

FROM THIS DESCRIPTION WE CAN FIND THE EXPLICIT OSCILLATOR REPRESENTATION OF $|\Xi\rangle$.

$$|\Xi_m\rangle = \exp\left(-\frac{1}{2}\hat{S}_{mn}\, a_m^{\mu\dagger} a_n^{\nu\dagger}\eta_{\mu\nu}\right)|0\rangle$$

$$\hat{S}_{mn} = -\frac{1}{\sqrt{mn}}\oint_0 \frac{dw}{2\pi i}\oint_0 \frac{dz}{2\pi i}\,\frac{1}{z^n w^m}\,\frac{f'(z)\,f'(w)}{\left(f(z)-f(w)\right)^2}$$

$$f(\xi) = \tan^{-1}\xi$$

LeClair, Peskin, Preitschopf

COMPARE THIS WITH

$$|\Psi_m\rangle \sim \exp\left(-\frac{1}{2}S_{mn}\, a_m^{\mu\dagger} a_n^{\nu\dagger}\eta_{\mu\nu}\right)|0\rangle$$

CAN BE COMPUTED NUMERICALLY

S_{mn} & $\hat{S}_{mn}$ AGREE TO GOOD ACCURACY

$$\Rightarrow |\Psi_m\rangle \simeq |\Xi_m\rangle$$