

# Orbital and Magnetic Physics in Vanadium Spinel

Yukitoshi Motome  
(Univ. of Tokyo)

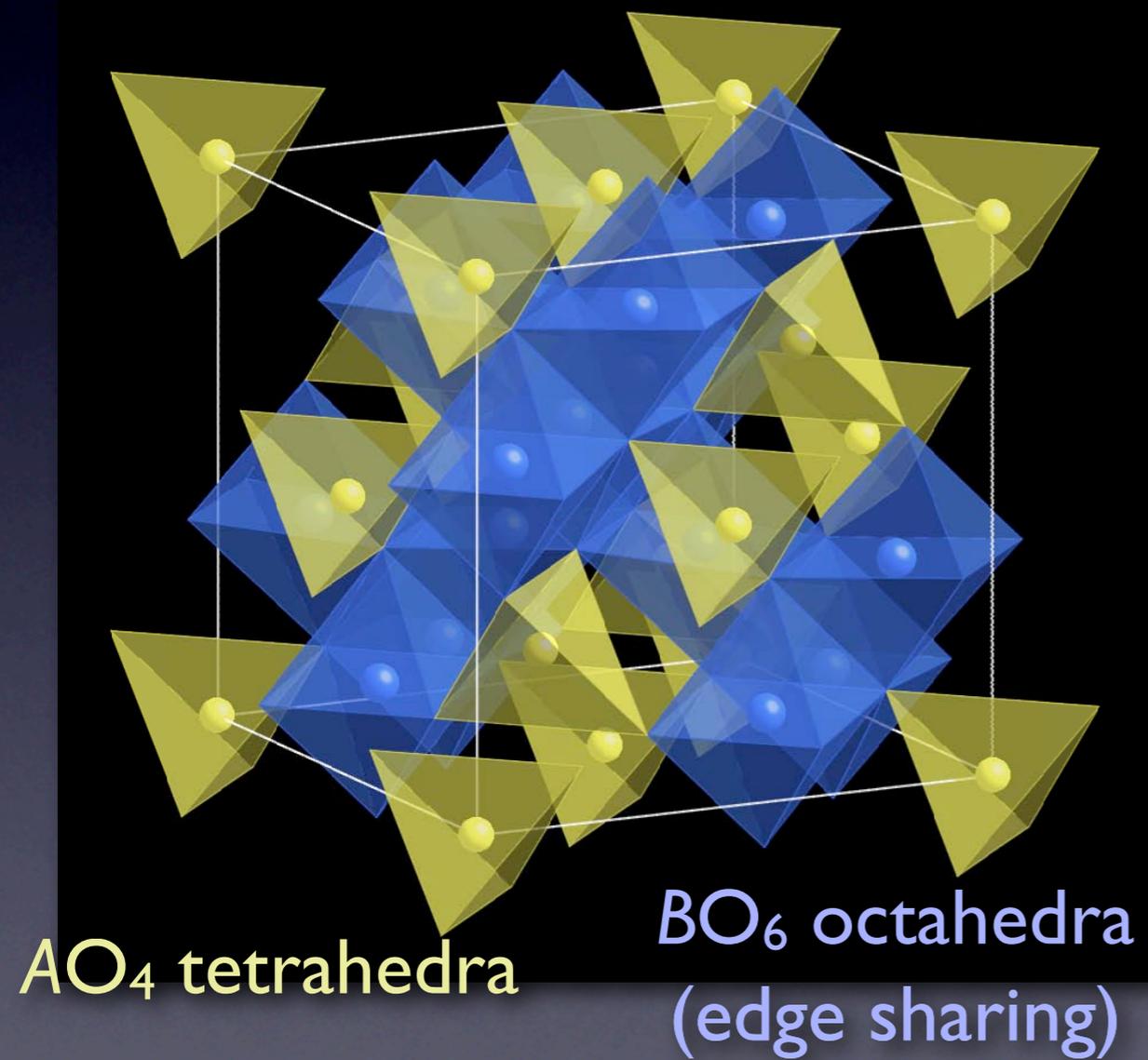
KITP Program  
*Moments and Multiplets in Mott Materials*  
Sep. 25, 2007

# Outline

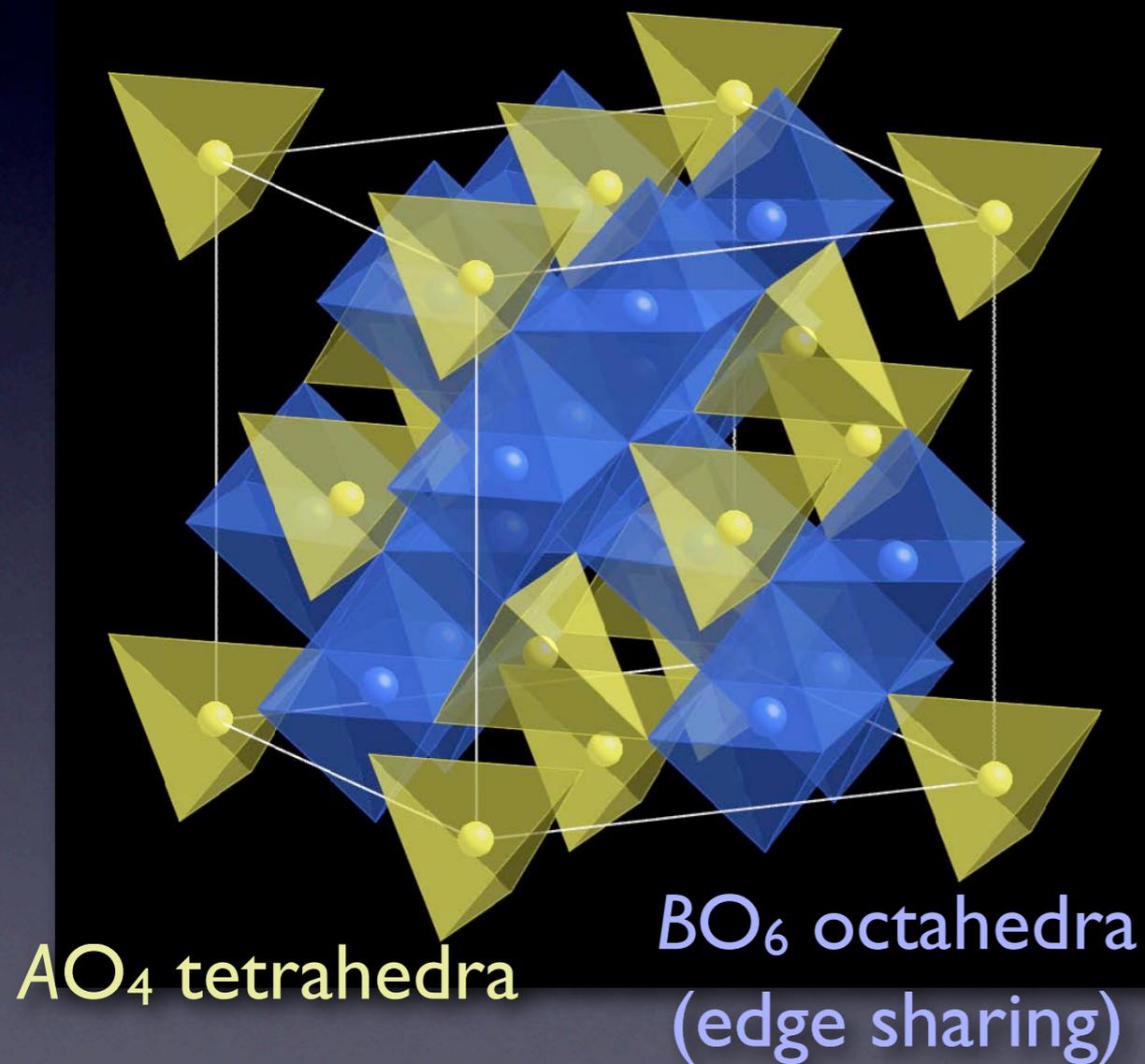
# Outline

- introduction to spinels and  $t_{2g}$  orbital physics
- controversy on orbital ordering in  $\text{ZnV}_2\text{O}_4$ 
  - different models for spin/orbital order in  $\text{ZnV}_2\text{O}_4$ : relative importance of Kugel-Khomskii superexchange, Jahn-Teller and relativistic spin-orbit couplings
  - symmetry analysis: lesson from experiments in  $\text{MnV}_2\text{O}_4$
- self-organized 7-site cluster (heptamer) in  $\text{AlV}_2\text{O}_4$ 
  - heptamer scenario: 'molecule' of bonding states with anisotropic  $t_{2g}$  orbitals
  - implication to heavy-fermion compound  $\text{LiV}_2\text{O}_4$

# Lattice Structure of Spinel $AB_2O_4$

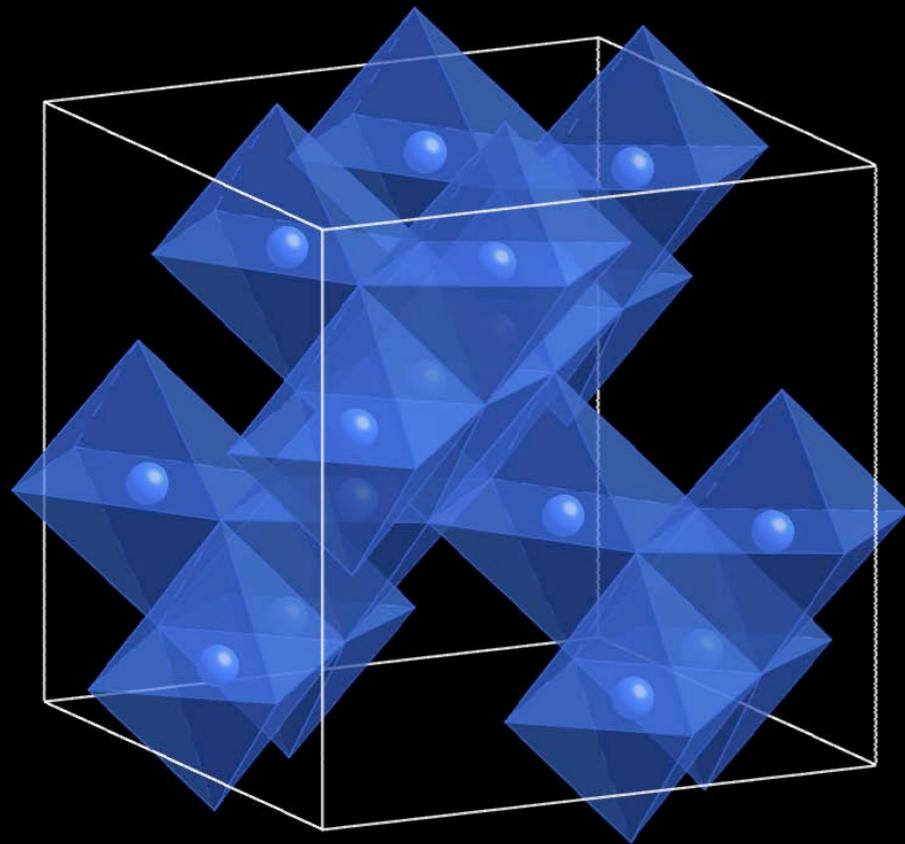


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- *B* spinels: A-site cations are nonmagnetic

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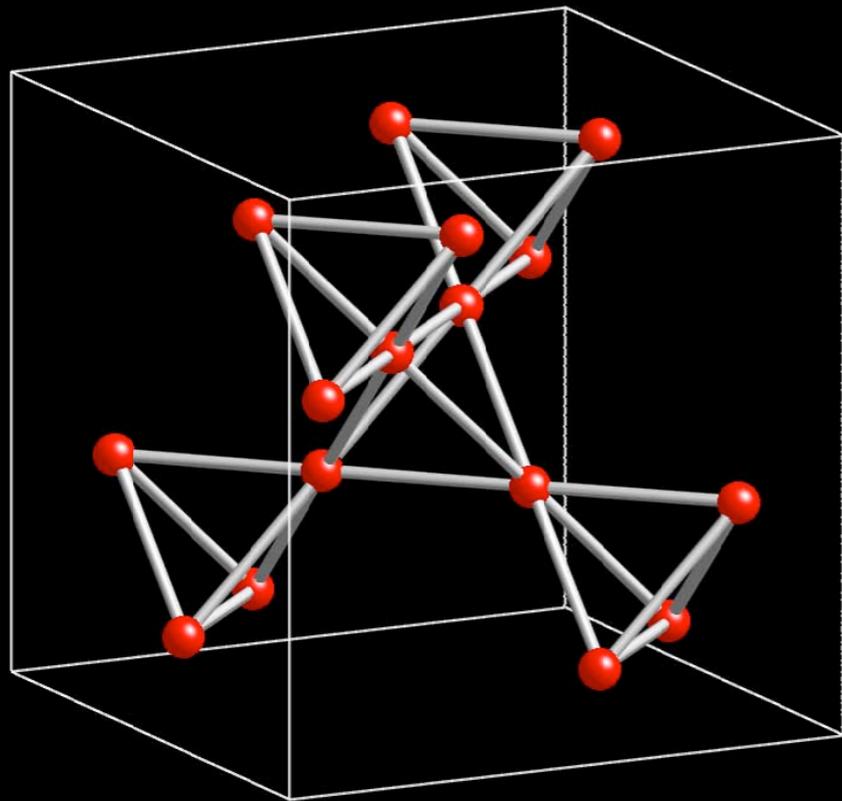


$BO_6$  octahedra  
(edge sharing)

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- 3D network of **edge-sharing**  $BO_6$  octahedra

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pyrochlore lattice



- $B$  spinels: A-site cations are nonmagnetic
- 3D network of **edge-sharing**  $BO_6$  octahedra
- 3D network of corner-sharing  $B_4$  tetrahedra → pyrochlore lattice: **strong geometrical frustration**

# B Spinelns with $t_{2g}$ Electrons

$d^1$ MgTi <sub>2</sub> O <sub>4</sub>	$d^2$ AV <sub>2</sub> O <sub>4</sub> (A=Zn,Mg)	$d^3$ ACr <sub>2</sub> O <sub>4</sub> (A=Cd,Hg,Zn)
<ul style="list-style-type: none"> <li>• metal-insulator transition</li> <li>• spin-singlet ground state</li> <li>• helical dimerization</li> <li>• orbital-Peierls scenario</li> </ul>	<ul style="list-style-type: none"> <li>• two successive transitions</li> <li>• complicated AF ordering</li> <li>• dimensionality reduction</li> <li>• competition between spin and orbital degrees of freedom</li> </ul>	<ul style="list-style-type: none"> <li>• single transition</li> <li>• half-magnetization plateau</li> <li>• spin-lattice coupling (spin Jahn-Teller mechanism)</li> <li>• self-organized 'hexamer' in high-T para phase</li> </ul>
$d^{0.5}$ LiTi <sub>2</sub> O <sub>4</sub>	$d^{1.5}$ LiV <sub>2</sub> O <sub>4</sub>	$d^{2.5}$ AlV <sub>2</sub> O <sub>4</sub>
<ul style="list-style-type: none"> <li>• superconductivity below 12.4 K (BCS mechanism)</li> </ul>	<ul style="list-style-type: none"> <li>• metallic down to 300 mK</li> <li>• absence of any transition</li> <li>• heavy-fermion behavior</li> <li>• metal-insulator transition by applying pressure</li> </ul>	<ul style="list-style-type: none"> <li>• structural transition with spin-singlet formation</li> <li>• self-organized 7-site cluster 'heptamer' ?</li> </ul>

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Three main questions related to the physics of orbital degrees of freedom came to the fore in the discussion of Wednesday (biased view --JvdB).

We know that orbitals can order and that they couple to the lattice, but the questions are:

1. is there any material in which the *quantum* character of orbital degrees of freedom become relevant?
2. are there any cases where orbital *fluctuations*, either quantum or classical are relevant?
3. does orbital ordering have interesting *textures*, *symmetries* and/or *excitations*?

Also 15 more or less detailed discussion topics came up:

1. What is the role of vibronic coupling in cooperative Jahn-Teller systems
2. The importance of relativistic spin orbit coupling in eg and t2g systems
3. Orbital and frustration: frustration due to orbital degrees of freedom --- orbitals in frustrated lattices
4. Relative importance of electron-lattice effects (Jahn Teller) versus electronic effects (superexchange).
5. Role of geometry: differences for the situation of 180 degree O-TM-O bonds, 90 degree O-TM-O bonds and edge sharing octahedra
6. Reduced dimensionality due to orbitals
7. Importance of direct d-d electronic hopping versus d-oxygen-d hopping, especially in t2g spinels
8. Orbitals in charge transfer insulators
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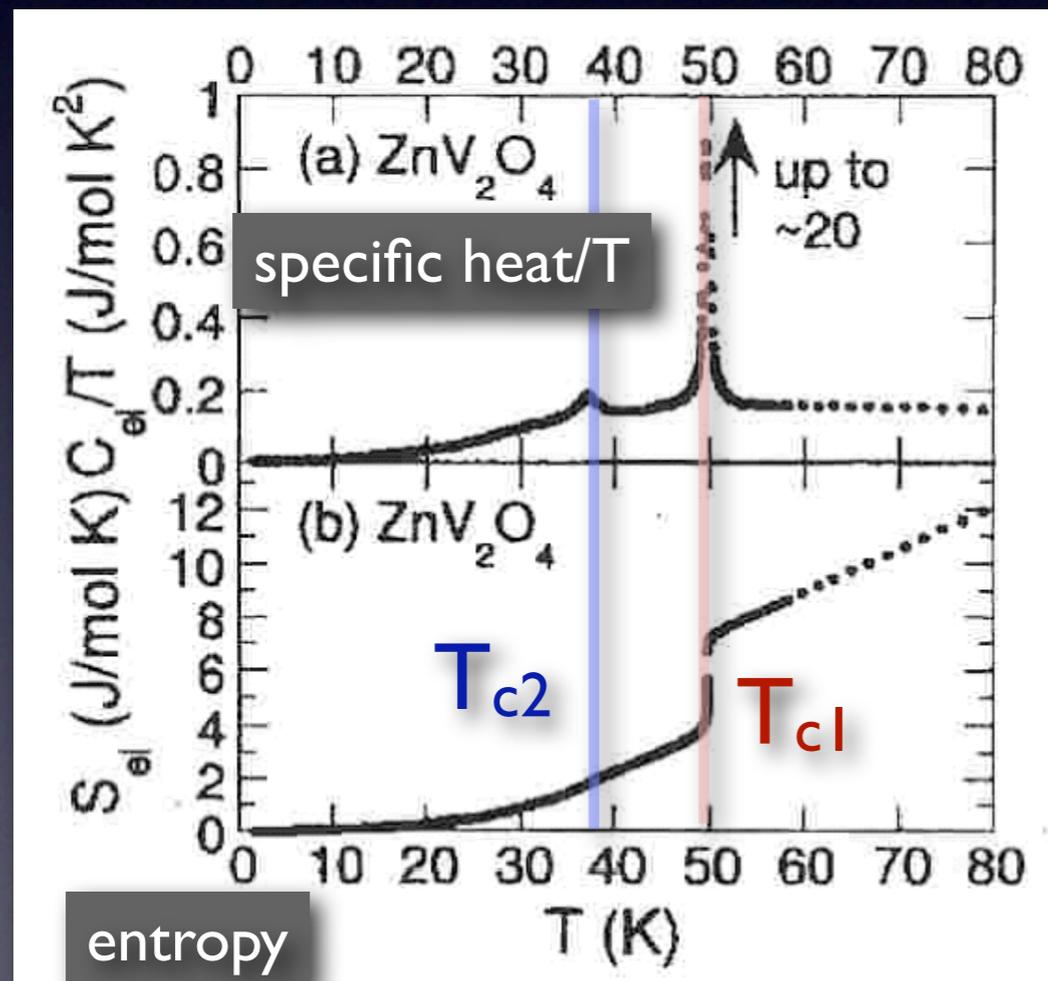
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# Two Transitions and Controversy on Orbital Ordering in $\text{ZnV}_2\text{O}_4$

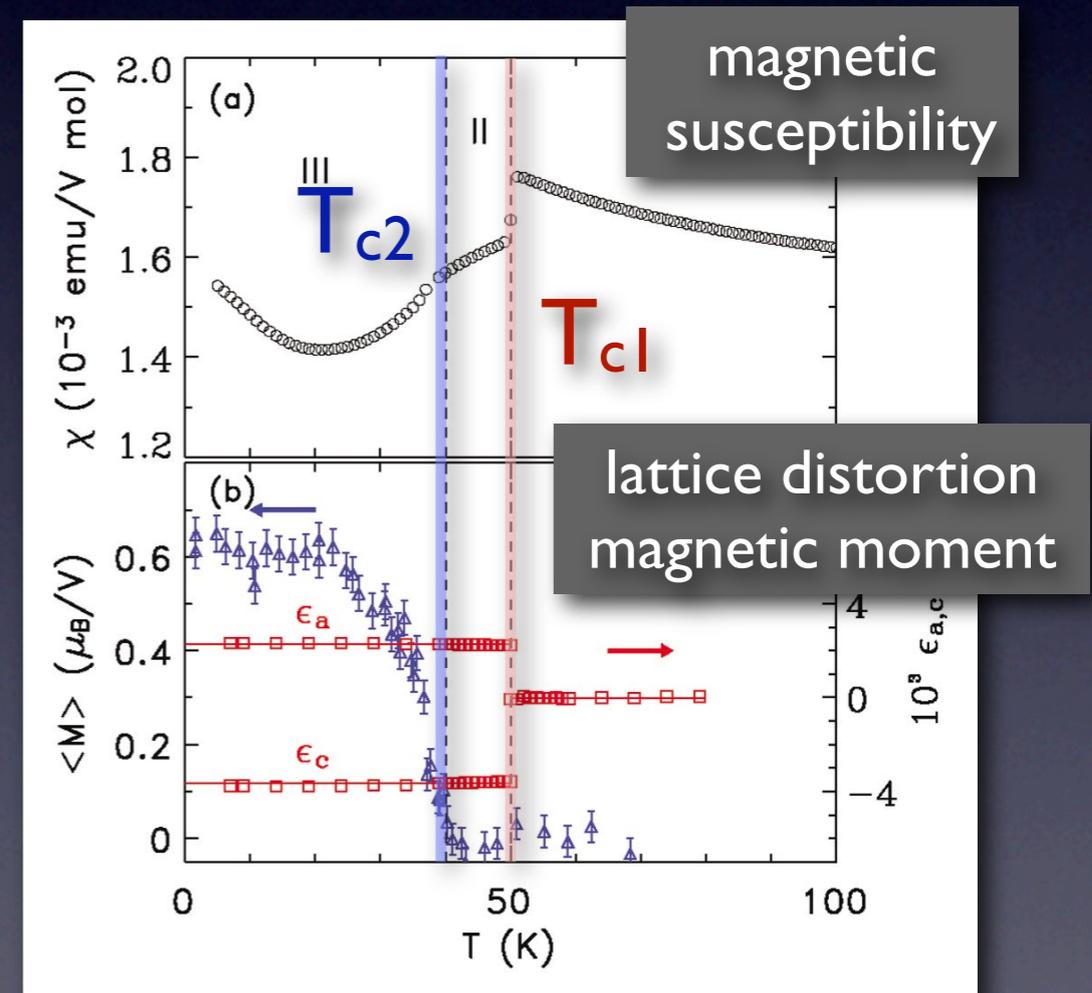
in collaboration with Hirokazu Tsunetsugu

# Two Transitions in $\text{ZnV}_2\text{O}_4$

- cubic to tetragonal transition at  $T_{c1} \sim 50\text{K}$  (1st order)
- antiferromagnetic transition at  $T_{c2} \sim 40\text{K}$  (2nd order)



Kondo et al., 2000

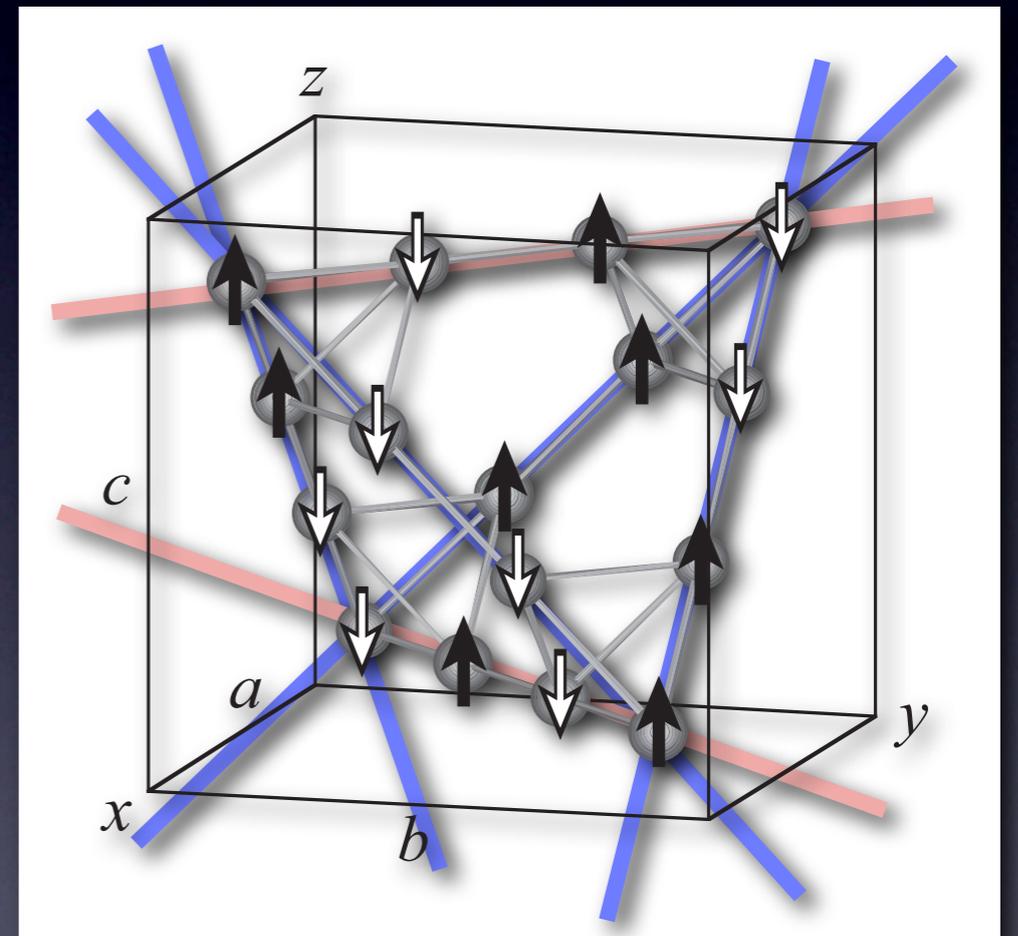


Ueda et al., 1997

Lee et al., 2004

# Lattice symmetry and Magnetic Order

- lattice symmetry:  $I4_1/amd$   
(powder sample)
- orbital order: undetermined
- spin order: antiferromagnetic  
 $\uparrow-\downarrow-\uparrow-\downarrow-\dots$  in the  $xy$  chains  
 $\uparrow-\uparrow-\downarrow-\downarrow-\dots$  in the  $yz/zx$  chains
- moment at  $T=0 \sim 0.6\mu_B$



Niziol, 1973

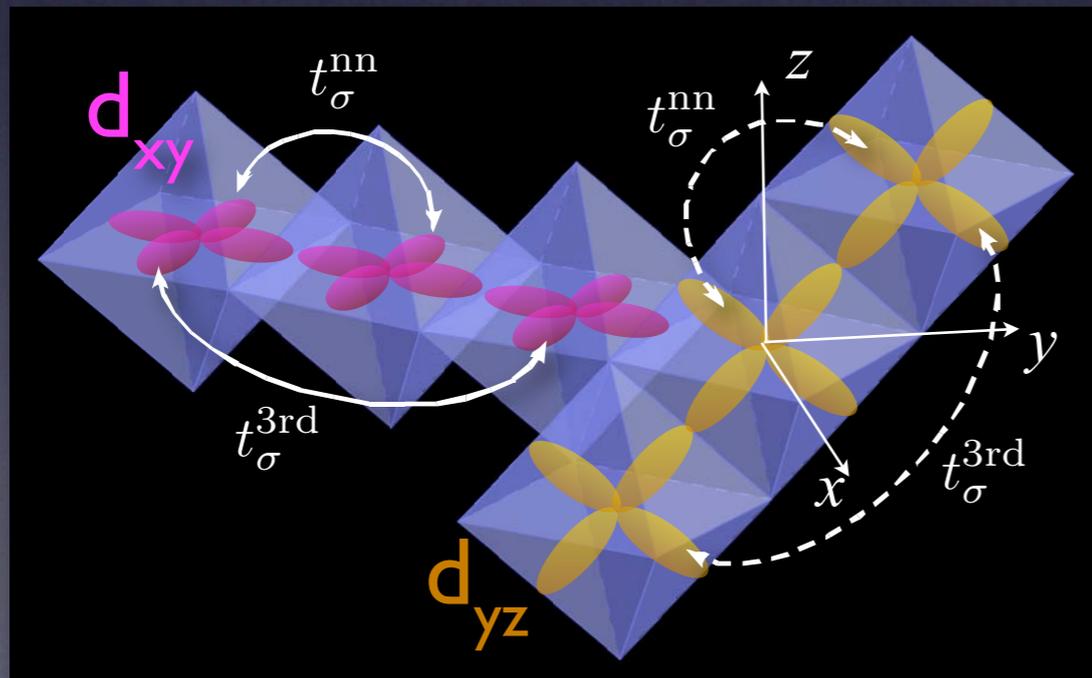
# Questions

- 📌 What is the microscopic mechanism of two transitions? Who is the main player? Kugel-Khomskii superexchanges, Jahn-Teller or relativistic spin-orbit coupling?
- 📌 How is the complex AF ordering stabilized? Why is the moment at  $T=0$  reduced so largely?
- 📌 What is the role of orbital degree of freedom? Is there orbital ordering? If yes, what type of ordering sets in?

# Model

Tsunetsugu and Motome (2003, 2004, 2005)

- Kugel-Khomskii type model derived from 3-fold multi-orbital Hubbard model + tetragonal Jahn-Teller coupling
- assumptions:  $\sigma$ -type transfer integrals only, classical phonon, neglecting spin-orbit coupling and trigonal distortion



$$t_{\sigma}^{nn} = \sim -0.32\text{eV}$$

$$t_{\sigma}^{3rd} = \sim -0.045\text{eV}$$

(Matsuno *et al.*, 1999: for LiV<sub>2</sub>O<sub>4</sub>)

# Model

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$$H_{\text{SO}}^{\text{nn}} = -J \sum_{\langle ij \rangle} [h_{\text{o-AF}}^{(ij)} + h_{\text{o-F}}^{(ij)}] \quad \text{: nearest neighbor term}$$
$$H_{\text{SO}}^{\text{3rd}} = -J_3 \sum_{\langle\langle ij \rangle\rangle} [h_{\text{o-AF}}^{(ij)} + h_{\text{o-F}}^{(ij)}] \quad \text{: 3rd neighbor term}$$
$$J = (t_{\sigma}^{\text{nn}})^2 / U \quad A = (1 - \eta) / (1 - 3\eta)$$
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$$\bar{n}_{i\alpha} = 1 - n_{i\alpha} \quad C = (1 + \eta) / (1 + 2\eta)$$
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$$h_{\text{o-AF}}^{(ij)} = (A + B \vec{S}_i \cdot \vec{S}_j) (n_{i\alpha(ij)} \bar{n}_{j\alpha(ij)} + \bar{n}_{i\alpha(ij)} n_{j\alpha(ij)})$$

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spin part: Heisenberg / orbital part: 3-state Potts

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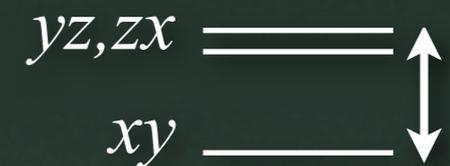
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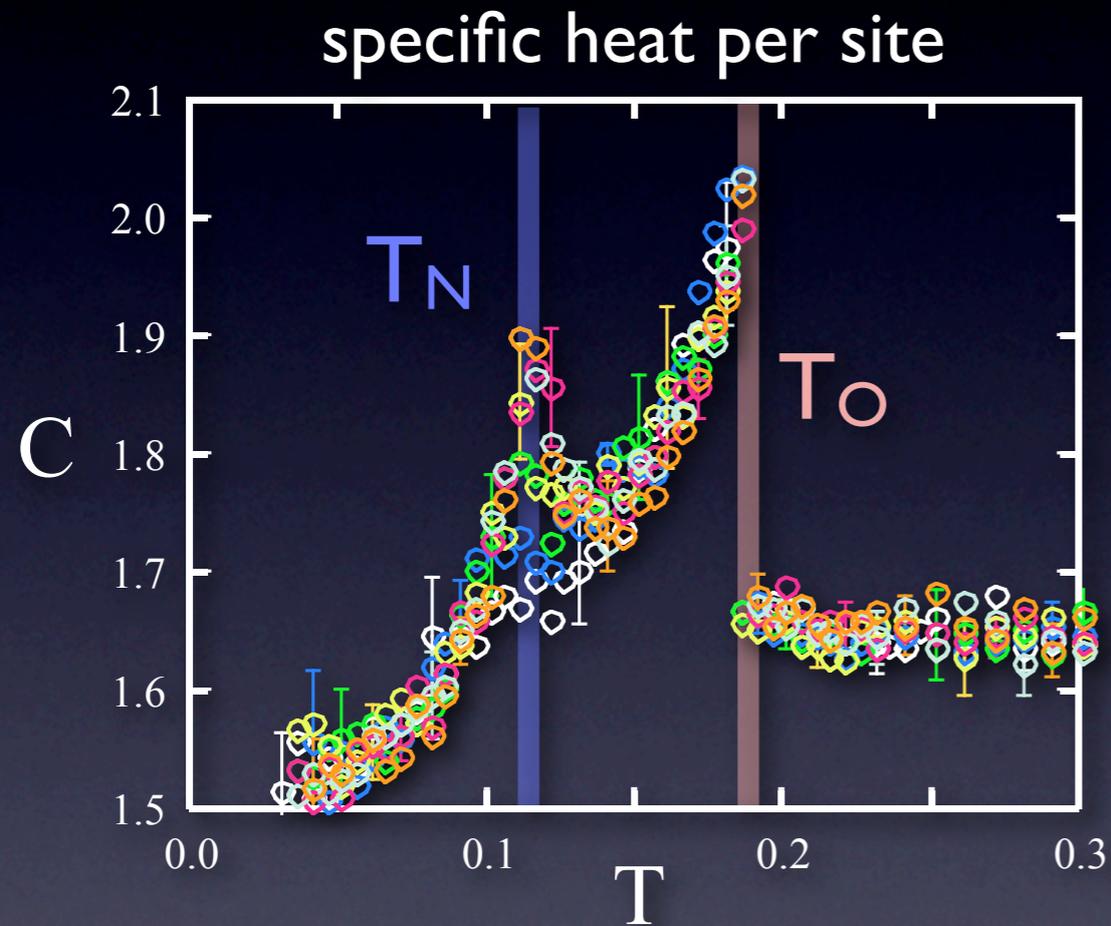
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+ tetragonal Jahn-Teller coupling

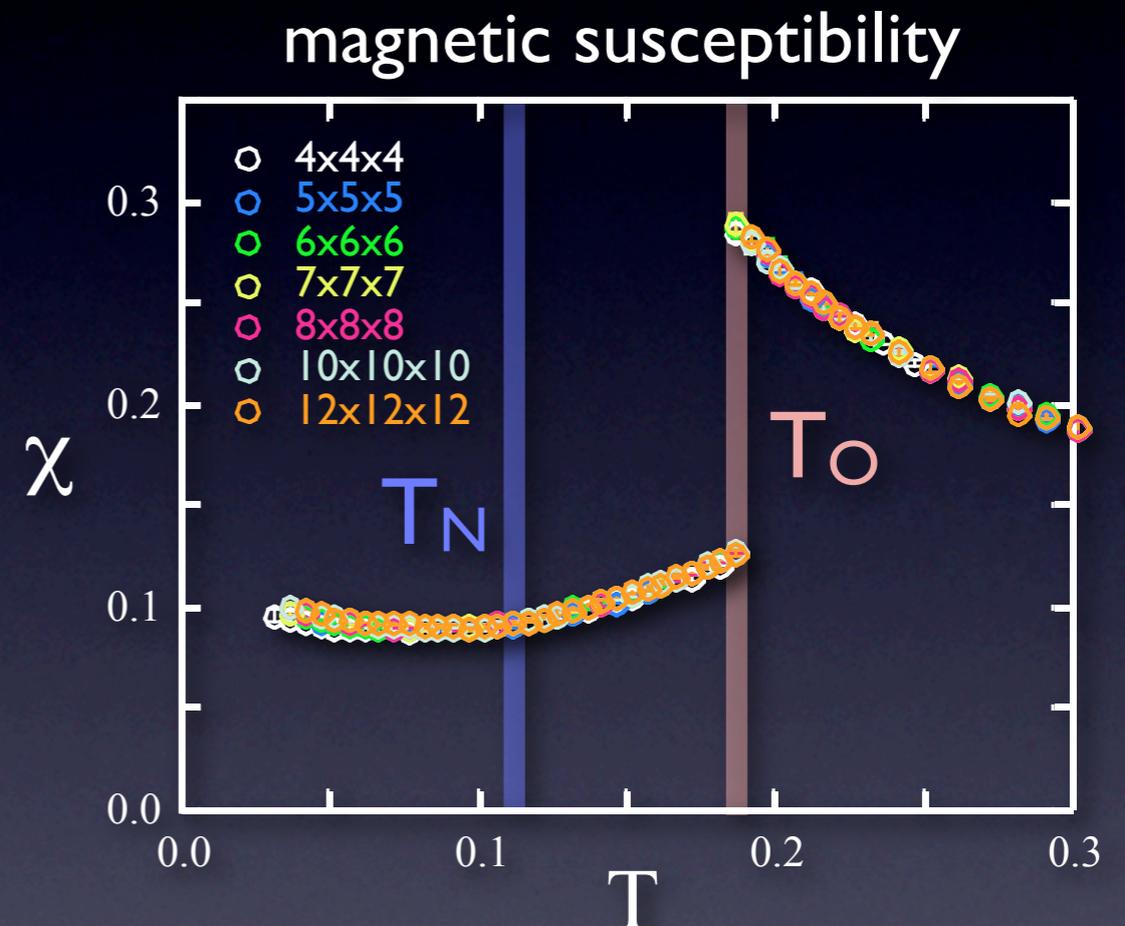
$$H_{\text{JT}} = g \sum_i Q_i (n_{i,yz} + n_{i,zx} - 2n_{i,xy}) + \sum_i Q_i^2 / 2 - \lambda \sum_{\langle ij \rangle} Q_i Q_j$$



# Monte Carlo Results

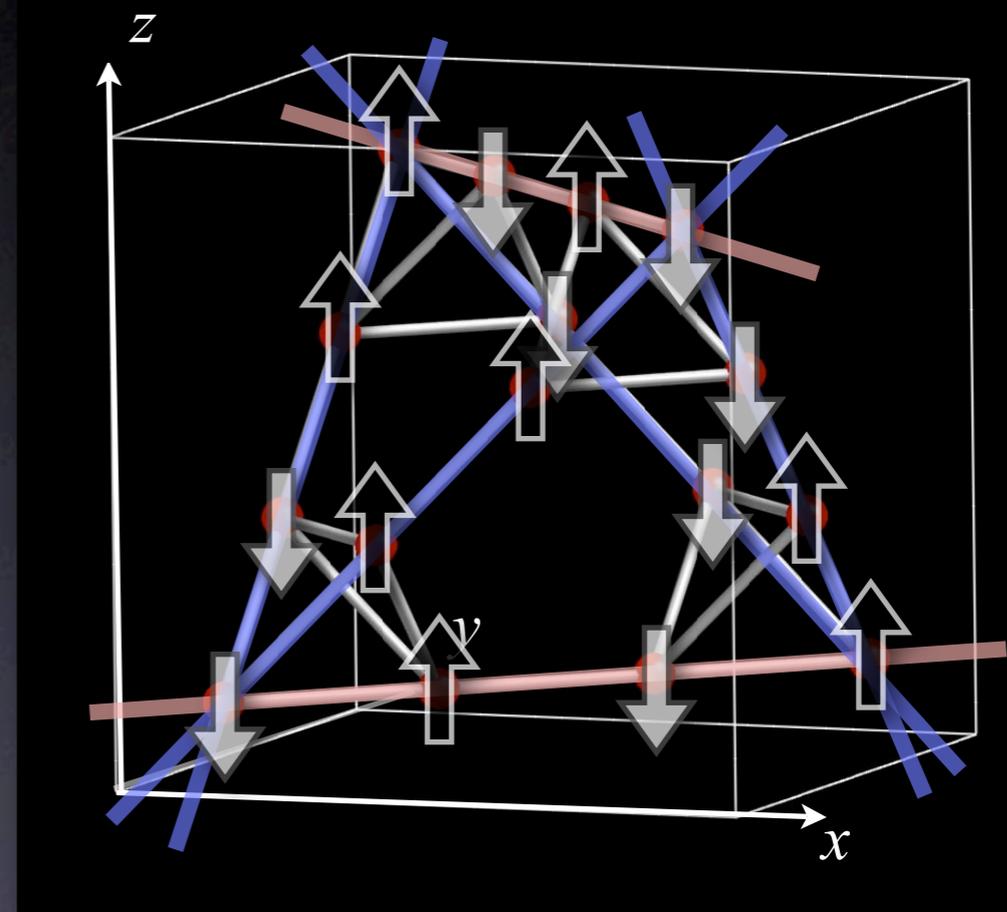
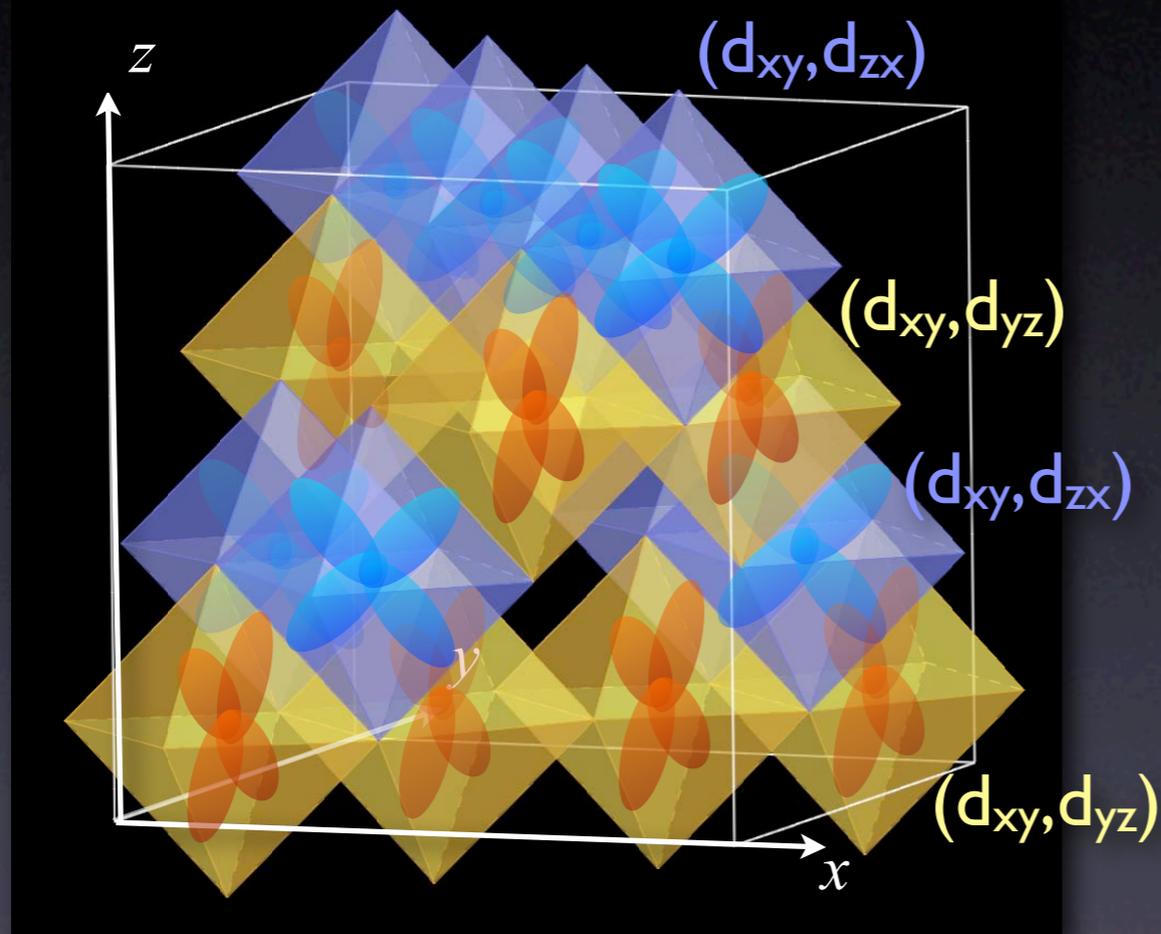


- 1st order at  $T=T_O$ ,  
2nd order at  $T=T_N$
- consistent estimates  
of entropy changes



- sudden drop at  $T=T_O$
- tiny change at  $T=T_N$

# Orbital and Spin Structure



- **orbital:** alternative stacking of  $(d_{xy}, d_{zx})$  and  $(d_{xy}, d_{yz})$  states
- **spin:**  $\uparrow-\downarrow-\uparrow-\downarrow-$  in the  $xy$  chains and  $\uparrow-\uparrow-\downarrow-\downarrow-$  in the  $yz/zx$  chains

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spin correlations hardly develop  
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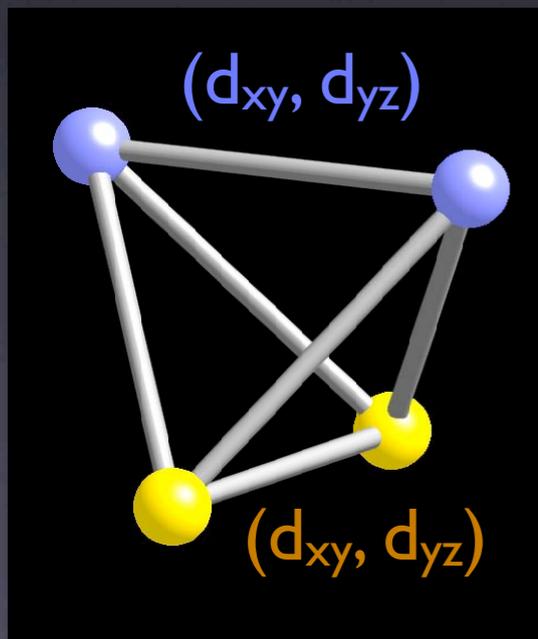
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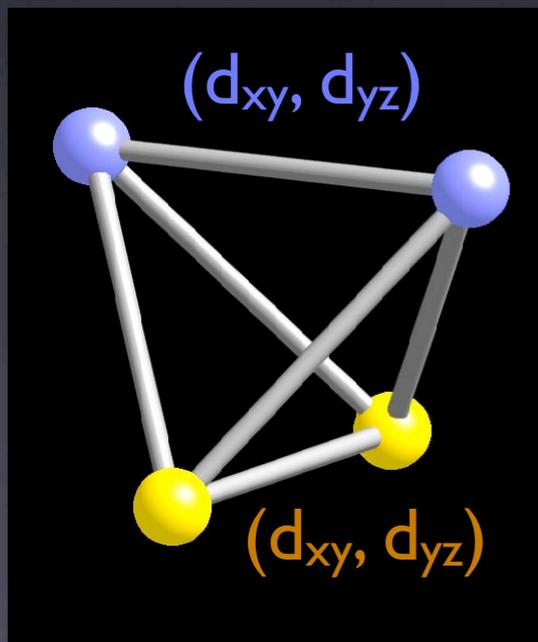
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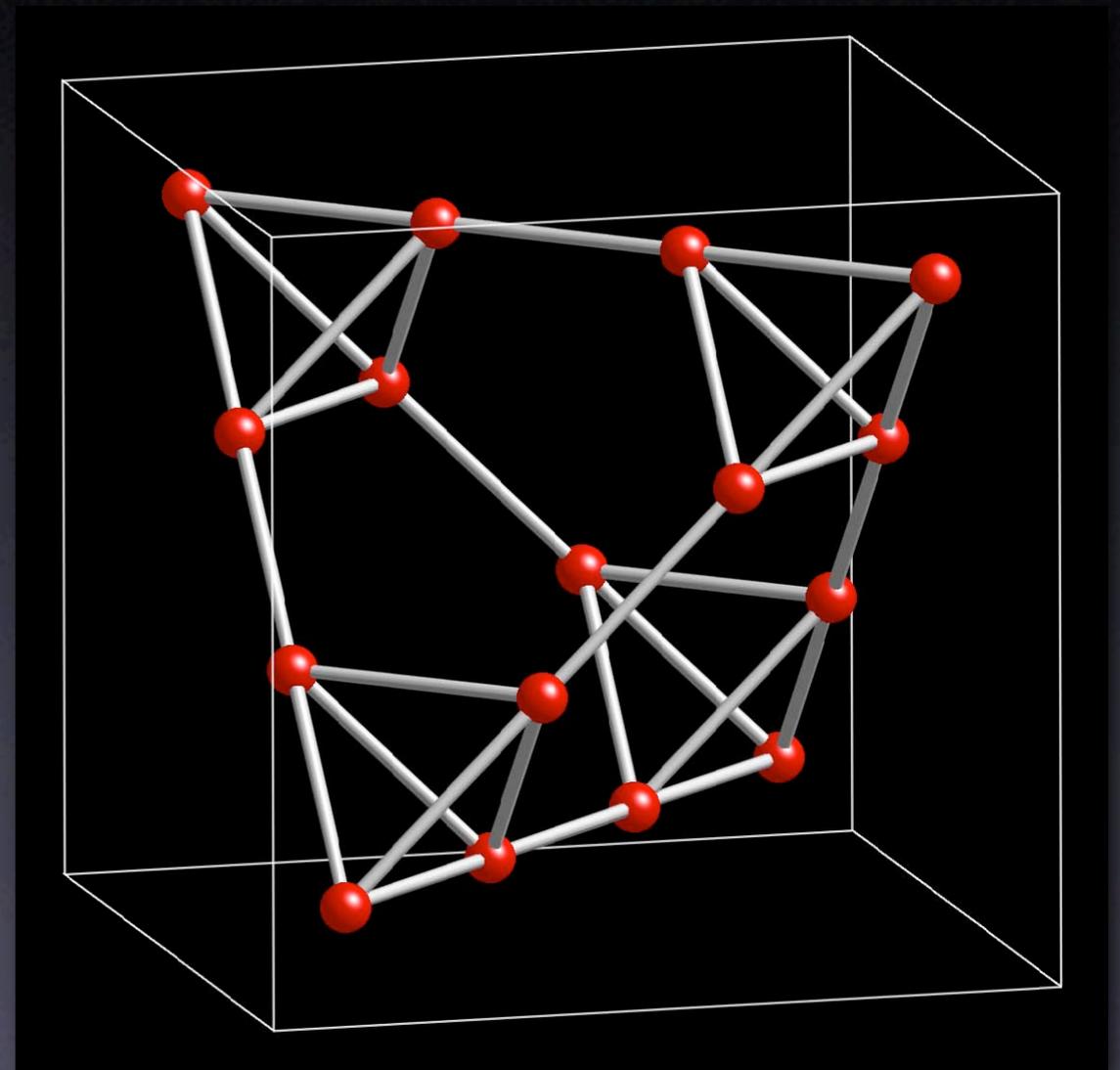
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tetragonal Jahn-Teller distortion assists to stabilize this orbital configuration

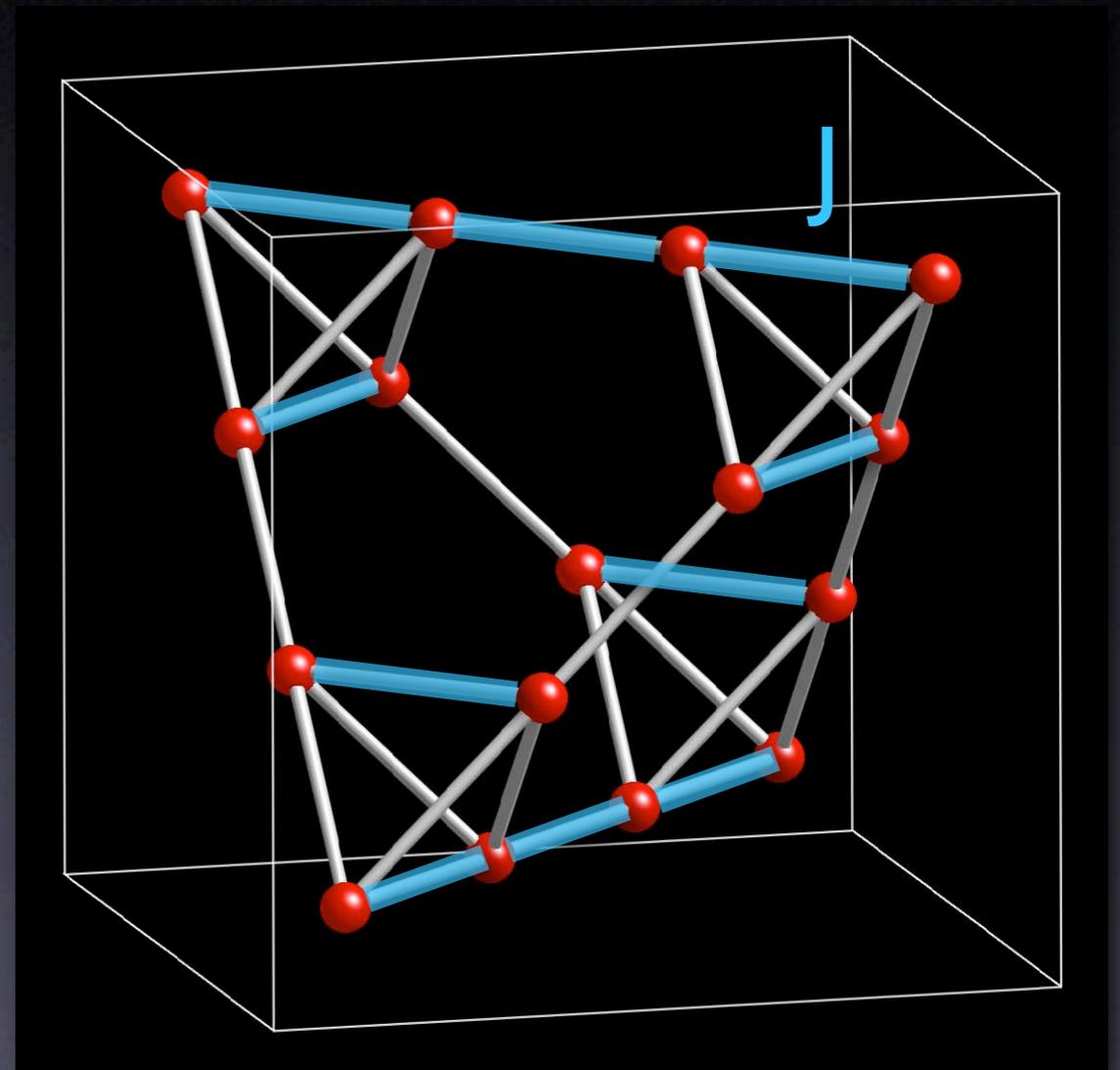
$$\begin{array}{l} yz, zx \text{ } \text{====} \\ xy \text{ } \text{====} \end{array} \Delta_{\text{JT}}$$

# Effective Spin Exchanges under the Orbital Order



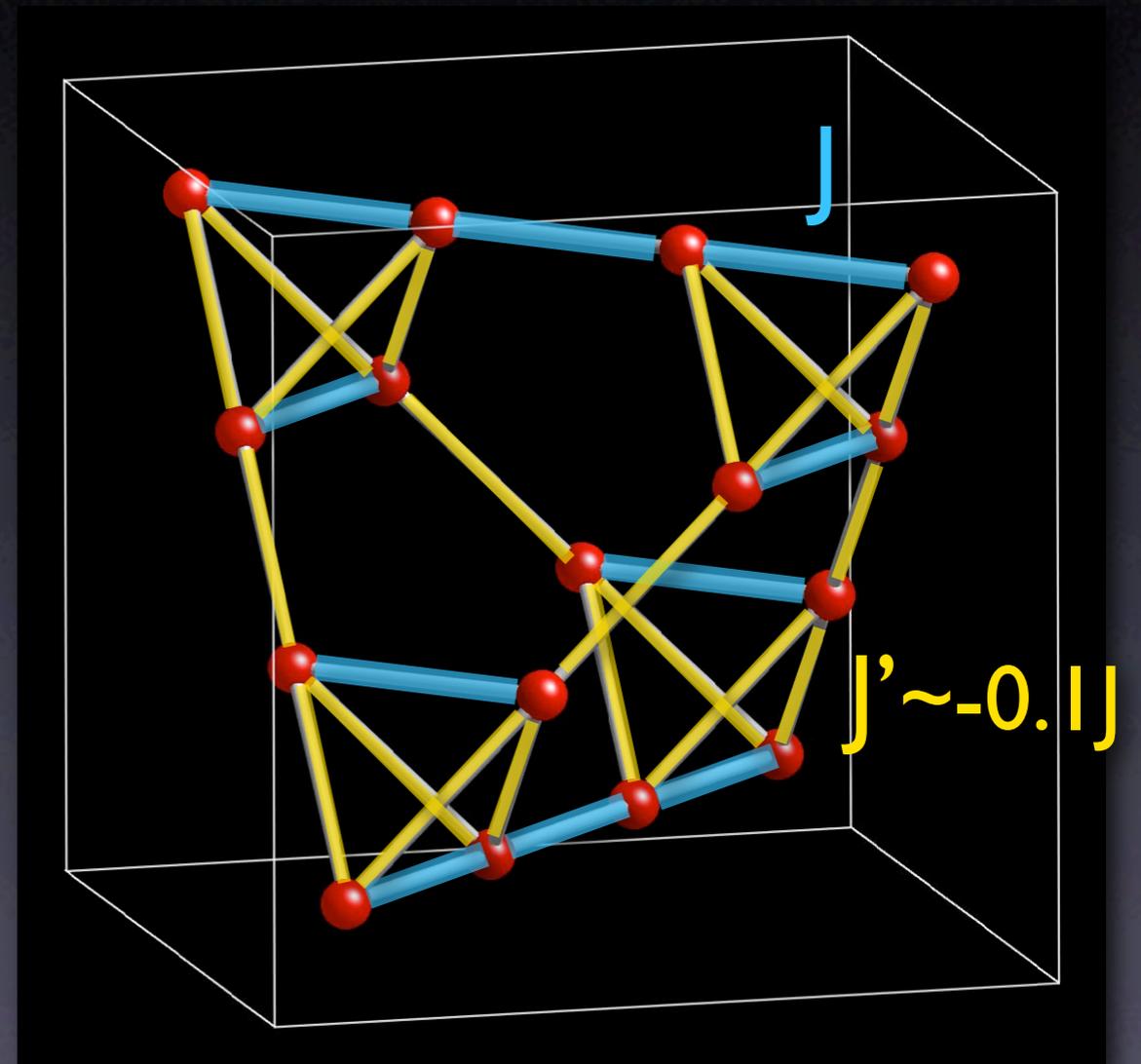
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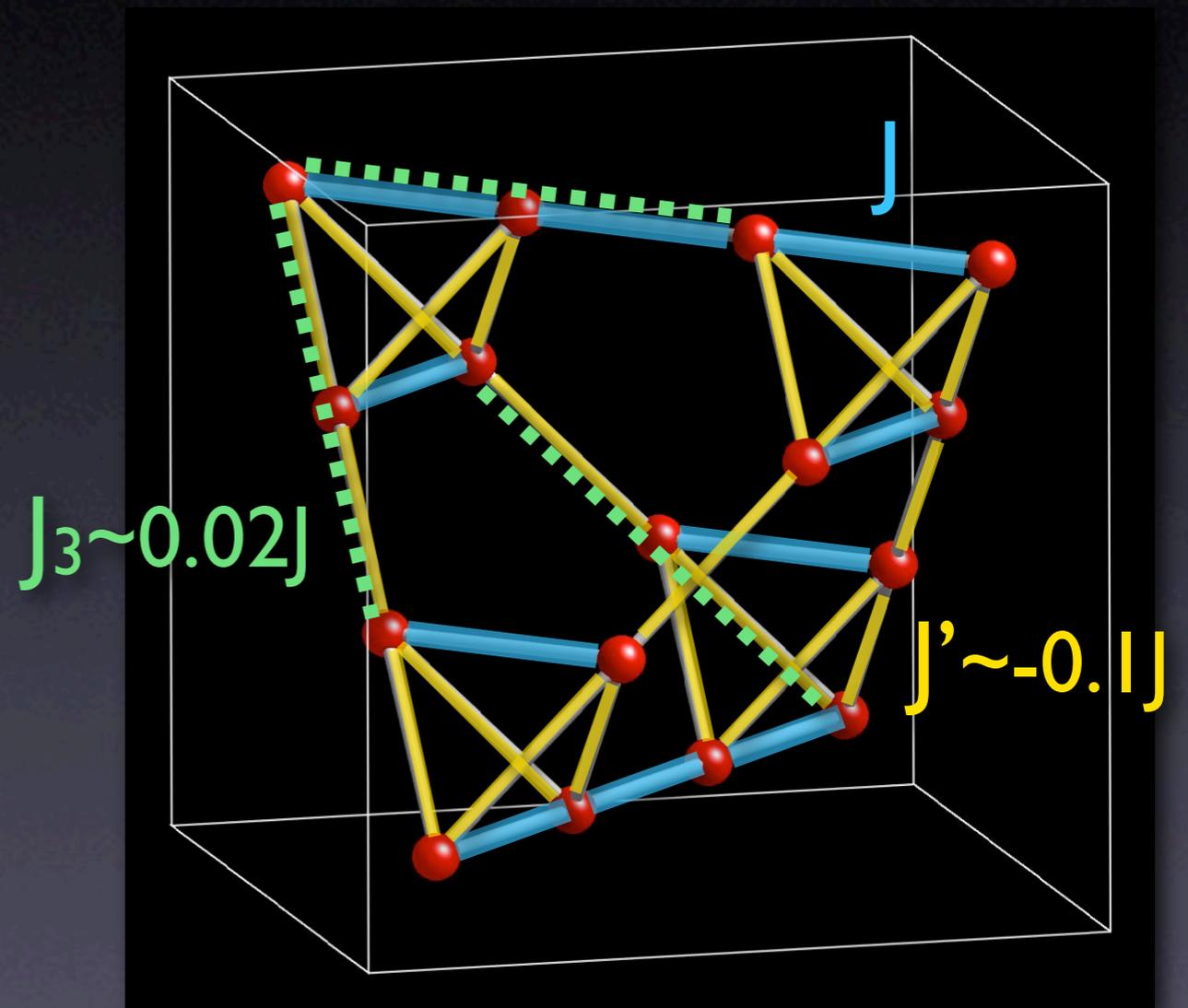
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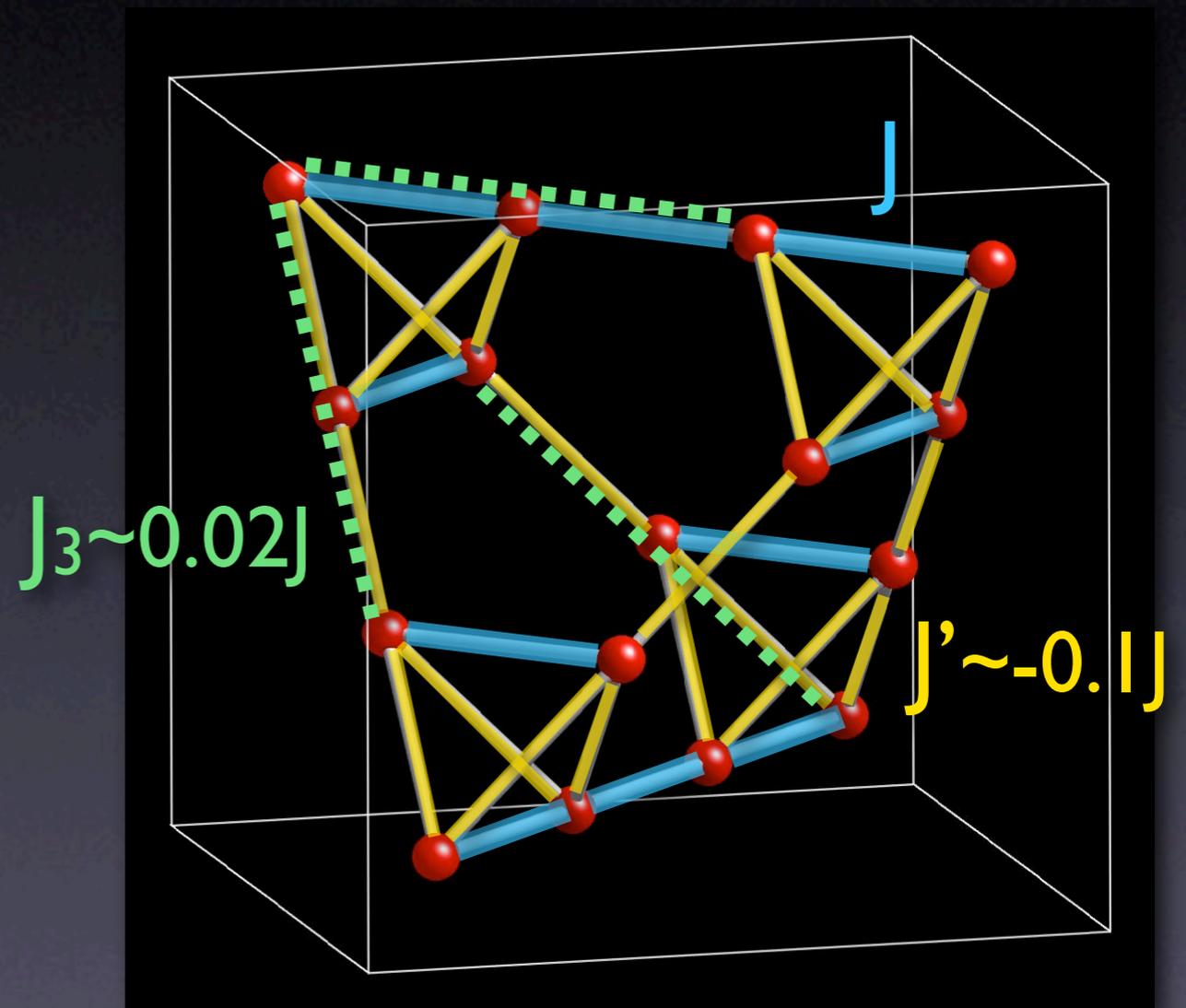
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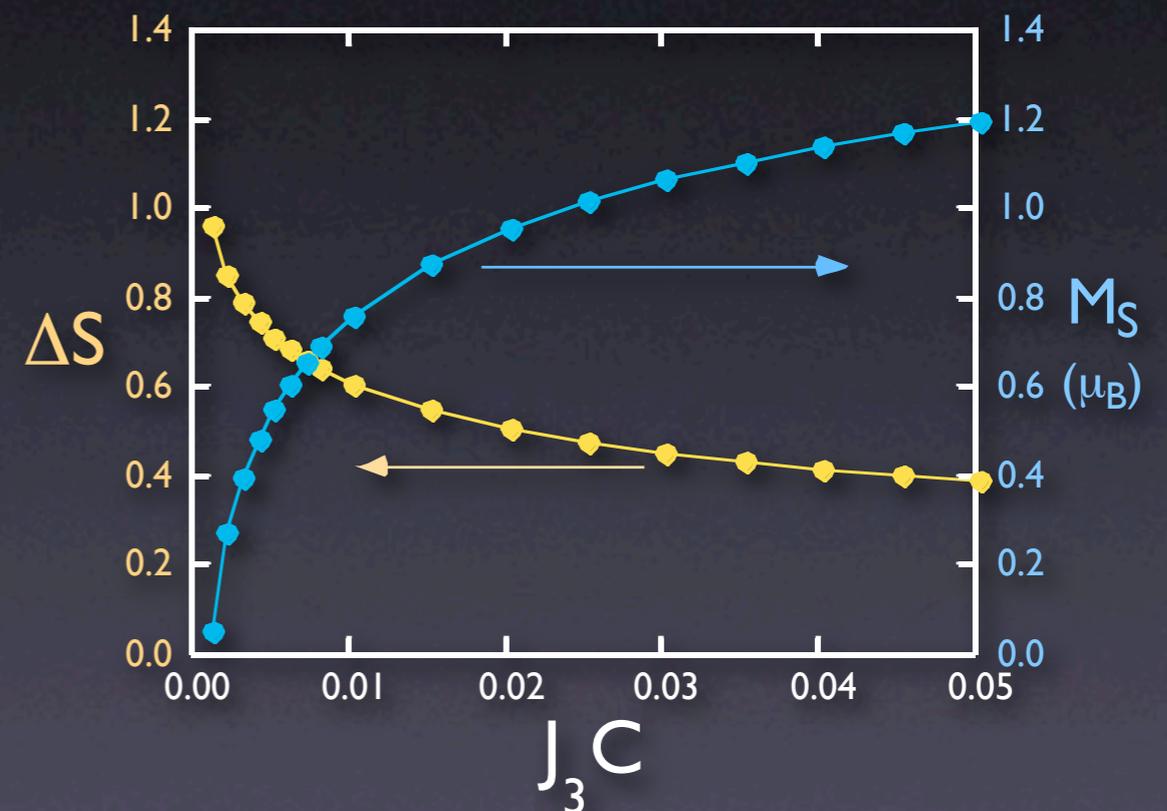
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weakly-coupled 1D spin chains (dimensionality reduction)

# Quasi-1D Quantum Fluctuation: Large Reduction of AF Moment

- linear spin-wave analysis for the spin and orbital ordered ground state
- moment reduction  $\Delta S$  diverges logarithmically at  $J_3=0$  due to the zero modes
- $\Delta S$  is large in the small  $J_3$  region:  
 $M_S \sim 1 \mu_B$  at  $J_3 \sim 0.02$   
consistent with the experimental result  $\sim 0.6 \mu_B$  (Lee *et al.*, 2004)



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- Kugel-Khomskii spin-orbital exchange + tetragonal Jahn-Teller
- classical Monte Carlo simulation and mean-field type analysis
- linear spin-wave analysis of effective spin model

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- Kugel-Khomskii spin-orbital exchange + tetragonal Jahn-Teller
- classical Monte Carlo simulation and mean-field type analysis
- linear spin-wave analysis of effective spin model
- ☑ two transitions with reasonable estimates of transition temperatures as well as entropy changes
- ☑ T-dep of magnetic susceptibility consistent with experiment
- ☑ magnetic order consistent with the neutron scattering result
- ☑ reduced magnetic moment at  $T=0$
- ☑ A-type antiferro orbital order with tetragonal distortion

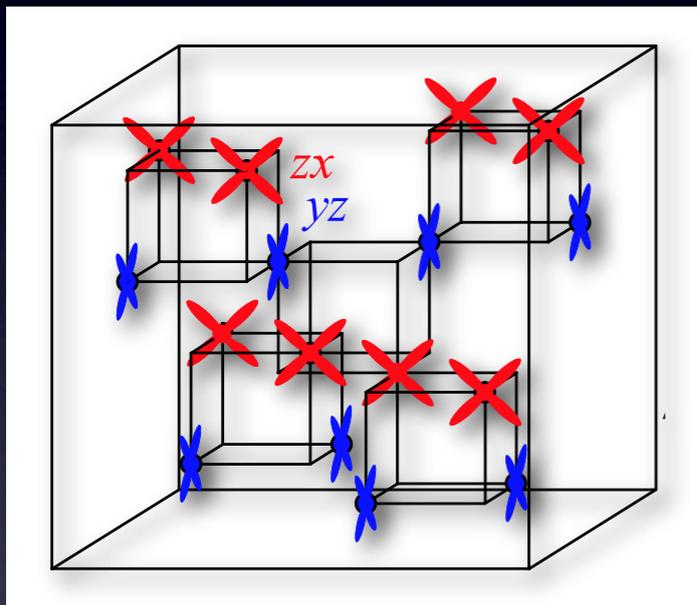
# Three Different Models

In all models,  $xy$  orbital is singly occupied at all the sites (not shown in the figures)

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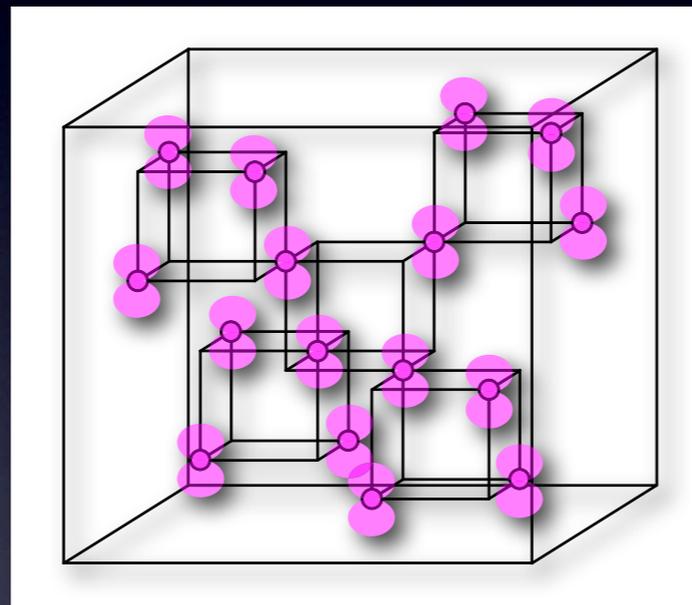
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Tsunetsugu-Motome, 2003



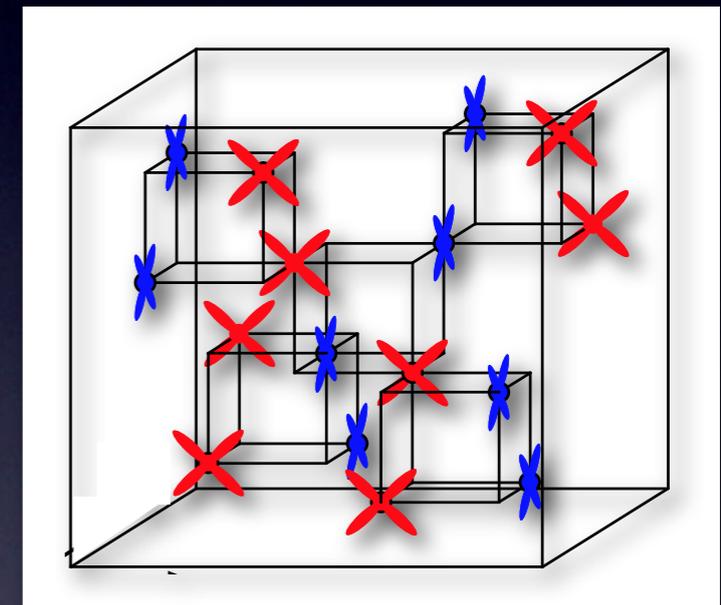
- A-type orbital order
- $I4_1/a$
- spin-orbital superexchanges

Tchernyshyov, 2004



- uniform orbital order
- $I4_1/amd$
- relativistic spin-orbit coupling

Khomskii-Mizokawa, 2005

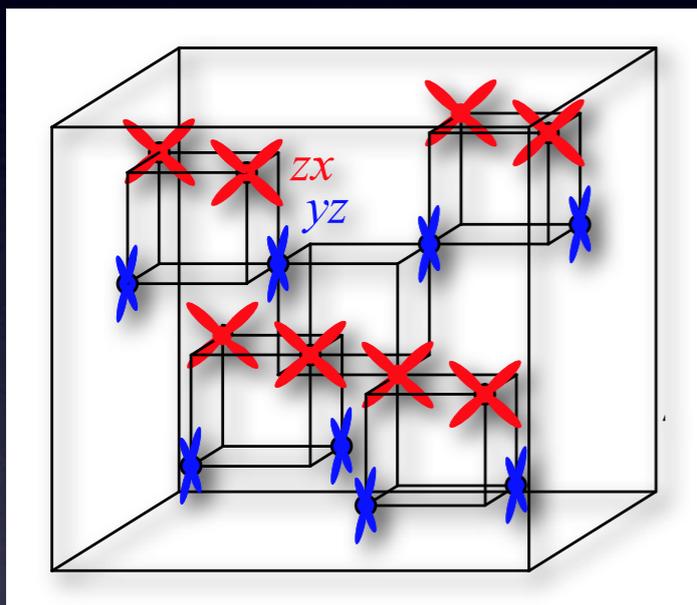


- orbitally-driven Peierls order
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- approach from itinerant picture (band Jahn-Teller)

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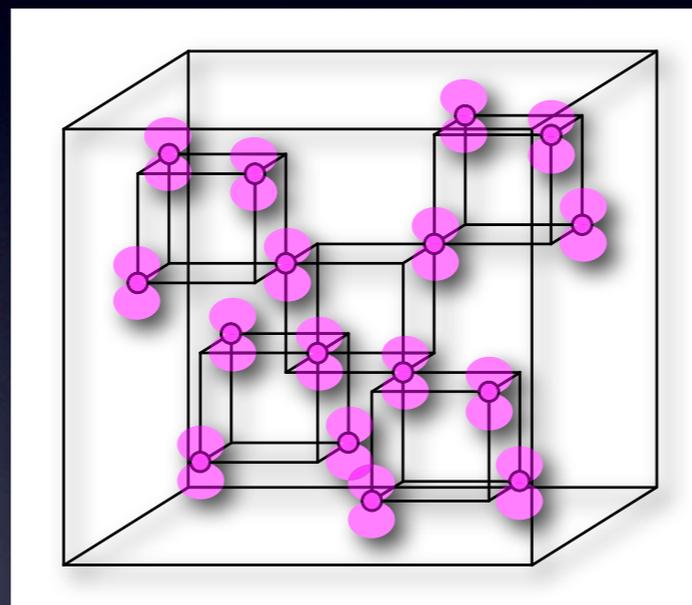
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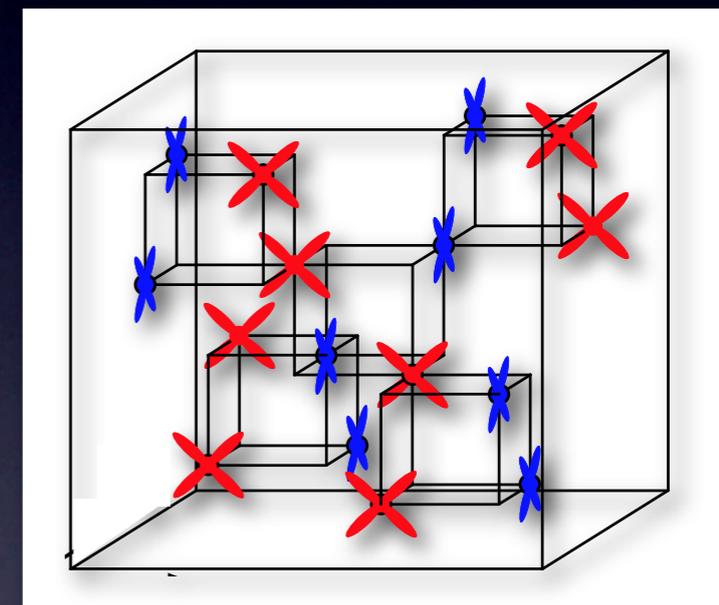
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- orbitally-driven Peierls order
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mean-field (Di Matteo *et al.*)  
LSDA+U+SO (T. Maitra and R. Valenti)

Issue...

# Issue...

-  role of relativistic spin-orbit interaction
  - orbital ordering at  $T=0$ : mean-field analysis and first-principle calculation suggest the relevant role
  - thermodynamics: single or two transitions? In general, systems with dominant spin-orbit coupling shows a single transition with concomitant ordering of spin and orbital.
  - reduced AF moment: due to dimensionality reduction and/or L-S coupling?

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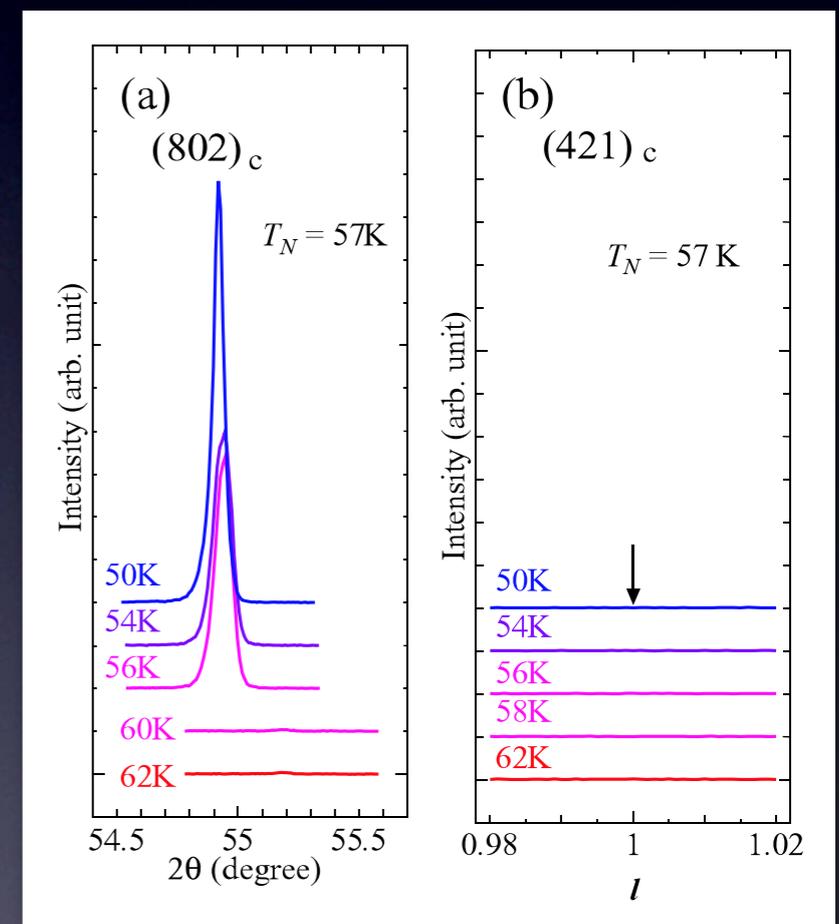
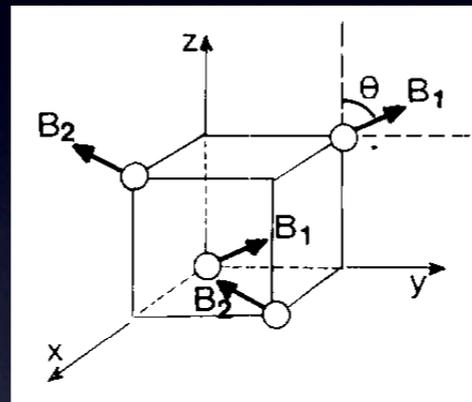
- 📌 role of relativistic spin-orbit interaction
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  - reduced AF moment: due to dimensionality reduction and/or L-S coupling?
- Remark: X-ray diffraction has been done only for powder samples...

# Lesson from Related Spinel

## MnV<sub>2</sub>O<sub>4</sub>

- Mn<sup>2+</sup> = (3d)<sup>5</sup>, V<sup>3+</sup> = (3d)<sup>2</sup>
- single transition at 57K
  - cubic → tetragonal
  - non-collinear ferri
- low-T phase: *I*4<sub>1</sub>/*a* (*large single crystal*)
  - diamond-glide symmetry is broken, but face-center symmetry is hold
  - ▶ peak intensity is ~10<sup>-4</sup> times smaller compared to the fundamental peaks, difficult to observe in powder samples

Plumier and Sougi, 1987



Suzuki et al., 2007

# Other Issues...

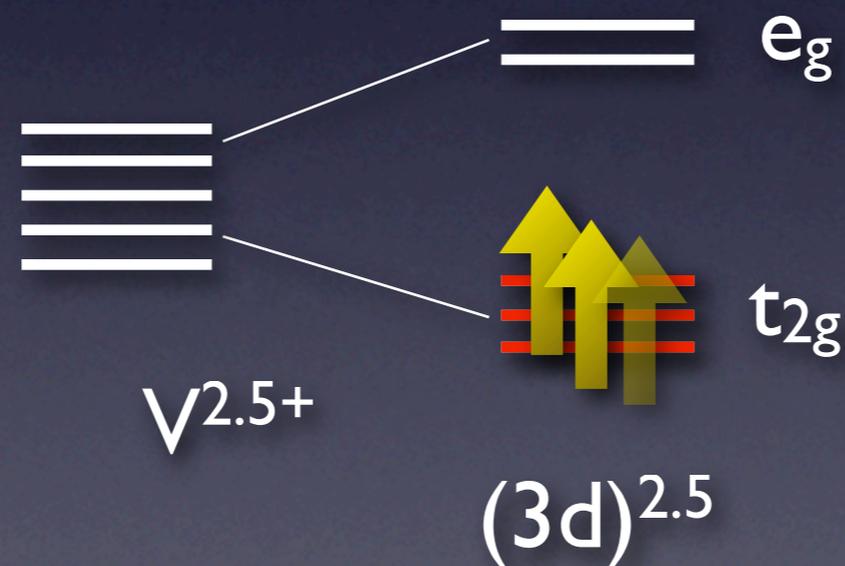
- 📌 role of trigonal distortion
  - quantitative difference in Cd compound
- 📌 d-d direct vs d-p-d (d-p-p-d) indirect transfers
- 📌 orbital and spin ordering in  $\text{MnV}_2\text{O}_4$
- 📌 single crystal of  $\text{ZnV}_2\text{O}_4$  !

# Self-organized 7-site Cluster (heptamer) in $\text{AlV}_2\text{O}_4$

in collaboration with Keisuke Matsuda and Nobuo Furukawa

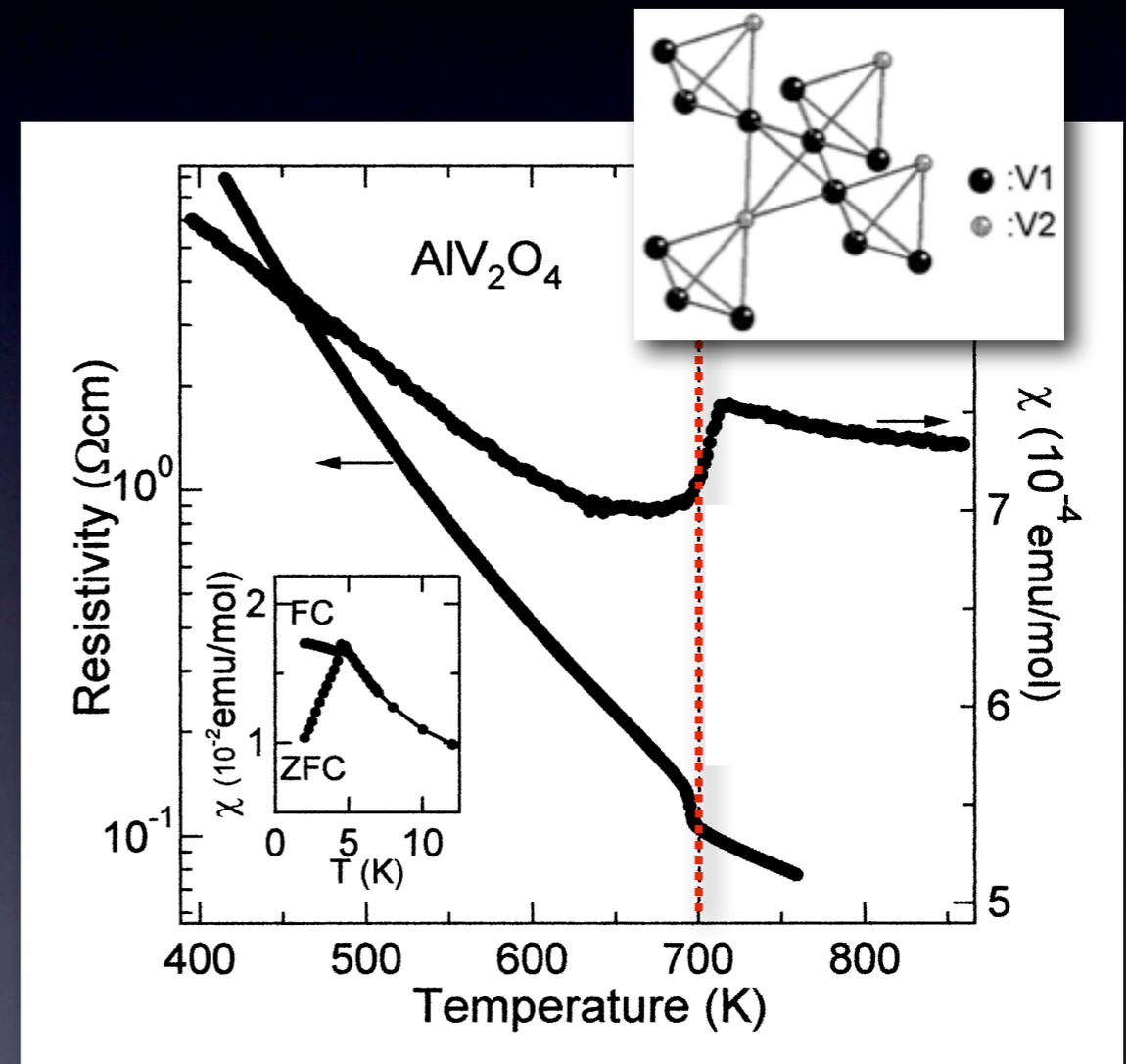
# (Atomic) Electronic Structure in $AlV_2O_4$

- mixed valence:  $V^{2.5+} = (3d)^{2.5}$
- charge, spin and orbital degrees of freedom are all active



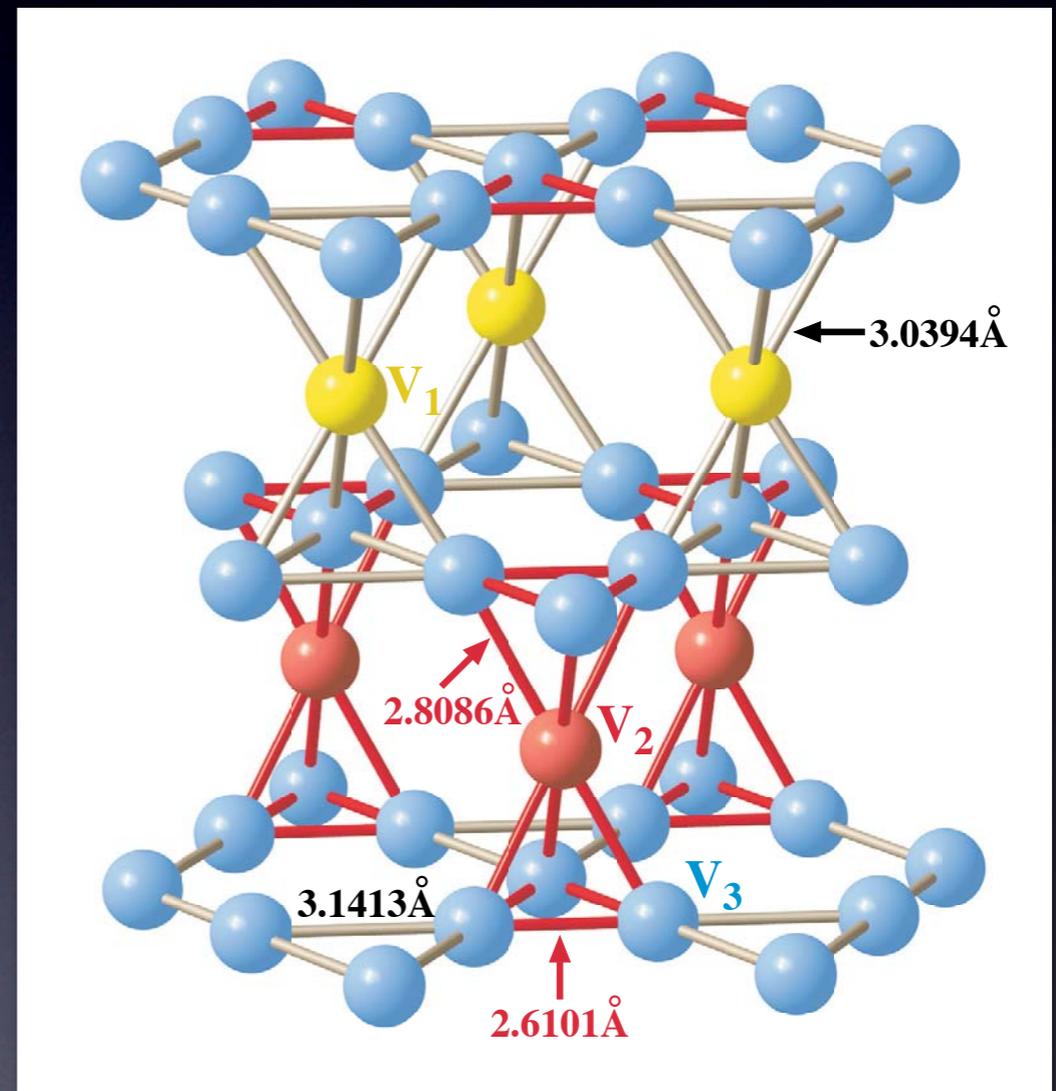
# Phase Transition at $T \sim 700\text{K}$

- structural change: doubling of the unit cell along the  $[111]$  direction
- shoulder in the resistivity
- sudden drop in the magnetic susceptibility followed by Curie behavior at lower temperatures
- valence-skipping-type charge ordering ?



K. Matsuno et al., 2001

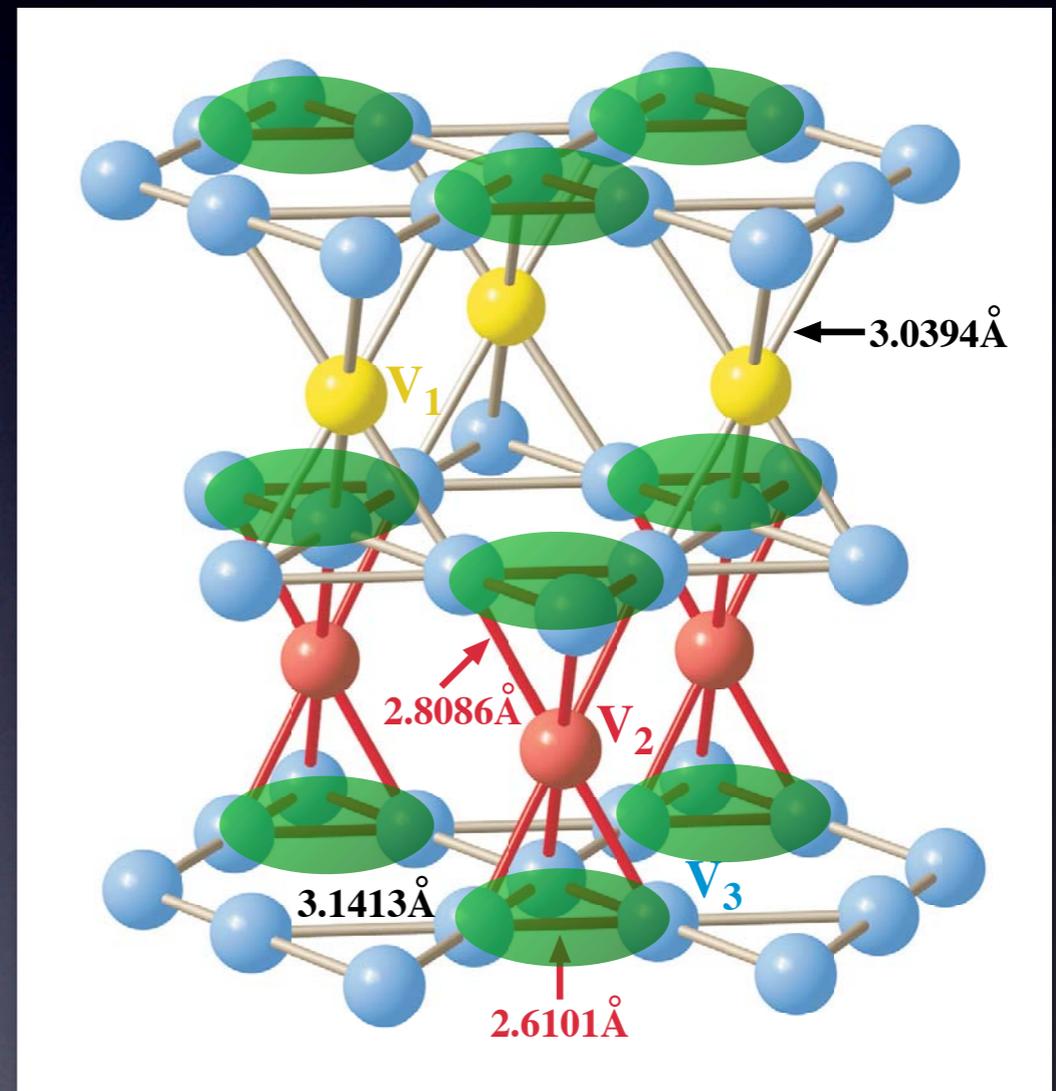
# Heptamer Scenario



Y. Horibe *et al.*, 2006

# Heptamer Scenario

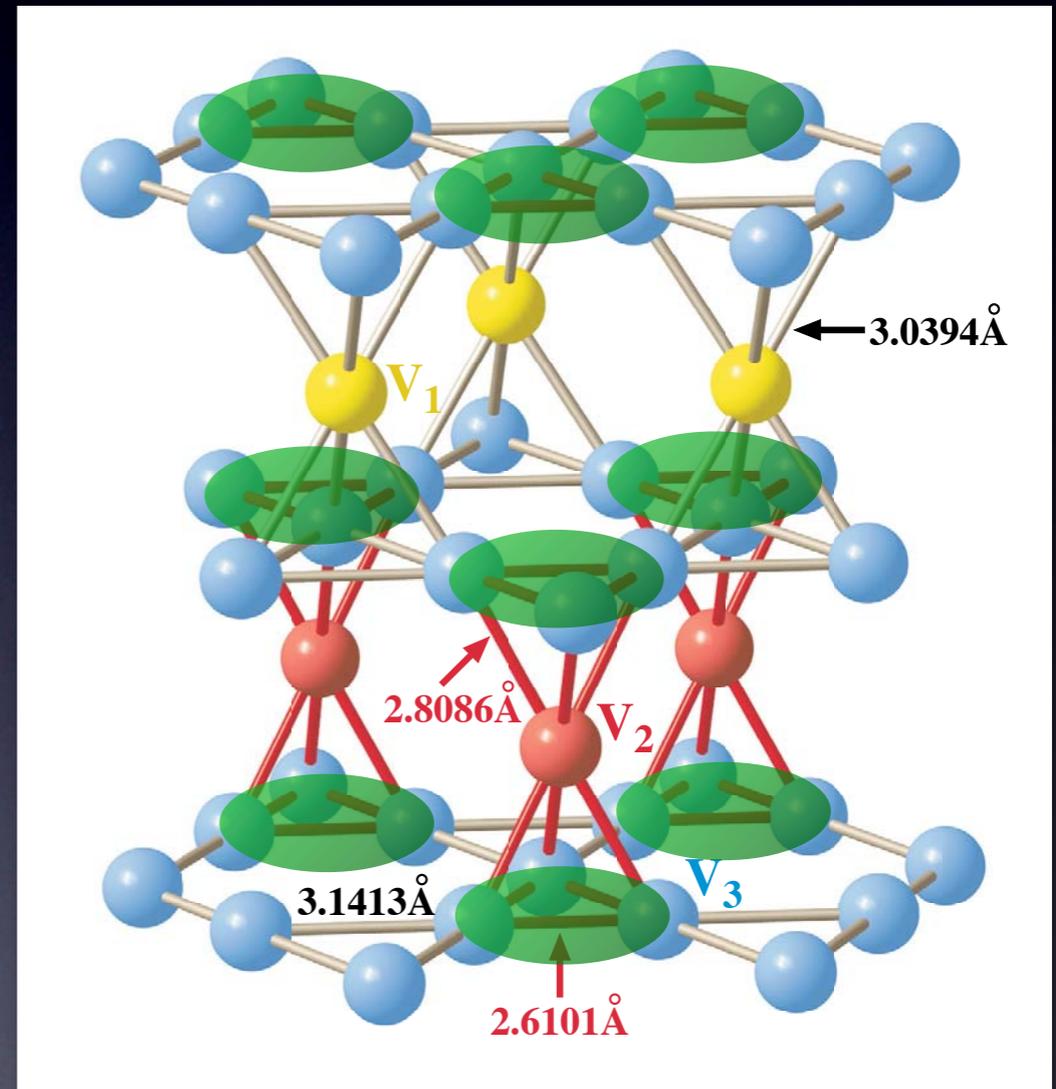
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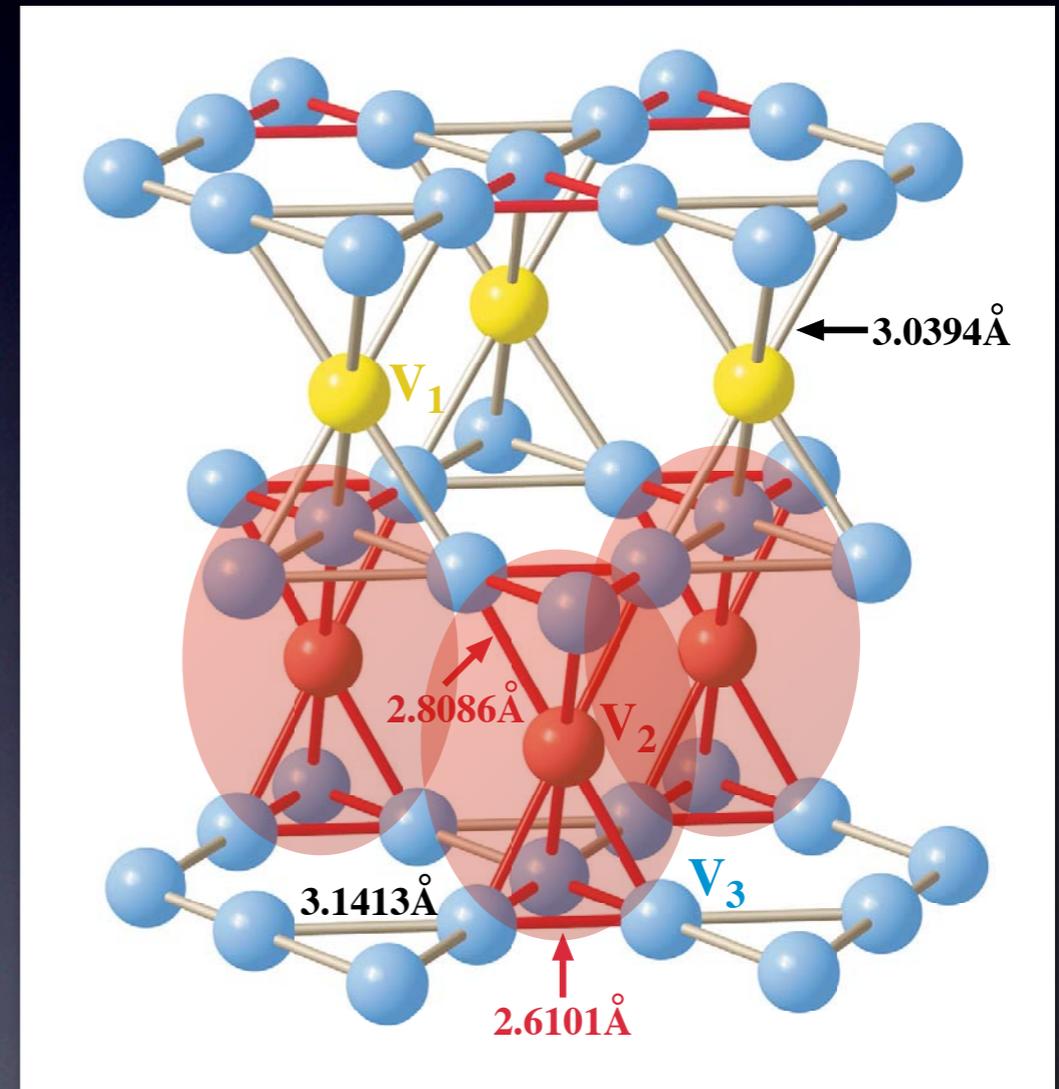
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# Heptamer Scenario

- new experimental finding: trimer formation in Kagome layers below  $T_c$
- spin-singlet formation in trimers?  $\rightarrow$  sharp drop of the magnetic susceptibility?
- We propose a singlet state emerging from the 7-site clusters (heptamers)

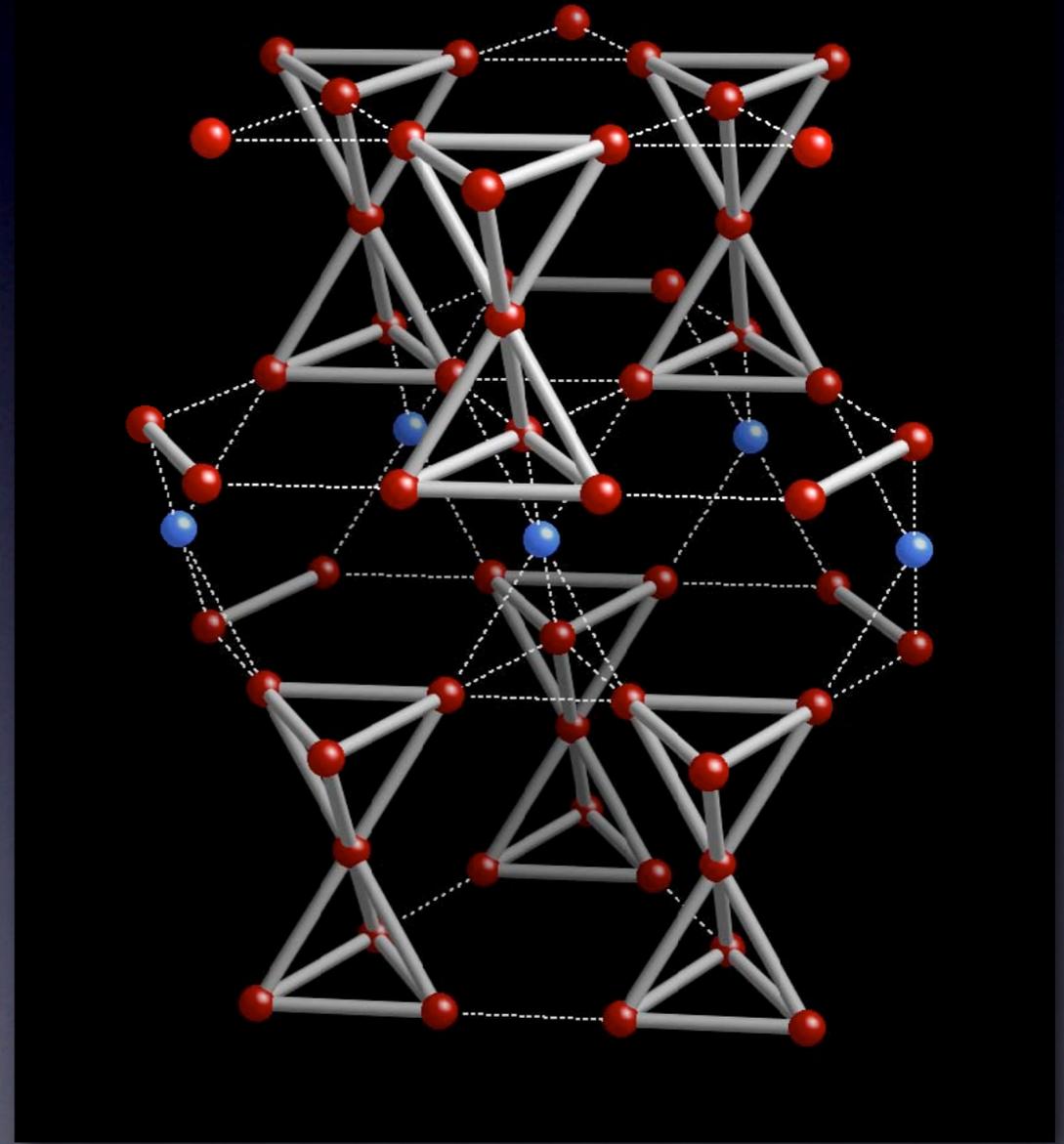


Y. Horibe *et al.*, 2006

# Questions

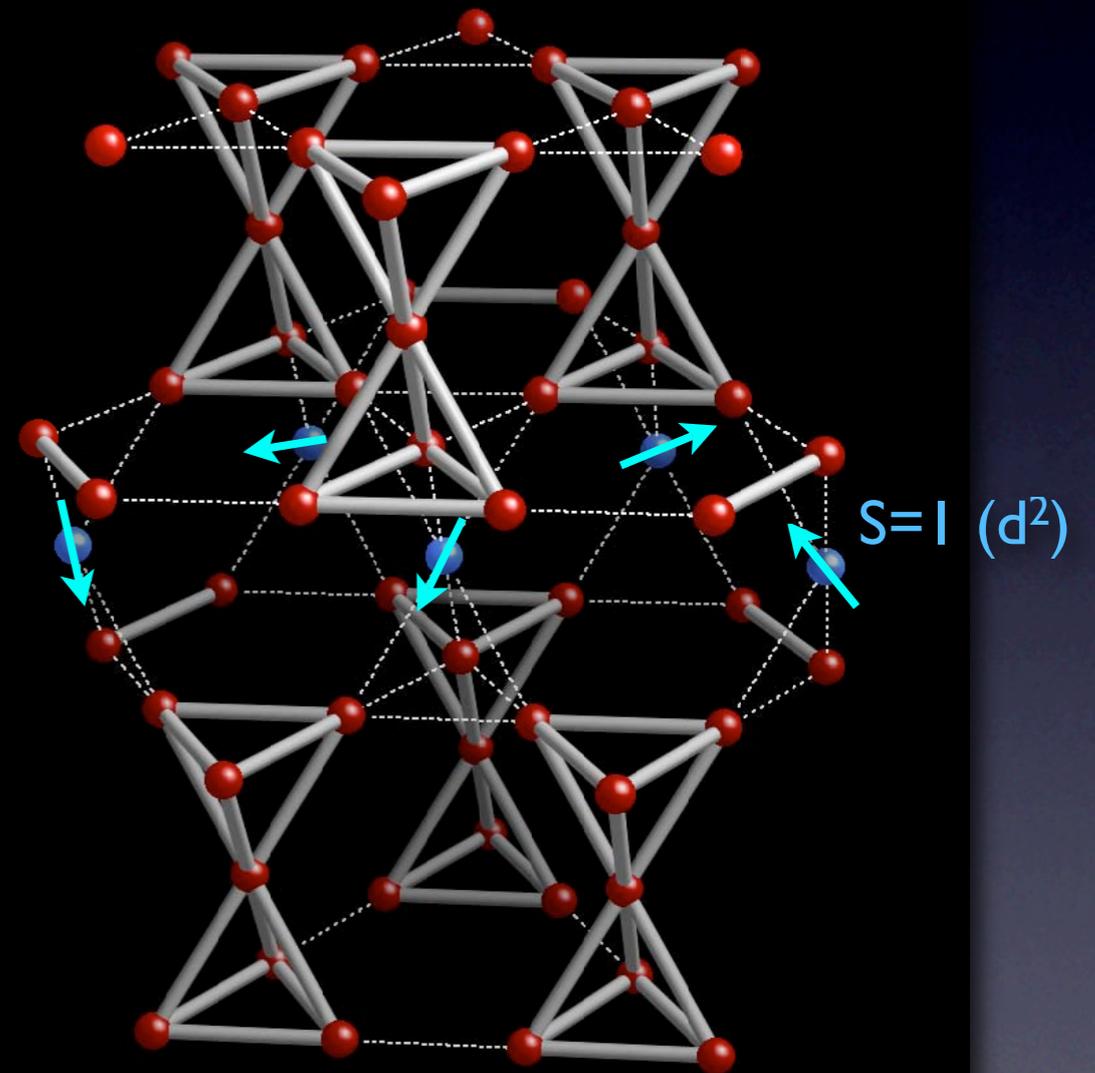
- 📌 What is the mechanism of the heptamer formation? How is the degeneracy in the frustrated pyrochlore system lifted?
- 📌 Is the heptamer in a spin-singlet state? How does the singlet state emerge in each heptamer?

# Multi-orbital Heptamer Model



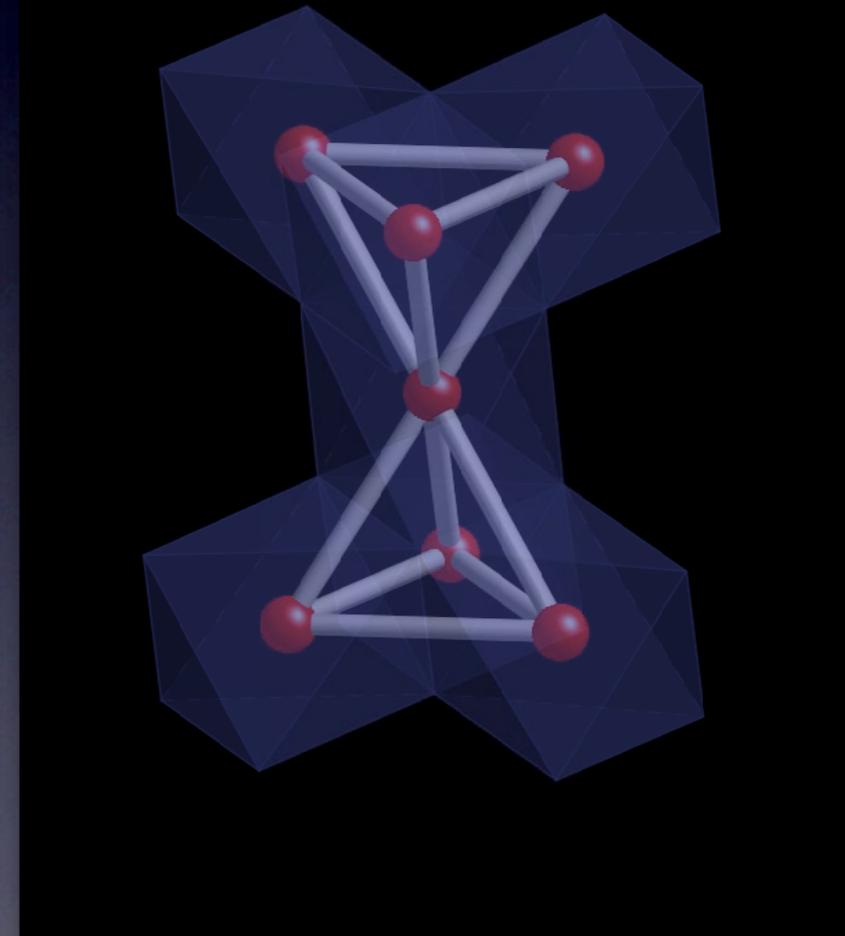
# Multi-orbital Heptamer Model

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→ 18 electrons per heptamer



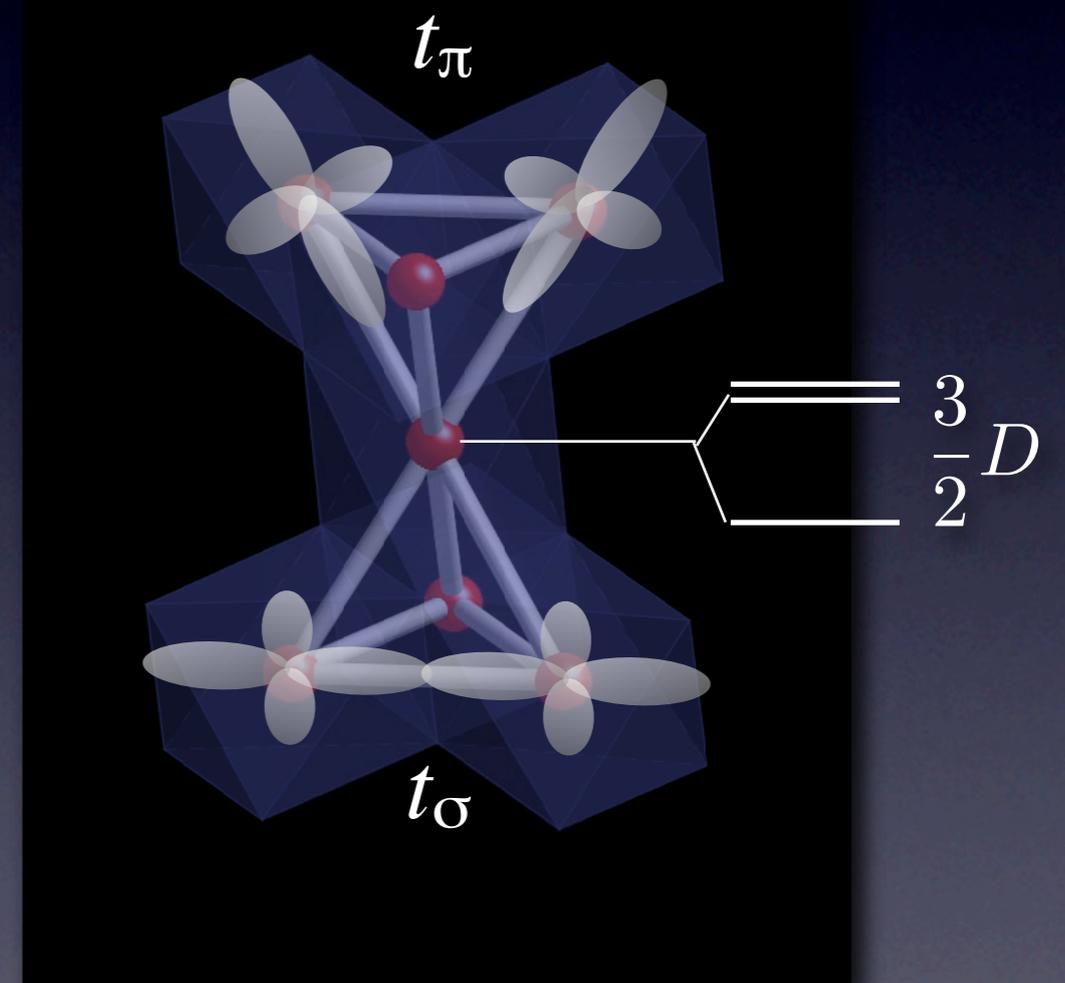
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  - $\sigma$  and  $\pi$  transfer integrals
  - trigonal lattice distortion at the central site
  - Coulomb interactions



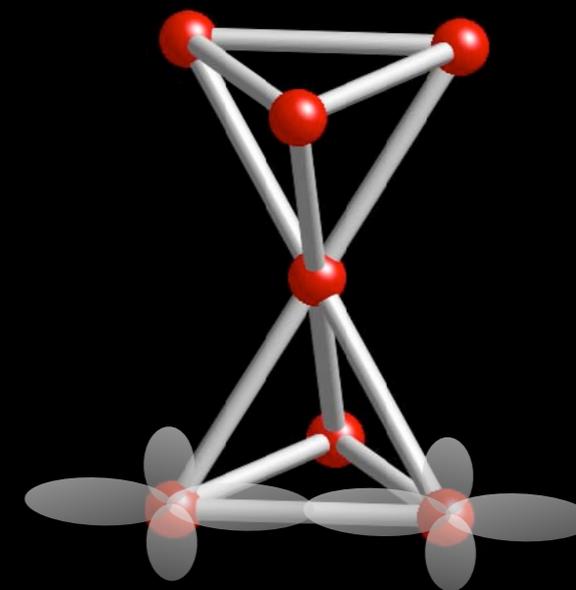
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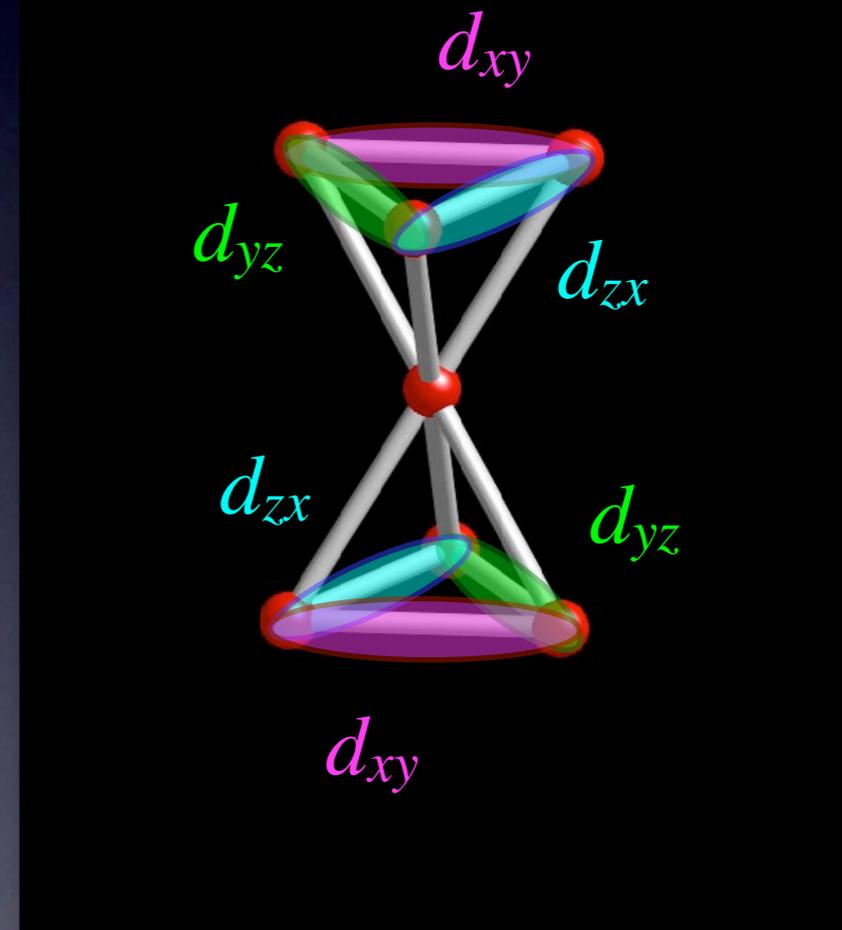
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bonding state:  
2 electrons per bond

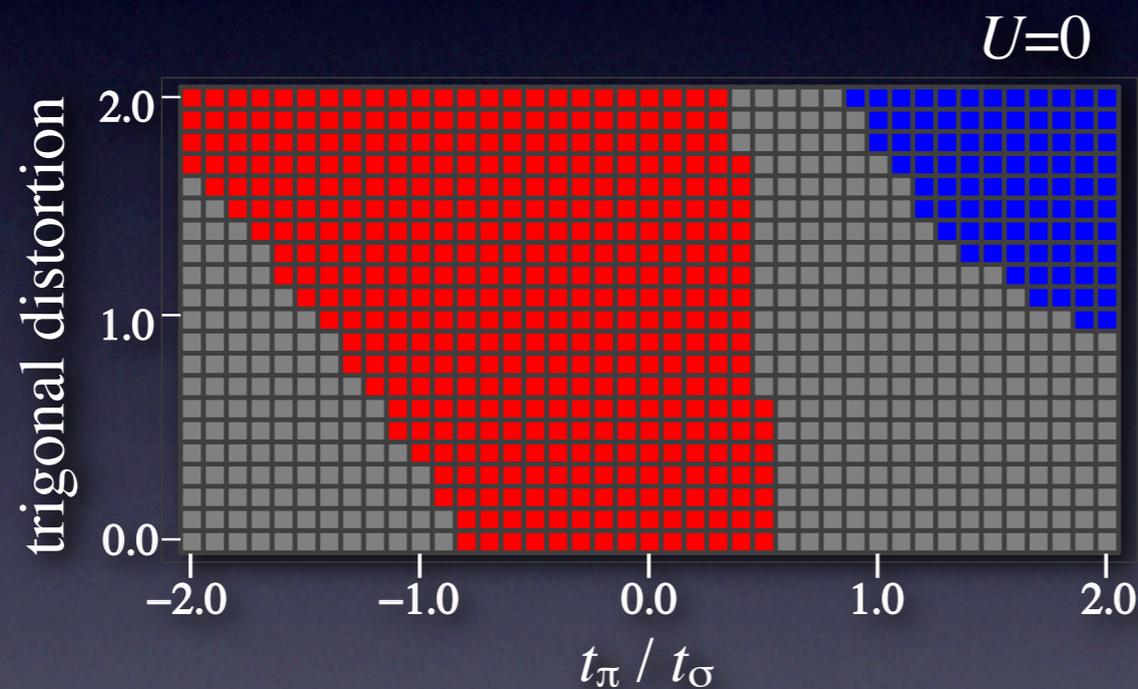
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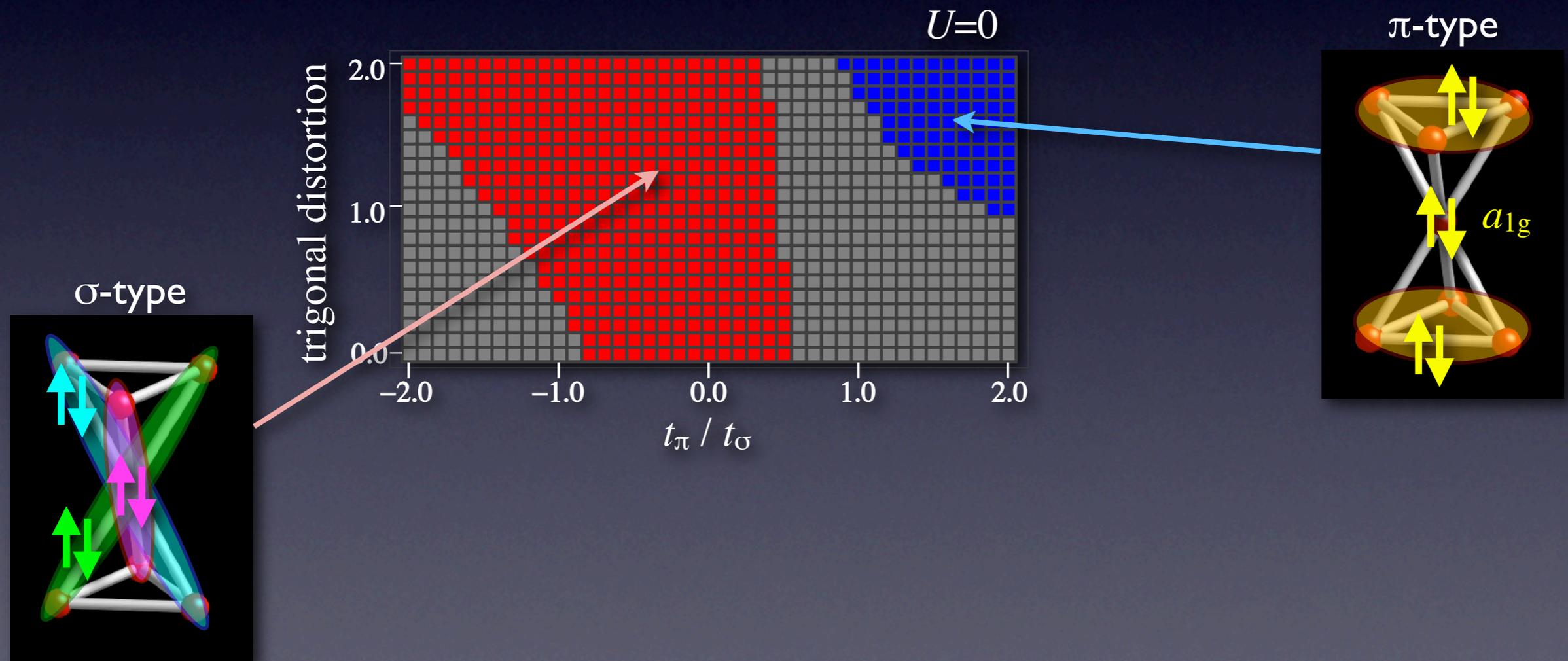
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- exact diagonalization of the effective heptamer model
- two different singlet regimes:  $\sigma$ -type and  $\pi$ -type



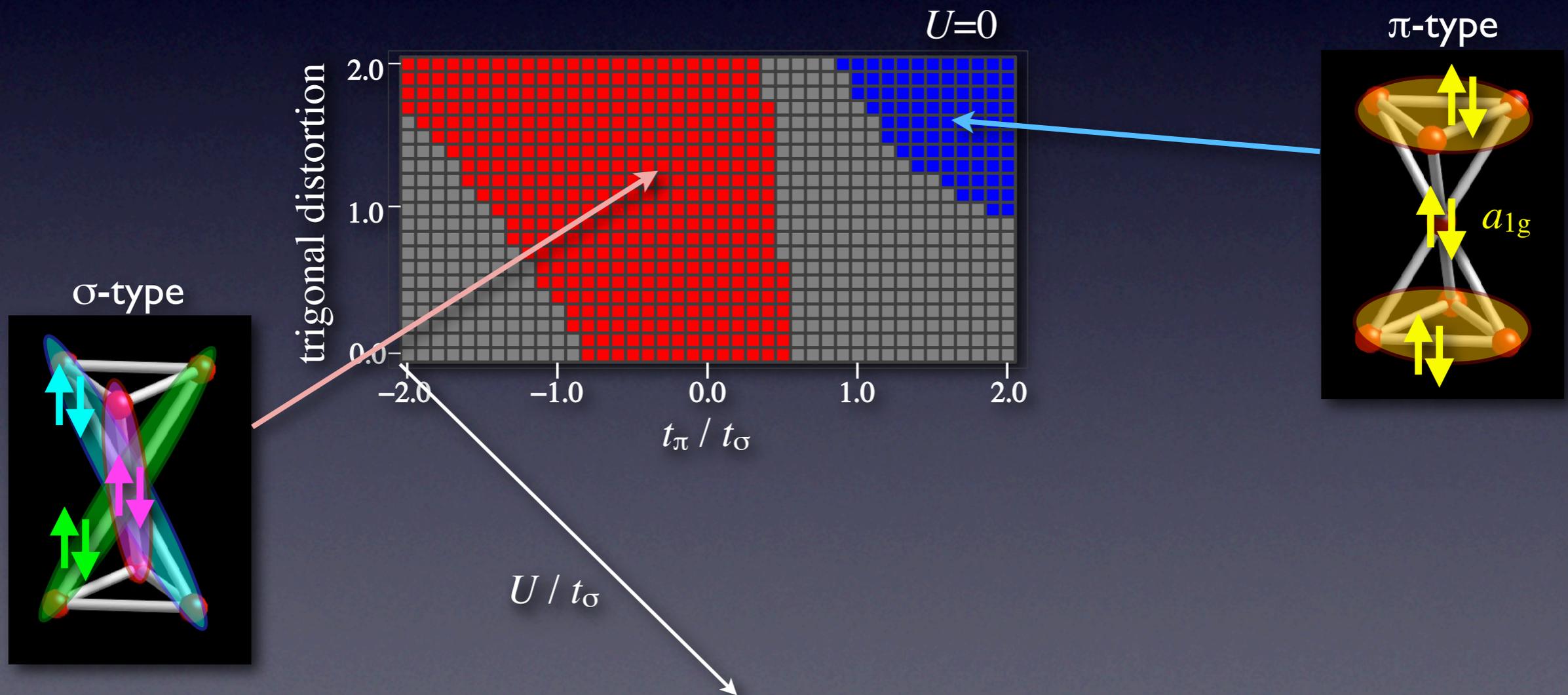
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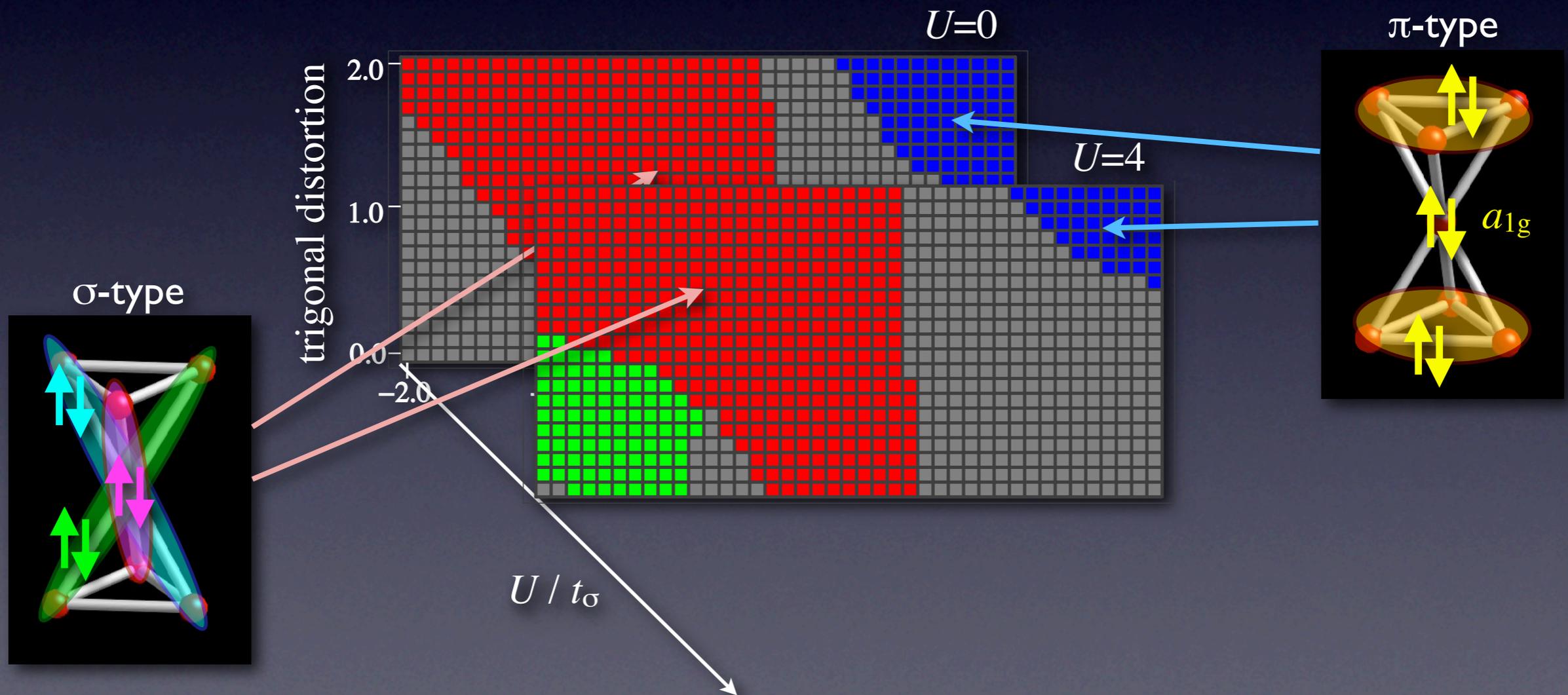
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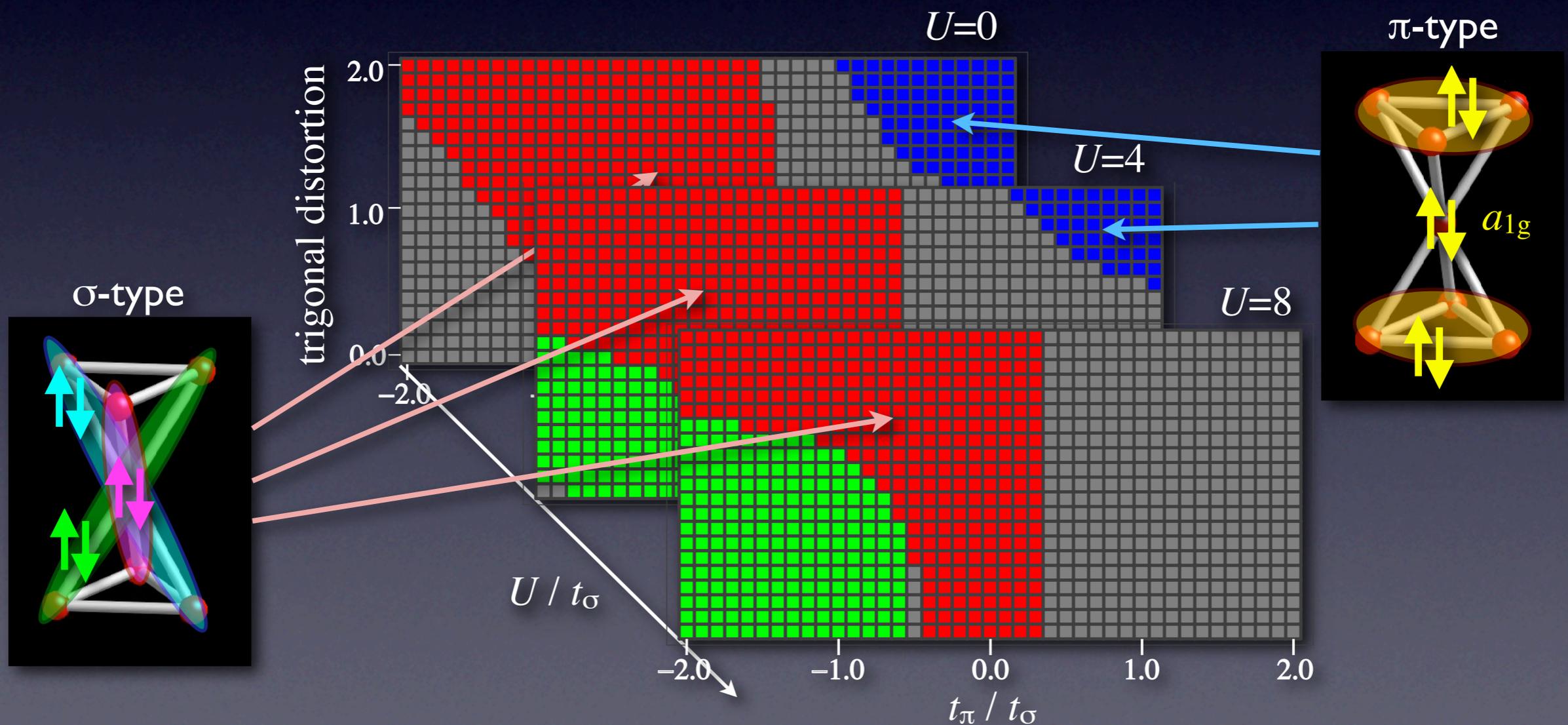
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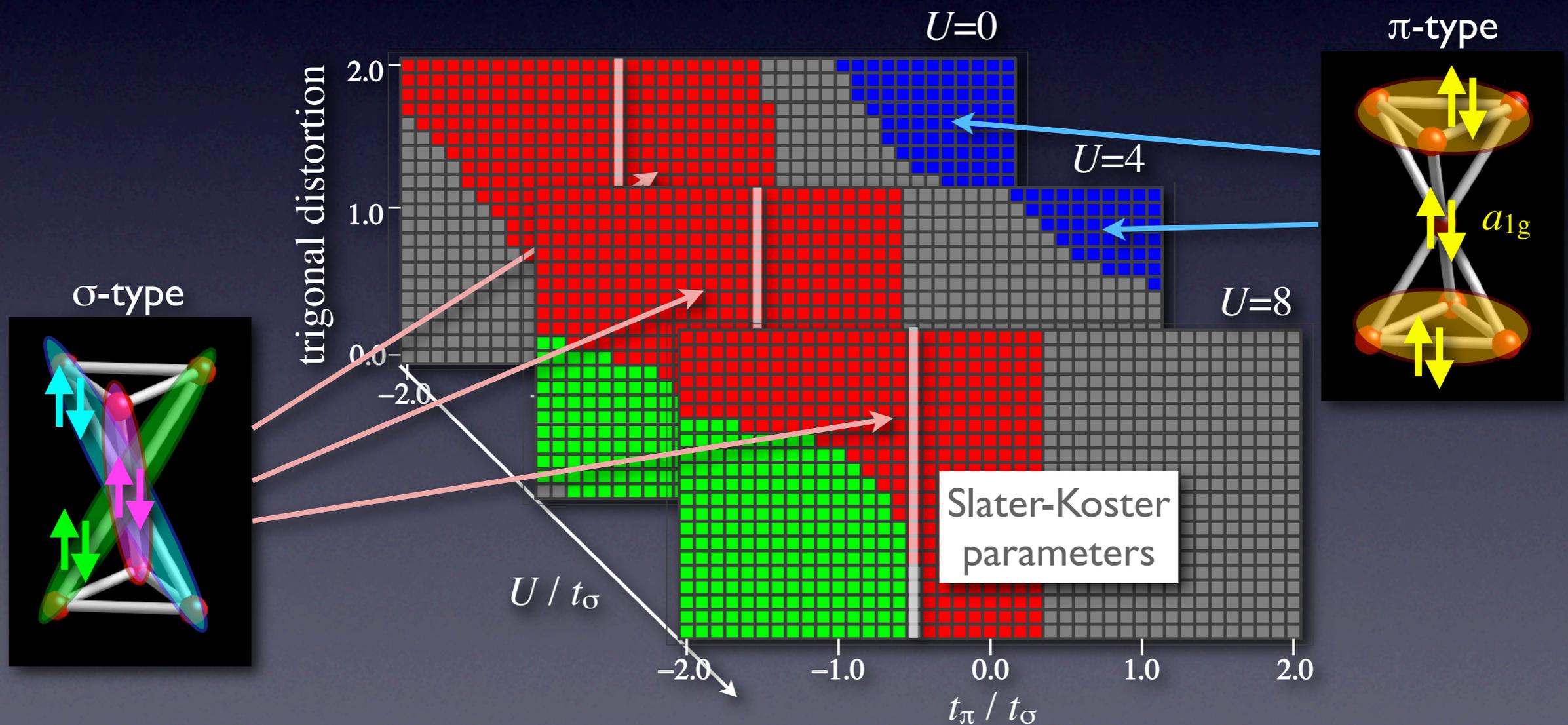
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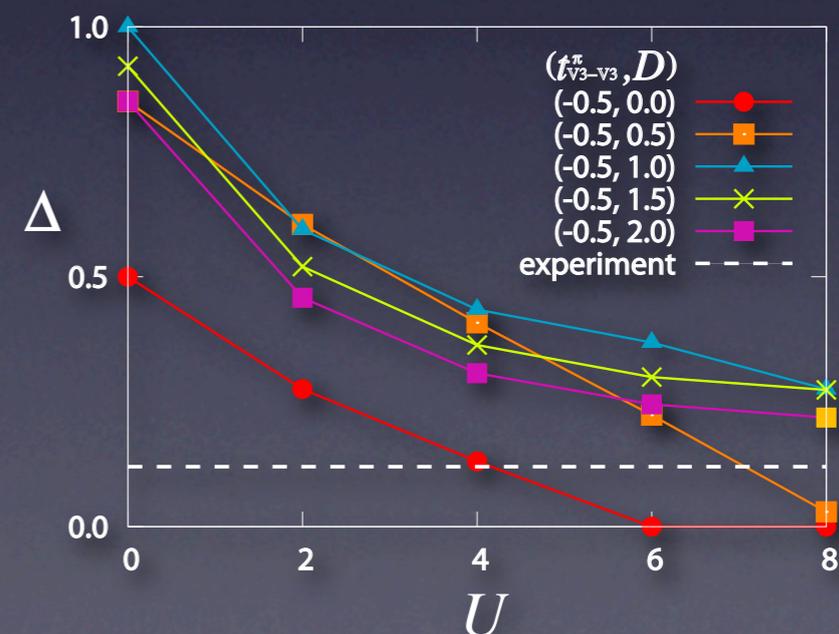
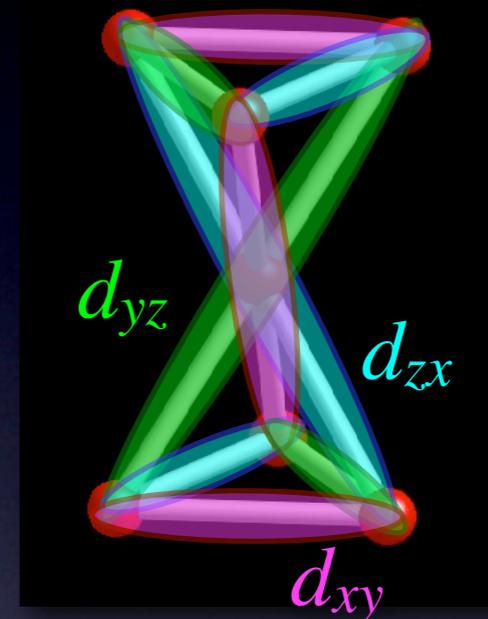
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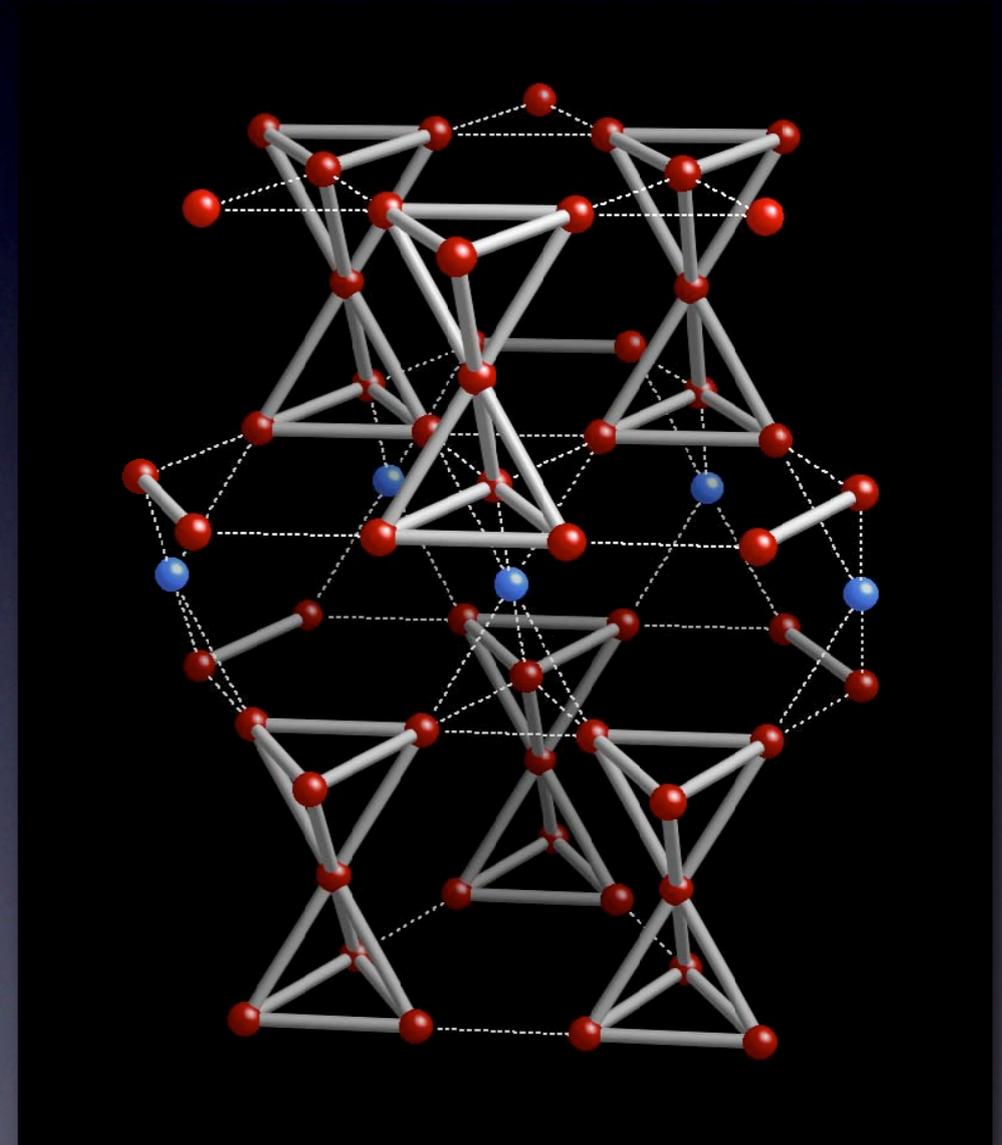
# Singlet State in Heptamer

- singlet ground state for realistic parameters =  $\sigma$ -type  
‘molecule’ of the bonding states with three  $t_{2g}$  orbitals
- estimate of the spin gap is larger than the experimental one: heptamer-heptamer coupling?
- comprehensive understanding of the T-dependence of magnetic susceptibility



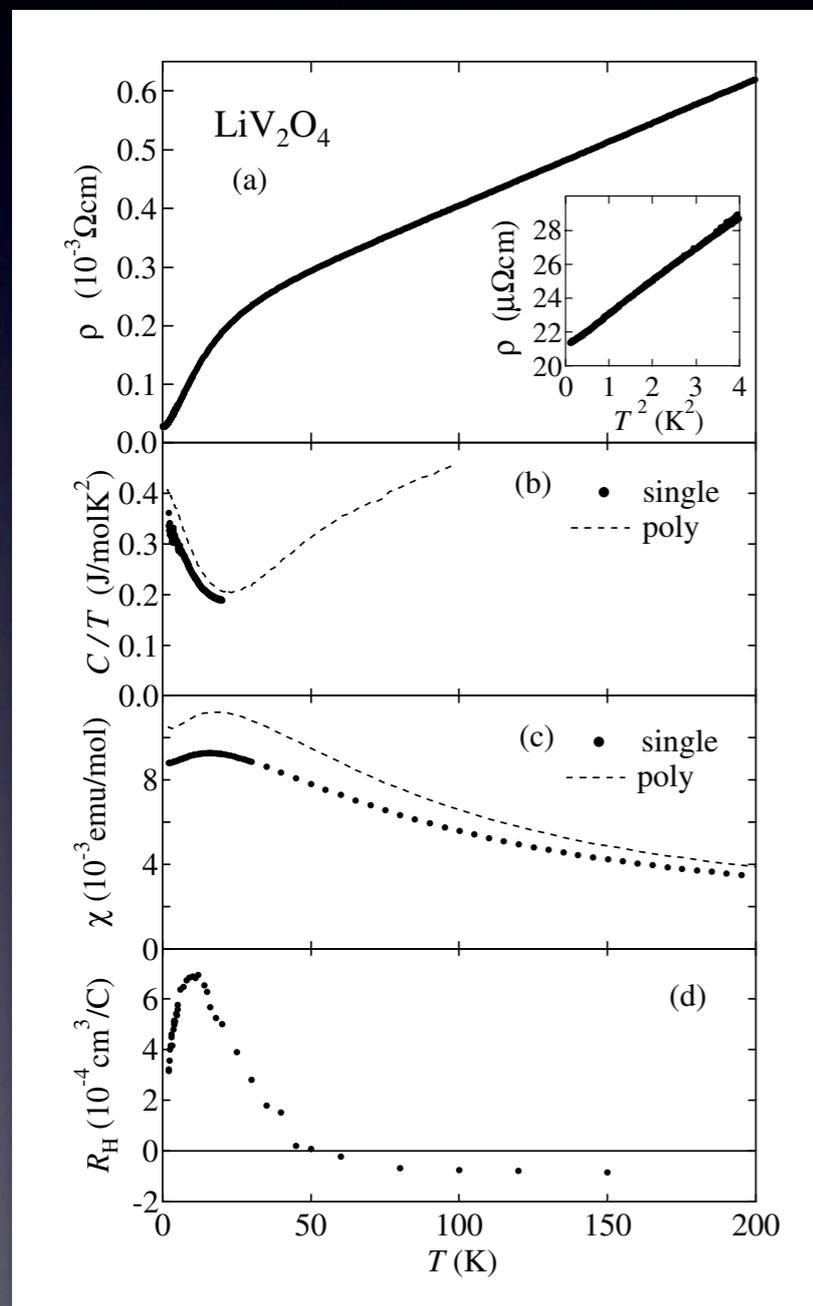
# Open Issues...

- Once the heptamers are assumed to be stable, experimental results at low-T phase are explained comprehensively.
- 📌 What is the mechanism of the heptamer formation? How is the degeneracy in the frustrated pyrochlore system lifted?
- 📌 Is similar phenomenon seen in other mixed-valence compounds?



# Another Mixed-Valence Compound

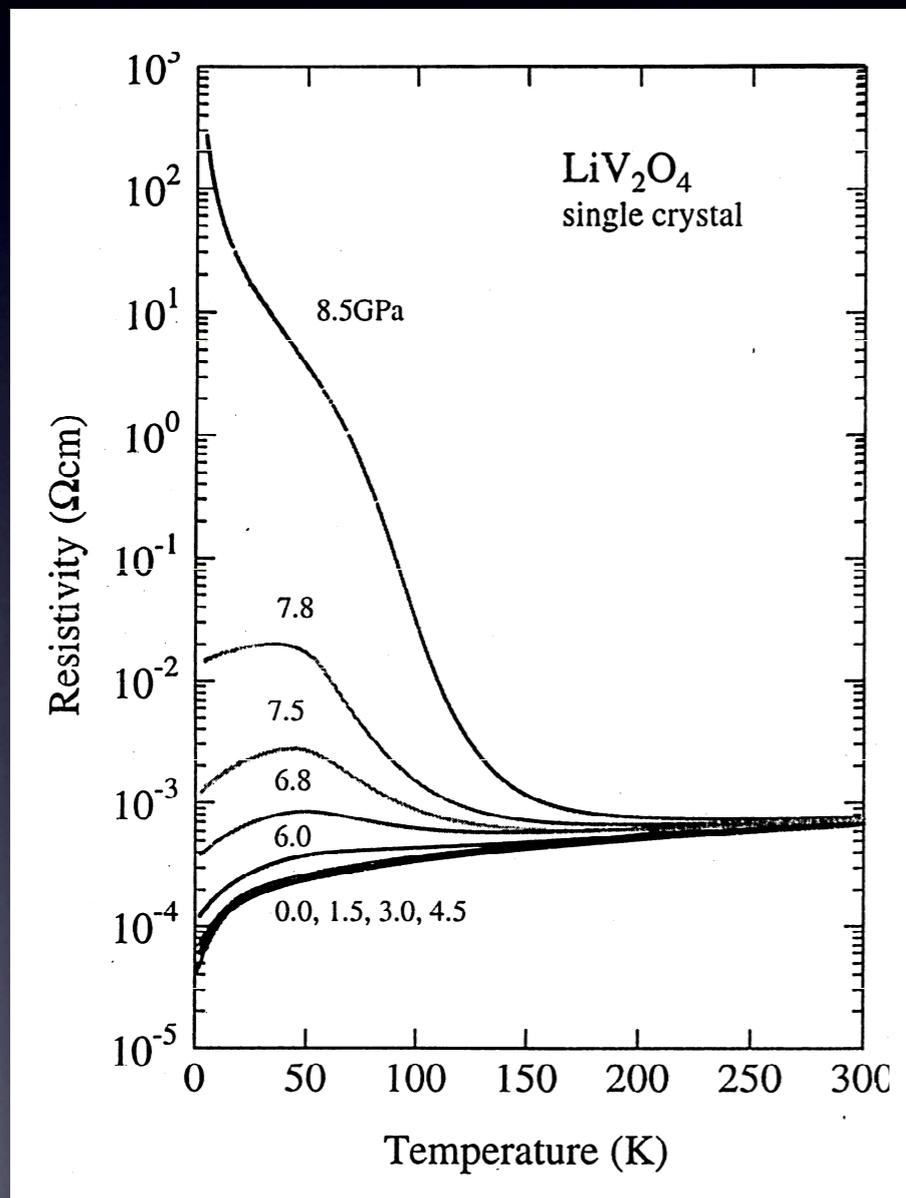
## $\text{LiV}_2\text{O}_4$ : Heavy-Fermion Behavior



Urano *et al.*, 2000

- mixed valence:  $\text{V}^{3.5+} = (3d)^{1.5}$
- heavy mass (Kondo *et al.*, 1997)
- cubic, metallic, no magnetic ordering (Rogers *et al.*, 1967; Chmaissem *et al.*, 1997; Mahajan *et al.*, 1997; Merrin *et al.*, 1998)
- only  $t_{2g}$  electrons: new mechanism for heavy fermion behavior?
  - Kondo scenario
  - geometrical frustration + correlation

# Implication of $\text{AlV}_2\text{O}_4$ ?



Urano, PhD Thesis

- metal-to-insulator transition by applying pressure: *opposite to usual pressure effect*
- short and long V-V bonds in the insulating state (EXAFS by Niitaka *et al.*, unpublished)
- possibility: some cluster formation similar to  $\text{AlV}_2\text{O}_4$

# Summary

- introduction to spinels and  $t_{2g}$  orbital physics
- controversy on orbital ordering in  $\text{ZnV}_2\text{O}_4$ 
  - different models for spin/orbital order in  $\text{ZnV}_2\text{O}_4$ : relative importance of Kugel-Khomskii superexchange, Jahn-Teller and relativistic spin-orbit couplings
  - symmetry analysis: lesson from experiments in  $\text{MnV}_2\text{O}_4$
- self-organized 7-site cluster (heptamer) in  $\text{AlV}_2\text{O}_4$ 
  - heptamer scenario: 'molecule' of bonding states with anisotropic  $t_{2g}$  orbitals
  - implication to heavy-fermion compound  $\text{LiV}_2\text{O}_4$