Quantitative predictions on auxin transport

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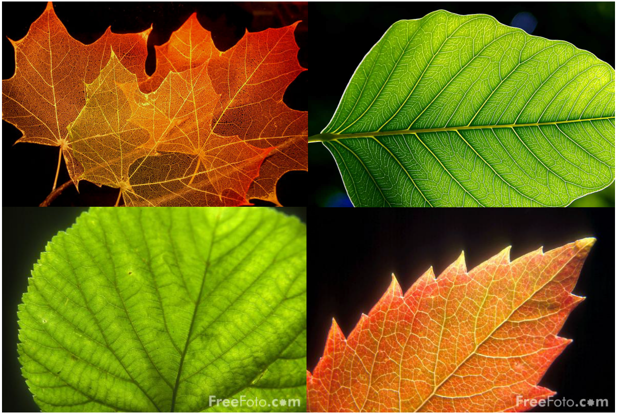
A manifold of leaf vein patterns shows conserved properties

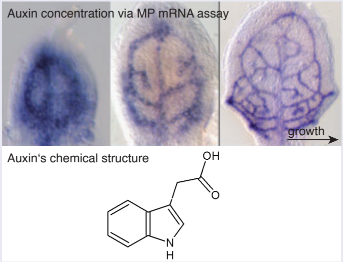


Conserved structures

- central midvein towards the petiole
- secondary veins spanning the leaf blade
- higher order veins reticulating the spacing

Auxin and its efflux carrier build patterning mechanism

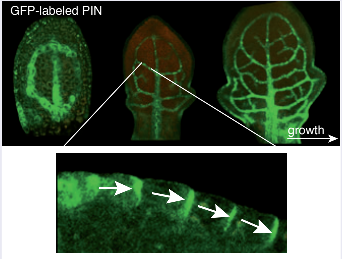
Plant hormone auxin



Wenzel et al. Plant J. (2007)

- auxin is produced in leaves and transported towards stem
- defective auxin transport results in disrupted veins → auxin induces vascular differentiation

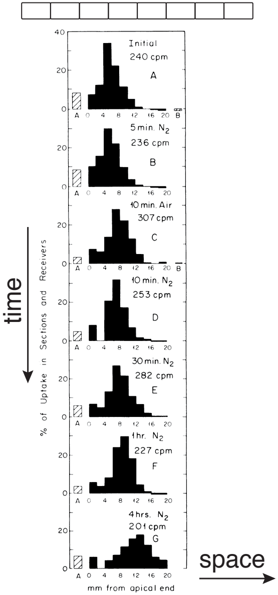
Efflux carrier PIN



Wenzel et al. Plant J. (2007)

- PIN membrane bound channel protein
- PIN knock out leads to undirected auxin transport
- PIN yields directed auxin transport in mammalian and yeast cells

In a string of cells auxin waves can be excited



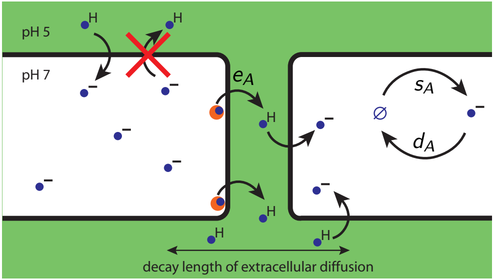
Goldsmith Plant Phys. (1967)

In vitro experiment

- strand of cells cut from plant
- apply source of auxin to one cell
- auxin wave travels down the cells
- radioactively labeled auxin becomes diluted
- observe spatial auxin distribution at different time steps
- velocity of $\approx \mu m / sec$

Outline

Auxin needs efflux carriers to exit cell

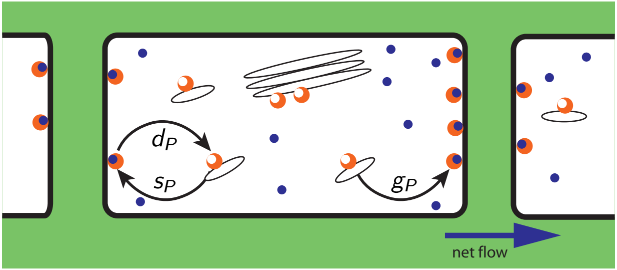


$$\dot{A}(n) = s_A - d_A A(n) - \frac{e_A}{\ell}[J(n) - J(n-1)], \ J(n) = A(n)P_r(n) - A(n+1)P_l(n+1)$$

Dynamics of the weak acid auxin

- protonated auxin follows concentration gradient into cell
- gets ionized at ph7 of cytoplasm, auxin is trapped
- efflux facilitated by carrier P_r, P_l with efficiency e_A
- extracellular diffusion suppressed by small interspace
- fast synthesis s_A and degradation d_A in cells

Efflux carrier mediate a self-enhancing flow of auxin



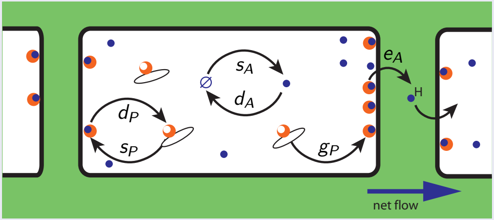
$$\dot{P}_r(n) = -d_P P_r(n) + [P_{tot} - P_r(n) - P_l(n)]\left[s_P + g_P J^2(n)\theta(J(n))\right]$$

Cycling of membrane bound PIN efflux carrier

- PIN proteins are membrane bound
- efflux carrier cycle between endosomes and cell membrane s_P, d_P
- auxin flux yields enhanced attachment g_P
- directed transport due to self-enhancing flow of auxin
- efflux carrier production too slow for wave $v \approx \mu m/sec$

Parameters are sensitive to environmental conditions

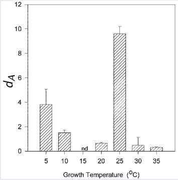
Full microscopic model



Varying parameters

auxin metabolism and transport affected by

- light
- temperature
- wind



Rapparini et al. Plant Phys. (2002)

Rescaling

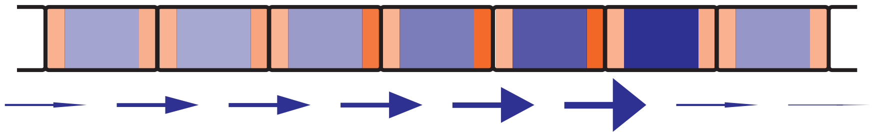
- normalize concentrations $a = A/A_{eq}$, $A_{eq} = s_A/d_A$ and $P_r = P_r/P_{tot}$
- transport efficiency e_A fastest time scale, others estimated slower
- measure time and rates in slowest estimated time-scale d_P

Questions

- What is the underlying nonlinear mechanism resulting in wave formation?
- How do parameters control wave properties?
- Are quantitative predictions possible?

Auxin traffic jam builds up wave

Auxin is piled up by yet undirected efflux carriers



- fast transport efficiency rate
- bottle neck due to slow directed attachment rate of efflux carriers

Microscopic equations

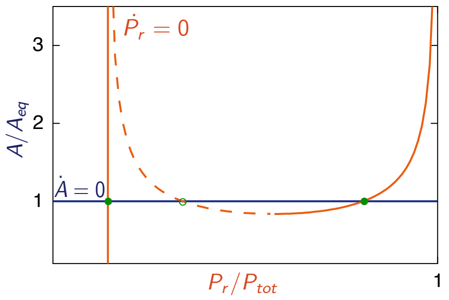
$$
\begin{aligned}
\dot{a}(n) &= d_A(1 - a(n)) - (e_A P_{\text{tot}}/\ell)\,[J(n) - J(n-1)] \\
\dot{p}_r(n) &= -p_r(n) + [1 - p_r(n) - p_l(n)]\left[s_P + g_P P_{\text{tot}}^2 A_{\text{eq}}^2 J^2(n)\Theta(J(n))\right] \\
\dot{p}_l(n) &= -p_l(n) + [1 - p_r(n) - p_l(n)]\left[s_C + g_P P_{\text{tot}}^2 A_{\text{eq}}^2 J^2(n-1)\Theta\right]
\end{aligned}
$$

$$J(n) = a(n)p_r(n) - a(n+1)p_l(n+1)$$

Excitability of a bistable system due to advection

Reaction-advection system

$$\dot{a} = g(a, p_r) + \tilde{g}(a, p_r, \nabla a, \nabla p_r)$$
$$\dot{p}_r = f(a, p_r) + \tilde{f}(a, p_r, \nabla a, \nabla p_r)$$



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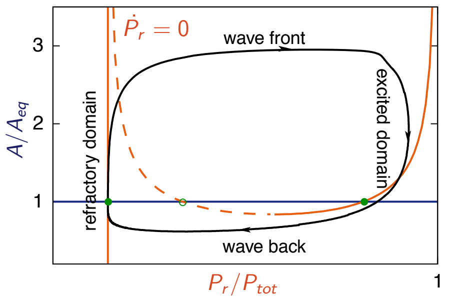
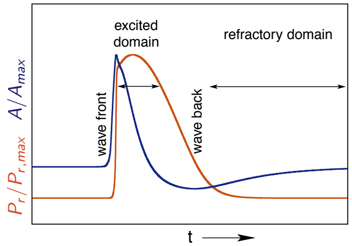


A wave developing

- bistable system
- excitation triggers wave
- advection drives system past second fixed point
- relaxation back



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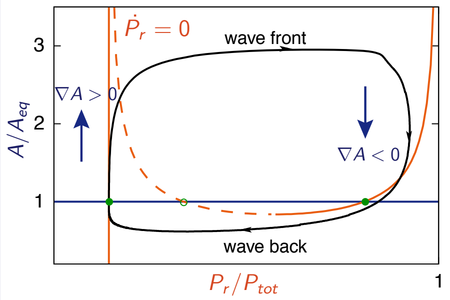

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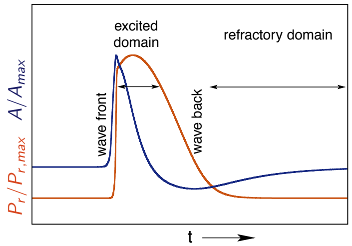
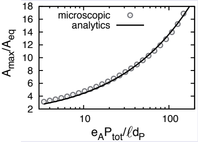
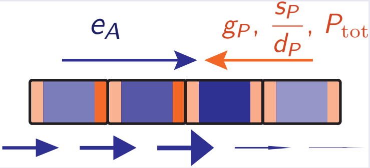


Theoretical predictions explain microscopic model

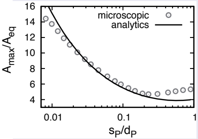
Varying transport efficiency



higher transport efficiency leads to more auxin accumulation

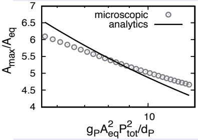


Change of carrier attachment



attachment rate sets $P_{r,eq}$ $\longrightarrow$ auxin is faster moved away

Change in enhanced attachment

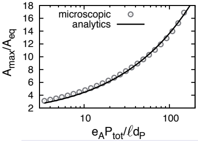


higher directed attachment $\longrightarrow$ auxin is faster moved away

$$A_{\max}^2 = \frac{2(d_P+2s_P)}{g_P P_{\rm tot}^2}\left(1+\sqrt{1+\frac{e_A^2 P_{\rm tot}^2 (d_P+s_P)^2}{\ell d_P^2 s_P^2}\left[1+\sin(\phi/3)-\frac{\cos(\phi/3)}{\sqrt{3}}\right]}\right), \phi = \arctan\left(-\frac{\sqrt{\frac{1}{4}\frac{e_A^4 P_{\rm tot}^4 (d_P+s_P)^4}{\ell^4 d_P^4 s_P^4} - 3^3}}{3\sqrt{3}}\right)$$

Theoretical predictions explain microscopic model

Varying transport efficiency



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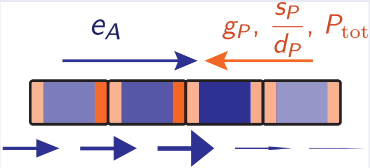

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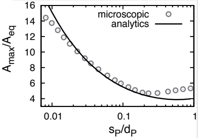

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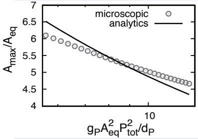
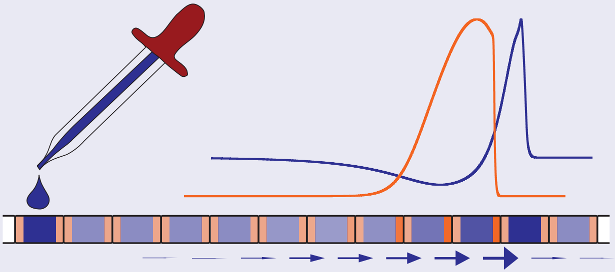


higher directed attachment $\longrightarrow$ auxin is faster moved away

$$A_{max}^2 \propto \frac{e_A P_{tot}}{\ell d_P} \frac{1}{\frac{g_P P_{tot}^2 A_{eq}^2}{d_P}} \frac{(1+\frac{s_P}{d_P})(1+2\frac{s_P}{d_P})}{\frac{s_P}{d_P}} \qquad v \propto e_A P_{tot}$$

Measure auxin amplitude in experiment

Experimental setup



Observing an auxin wave

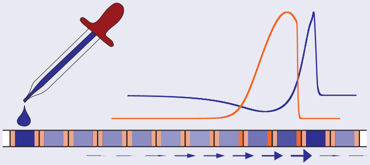
- quantify radioactively labeled or deuterated auxin
- GFP-labeled carriers mark position of wave front
- avoid boundary layer effects > 100 cells
- vary transport parameters by external stimuli (wind, light) or by use of mutants

Conclusion

Unveil nonlinear mechanism behind auxin wave

- found bistable system that is driven excitable due to advection terms
- analyzed influence of parameters on wave properties

Quantitative predictions

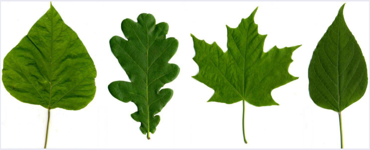


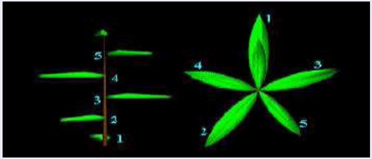
- able to solve dynamics analytically and make quantitative forecasts
- hope to inspired experiments to test predictions

Outlook

Stationary flow pattern in two dimensions

- by what mechanism do 2D patterns arise
- how do 2D pattern depend on the parameters and form
- how may plants regulate 2D patterns





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- financial support

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