New Methods for Galaxy Modelling

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Outline

- Why do we need good models?
- How are models made now?
- Quasiperiodicity & integrability
- The torus technique
- Secular perturbation theory revised
- Application to tidal debris

Why we need dynamical models

- Dynamics connects measurements made at different places
- It connects velocity space to real space
- It connects stars to DM
- Dynamics reduces the dimensionality of the Galaxy from 6 to 3

Why upgrade now?

- Advances in observational technique:
  - Integral-field spectroscopy
    - (SAURON, OASIS, KMOS, WFMOS, ..)
  - Photometric & radial-velocity surveys
    - (2Mass, SDSS, SEGUE, RAVE, VHS, Pan-Starrs, ..)
  - Astrometric satellites
    - (Hipparcos, Gaia, Jasmine,..)
- Only dynamical models can adequately exploit these large data sets

Science requirements

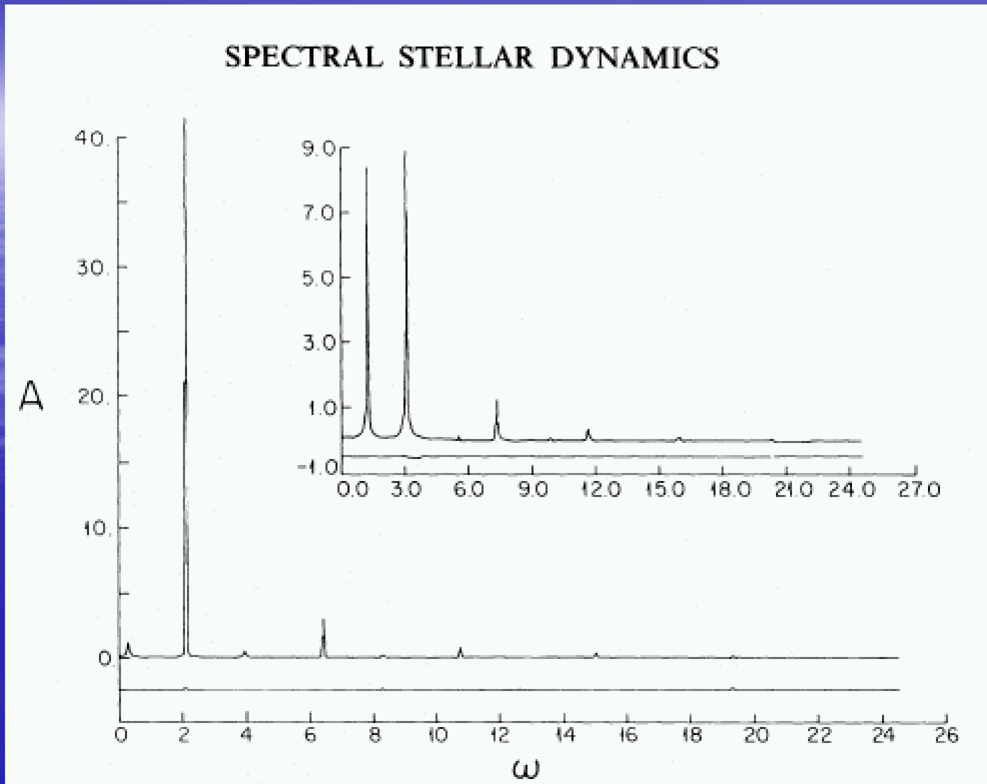
- MW complex, non-equilibrium system
  - The bulge-bar, spiral arms, streams, SFR(t,Z,α) at many locations, secular heating, radial migration, chemical evolution, …..
- We don't even want a definitive model
- We must model hierarchically:
  - axisymmetric model $\rightarrow$ barred model $\rightarrow$ spiral structure $\rightarrow$ warped model $\rightarrow$ metallicity tagging $\rightarrow$ …
- We need the DF so we can sample at will & calculate likelihoods
- We must be able to compute secular evolution
  - Possible if we use analytic DF $f_k(E,L_z,I_3)$.. for each of K populations

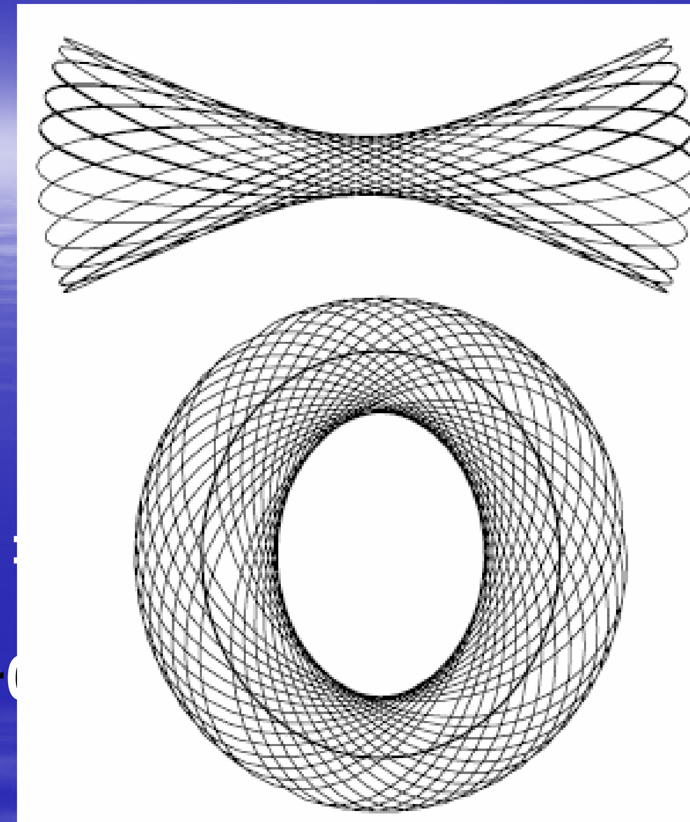
Galaxy modelling now

- N-body modelling
  - Operationally straightforward
  - Limitations
    - Lack of control of configuration (but M2M)
    - Hard to characterise configuration (no DF)
    - Poisson noise and spurious relaxation
    - Sampling problem (must have many low-L stars, but nearly all invisible)
    - Hard to add stellar populations, secular & chemical evolution etc

Schwarzschild modelling

- Standard for BH searches
- Given Φ(x) and ρ(x) and $\langle$v$\rangle$(x) etc
- Integrate orbits in Φ & save p_α(x,v)
- Seek $w_\alpha \geq 0$ s.t. $\rho(x)=\sum_{\alpha\ v} w_\alpha\ p_\alpha(x,v)$, etc
- Limitations
  - Messy: need to store M phase-space p_α for N orbits $\rightarrow$ N*M matrix to invert
  - Orbits not naturally characterised
  - Poisson noise
  - Eqs under-determined so no unique soln; should count # of solutions Magorrian (06)
  - Sampling problem
- Solution: replace time-series orbits with orbital tori





- Orbits come in families
- Time series x(t) etc are quasiperiodic

Angles & actions

- Quasiperiodic orbits $\Rightarrow$ exist magic integrals J_1, J_2, J_3 that can be complemented by coordinates θ_1, θ_2, θ_3 with trivial eqns of motion J_i = constat and $\theta_i = \Omega_i$ t + const
- Orbits 3-tori labelled by J with θ defining position on torus
- Torus null is sense $\int_{\rm torus} dx\cdot dv=0$
- Question is: how to find (x, v)(J,θ) for given Φ?

Analytic models
(de Zeeuw MNRAS 1985)

- Most general:
  - Staeckel Φ defined in terms of confocal ellipsoidal coordinates
- Φ separable in x,y,z and Φ(r) limiting cases
- Staeckel Φ yields analytic I_i but numerical integration required for J_i, θ_i
- everything analytic for 3d harmonic oscillator and isochrone

$$\Phi(r) = \frac{1}{b + \sqrt{b^2 + r^2}}$$

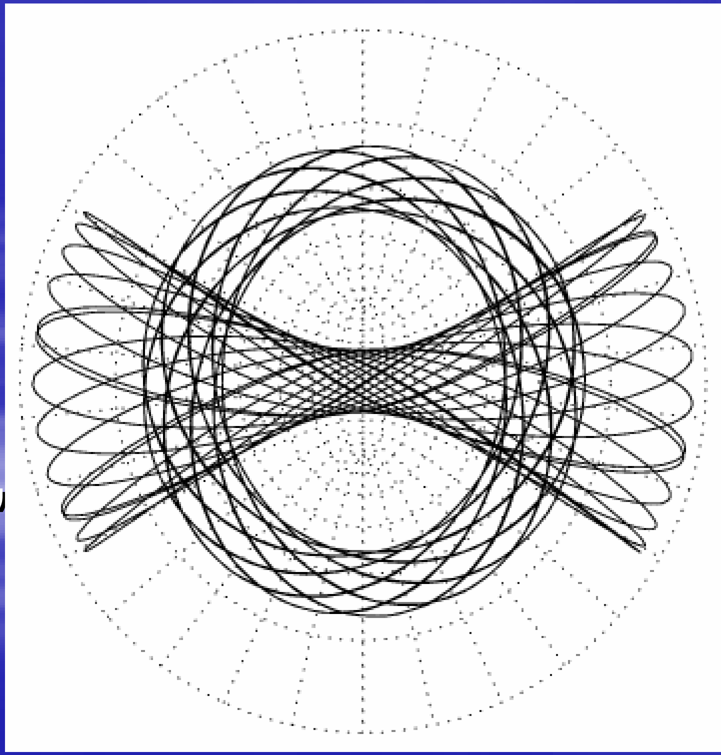
Torus programme

- Map toy torus from harmonic oscillator or isochrone into target phase space
- Use canonical mapping, so image is also null
- Adjust mapping so H = const on image

e.g. Box orbits

(Kaasalainen & Binney 1994)

- Orbits $\sim$ bounded by confocal ellipsoidal coords (u,v)
- $x' = \Delta \sinh(u) \cos(v)$;
  $y' = \Delta \cosh(u) \sin(v)$
- When (u,v) cover rectangle, (x',y') cover realistic box orbit



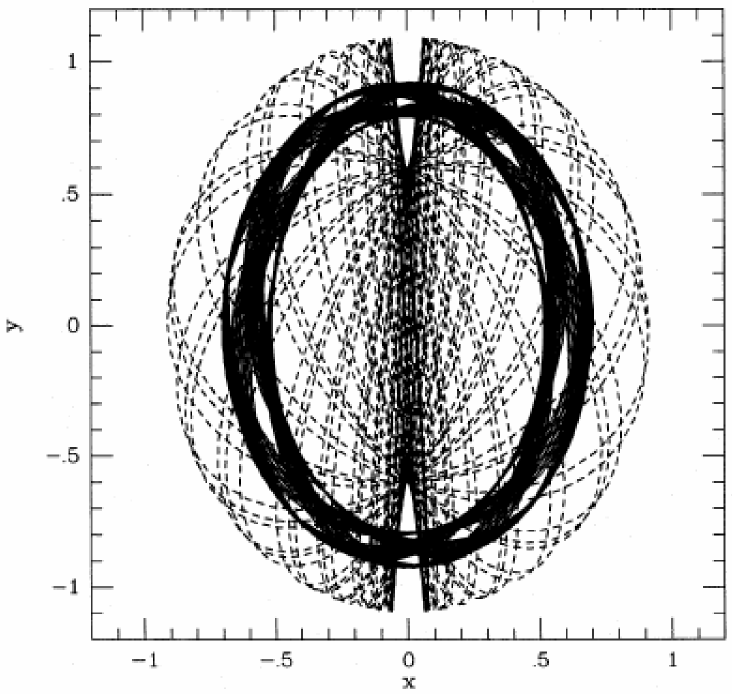
Box orbits (cont)

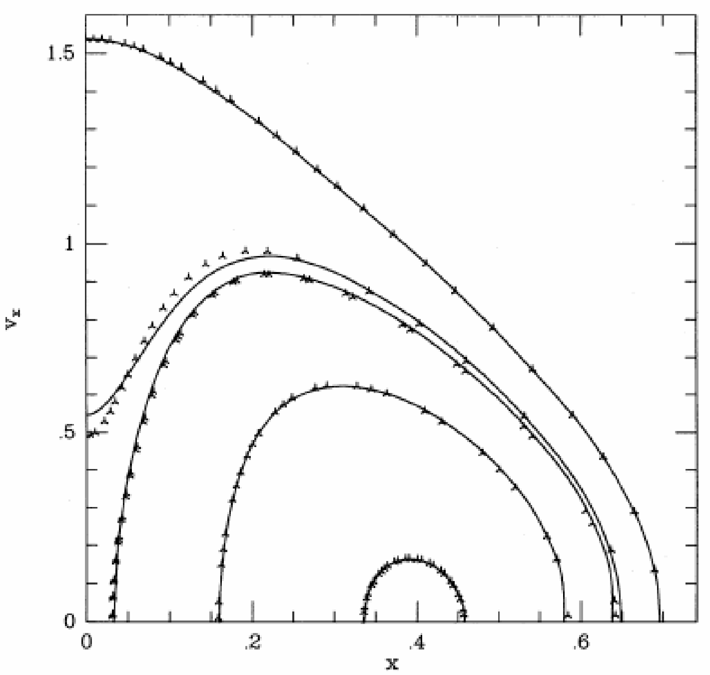
- Drive (u,v) with equations of motion when x=f(u), y=g(v) execute s.h.m.
- $p_u(x,p_x)$=df/du p_x ; p_v=dg/dv p_y
- $x=(2J_x/\omega_x)^{1/2}\sin(\theta_x)$, p_x= etc
- So $(J,\theta) \to (x,p_x,..) \to (u,p_u,..) \to (x',p_x',..)$
- Requires orbit to be bounded by ellipsoidal coord curves – insufficiently general

Box orbits (cont)

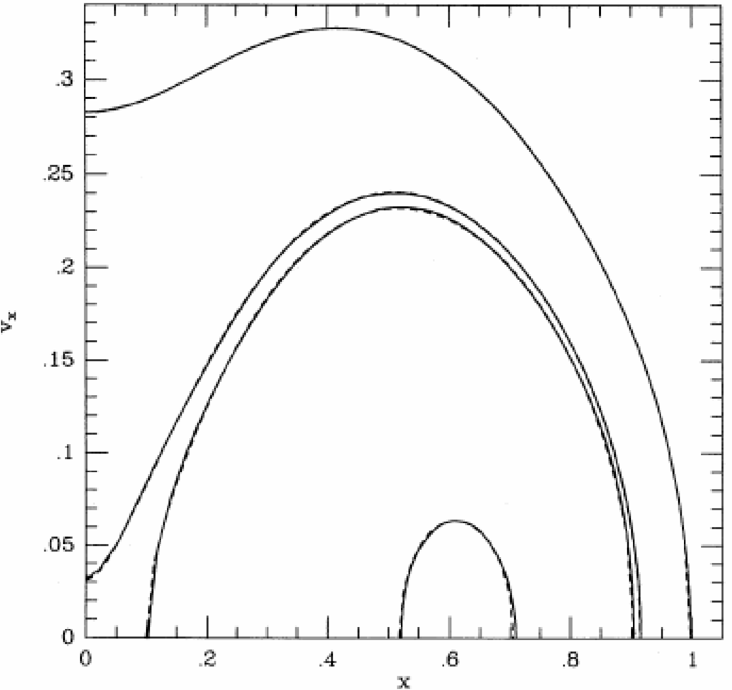
- So make transformation $(J',\theta) \rightarrow (J,\theta)$ by
- $S(\theta,J') = \theta.J' + 2\sum S_n(J') \sin(n.\theta)$
- $J = \partial S/\partial\theta = J' + 2\sum nS_n(J') \cos(n.\theta)$
- The overall transformation $(J',\theta) \rightarrow (x',p_x',..)$ is now general
- (x,y) are not quite bounded by a rectangle, so (x',y') are not quite bounded by ellipsoidal coordinates
- Determine Δ, S_b and parameters in f(u), g(v) to minimize $\langle(H-\langle H\rangle)^2\rangle$ over torus

Kaasalainen & B (1994)





Log Φ

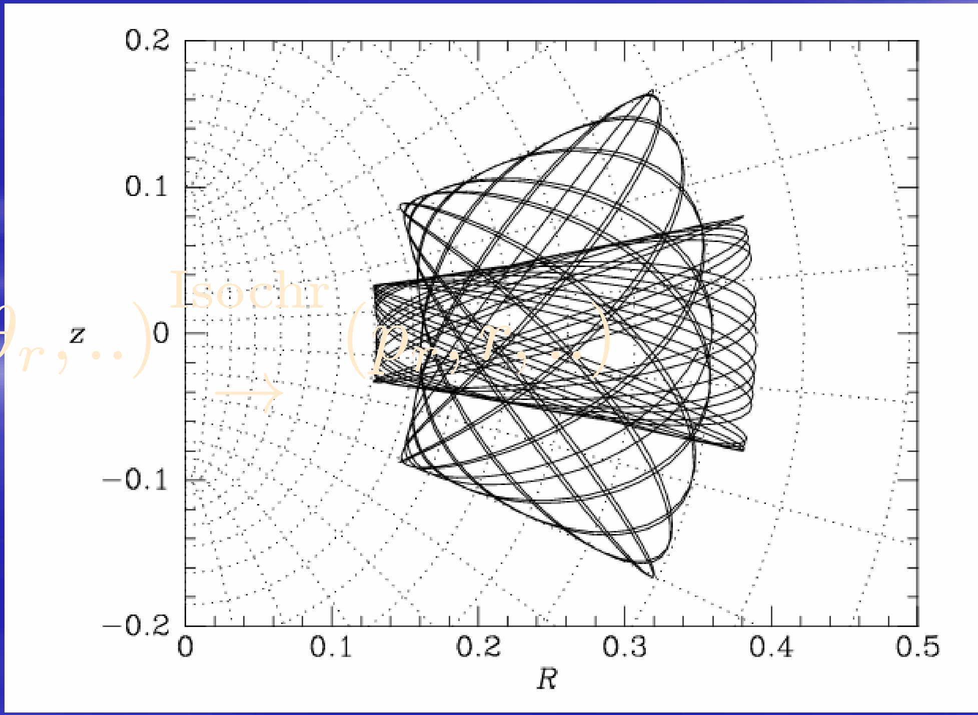


Staeckel Φ

Orbits in Φ(R,z)

- Ignorable $\phi \rightarrow$ motion in (R,z) with $H = p^2/2 + L_z{}^2/2R^2 + \Phi$
- Orbits nearly bounded by (u,v) so can proceed as above
- Or do

$$(J'_r, \theta'_r, ..) \xrightarrow{S=J\theta'+\cdots} (J_r, \theta_r, ..) \xrightarrow{\text{Isochr}} (p_r, r, ..)$$



General $\Phi(x,y,z)$

- No significant modifications required for general Φ (including rotating frame of reference; Kaasalainen 1995)

What have we achieved?

- Analytic formulae $x(J,\theta)$ and $v(J,\theta)$
- So can find at what θ star is at given x & get corresponding v
- If orbit integrated in t, star will just come close, & we have to search for closest x
- Orbit characterized by actions J – essentially unique unlike initial conditions
- Sampling density apparent because $d^6w=(2\pi)^3d^3J$
- The J are adiabatic invariants – useful when Φ slowly evolving (mass-loss, 2-body relax, disc accretion…)

What have we achieved (cont)

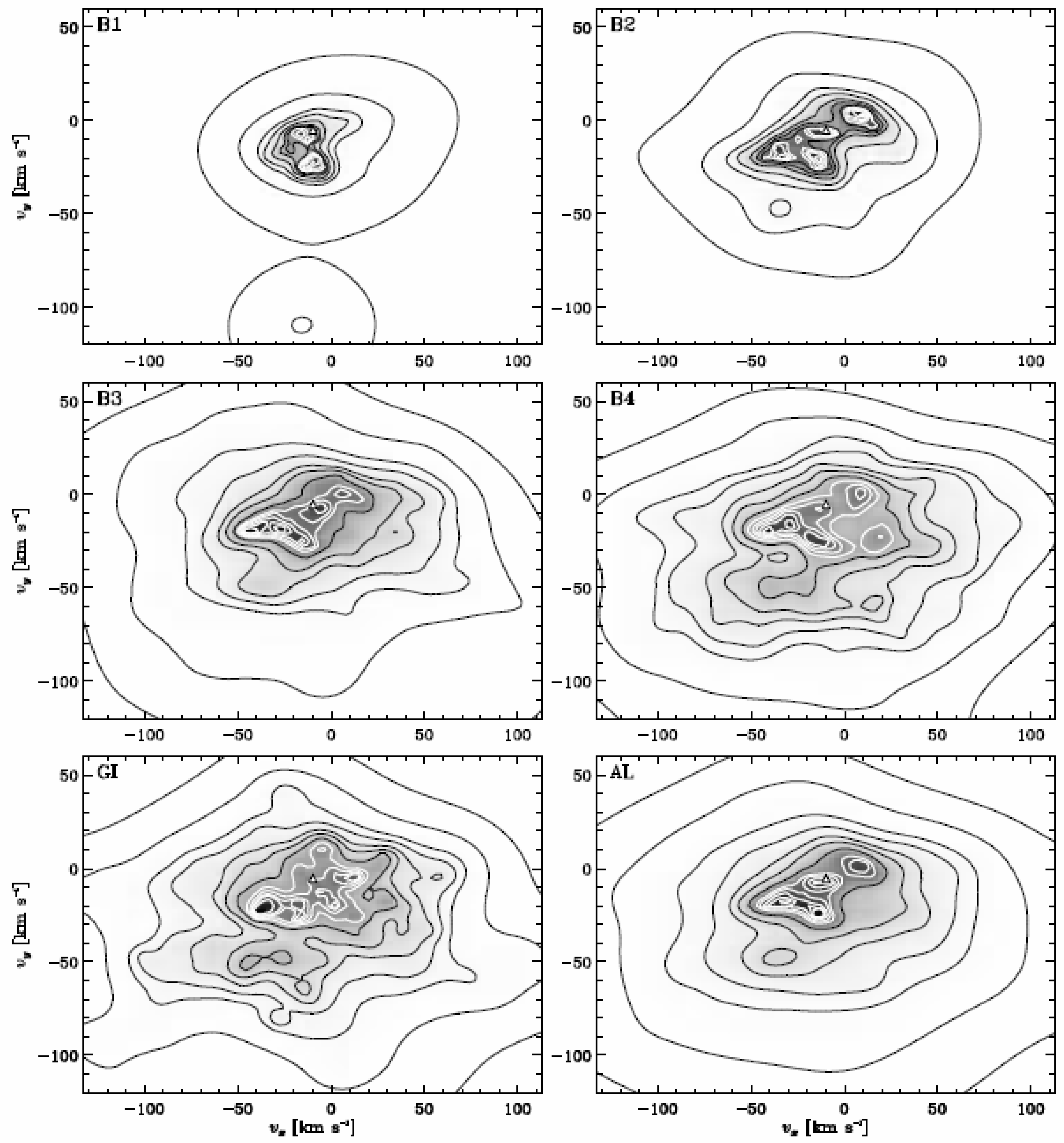
- Real-space characteristics of orbits naturally related to J so can design DF f(J) to give component of specified shape & kinematics (GDII sec 4.6)
- Numerically orbit given by parameters of toy plus point transformations plus <~100 S_n (cf 1000s of $(x,p)_t$ if orbit integrated in t)
- S_n are continuous fns of J, so we can interpolate between orbits
- The likelihood of arbitrary data given a model can be calculated by doing 1-d integral for each star
- Given f(J) have a stable scheme for determining self-consistent Φ
- Fokker-Planck eqn exceptionally simple in a-a coordinates
- We are equipped to do Hamiltonian perturbation theory

Resonances & topology

- Orbit family determined a priori by gross structure of mapping
- Can foliate phase space with tori at will
- Then define integrable $H_0(J)=\langle H\rangle_{\mathcal{J}}$
- $\delta H \equiv H - H_0$ may cause qualitative change when ω_i rationally related
- Orbit said to be "trapped" by resonance

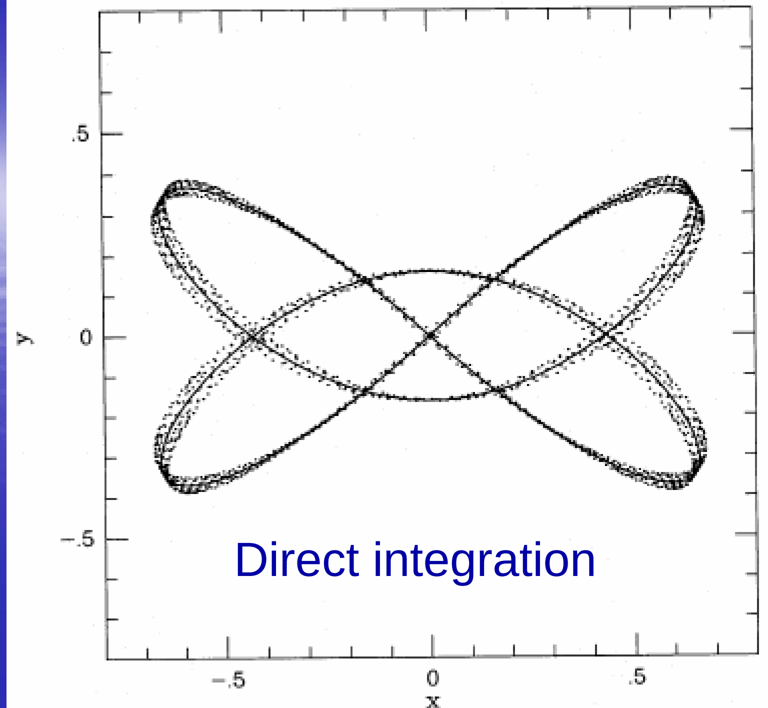
Observ

- Dehnen (1998)
- Trapping by spiral Φ?

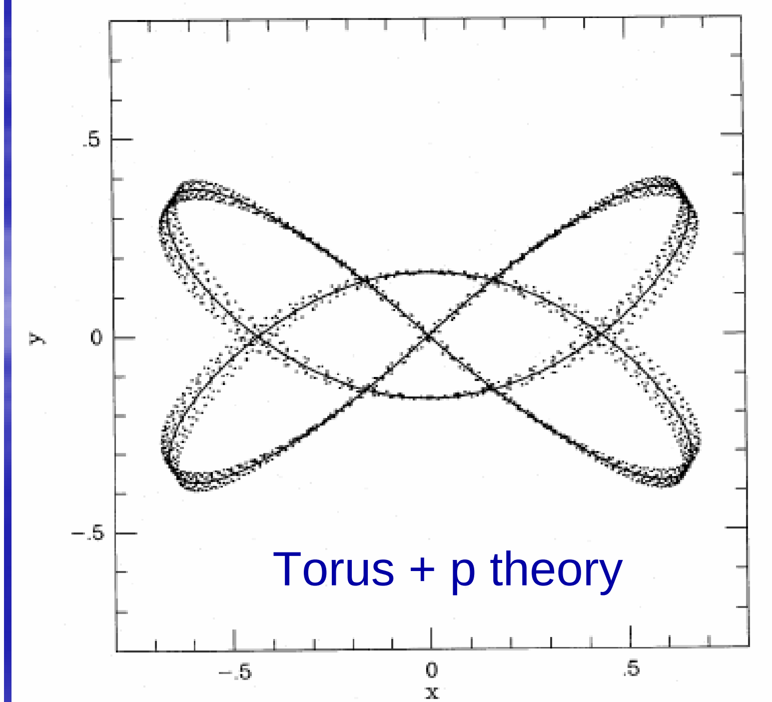


Kaasalainen (1994)

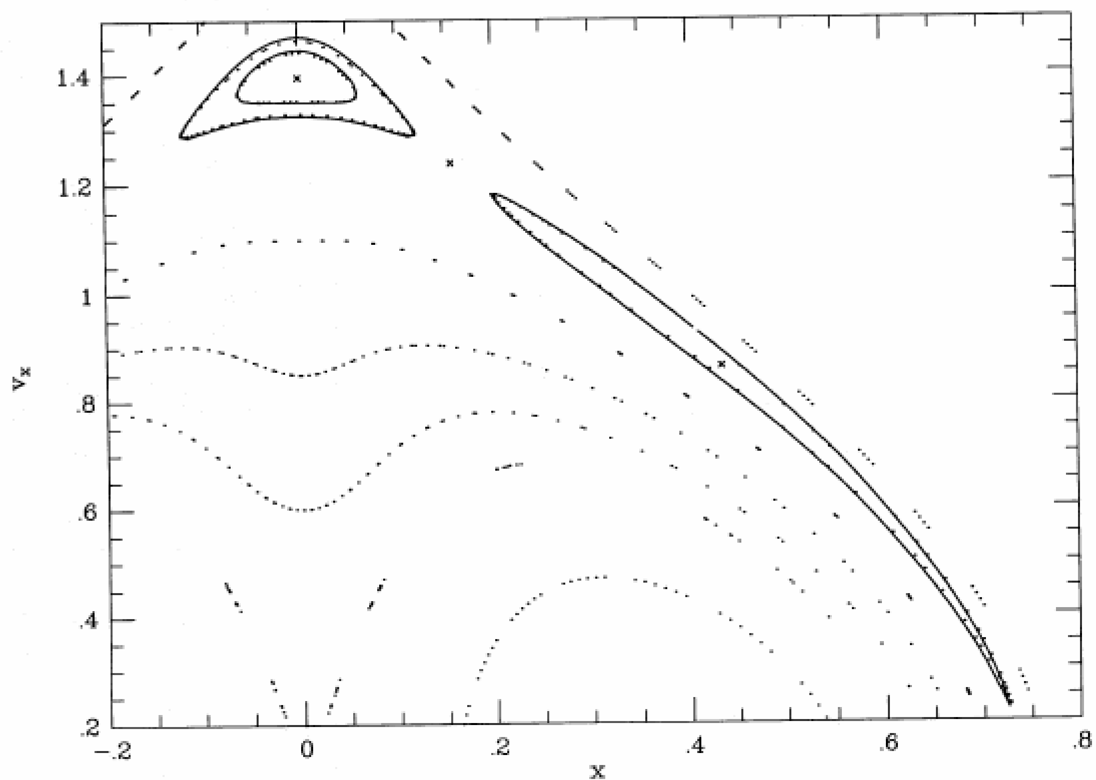
- Standard Hamiltonian theory doesn't work too well



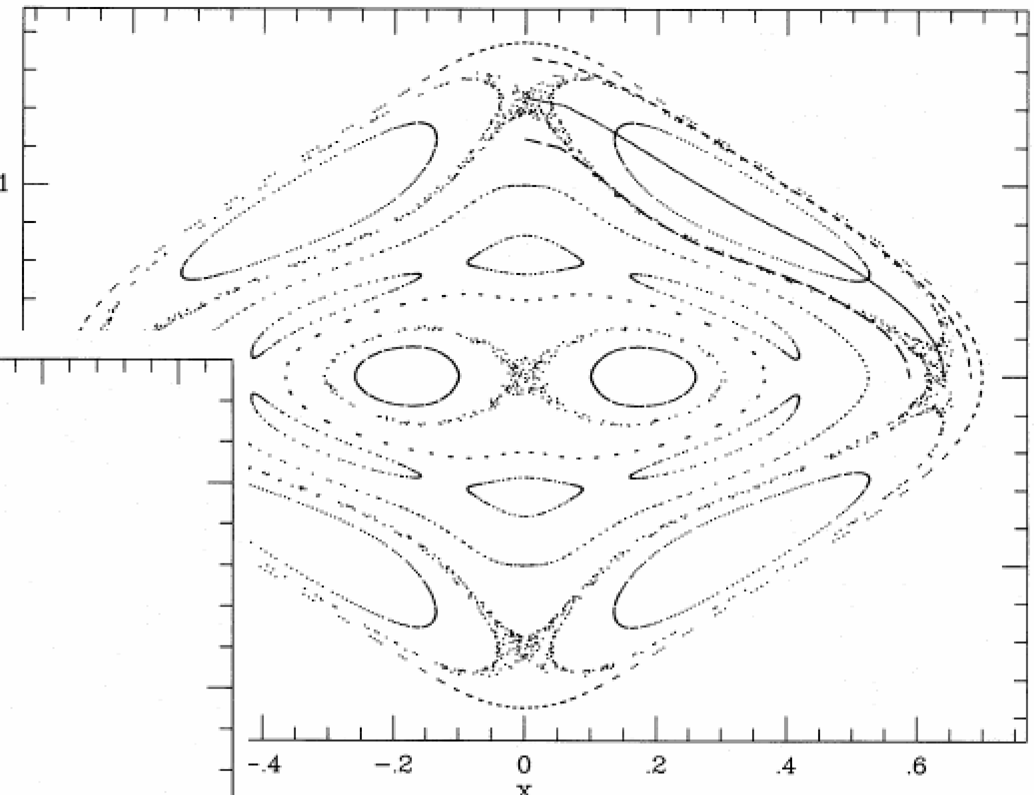
Direct integration

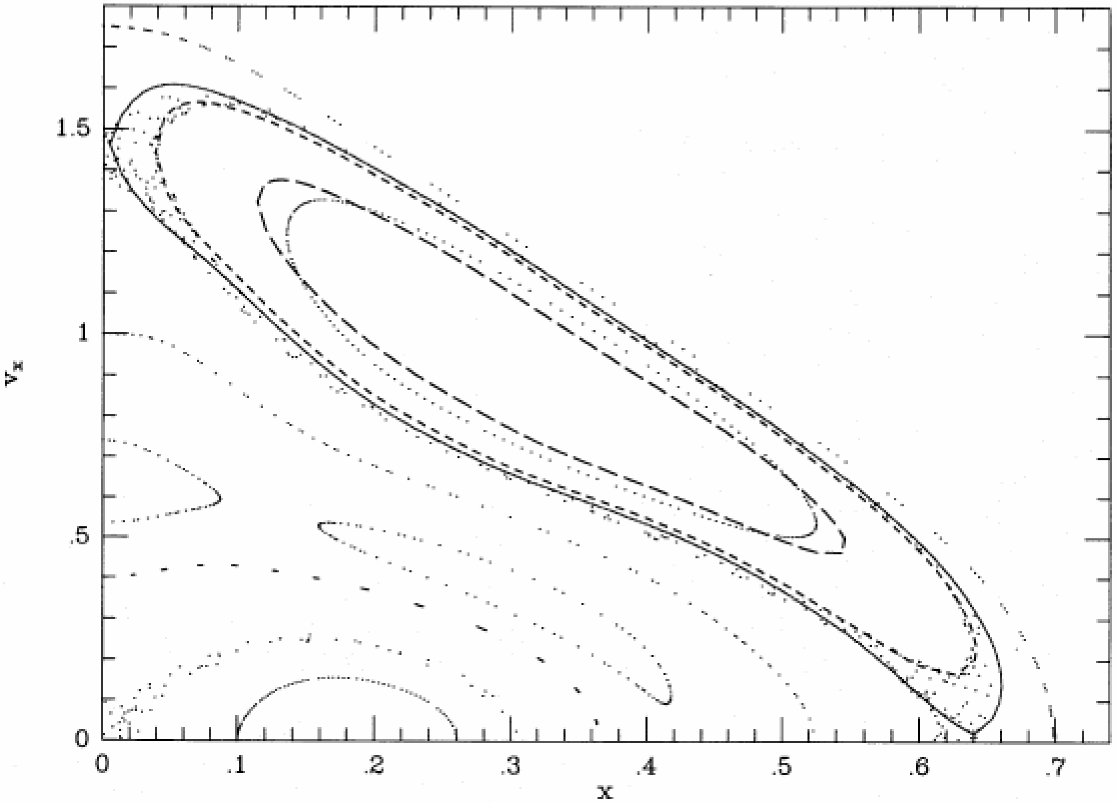


Torus + p theory



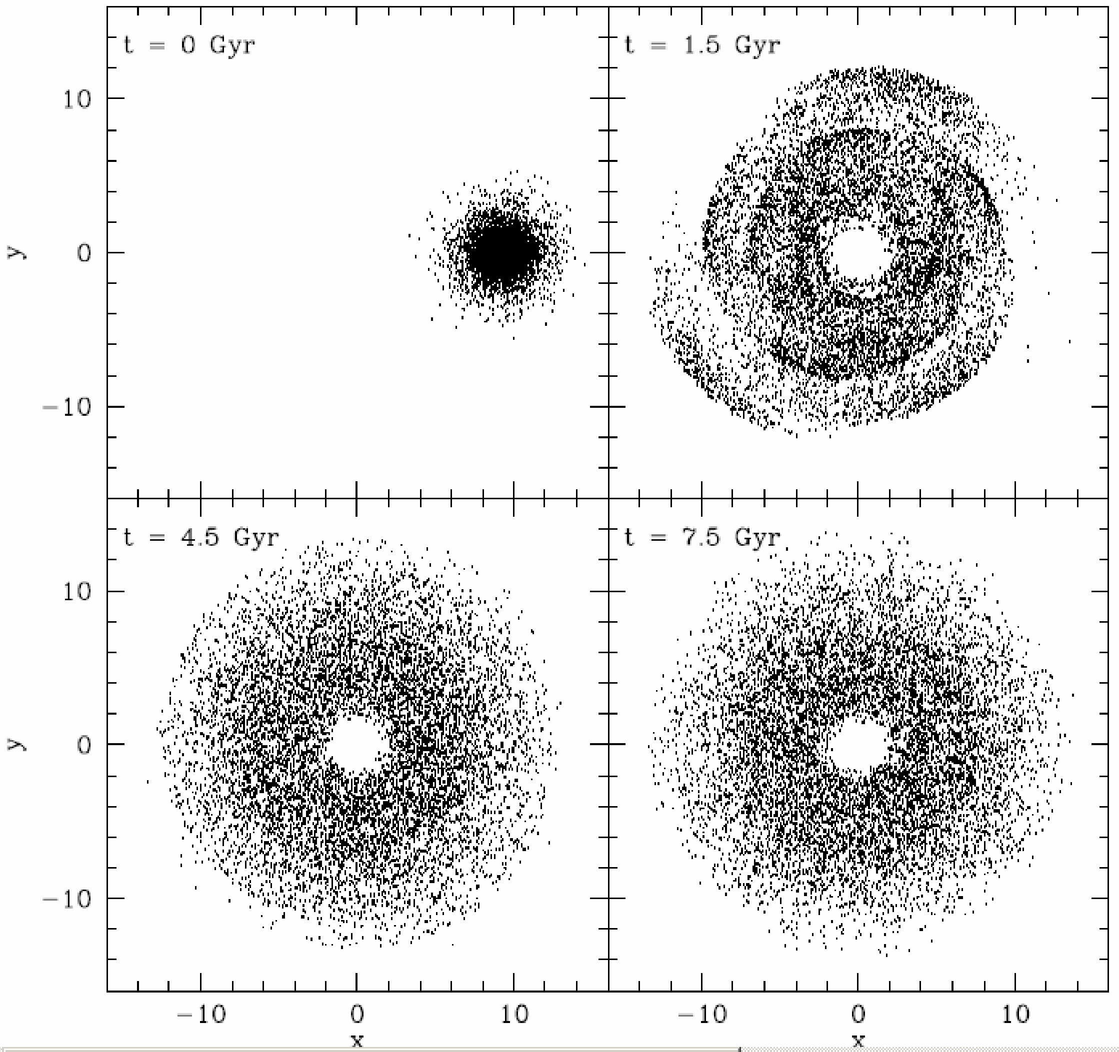
Kaasalainen 1994





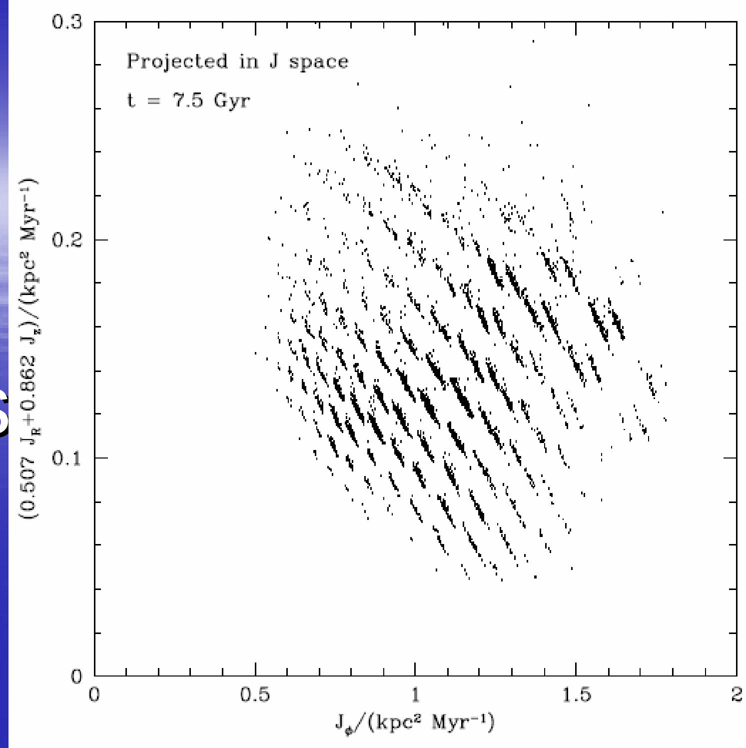
Application to tidal debris
(McMillan & B 08)

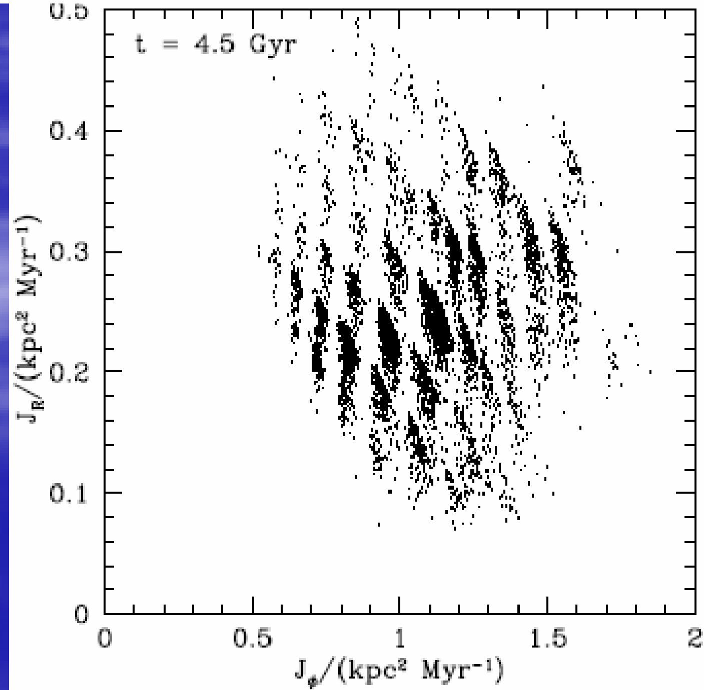
- Helmi et al (06) conjecture for origin of Arcturus stream
- Get (x,v) of stars with d<1.5 kpc
- Obtain $(J_i, \theta_i, \Omega_i)$ for these stars



McMillan & B (cont)

- In phase-phase plane stars clumped
- Signals common origin
- Leads to similar clumping in (J_ϕ, J_R)
- Can sharpen by viewing on axis inclined to J_z axis
- Can use sharpness of clumping to identify Φ





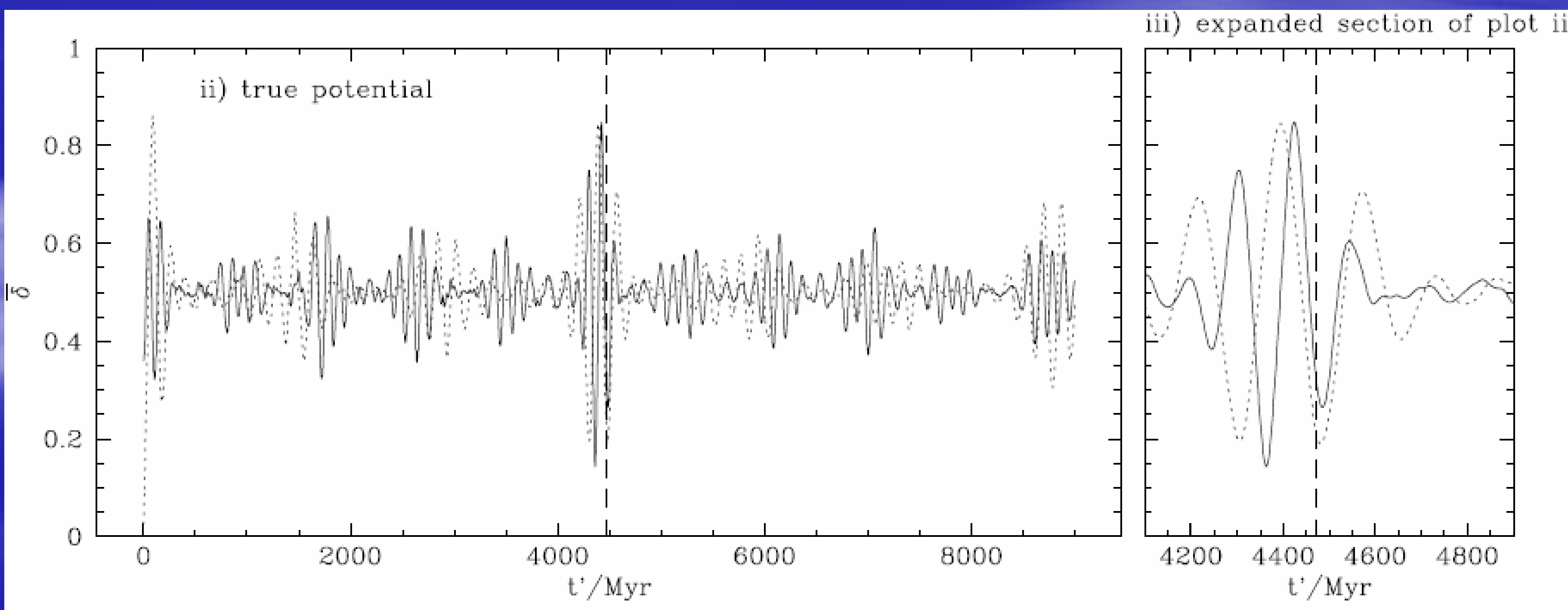
McMillan & B (cont)

- Define

$$\delta_{R,\alpha} = \left| \frac{\Omega_{R,\alpha} t' - (\theta_{R,\alpha} - \theta_{R,0}) - 2\pi m_{R,\alpha}}{\pi} \right|$$

$$\delta_{\phi,\alpha} = \left| \frac{\Omega_{\phi,\alpha} t' - (\theta_{\phi,\alpha} - \theta_{\phi,0}) - 2\pi m_{\phi,\alpha}}{\pi} \right|,$$

- By minimising means of δ_R and δ_ϕ over particles, determine time since cluster disrupted



Conclusions

- Existing analytic or particle based methods inadequate for existing and future surveys
- Particle models seriously limited by Poisson noise, poor characterisation of orbits and sampling problem
- All these difficulties eliminated if time series replaced by tori
- With tori can also
  – use perturbation theory to study fine structure and develop deeper understanding
  – Identify tidally destroyed clusters and determine date of disruption
  – Characterise populations by analytic DFs that evolve in time to reflect SF and secular heating