Skyrmions in quasi-2D chiral magnets

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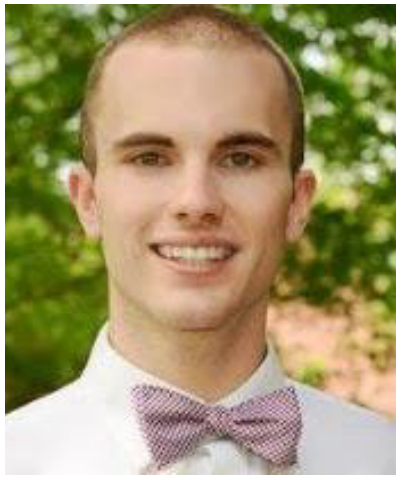
August 2015

MRSEC



James Rowland

Sumilan Banerjee
(now at Weizmann)

Onur Erten
(now at Rutgers)

cem
Center for Emergent Materials
MRSEC

* Banerjee, Erten & MR,
Nature Physics **9**, 626 (2013)

* Banerjee, Rowland, Erten & MR,
Phys. Rev. **X 4**, 031045 (2014)

* Rowland, Banerjee & MR (in preparation, 2015)

Outline:

- Introduction
  * Skyrmions
  * Materials
  * Properties

- How to stabilize skyrmion phases?
  * Role of Dresselhaus vs. Rashba SOC
  * Role of magnetic anisotropy

- Conclusions

- Skyrmions – topological solitons in field theory

A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME †
A.E.R.E., Harwell, England
Nuclear Physics **31** (1962) 556—569

- Skyrmions in magnetism – theory

Metastable states of two-dimensional isotropic ferromagnets

A. A. Belavin and A. M. Polyakov
Gor'kii State University
(Submitted October 4, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **22**, No. 10, 503–506 (20 November 1975)

Thermodynamically stable "vortices" in magnetically ordered crystals. The mixed state of magnets

A. N. Bogdanov and D. A. Yablonskiĭ
Physicotechnical Institute, Donetsk, Academy of Sciences of the Ukrainian SSR
(Submitted 20 April 1988)
Zh. Eksp. Teor. Fiz. **95**, 178–182 (January 1989)

Stabilized by DM interaction & H field

- Skyrmions in magnetism – experiments (~ 2006 onwards)

C. Pfleiderer (Munich), Y. Tokura (Tokyo)

Chiral Magnets

- Ferromagnetic Exchange $-J\ \mathbf{S}_i \cdot \mathbf{S}_j$

- Chiral DM interaction (Dzyaloshinskii-Moriya)

$$\mathbf{D} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

Material constraints for DM:

1. Broken Inversion Symmetry → direction of **D**
2. Spin-orbit coupling (SOC) → magnitude of **D**

Chiral Magnets → Spin textures

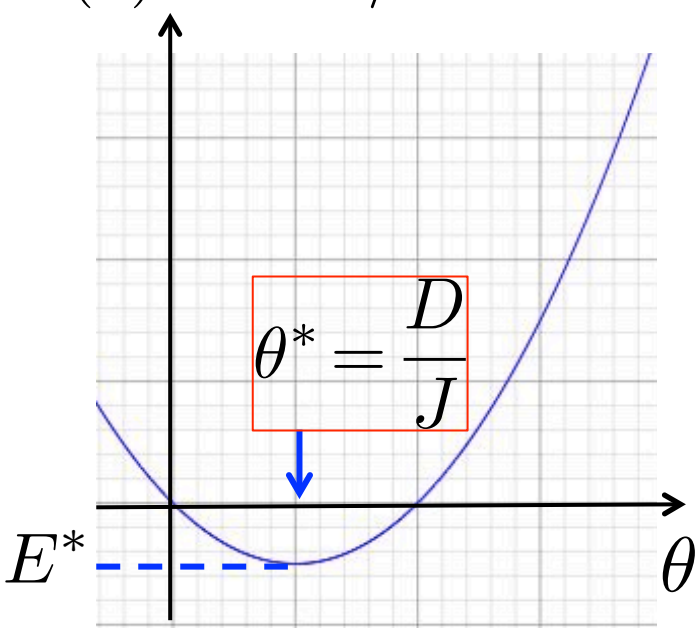
- Ferromagnetic Exchange $-J\ \mathbf{S}_i \cdot \mathbf{S}_j$

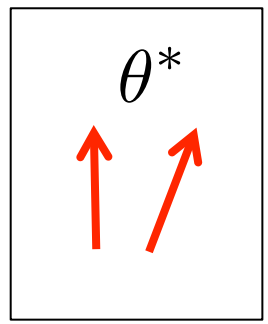
$$\sim -J\cos\theta \sim J\theta^2$$

- DM interaction (Dzyaloshinskii-Moriya)

$$\mathbf{D}\cdot(\mathbf{S}_i \times \mathbf{S}_j) \simeq D\sin\theta \simeq D\theta$$

$$E(\theta) = J\theta^2/2 - D\theta$$





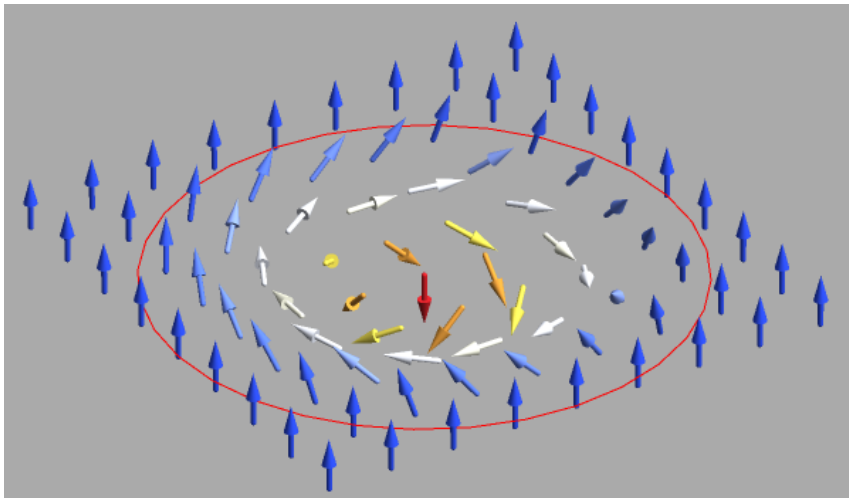
Spontaneous spin textures like Spirals or Skyrmions

Length scale $\left(\frac{J}{D}\right) a \gg a$

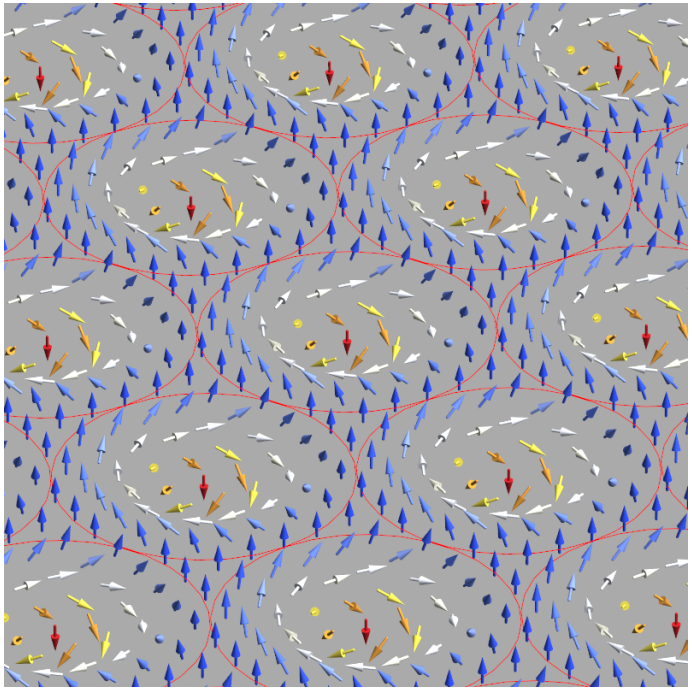
E^* Energy scale D^2/J

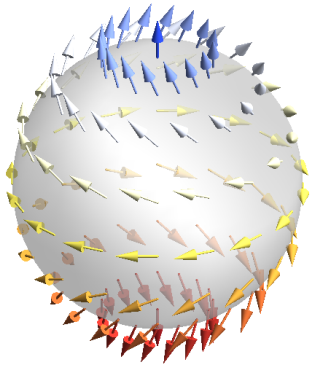
Skyrmion:

Topological spin texture in magnetization $\mathbf{M}(\mathbf{r}) = \mathrm{M}\ \widehat{\mathbf{m}}(\mathbf{r})$



Skyrmion Crystal (SkX)





"Winding Number" on unit sphere in spin-space

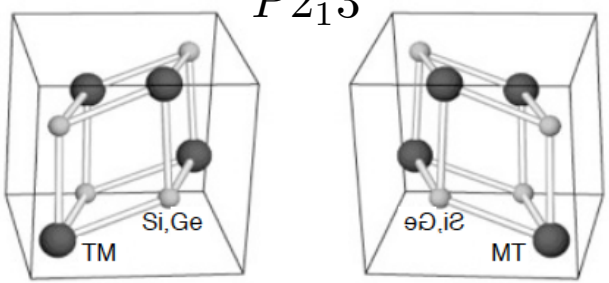
$$Q = \frac{1}{4\pi} \int d^2\mathbf{r}\ \widehat{\mathbf{m}} \cdot (\partial_x \widehat{\mathbf{m}} \times \partial_y \widehat{\mathbf{m}}) = 0, \pm 1, \pm 2, \ldots$$

Topological Invariant

Chiral Magnetic Materials
with broken <u>bulk inversion</u> symmetry

- Non-centrosymmetric Crystals

$P2_13$



B20: no inversion center

B20 structure

Metals: MnSi, $Fe_{1-x}Co_xSi$, FeGe, …

Insulators: Cu_2OSeO_3 …

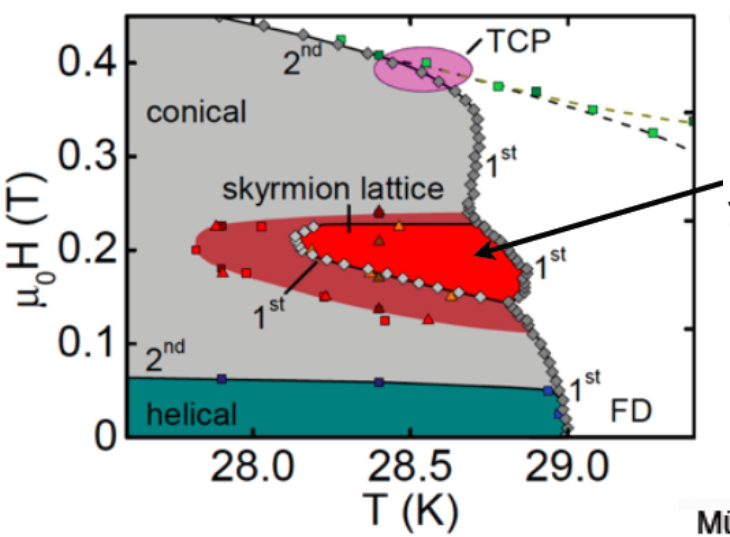
Experiments: Pfleiderer; Tokura …

Theory: Bogdanov; Nagaosa; Rosch …

Spin textures in chiral magnets

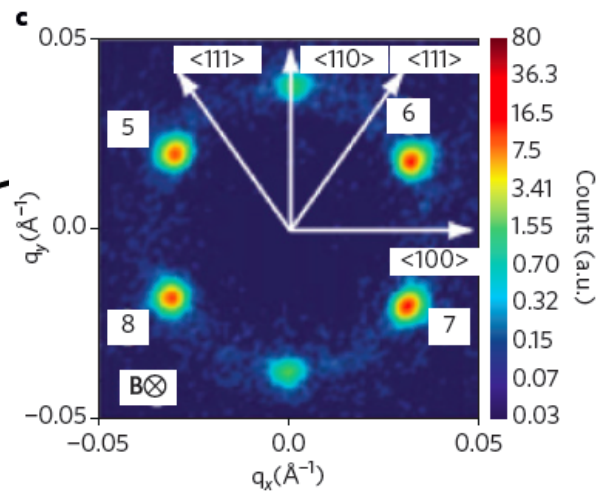
Skyrmion crystal (SkX) probed in **q**-space and in **r**-space

MnSi



Bauer et al, PRL **110**, 177207 (2013)

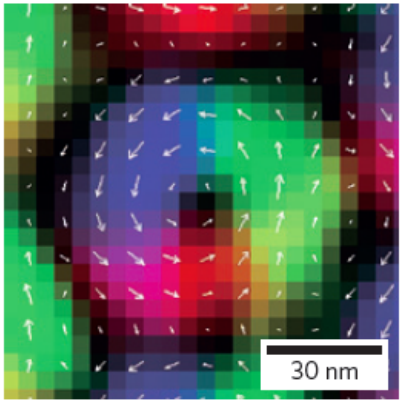
MnSi – SANS



Mühlbauer et al, Science **323**, 915 (2009)

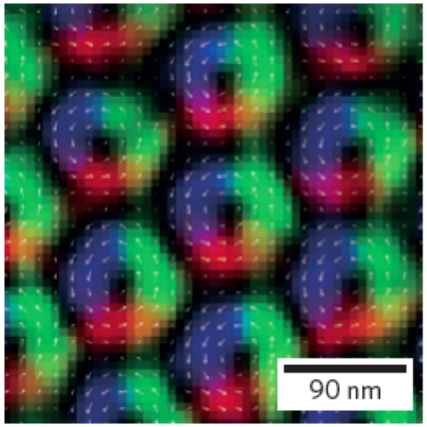
$\lambda \sim 200 \mathring{A}$

FeCoSi – Lorentz TEM

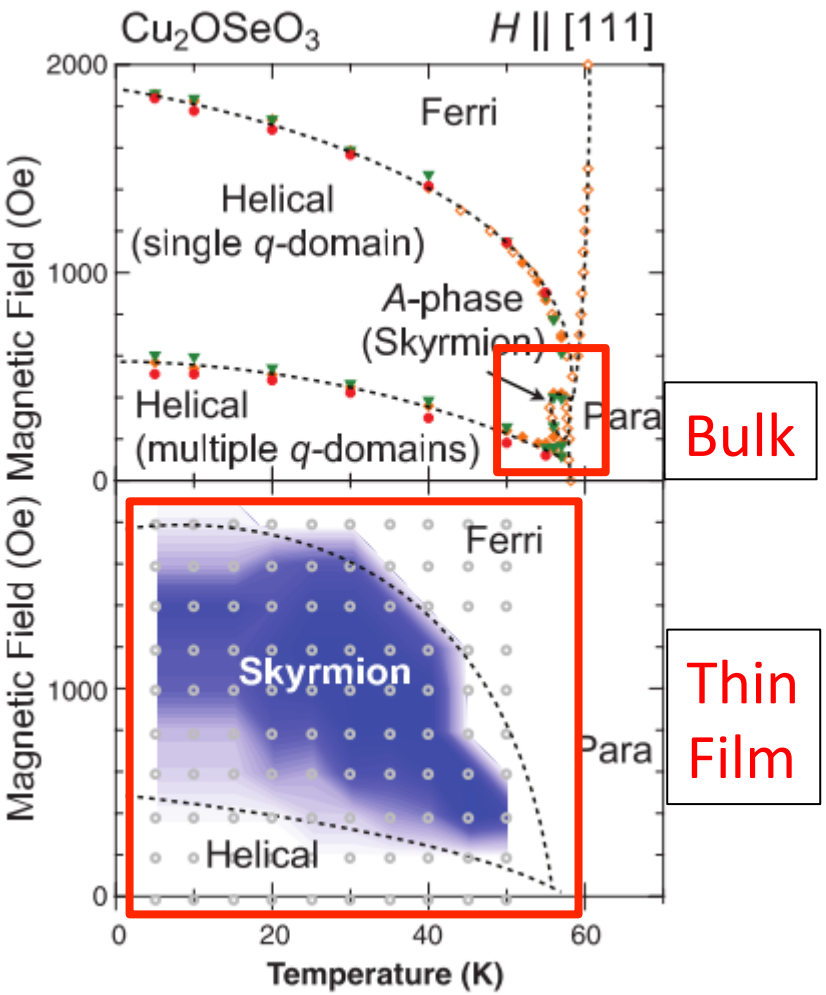


X. Z. Yu et al.,
Nature 465, 901 (2010)

$Fe_{0.5}Co_{0.5}Si$

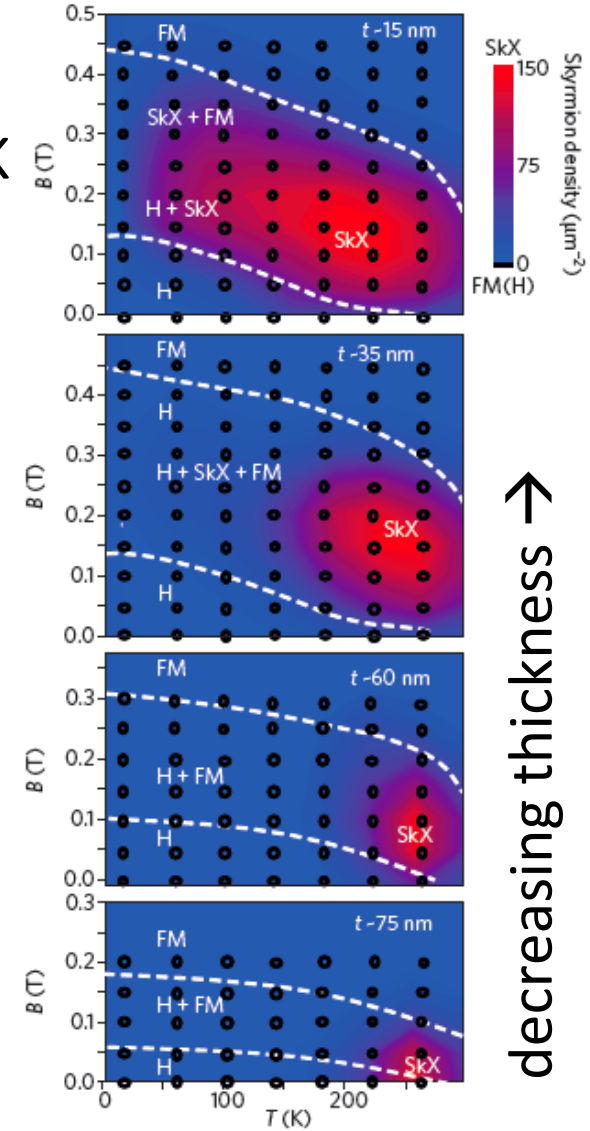


Skyrmions are more stable in thin films



Seki, et. al., Science 336, 198 (2012)

FeGe
Tc = 278K



Multiferroic: Seki et. Al. PRBB 86, 060403 (2012)

Z. Yu et. al., Nature Mat 10, 106 (2011)

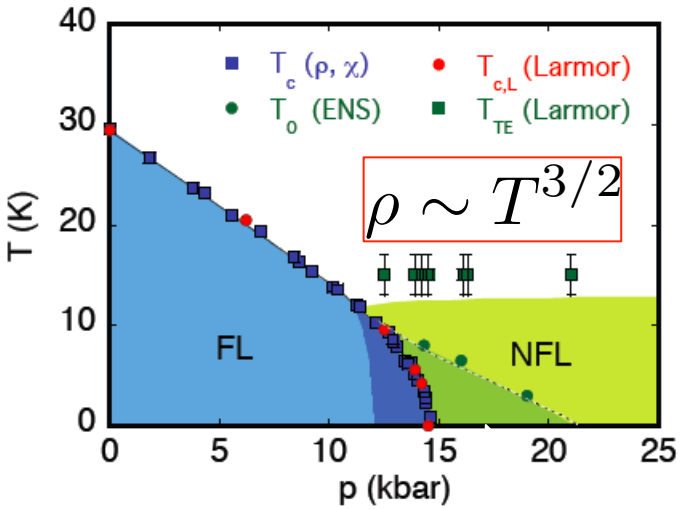
Unusual Properties of Skyrmion phases

Review: Nagaosa & Tokura, Nature Nano **8**, 899 (2013)

* Skyrmions in metallic magnets

- "Emergent electromagnetism"
- Topological Hall effect
- Non-Fermi liquid <u>phase</u>

$T^{3/2}$ resistivity above p_c

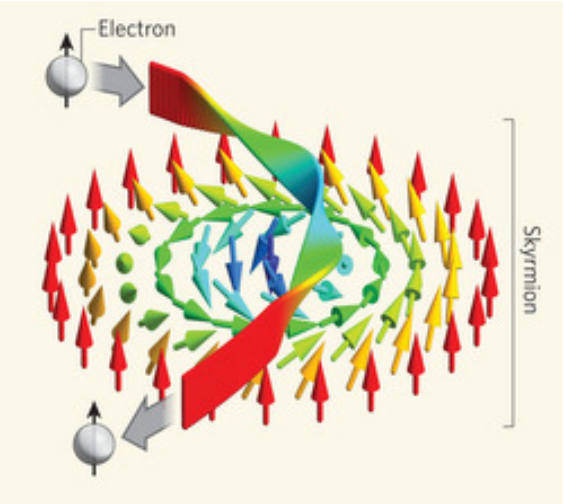


Ritz et al, Nature 497, 231 (2013)

Conduction electrons moving in Skyrmion texture acquire Berry phase

→ Effective E & B fields

For a 10 nm Skyrmion
Effective B ~ 100 T(!)



Potential for novel applications in memory and logic

→ Low pinning compared to domain walls

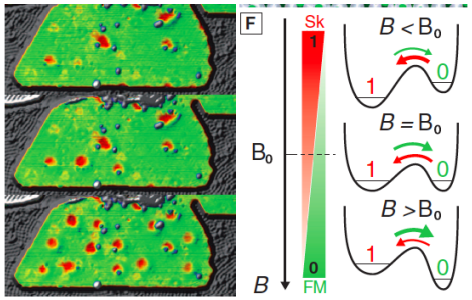
$J_C \sim 10^6 A/m^2$ Skyrmions

vs. $J_C \sim 10^{11} A/m^2$ Domain walls

Jonietz , et al., Science 330, 1648 (2010)
Yu, et al., Nature Comm. 3, 988 (2012)

→ Reading & writing single Skyrmions

Romming et al., Science 341, 636 (2013)



* Proposals for Memories:

Skyrmion "slide cell" magnetic tunnel junction (MTJ) memory

Rahman et al, JAP 111, 07C907 (2012)

Skyrmion "race-track"memory

Koshiabe et al (Tokura-Nagaosa), arXiv:1501.07650 (2015)

Chiral Magnetic Materials
with broken "surface inversion" symmetry

- Thin films of non-centrosymmetric crystals
  -- both bulk & surface inversion broken in general

- 3d Magnetic monolayer on 5d metals with large SOC
  -- nano-skyrmions from competing exchange

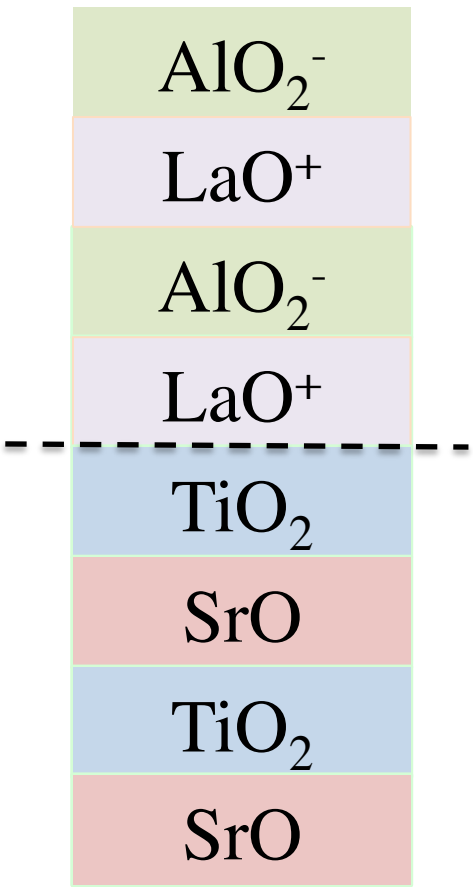
  Experiments: Weisendanger
  Theory: Blugel

- Bulk materials with broken z → -z mirror symmetry
  -- examples with magnetism?

- Oxide interfaces: e.g. LAO/STO
  -- tantalizing hints of magnetism

A quick introduction to oxide interfaces

Emergent phenomena at the interface leads to properties totally different from either bulk material



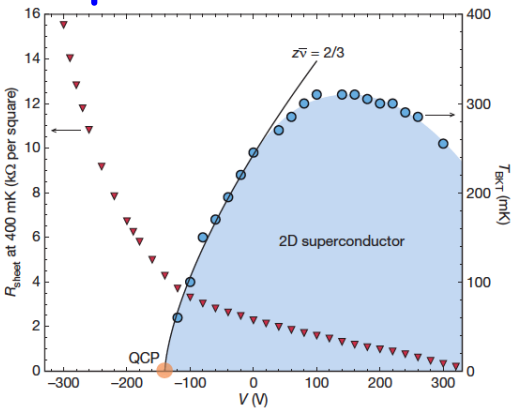
Interface

$LaAlO_3$ (LAO)
E_g = 5.6 eV

$SrTiO_3$ (STO)
E_g = 3.2 eV

Ohtomo & Hwang, Nature (2004)

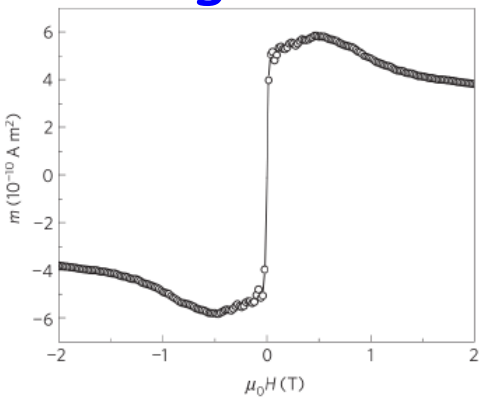
Superconductivity



Caviglia et al, Nature (2008)

$T_c(n)$

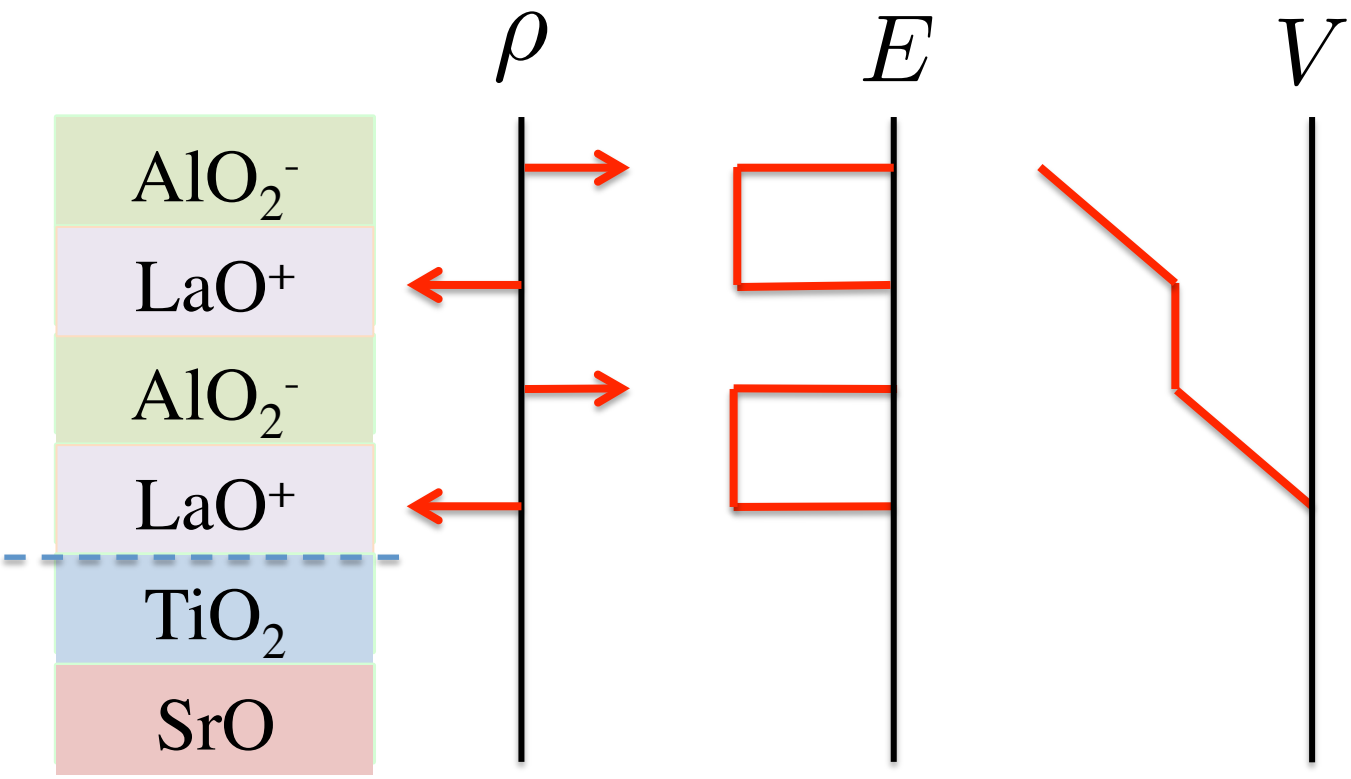
Magnetism



Li et al, Nat. Phys (2008)

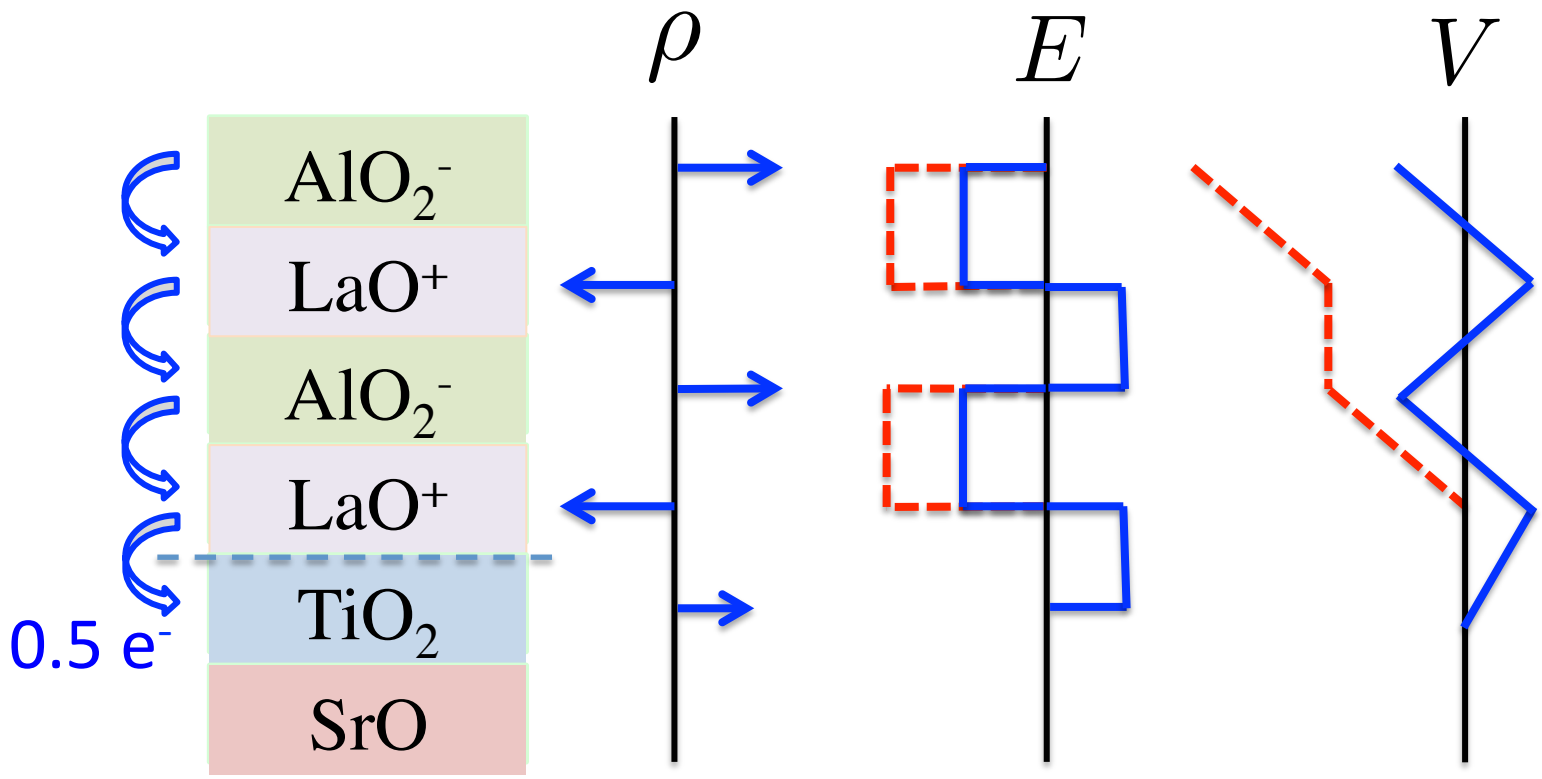
M(H)

Where do the electrons come from?



Polar Catastrophe → Electronic reconstruction

Where do the electrons come from?

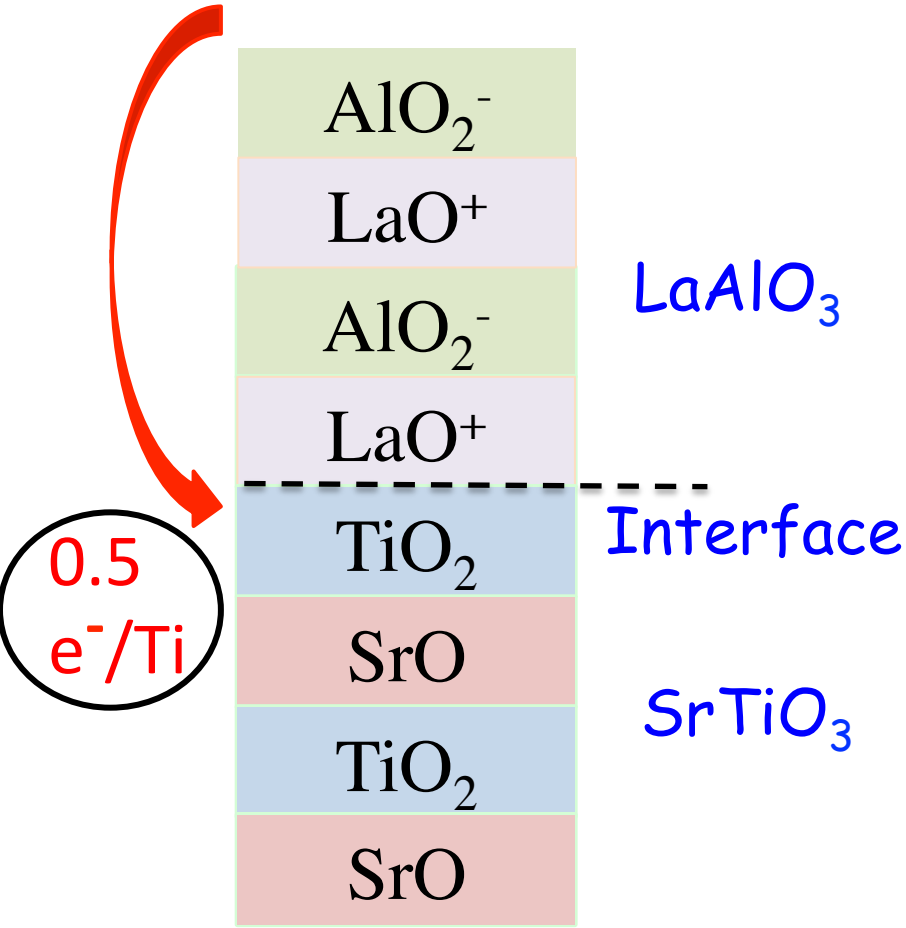


Polar Catastrophe → 0.5 e/Ti → n_{2D}~ 3.3 x 10^{14} cm^{-2}

+ additional carriers from Oxygen vacancies

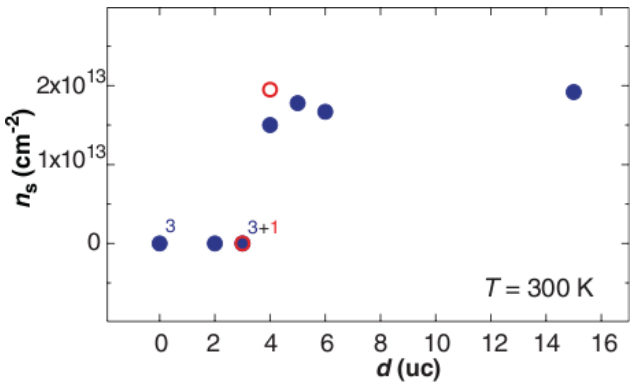
LAO/STO Interface

Polar catastrophe



Ohtomo & Hwang, Nature (2004)

2DEG



Thiel et al, Science (2006)

Hall → n

Polar Catastrophe
= $n_{2D} \sim 3.3 \times 10^{14}$ cm^{-2}

+ <u>additional</u> electrons from O-vacancies

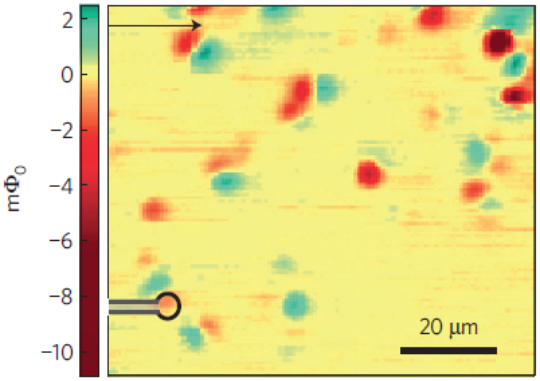
Hall → mobile carriers
$n_{2D} = 2 \times 10^{13}$ cm^{-2}
~ 10% of polar catastrophe

Where are the missing electrons?

If the polar catastrophe does occur in LAO/STO, where are the electrons not seen in transport?

Answers -- not certain
-- seem to be sample & probe dependent

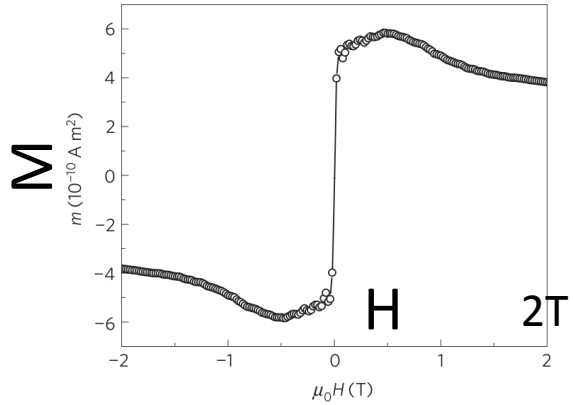
However, many experiments indicate large density of Localized electrons that behave like local moments



Scanning SQUID

Bert et al. Nature Phys. (2011)

* Inhomogeneous; M=0
* Isolated micron-size patches of in-plane FM
* Susceptometry: local moments ~ 0.5 e/Ti



Torque Magnetometry

Li et al, Nature Phys (2011)

* M ≈ 0.3 - 0.4μ_B /Ti
* Exchange scale ~ 100K
* No hysteresis

Spectroscopy: Ti d^1 states

XPS Sing et al, PRL (2009)

XMCD ~ 0.1 e/Ti
Lee et al, Nature Mat. (2013)

MFM: Room Temp FM

Bi et al,
Nature Comm. (2014)

LAO/STO summary

* 2D interface

* Broken inversion z → -z

* Rashba spin-orbit coupling $\mathcal{H}_{\rm soc} = \lambda_{\rm soc}\, \widehat{\mathbf{z}} \cdot (\mathbf{k} \times \sigma)$
  tunable by gate voltage [Expt: Caviglia et al., PRL (2010)]

*Many experiments see evidence for magnetism

* Disorder & inhomogeneity

* sample-to-sample variations & many open questions!

<u>If</u> there is magnetism at oxide interfaces,
then we have all the ingredients for Spirals & Skyrmions.

Outline:

- Introduction
  * Skyrmions
  * Materials
  * Properties

- How to stabilize skyrmion phases?
  * Role of Dresselhaus vs. Rashba SOC
  * Role of magnetic anisotropy

- Conclusions

Form of the DM Interaction $\mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$

Symmetry → direction of DM vector → Form of DM terms in continuum "Ginzburg-Landau" Theory

- broken bulk inversion
  → Dresselhaus SOC

  → $\widehat{\mathbf{D}}_{ij}^{D} = \widehat{\mathbf{r}}_{ij}$ → $-D_D \; \widehat{\mathbf{m}} \cdot (\nabla \times \widehat{\mathbf{m}})$

- broken surface inversion
  → Rashba SOC

  → $\widehat{\mathbf{D}}_{ij}^{R} = \widehat{\mathbf{z}} \times \widehat{\mathbf{r}}_{ij}$ → $-D_R \; \widehat{\mathbf{m}} \cdot [(\widehat{\mathbf{z}} \times \nabla) \times \widehat{\mathbf{m}}]$

Continuum field theory

$$\mathcal{F}\left[\widehat{\mathbf{m}}(\mathbf{r})\right] = \frac{J}{2}|\nabla\widehat{\mathbf{m}}|^2$$ FM exchange

$$-D_D\ \widehat{\mathbf{m}}\cdot(\nabla\times\widehat{\mathbf{m}})$$ DM -- Dresselhaus

$$-D_R\ \widehat{\mathbf{m}}\cdot\left[(\widehat{\mathbf{z}}\times\nabla)\times\widehat{\mathbf{m}}\right]$$ DM -- Rashba

$$+\ A\ m_z^2$$ Anisotropy

$$-\ H\ m_z$$ Field

Minimize $\mathcal{F}$ subject to the constraint $|\widehat{\mathbf{m}}(\mathbf{r})|^2 = 1$

→ Optimal $\widehat{\mathbf{m}}(\mathbf{r})$

Magnetic Anisotropy

$$+ A\, m_z^2$$

A > 0 Easy-plane
A < 0 Easy-axis

Sources of Anisotropy: Single-ion + Dipolar + … + Rashba SOC

Broken z-inversion → Rashba SOC

* Symmetry allows an easy-plane Compass-Kitaev term

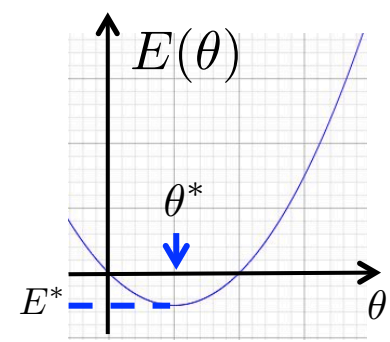
$$-A_c \sum_i (S_i^y S_{i+\hat{x}}^y + S_i^x S_{i+\hat{y}}^x)$$

* Easy-plane anisotropy with <u>same</u> microscopic origin as DM

Ignored in literature $A_c \sim \mathcal{O}\left(\lambda_{\rm soc}^2\right) \ll D \sim \mathcal{O}\left(\lambda_{\rm soc}\right)$

However, both A_c and D impact energy at the <u>same</u> order $A_c \sim D^2/J$

→ Cannot ignore A_c!



Microscopic Interlude I: Rashba SOC + Double exchange

$$\mathcal{H}_0 = -t \sum_{<ij>,\alpha} c^\dagger_{i\alpha} c_{j\alpha} - i\lambda \sum_{<ij>,\alpha\beta} \boldsymbol{\sigma}_{\alpha\beta}.\hat{\mathbf{d}}_{ij} c^\dagger_{i\alpha} c_{j\beta} + \text{h.c.}$$

with $\hat{\mathbf{d}}_{ij} = \hat{\mathbf{z}} \times \hat{\mathbf{r}}_{ij}$

$$\mathcal{H}_{\text{int}} = -\frac{J_{\text{H}}}{2} \sum_{i\alpha\beta} c^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta} \cdot \mathbf{S_i}$$

To derive effective H, consider two-site problem:

$$\tilde{t} = \sqrt{t^2 + \lambda^2}$$

$$\mathcal{H}_0 = -\tilde{t} \sum_{\alpha\beta} (c^\dagger_{i\alpha} [e^{i\vartheta \boldsymbol{\sigma}.\hat{d}_{ij}}]_{\alpha\beta} c_{j\beta} + \text{h.c.})$$

$$\tan\vartheta = \lambda/t$$

SU(2) "Gauge Transformation" $\widetilde{c}_{i\alpha} = [e^{-i(\vartheta/2)\boldsymbol{\sigma}.\hat{d}_{ij}}]_{\alpha\beta} c_{i\beta}$ etc.

In the "rotated" basis

$$\mathcal{H}_0 = -\widetilde{t} \sum_{<ij>,\alpha} \widetilde{c}^\dagger_{i\alpha} \widetilde{c}_{j\alpha} \qquad \mathcal{H}_{\text{int}} = -J_{\text{H}} \sum_i \widetilde{\mathbf{s}}_i \cdot \widetilde{\mathbf{S}}_i$$

Microscopic Interlude II: Rashba SOC + Double exchange

$$\mathcal{H}_0 = -\widetilde{t} \sum_{<ij>,\alpha} \widetilde{c}^\dagger_{i\alpha} \widetilde{c}_{j\alpha} \qquad \mathcal{H}_{\rm int} = -J_{\rm H} \sum_i \widetilde{\mathbf{s}}_i \cdot \widetilde{\mathbf{S}}_i$$

This is the standard Anderson-Hasegawa problem -- in the transformed variables

$$J_{\rm H} \to \infty$$

Classical $\mathbf{S}'_i s$

$$\mathcal{H}_{\rm DE} = -J_{\rm F} \sum_{<ij>} \left[1 + \mathbf{S_i} \cdot \mathcal{R}(\mathbf{2}\vartheta \hat{\mathbf{d}}_{\mathbf{ij}}) \mathbf{S_j} \right]^{1/2}$$

With $J_F \sim \widetilde{t}$

Expand square root to get three terms:

FM exchange $J = J_{\rm F} \cos 2\vartheta$

Rashba-DM $D_{\rm R} = J_{\rm F} \sin 2\vartheta$

Compass Anisotropy $A_c = J_{\rm F}(1 - \cos 2\vartheta)$

$$\boxed{A_c J / D^2 = 1/2}$$

$$\lambda \ll t$$

Microscopic Interlude: summary

Rashba SOC → significant easy-plane anisotropy

For a wide variety of exchange mechanisms
in both metals & insulators with Rashba SOC

$$\boxed{A_c|J|/D^2 = 1/2} \qquad (\lambda_{\rm soc} \ll t)$$

Exchange = J Rashba DM = D_R compass anisotropy = A_c

for "pair-wise exchange" mechanisms
Banerjee, Rowland Erten & MR, PRX 4, 031045 (2014)

* AFM Superexchange + SOC Moriya, PR (1960); Shekhtman et al, PRL (1992)
* RKKY + SOC Imamura et al, PRB (2004)
* Double exchange + SOC Banerjee, Erten & MR, Nature Phys. (2013)
* Bosons + SOC Cole, Zhang, Paramekanti & Trivedi, PRL (2012)

T=0 Phase Diagram as function of

* Anisotropy AJ/D^2

* Field HJ/D^2

* Rashba/Dresselhaus

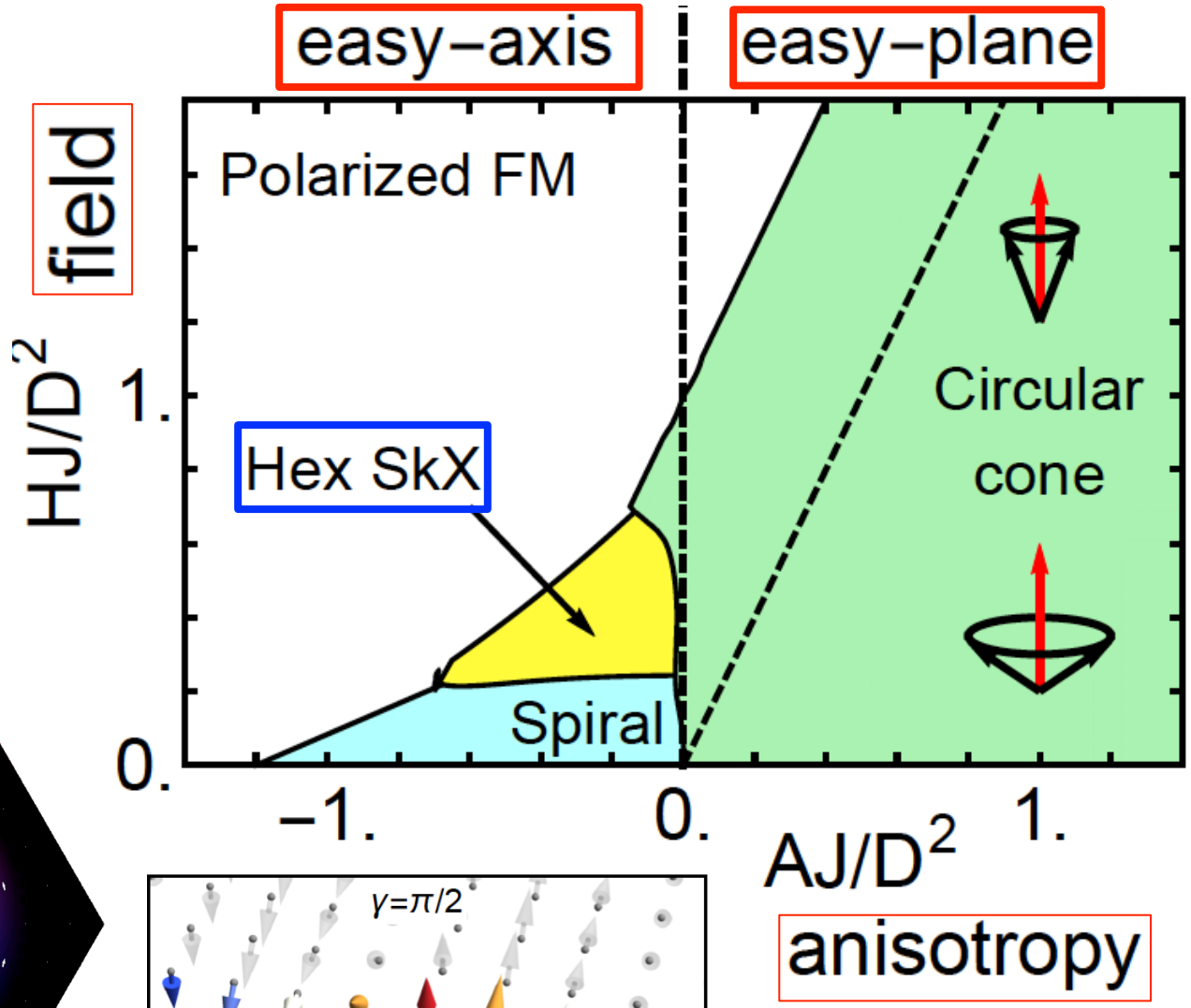
Minimize continuum Energy functional $\mathcal{F}$

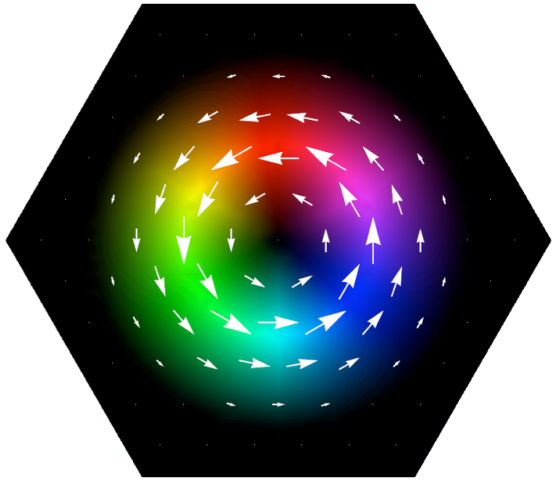
* Variational calculations -- analytical
* Numerical minimization -- conjugate gradient

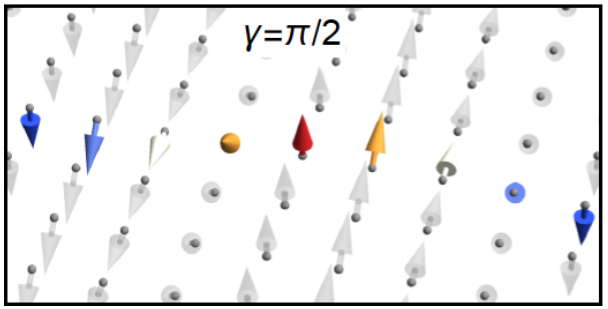
* "Classical" analysis

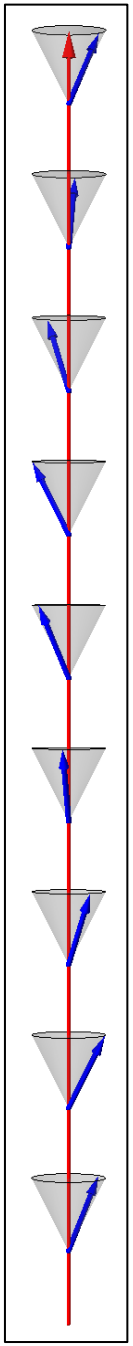
Broken Bulk Inversion: Dresselhaus SOC

Wilson, Butenko, Bogdanov & Monchesky, PRB 89, 094411 (2014).







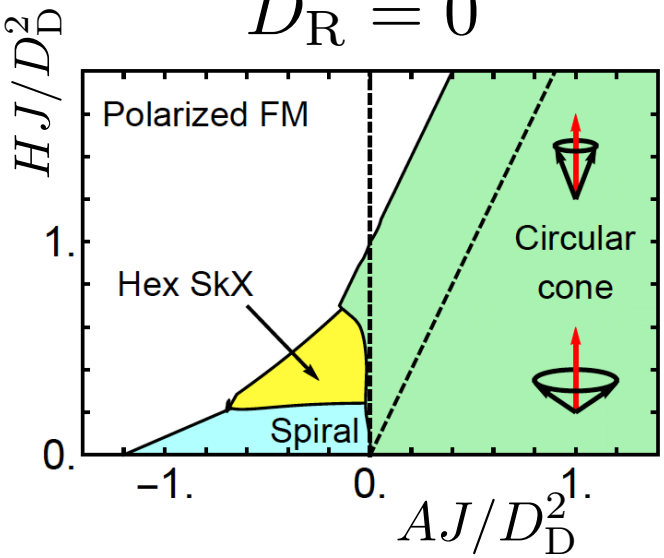


Stability of Spiral & Skyrmion crystal enhanced with Rashba

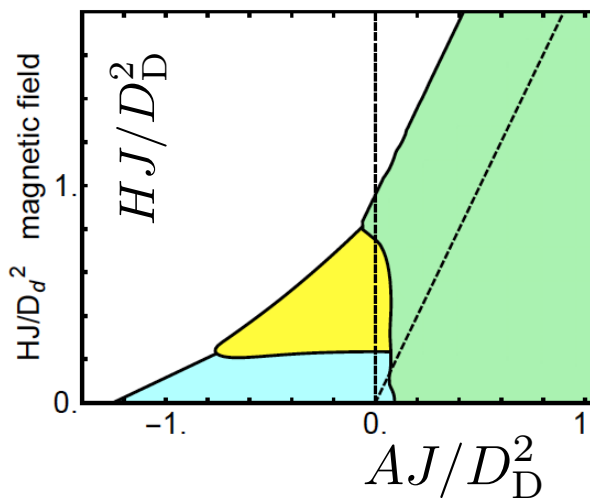
Dresselhaus limit

Increasing Rashba →

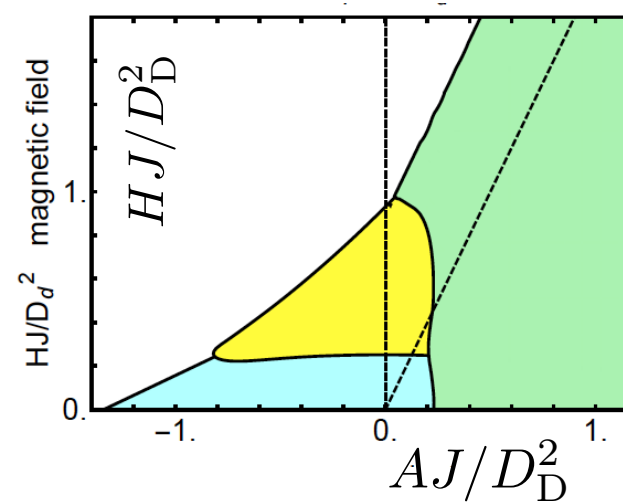
$D_{\mathrm{R}} = 0$



$D_{\mathrm{R}} = 0.3D_{\mathrm{D}}$



$D_{\mathrm{R}} = 0.5D_{\mathrm{D}}$



Circular cone: $\mathbf{m}(z)$ → gains energy <u>only</u> from D_{D}

$$\left.\begin{array}{r}\text{Spiral: } \mathbf{m}(x) \\ \text{Skyrmion: } \mathbf{m}(x,y)\end{array}\right\} \rightarrow \; D = \sqrt{D_{\mathrm{D}}^2 + D_{\mathrm{R}}^2}$$

Analysis of Spiral & Skyrmion Phases where $\partial_z \equiv 0$

$$\mathbf{m}(x, y)$$

Define: $D_{\mathrm{D}} = D \sin\gamma$

$D_{\mathrm{R}} = D \cos\gamma$

Rewrite the sum of two DM terms
in a simple form via change of variables

$$\mathbf{m} \to \mathbf{n} \qquad \mathbf{n} = \mathcal{R}_{\hat{z}}\left(\pi/2 - \gamma\right)\mathbf{m}$$

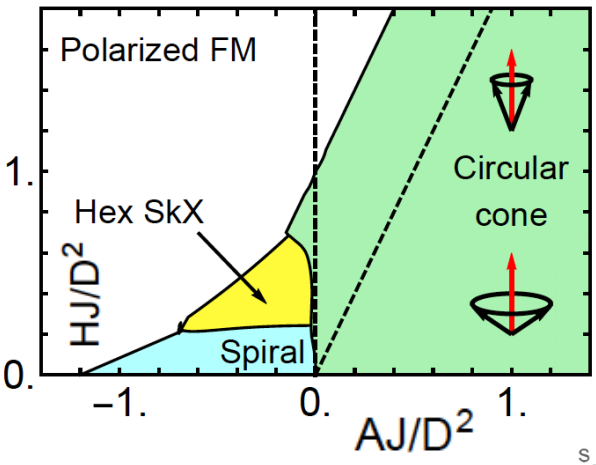
$$\boxed{\begin{aligned} -D_{\mathrm{D}}\ \mathbf{m}\cdot(\nabla\times\mathbf{m}) - D_{\mathrm{R}}\ \mathbf{m}\cdot\left[(\widehat{\mathbf{z}}\times\nabla)\times\mathbf{m}\right] \\ = -D\,\mathbf{n}\cdot(\nabla\times\mathbf{n}) \end{aligned}}$$

With $D = \sqrt{D_{\mathrm{D}}^2 + D_{\mathrm{R}}^2}$ and $\gamma = \tan^{-1}\left(\dfrac{D_{\mathrm{D}}}{D_{\mathrm{R}}}\right)$ = "helicity"

Evolution from Dresselhaus → Rashba

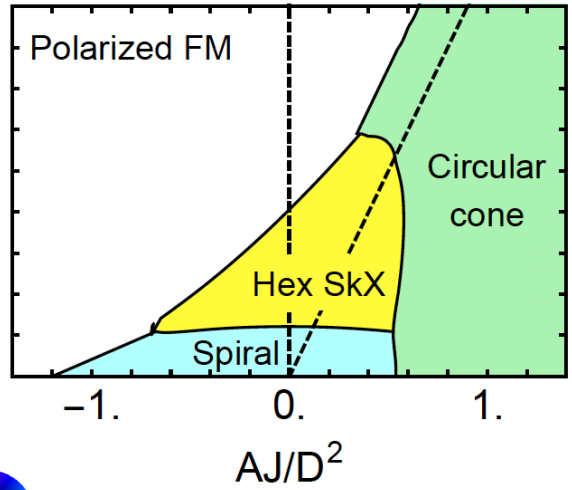
$\gamma = \pi/2$

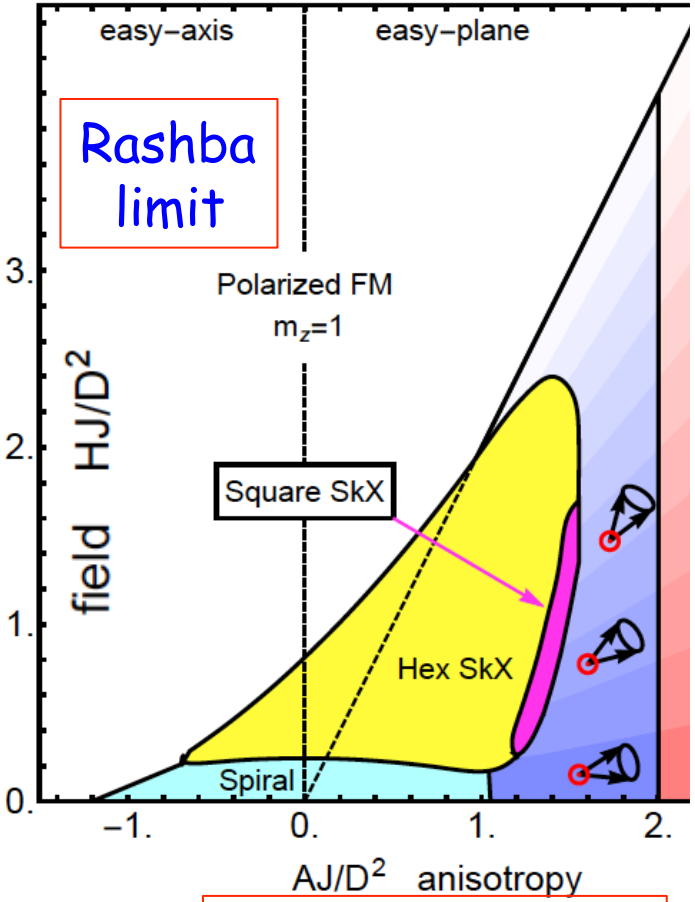
Dresselhaus limit



$\gamma = \pi/4$

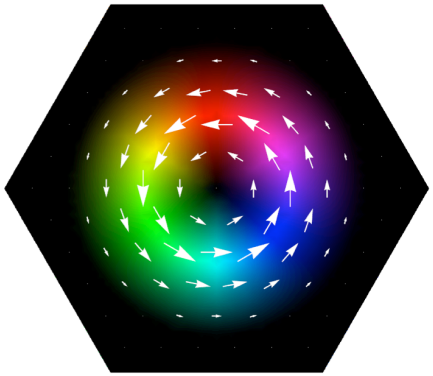
Rashba = Dresselhaus

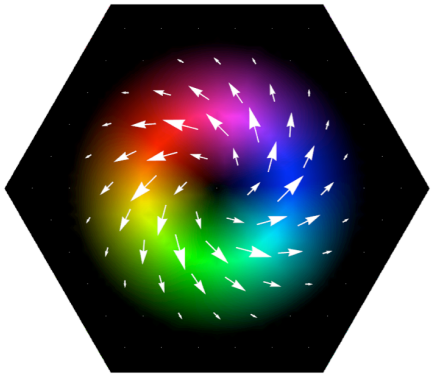


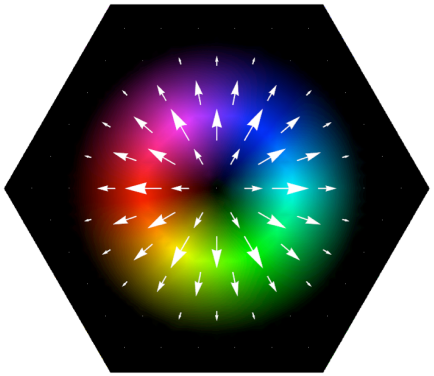


Vortex-like / Bloch

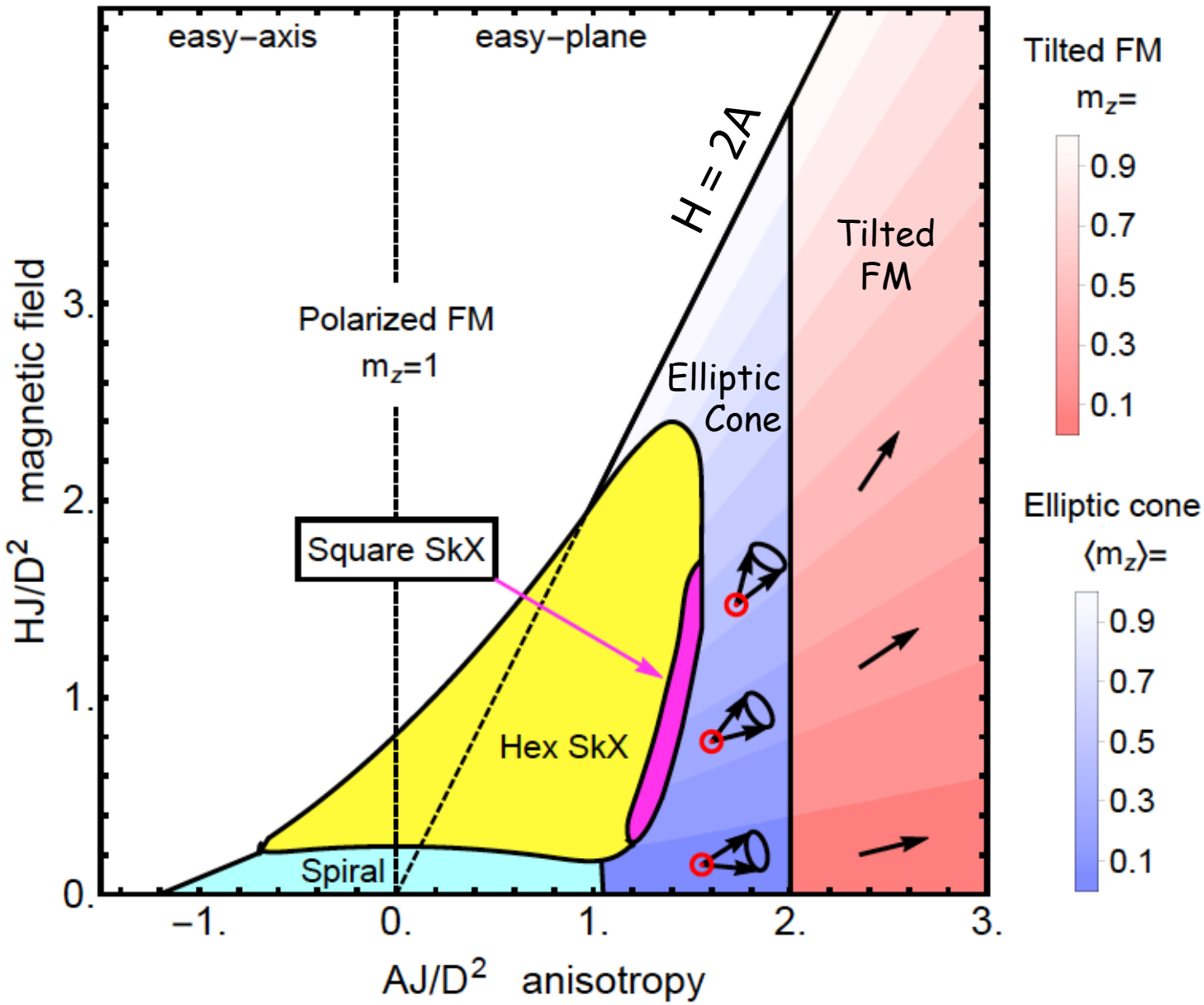
Hedgehog / Neel



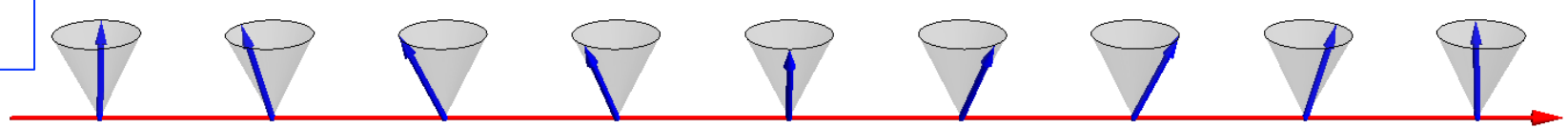




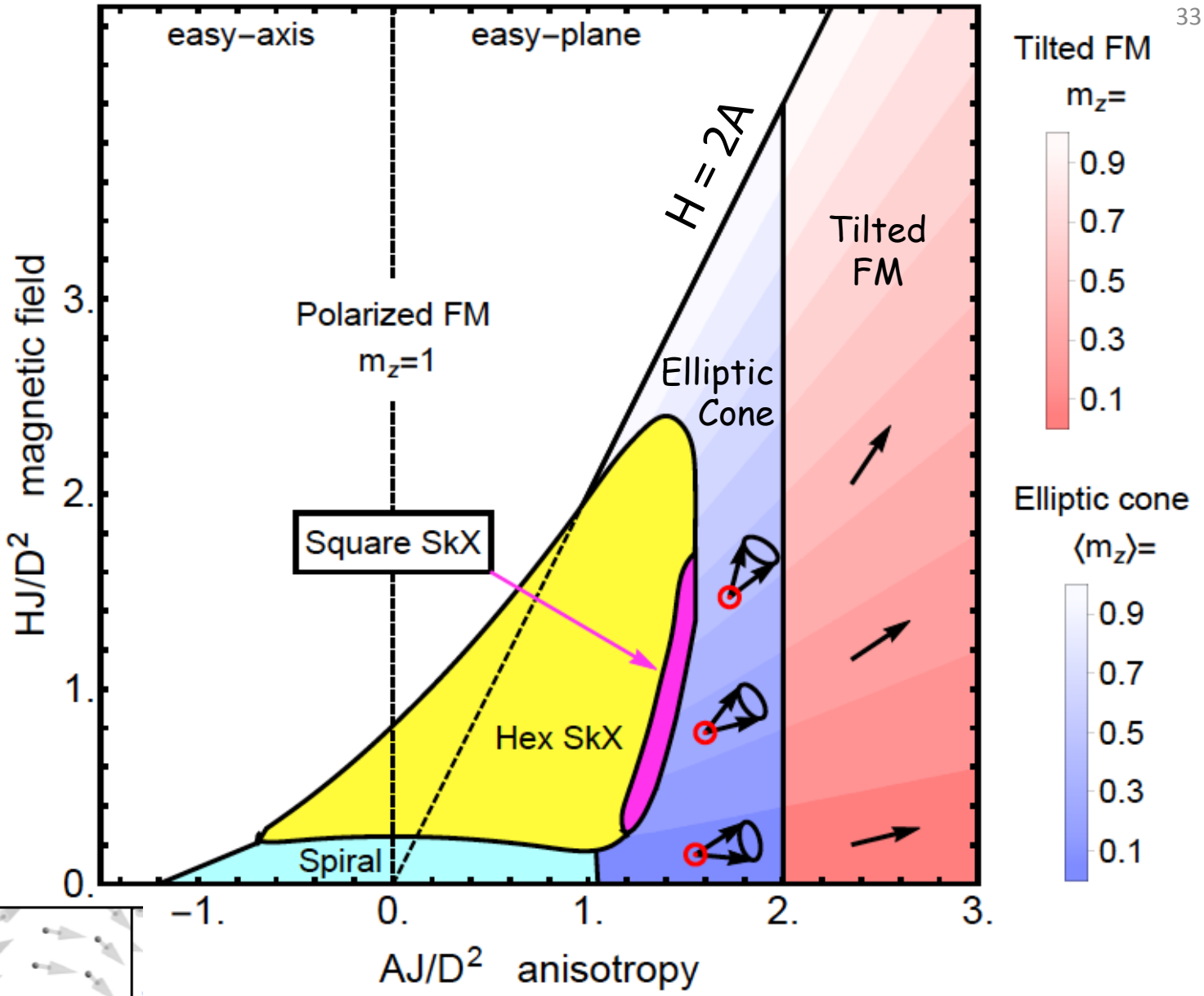
Phase Diagram For Rashba Limit

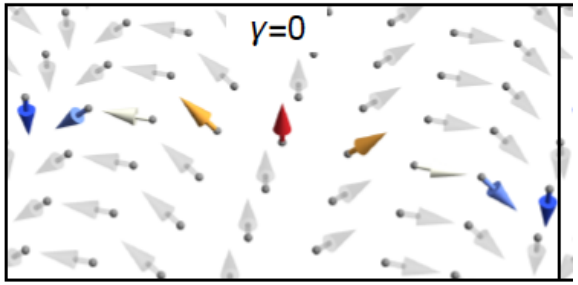


Elliptic Cone



Phase Diagram For Rashba Limit



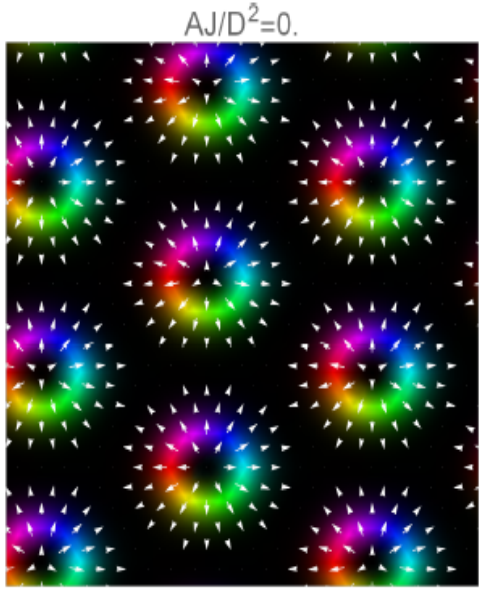


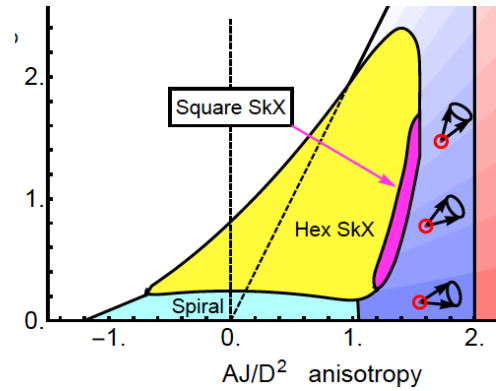
H=0, A>0 results consistent with
Li, Liu, & Balents [PRL 112, 067202 (2014)], who focus on T>0

Evolution of spin-texture in Rashba limit with A at fixed H

A=0.0, H=0.7

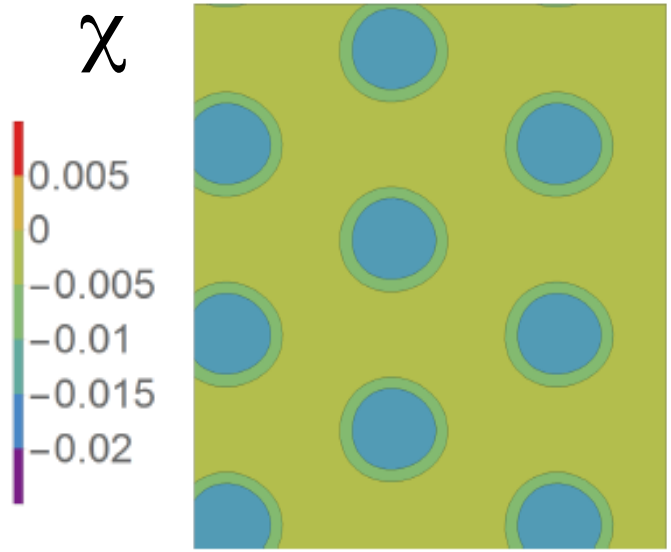
Spin texture





χ

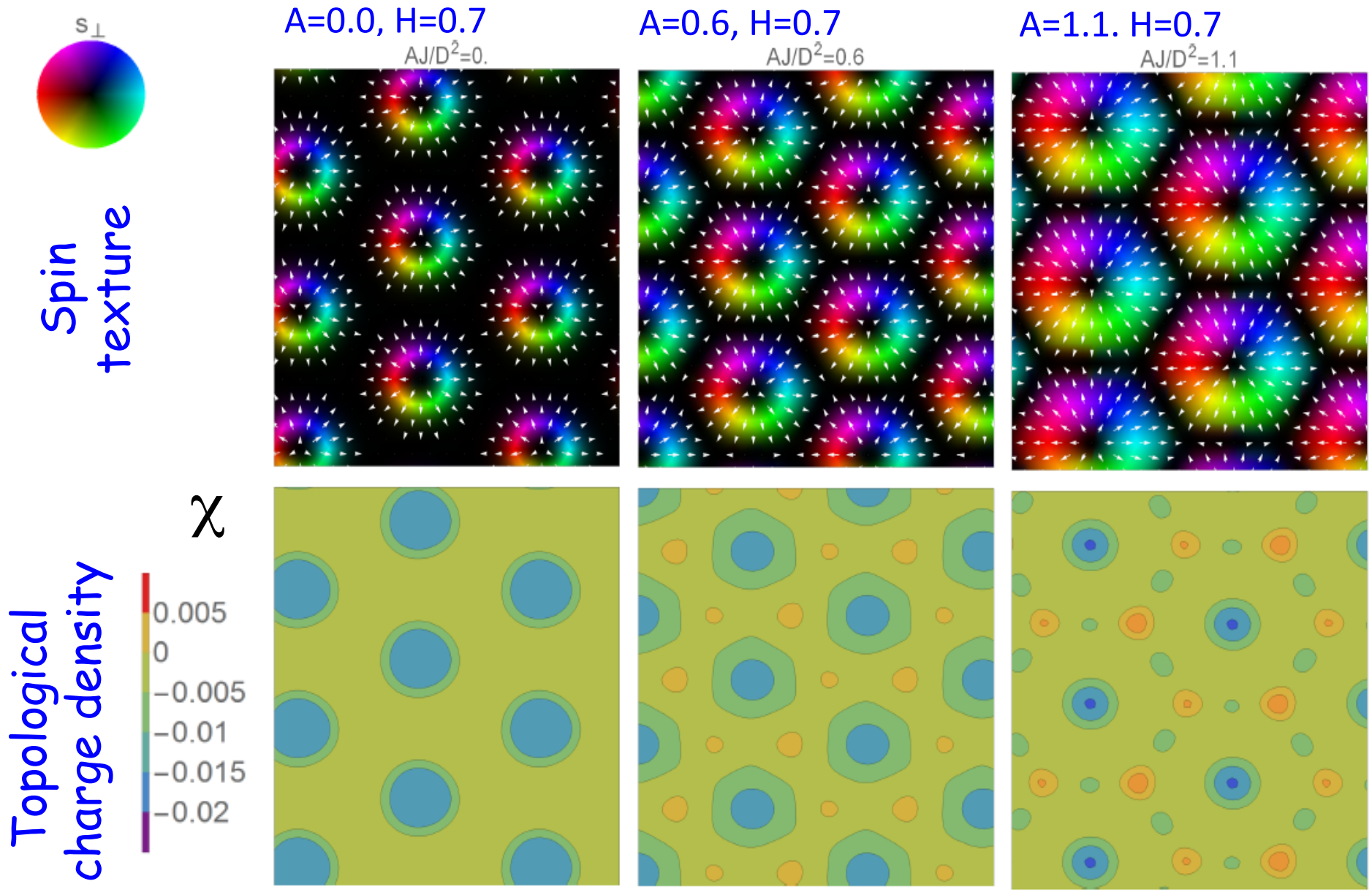
Topological charge density



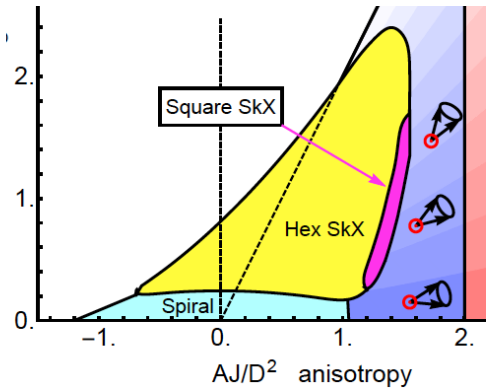
$$\chi = \frac{1}{4\pi}\hat{\mathbf{m}}\cdot(\partial_x\hat{\mathbf{m}}\times\partial_y\hat{\mathbf{m}})$$

$$\int_{\text{unit cell}} d^2\mathbf{r}\ \chi = -1$$

Evolution of spin-texture in Rashba limit with A at fixed H



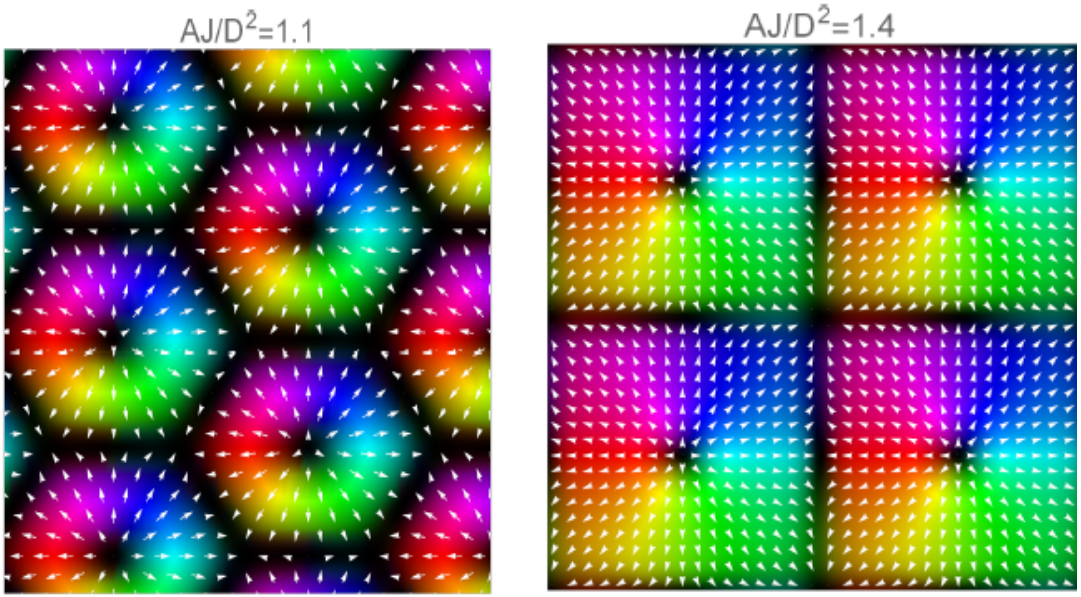
Evolution of spin-texture in Rashba limit with A at fixed H



A=1.1, H=0.7

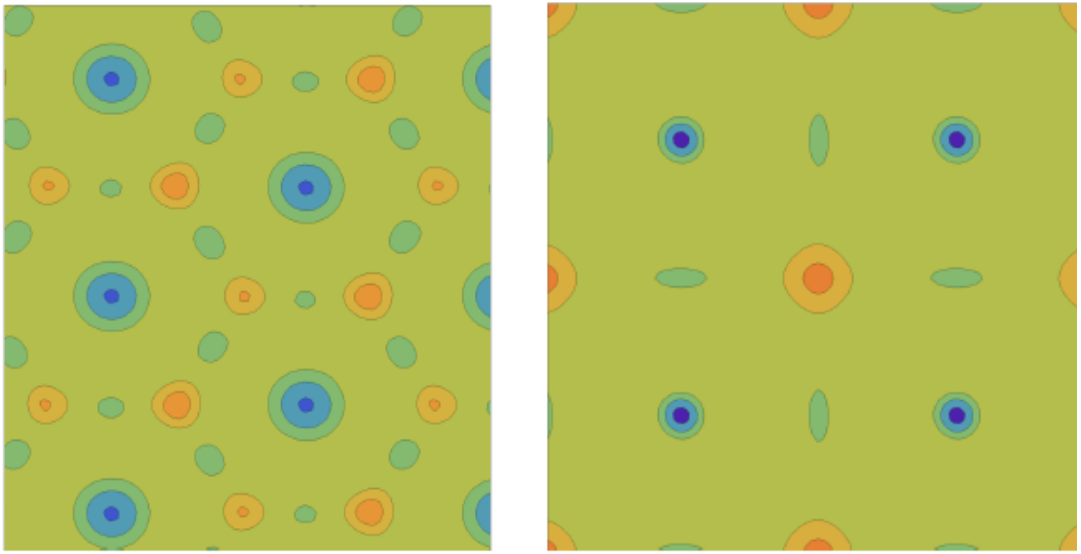
A=1.4, H=0.7

Spin texture



χ

Topological charge density

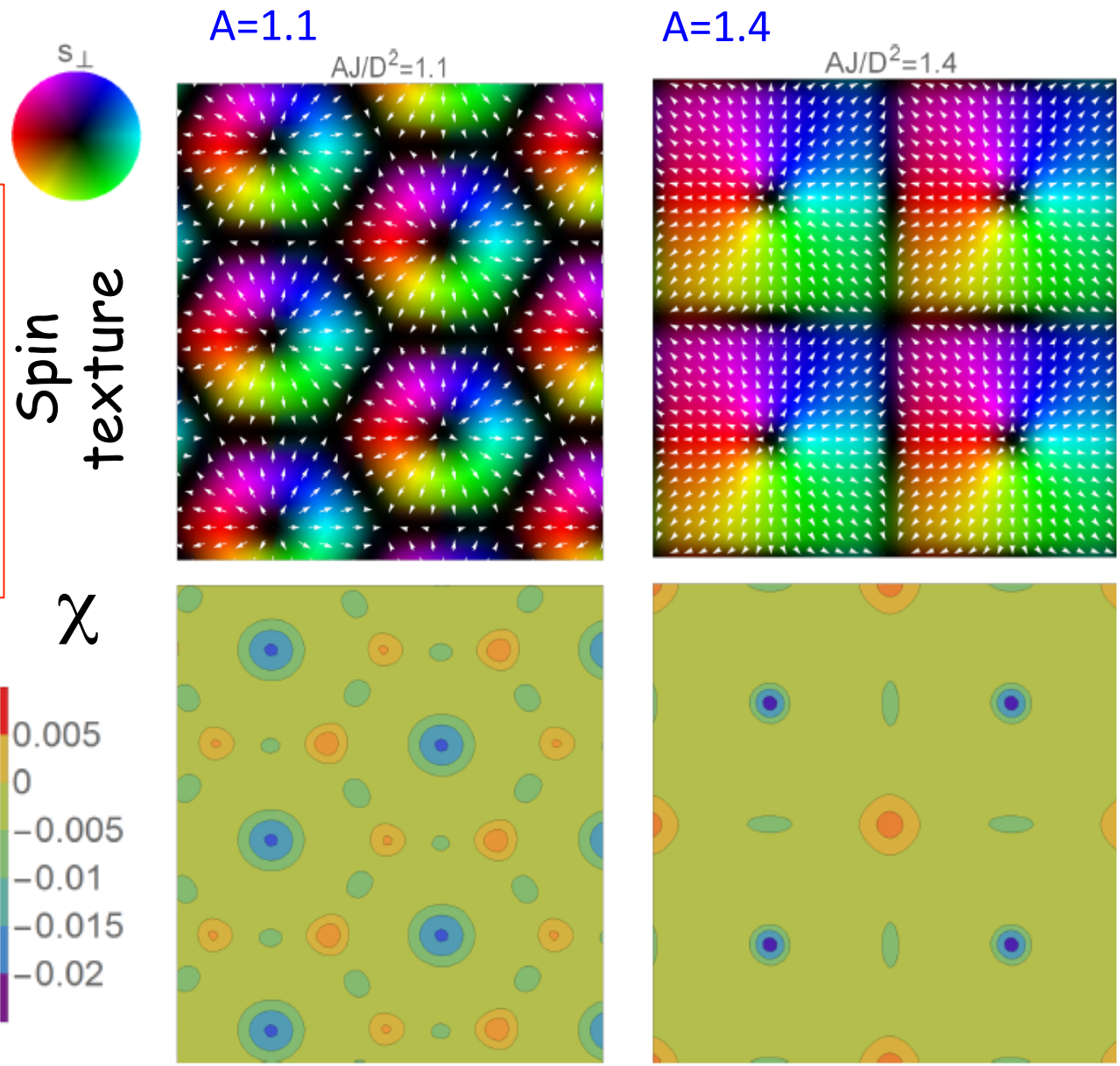


Evolution of spin-texture in Rashba limit with A at fixed H

Cannot use Homotopy group $\Pi_2(S^2) = \mathbb{Z}$

Use Chern Number for maps from 2-Torus → 2-Sphere

→Each unit cell Q = -1



Outline:

- Introduction
  * What is a Skyrmion?
  * Skyrmion materials & properties

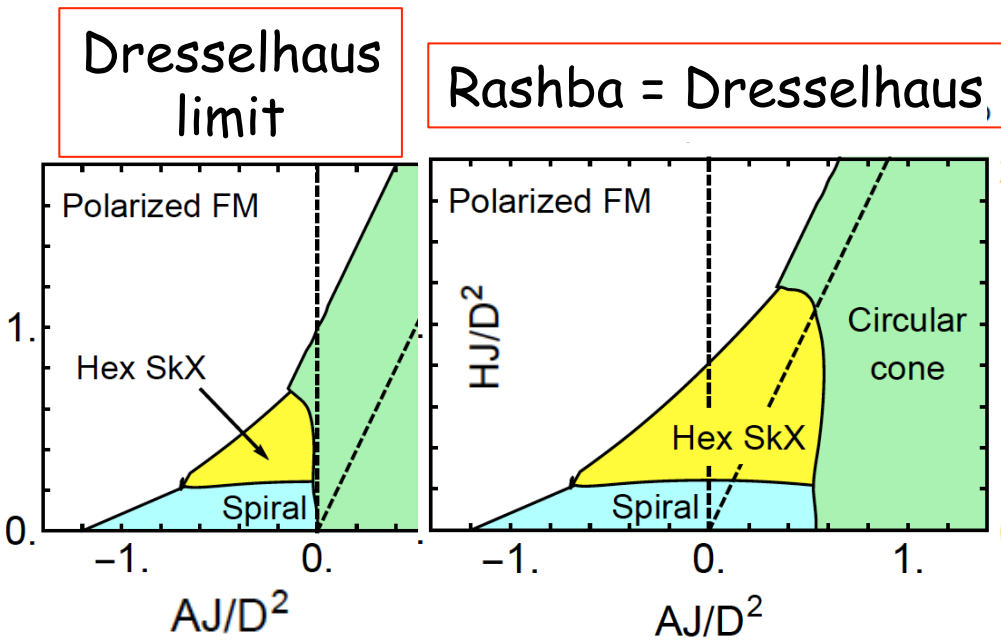
- How to stabilize skyrmion phases?
  * Role of Dresselhaus vs. Rashba SOC
  * Role of magnetic anisotropy

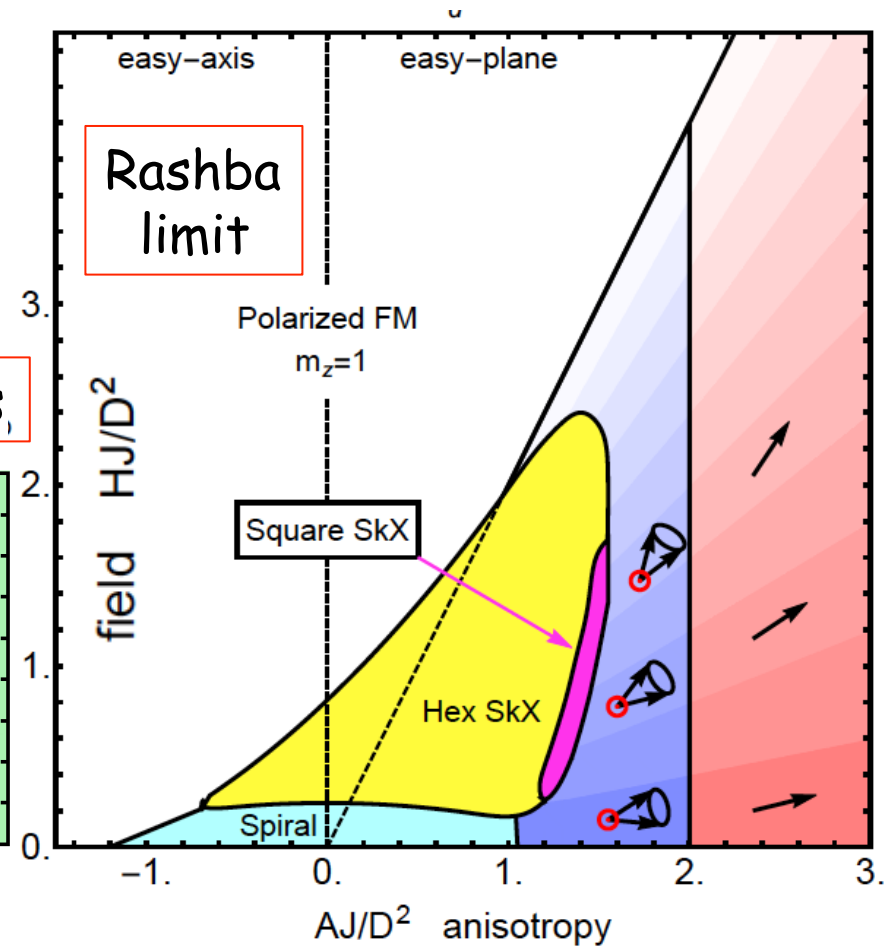
- Conclusions

Summary

Importance of
* Broken "surface" inversion
* Rashba SOC
* easy-plane anisotropy

→ Predict enhanced stability of Skyrmions





* Nature Physics **9**, 626 (2013)
* Phys. Rev. **X 4**, 031045 (2014)
* In preparation (2015)

Could not discuss this for lack of time …

Related Work in Cold Atoms:

Bose Hubbard Model with Rashba SOC

FM + Rashba DM + Compass Anisotropy

Arbitrary D/J

References:

Cole, Zhang,Paramekanti, & Trivedi PRL 109, 085302 (2012)

See also: Radic, Di Ciolo, Sun & Galitski, PRL (2012)