Quasiparticle Tunneling and Interferometry in Possible Non-Abelian FQH States

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W. Bishara, A. Feiguin, P. Fendley, M.P.A. Fisher,
K. Shtengel, P. Bonderson, J. Slingerland ,

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- In a way, this physics is still interesting today.

  Much of the discussion of the 5/2 state and the MR and anti-Pfaffian wavefunctions is couched in the language of the Ising model (e.g. $1, \sigma, \psi$) and the *Ising TQFT*.

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  Why?

  Does the Ising model tell us anything about 5/2?

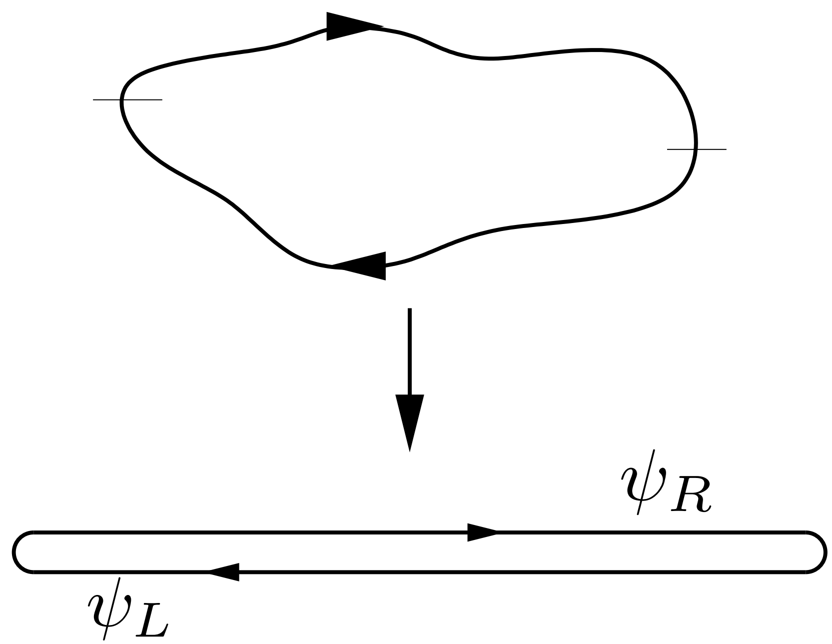
Effective Theory for the MR Pfaffian Edge

$$\mathcal{L}^{\text{edge}} = \frac{1}{4\pi}\partial_x\phi_c(\partial_t + v_c\partial_x)\phi_c + \frac{1}{2\pi}\psi(\partial_t + v_n\partial_x)\psi$$

Milovanovic and Read ’95

The neutral sector is a *chiral Majorana fermion*.
= chiral part of the *critical 2D Ising model*
and the 1+1-D transverse field Ising chain.

subdivide edge into halves, call them right/left moving

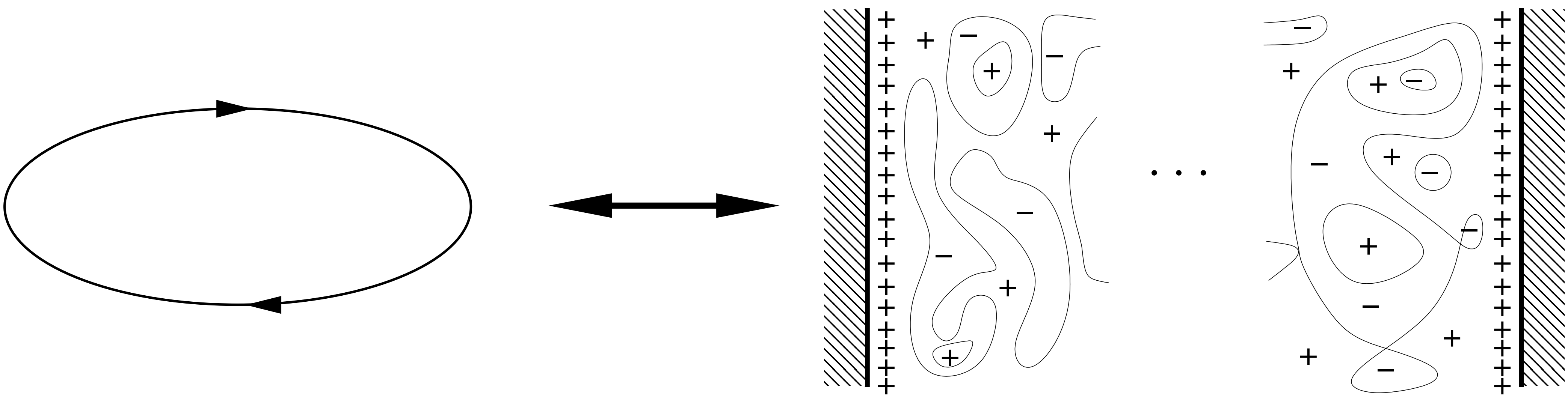


If we represent the edge of an MR droplet by a non-chiral Majorana fermion on an interval, then the boundary conditions at the ends of the interval must be *conformally-invariant*.

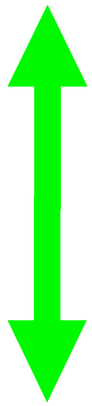
Cardy, late 80's: boundary CFT.

Conformally-inv. b.c. of Ising model: fixed+, fixed-, free

With no qps. in bulk, the droplet is mapped to a strip with fixed b.c. at both ends.



Different possible
Ising boundary cond.



Different possible
bulk quasiparticles.

Different possible
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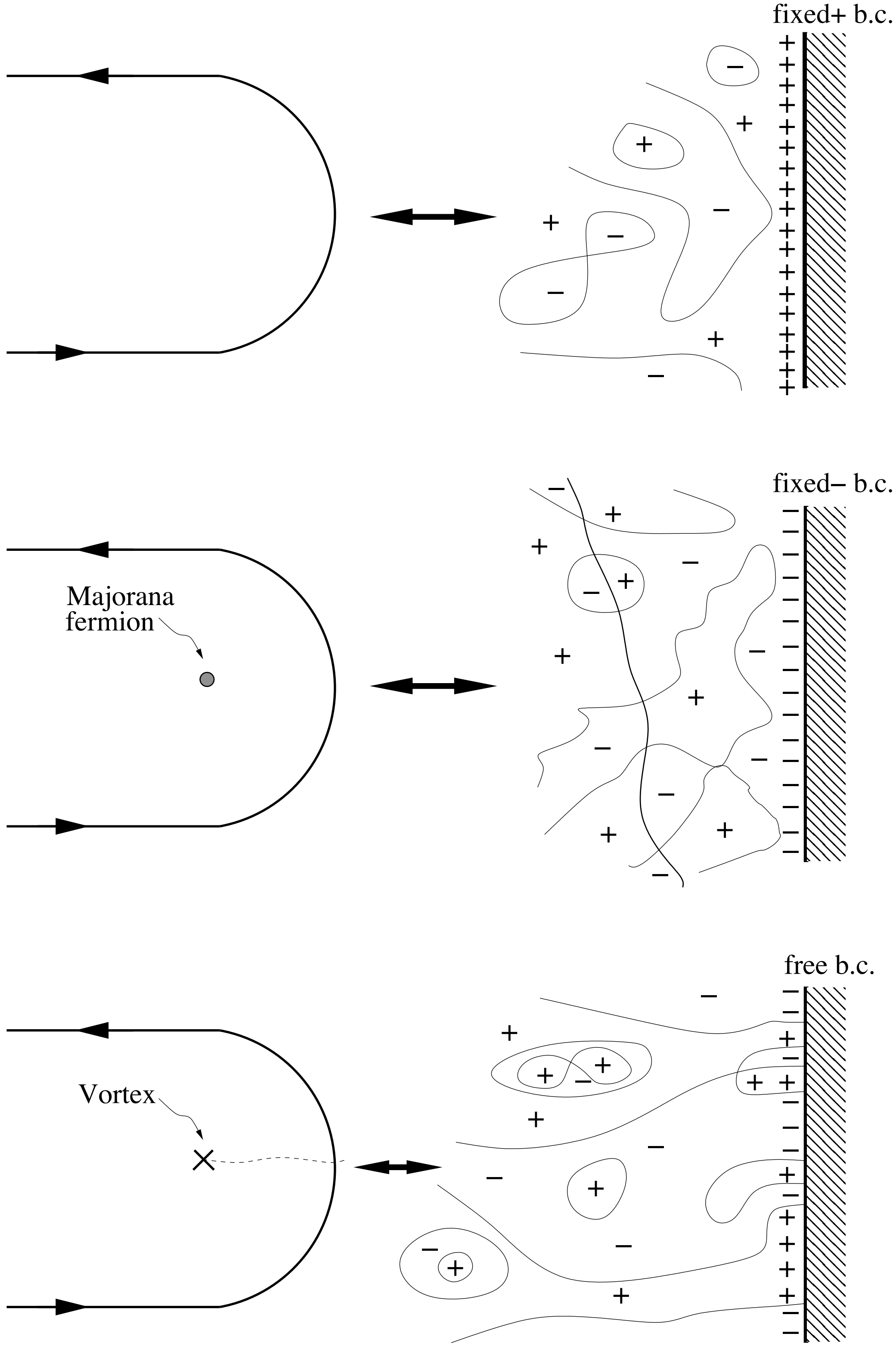
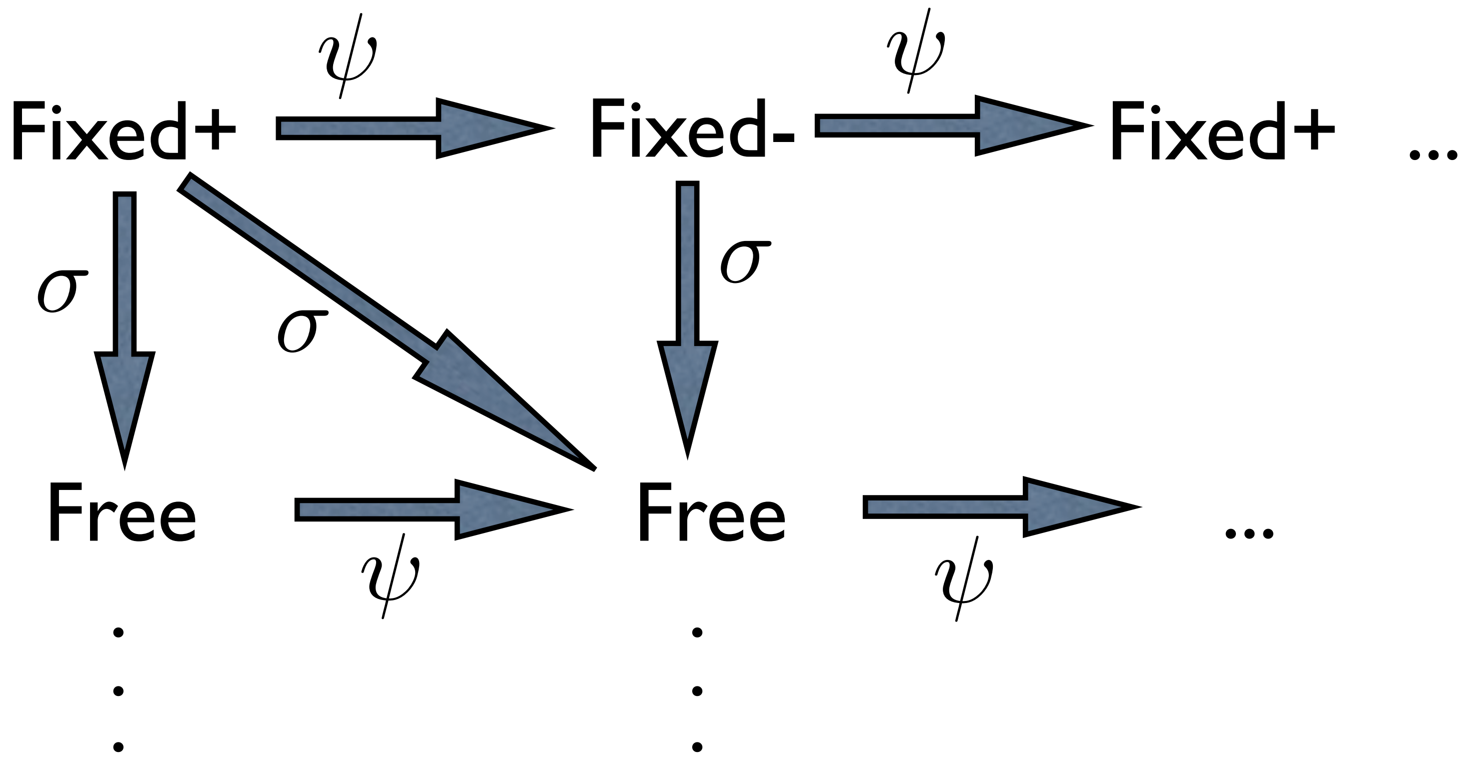


- We can obtain any conf. invar. b.c. at one end of the strip by adding the corresponding quasiparticle in the bulk.



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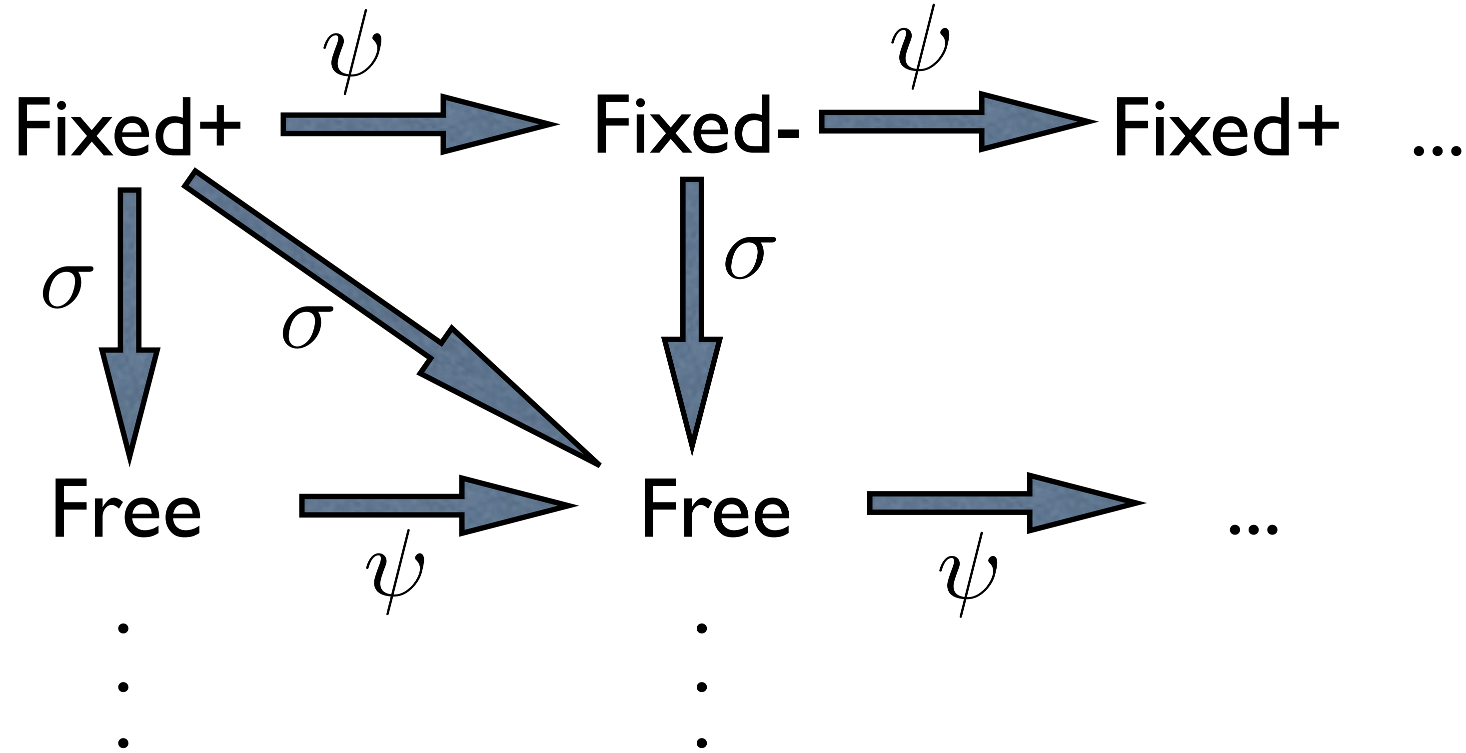


- At which end of the strip should the b.c. be changed? Can be switched by a $\mathbb{Z}_2$ gauge choice = Ising K-W duality

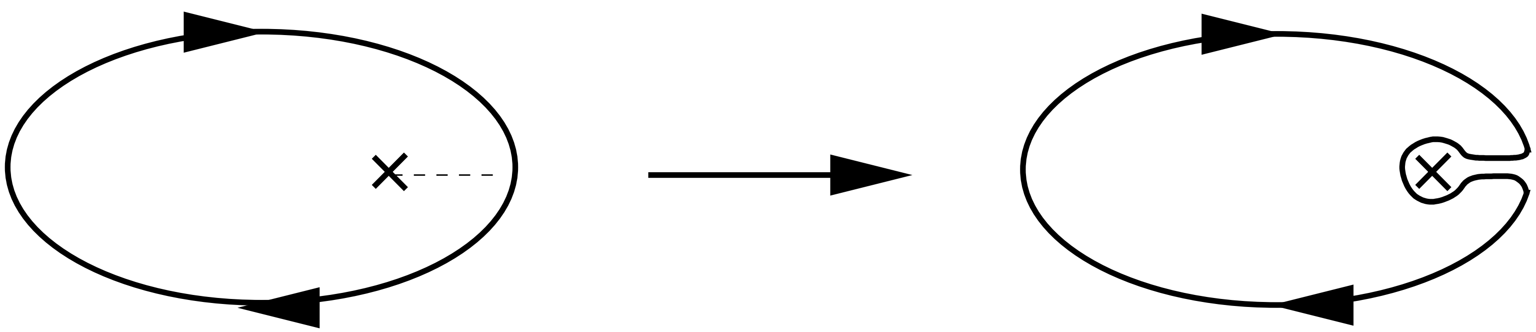
Coupling a bulk vortex to the edge corresponds to applying a boundary magnetic field when the b.c. is 'free'.

$$L = \int dt\,(i\psi_R(\partial_t + v_n\partial_x)\psi_R + i\psi_L(\partial_t - v_n\partial_x)\psi_L)$$
$$+ i\psi_0\partial_t\psi_0 + ih\,\psi_0[\psi_R(0) + \psi_L(0)]$$

Flow from free to fixed b.c. ⟷ The vortex is absorbed by the edge

Entropy loss: $\Delta S = -\ln\sqrt{2}$

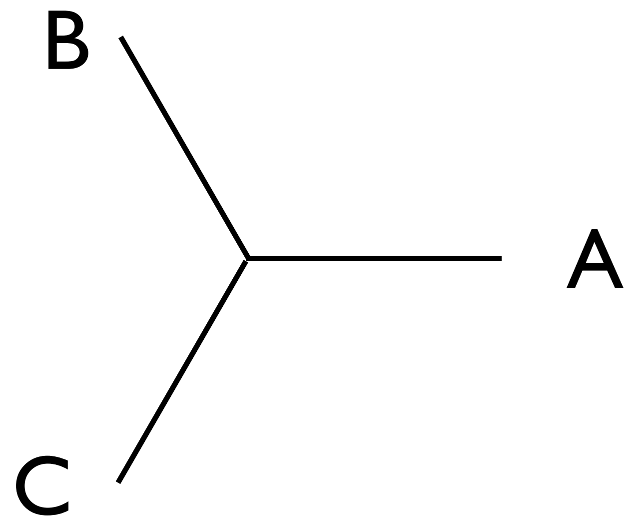
(similar to 2-channel Kondo e.g. Affleck+Ludwig early 90's)



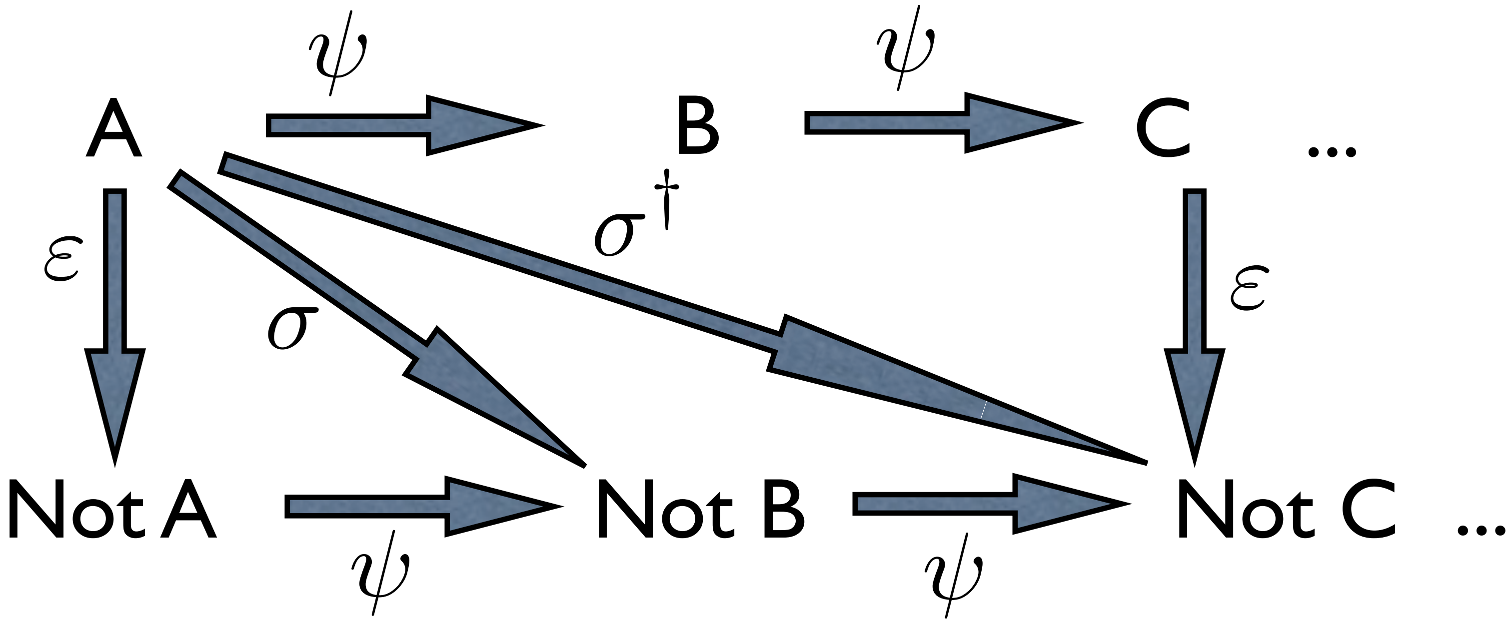
A similar analysis holds for $\mathbb{Z}_3$ parafermions, the critical 3-State Potts model, and the k=3 RR state.

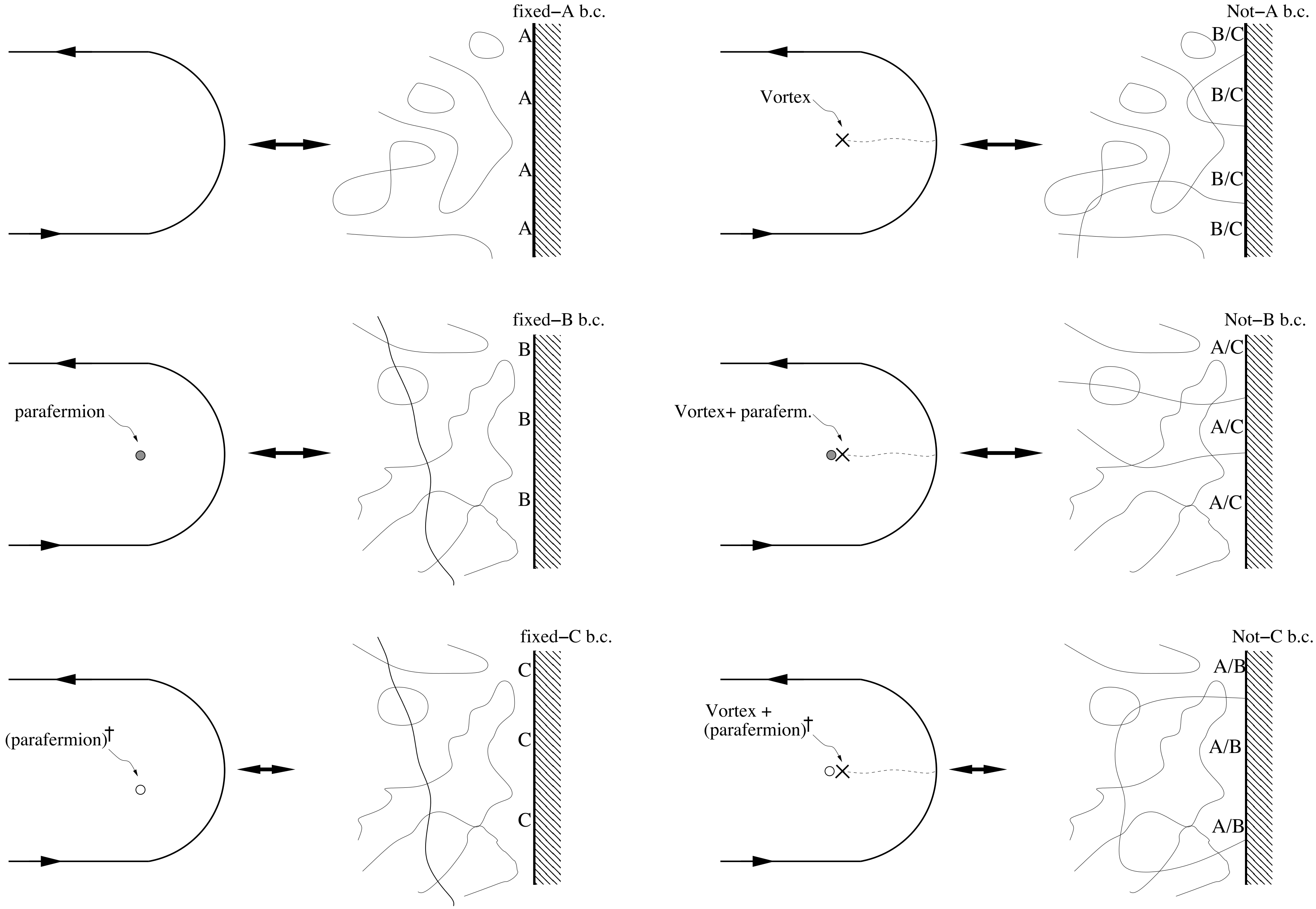
Read, Rezayi '99

$$H = -J \sum_{\langle i,j \rangle} \delta_{s_i s_j}$$



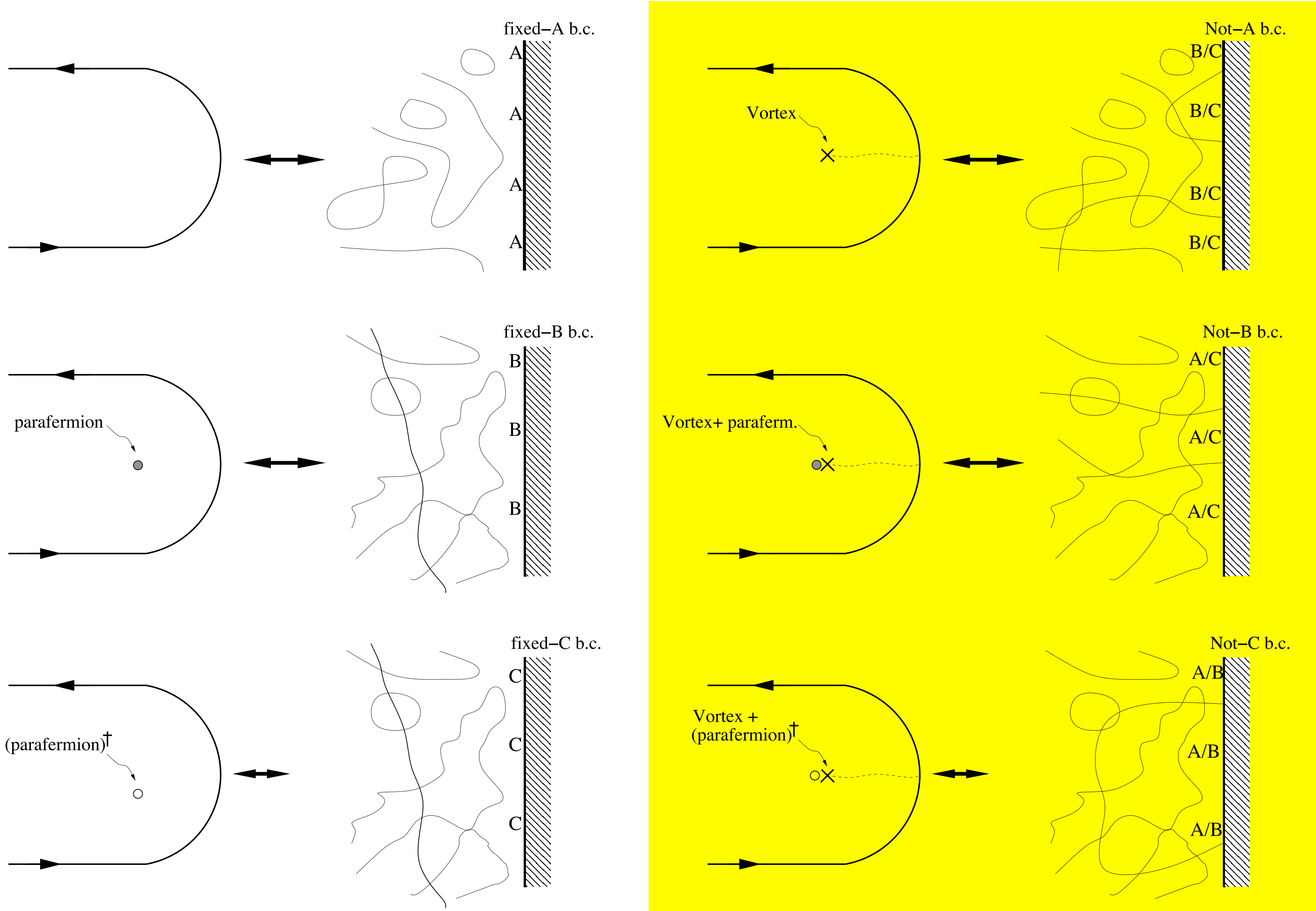
Primary fields: $1, \psi, \psi^\dagger, \sigma, \sigma^\dagger, \varepsilon$





$$\sigma = \varepsilon \times \psi$$

$$\sigma^\dagger = \varepsilon \times \psi^\dagger$$



$$\sigma = \varepsilon \times \psi$$

$$\sigma^{\dagger} = \varepsilon \times \psi^{\dagger}$$

Higher entropy by

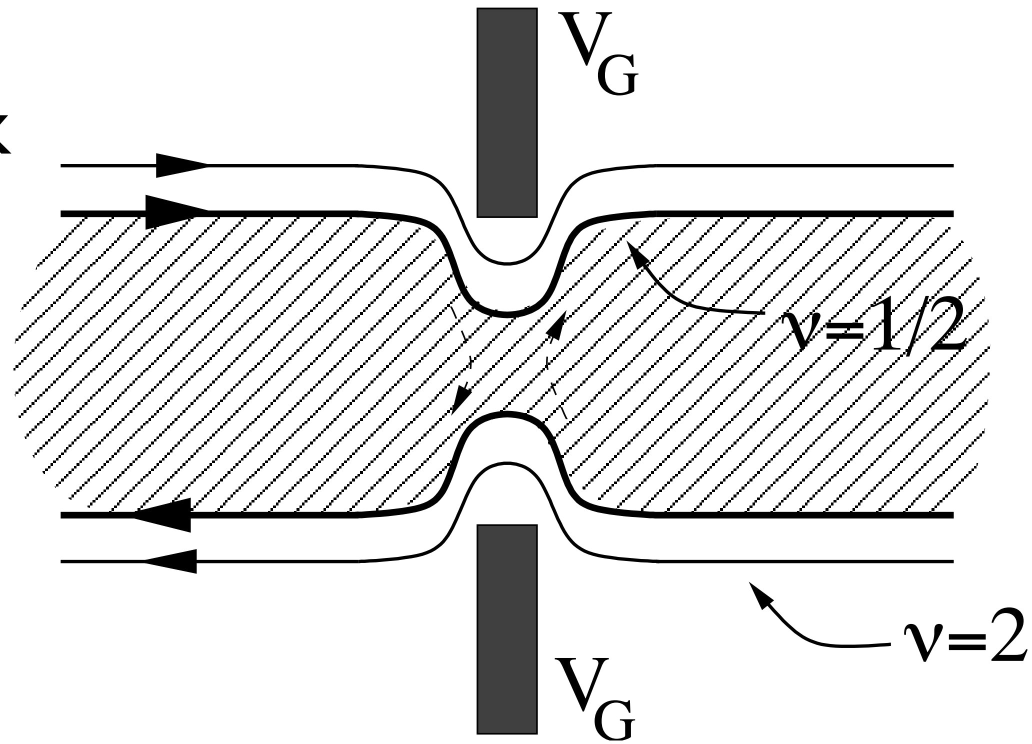
$$\tau = (1+\sqrt{5})/2$$

Point Contacts

Point contacts are a useful probe of the edge excitations (and edge-bulk interplay) of a topological state.

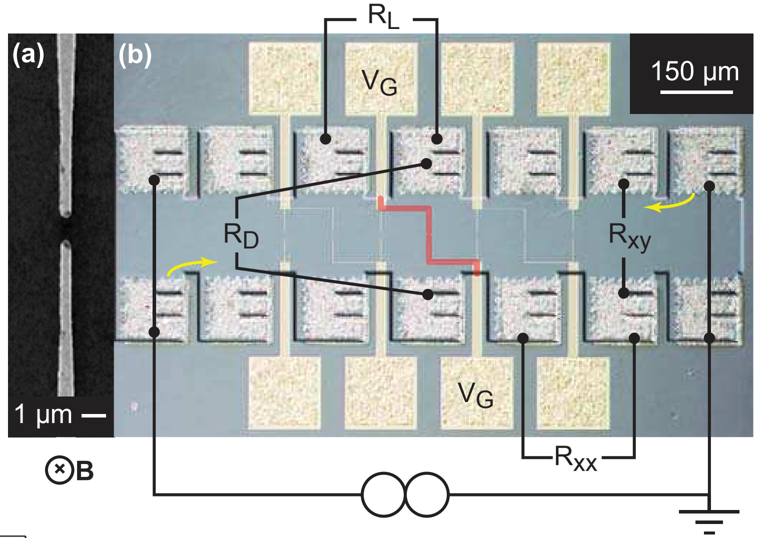
Tunneling through the bulk selects quasiparticles.

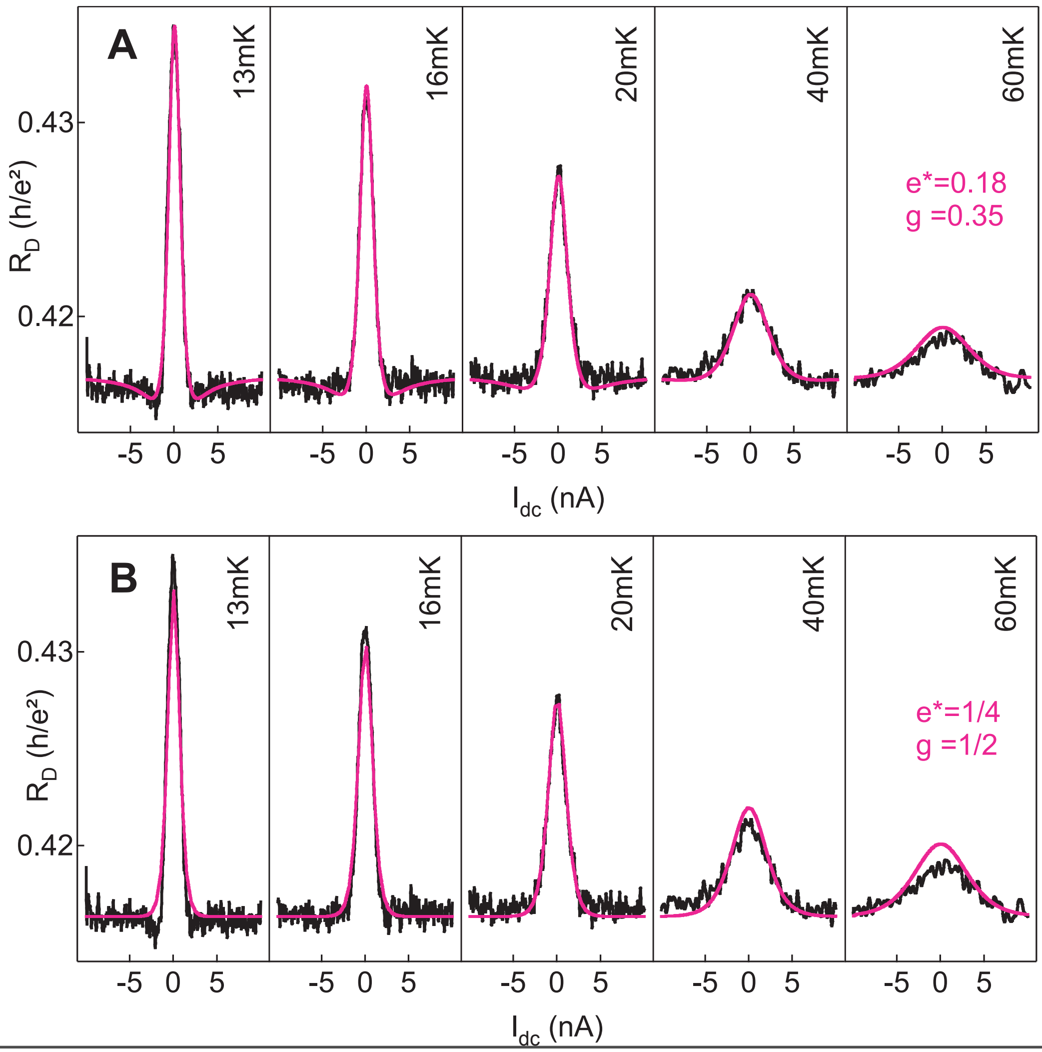
Therefore, the scaling exponents revealed by transport through a pt. can tell us about the qps. supported by the state.

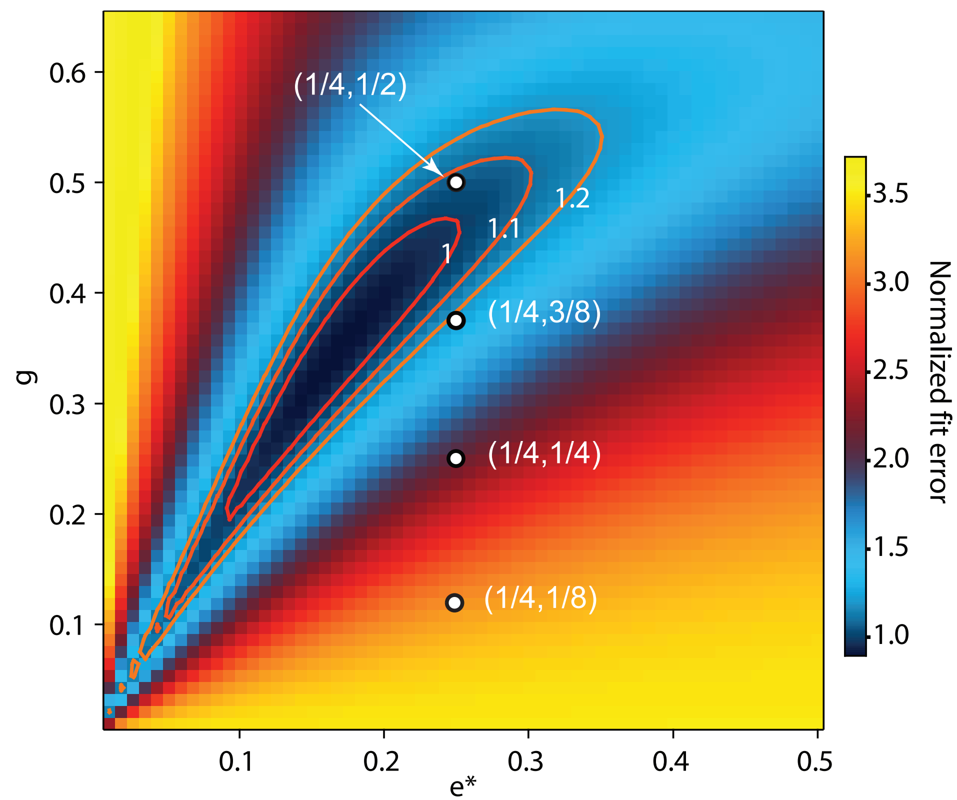


Radu et al. ‘08:

Point contact
in 5/2 state

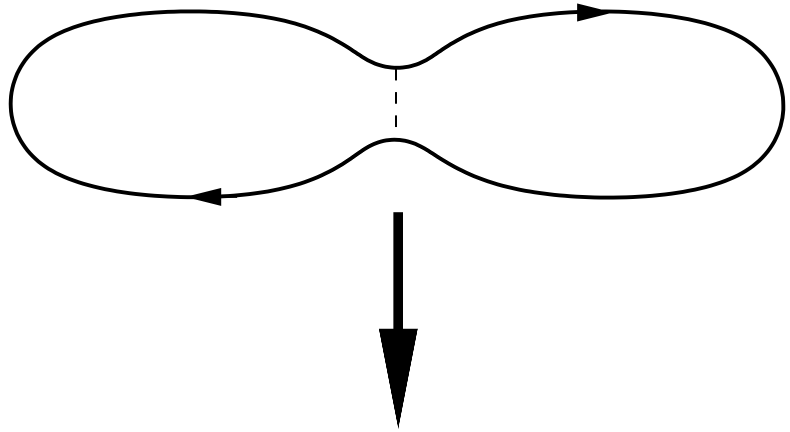




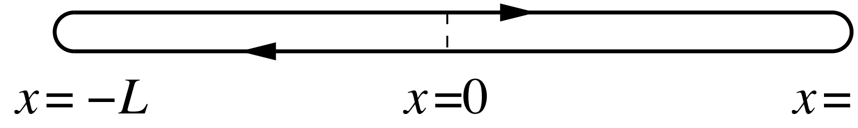


Point Contacts and Perturbed Boundary CFT

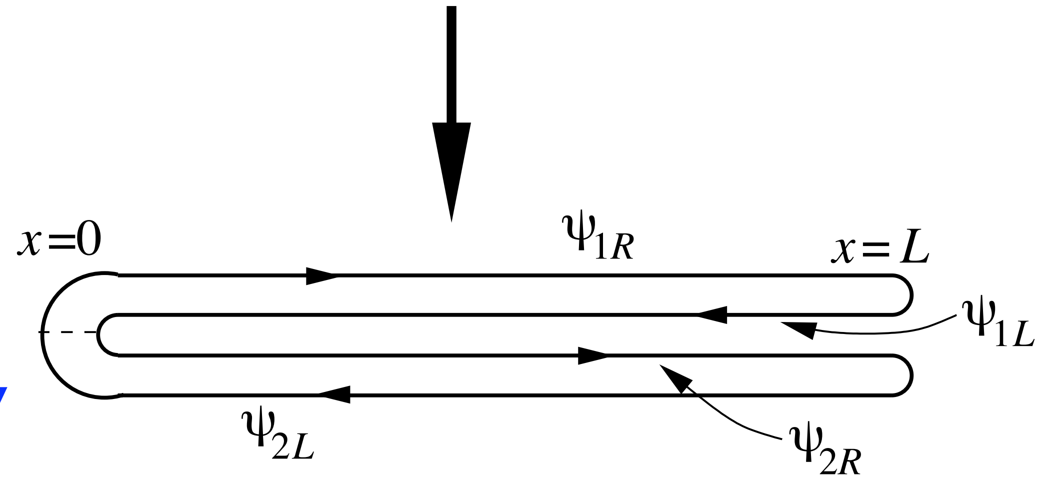
Quantum Hall Droplet
with a Point Contact



Non-chiral Majorana
Fermion with a defect



Two copies of a Majorana
fermion coupled at boundary



Ising Defect Conformal B.C.

The Ising model with a defect line has 4 possible conformally-invariant b.c.

'Continuous Neumann'

(Free, Fixed)

'Continuous Dirichlet'

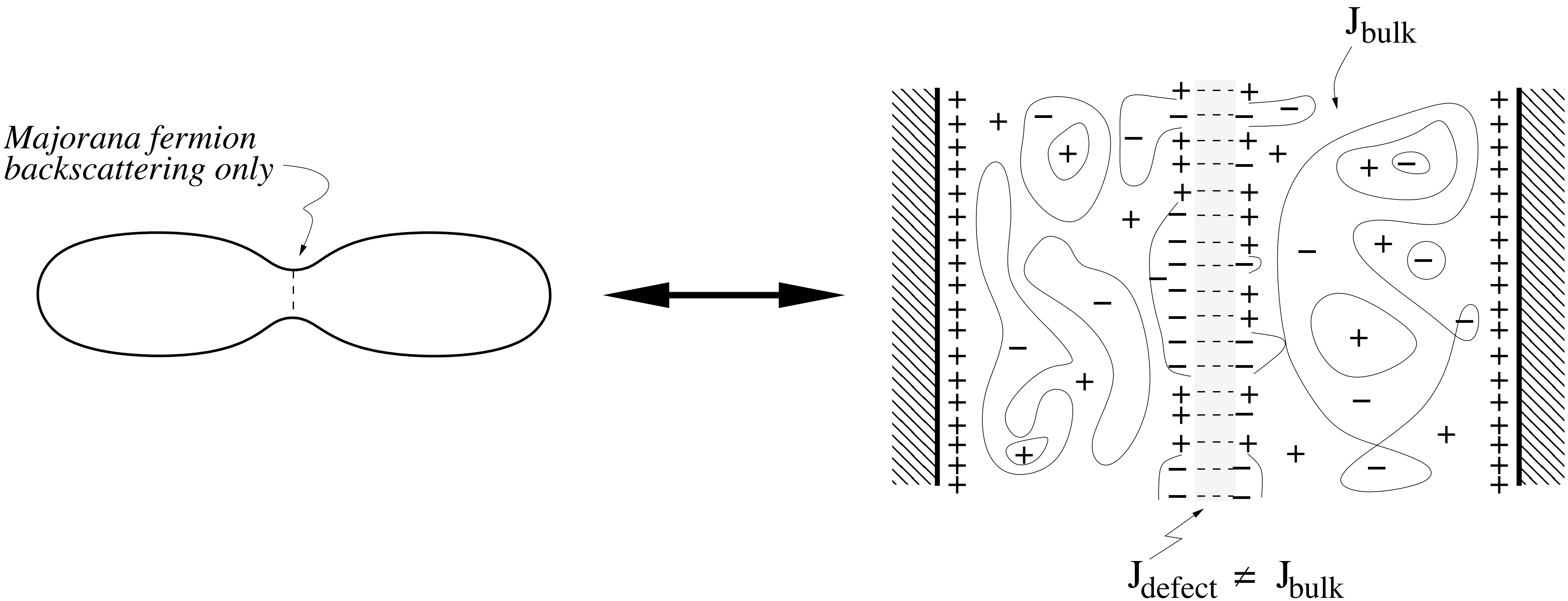
(Fixed, Fixed)

fixed lines

product boundary conditions

Oshikawa and Affleck '97

Majorana Fermion Backscattering



Continuous Dirichlet Line:

Fermion backscattering only (no vortex tunneling) = Column of bonds weakened/strengthened

Special Points on C.D. Line

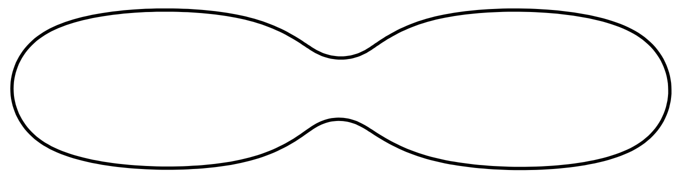
- Define a Dirac fermion, bosonize:

$$\begin{cases} e^{i\varphi_R} = -\psi_{1R} + i\psi_{2R} \\ e^{i\varphi_L} = \psi_{1L} + i\psi_{2L} \end{cases}$$

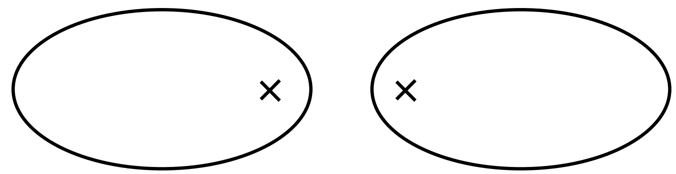
- Parametrize by phase shift/boson value at boundary:

$$\varphi(0) = \frac{\delta}{2} + \frac{\pi}{4}$$

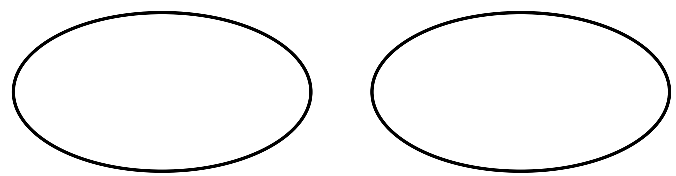
Transmitting: $\delta = 0$



(Free,Free): $\delta = \pi/2$



$(\pm, \pm): \ \delta = -\pi/2$



Vortex Tunneling

- Vortex Tunneling = Magnetic Field at Defect Line

- Causes flow to (Fixed, Fixed) B.C.

  Entropy loss: (Free,Free) $\longrightarrow$ (Fixed,Fixed) = ln 2

- Scaling dim. depends on pt. on C.D. line:

$$\Delta = \frac{1}{8}\left(1 + \frac{2\delta}{\pi}\right)^2 + \frac{1}{8}$$

Implications for transport through a pt. contact:

$$R_{xx} \sim T^{2\Delta - 2}$$

see also LeClair and Ludwig ’99

Non-Perturbative Treatment

Standard bosonization:

$$\psi_a + i\psi_b \sim e^{i\phi}$$
$$i\psi_a\psi_b \sim \partial\phi$$

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bookkeeping

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bookkeeping

Fendley, Fisher, Nayak PRL, PRB '06.

$$\mathcal{H}_{5/2} = \int dx \left(\frac{v_c}{2\pi} \left(\partial_x \phi_\rho\right)^2 + \frac{v_n}{2\pi} \left(\partial_x \phi_\sigma\right)^2 \right) + \lambda_{1/4} \left(S^+ e^{-i\phi_\sigma(0)/2} + S^- e^{i\phi_\sigma(0)/2} \right) \cos(\phi_\rho(0)/2) + \lambda_{1/2} \cos\phi_\rho(0) + \frac{\lambda_{\psi,0}}{2\pi} \partial_x \phi_\sigma(0),$$

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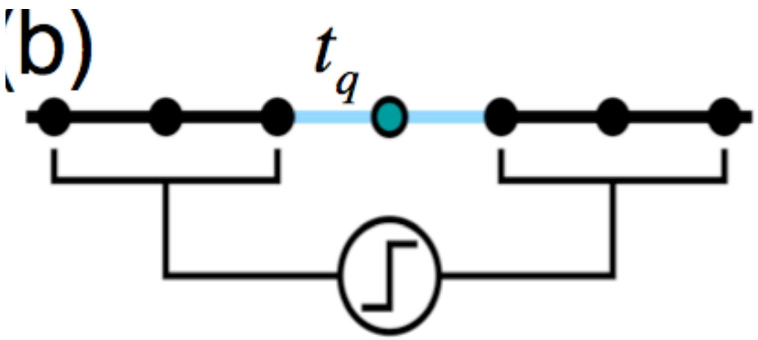
$$+ \lambda_{1/2} \cos\phi_\rho(0) + \frac{\lambda_{\psi,0}}{2\pi} \partial_x \phi_\sigma(0),$$

C. Kane: S^z

Crossover from Trans. to (Fixed, Fixed)

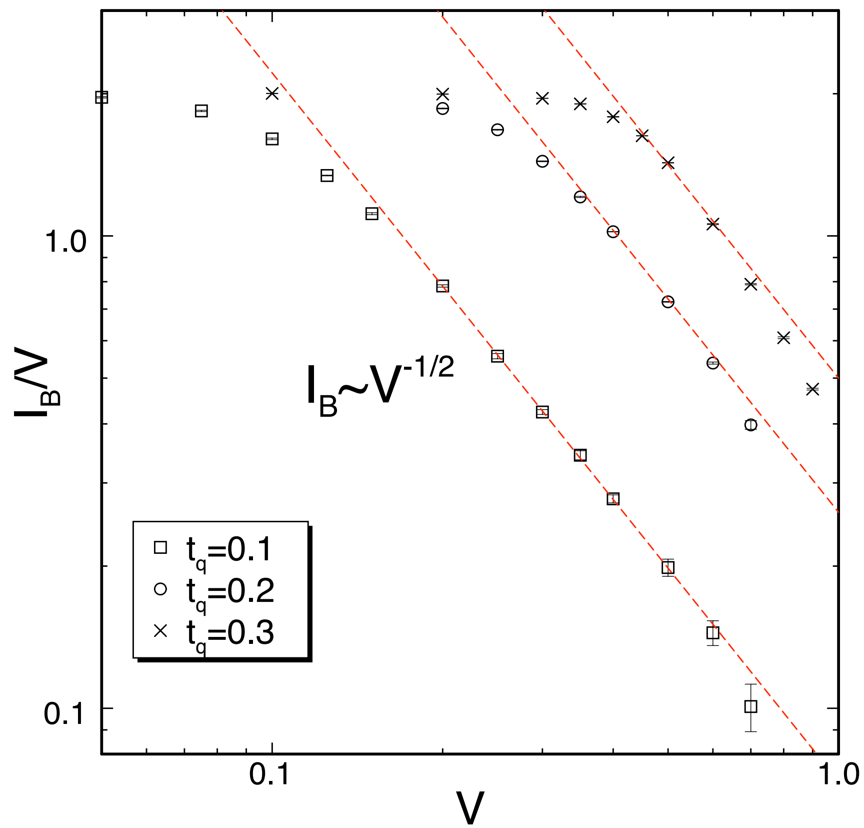
- Pf. point contact can be rewritten as resonant tunneling between Luttinger liquids

$$\mathcal{H}_{\rm res} = \int_0^\infty dx\,\frac{v}{2\pi}\left((\partial_x\phi_a)^2 + (\partial_x\phi_b)^2\right)$$
$$+t\,d^\dagger e^{i\phi_a(0)/\sqrt{g}} + t\,d^\dagger e^{i\phi_b(0)/\sqrt{g}} + \mathrm{h.c.}$$



- Tunneling current can be computed by time-dependent DMRG.

- Agrees with perturbative calculations around the weak- and strong-backscattering limits. Only way to compute the current in the crossover regime.
  Agrees with Bethe ansatz for 1/3 point contact.



Feiguin, Fendley, Fisher, Nayak '08

- Future: time-dep. DMRG for anti-Pfaffian, 331.

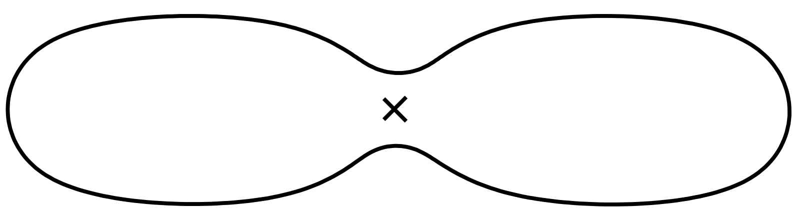
Continuous Neumann B.C.

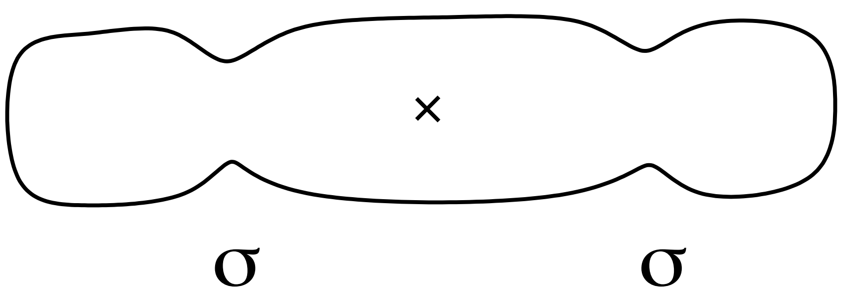
- Dirichlet b.c. for the dual boson: $\widetilde{\varphi} = \varphi_R - \varphi_L$

- TFIM with a defect: $H = -\sum_{n \neq 0} \sigma_n^x - \sum_{n \neq 0} \sigma_{n-1}^z \sigma_n^z - b\sigma_{-1}^z \sigma_0^x$

  - Same as C.D. line, but with a vortex pinned at the pt. contact

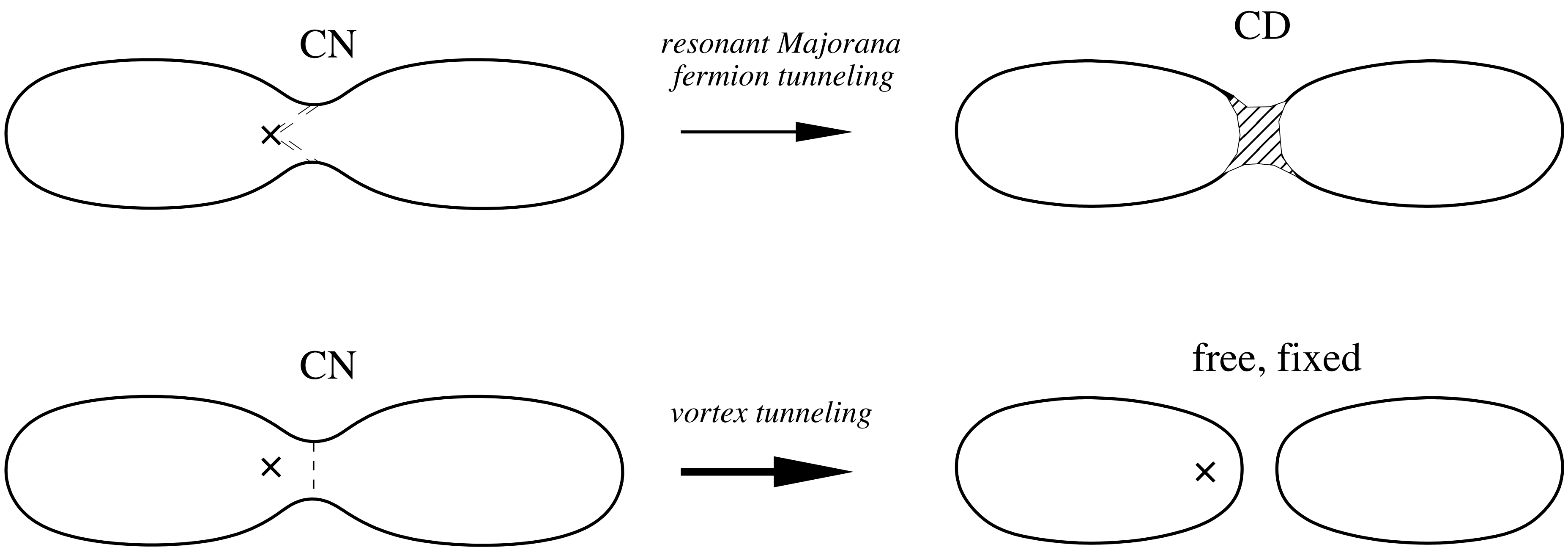
will think about this context soon:



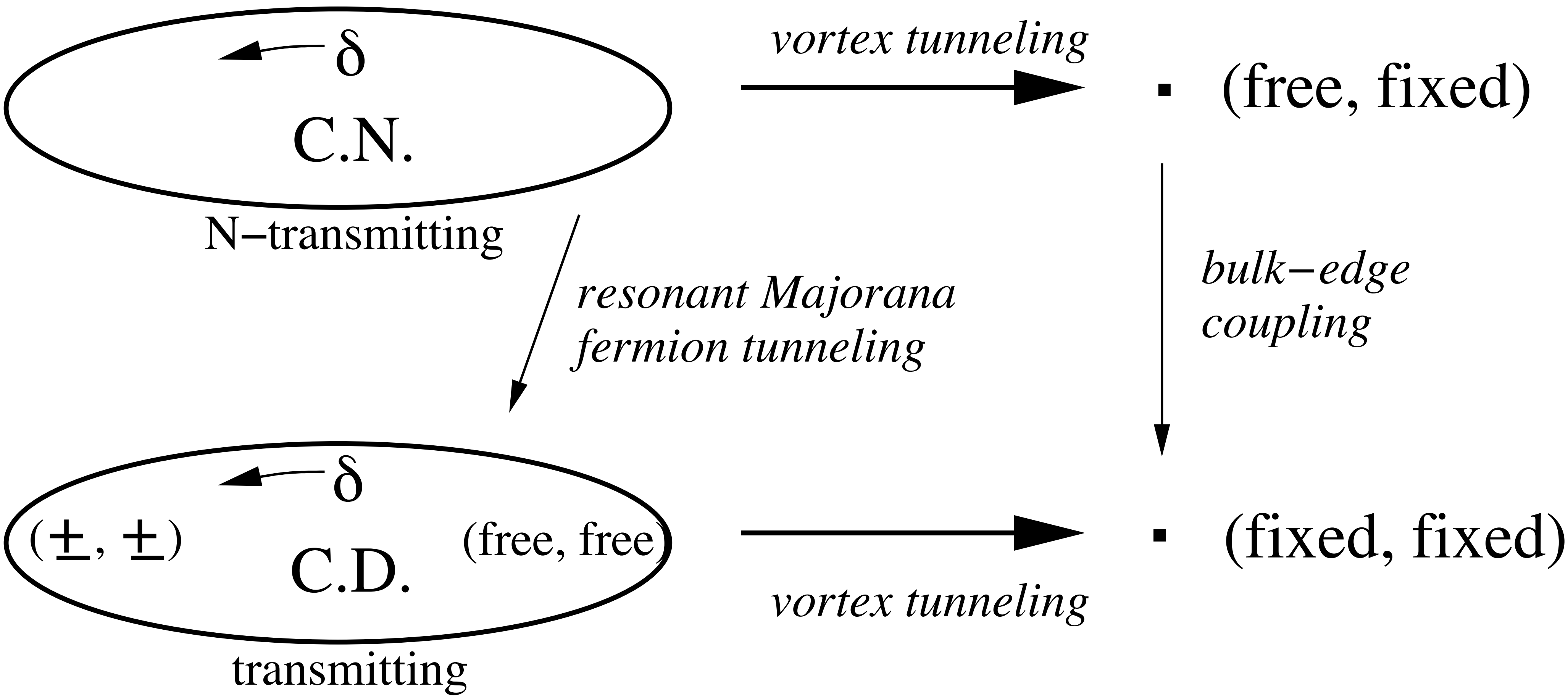


Perturbing Continuous Neumann b.c.

- If the bulk vortex is coupled to the edge, the system flows to the C.D. line.
- Vortex tunneling takes the system to (Free,Fixed) because one of the droplets contains a vortex



Summary: Fixed Pts. and Flows

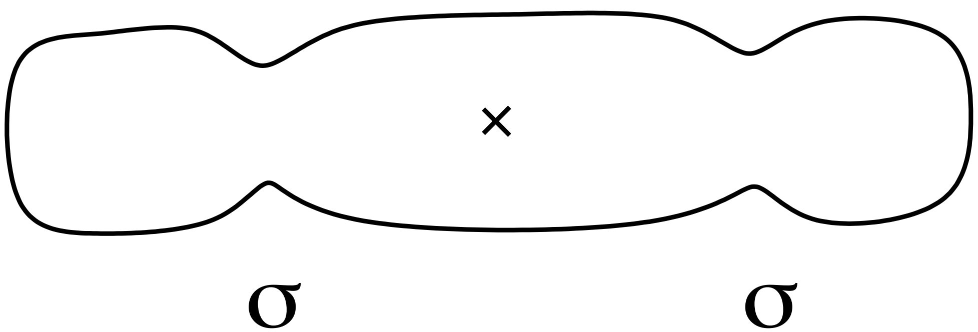


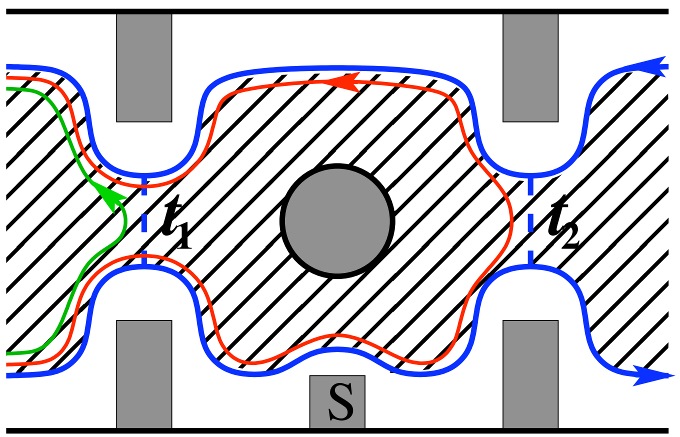
Continuous Neumann b.c. and Two-Pt. Contact Interferometers

Along the C.N. line, correlation functions have the following property (Oshikawa+Affleck '97):

$$\langle \sigma(x<0)\,\sigma(x'>0)\rangle = 0$$

Same as the odd-even effect (Bonderson, Kitaev, Shtengel '06; Stern-Halperin '06) in an interferometer:

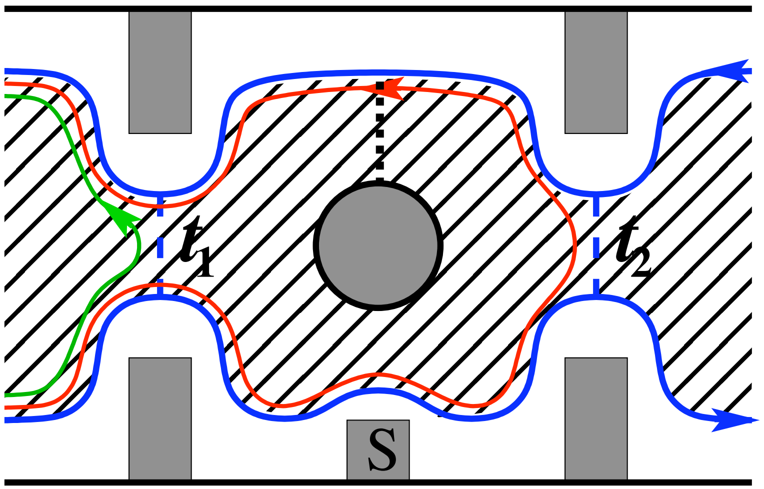




Bulk-Edge Coupling in an MR Interferometer

The corr. fcn. can be computed along the flow from C.N. to C.D.

$$\langle \sigma(x,t)\sigma(0,0)\rangle = \Big(2\lambda^2(x - v_n t)\Big)^{1/4} \Psi(1/2, 1, \lambda(x - v_n t))$$



Confluent hypergeometric function:

$$\Psi(a,c,x) = \frac{1}{\Gamma(a)} \int_0^\infty ds\, e^{-xs} s^{c-1} (1+s)^{a-c-1}$$

$$\lambda = 4\pi h^2 / {v_n}^2$$

Chamon et al. '97; Fradkin et al. '98
Bonderson, Kitaev, Shtengel '06
Stern, Halperin '06

similar to free-to-fixed flow,
see Chatterjee and Zamolodchikov, '94

This leads to an interference term in the backscattered current of the form:

(assuming equal charge/neutral velocities)

$$I_{12} = \frac{e}{4}\,|t_1 t_2|\,2^{5/4}\sqrt{\pi\lambda}\,\cos(2\pi\Phi/4\Phi_0) \times$$

$$\cos(xe^* V/v)\;\frac{1}{\left[e^* V(v\lambda + e^* V)\right]^{1/2}}$$

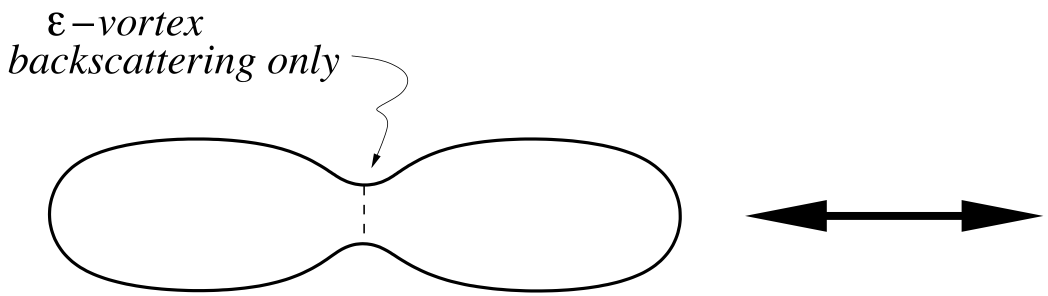
more complicated for unequal velocities, but similar physics and scaling

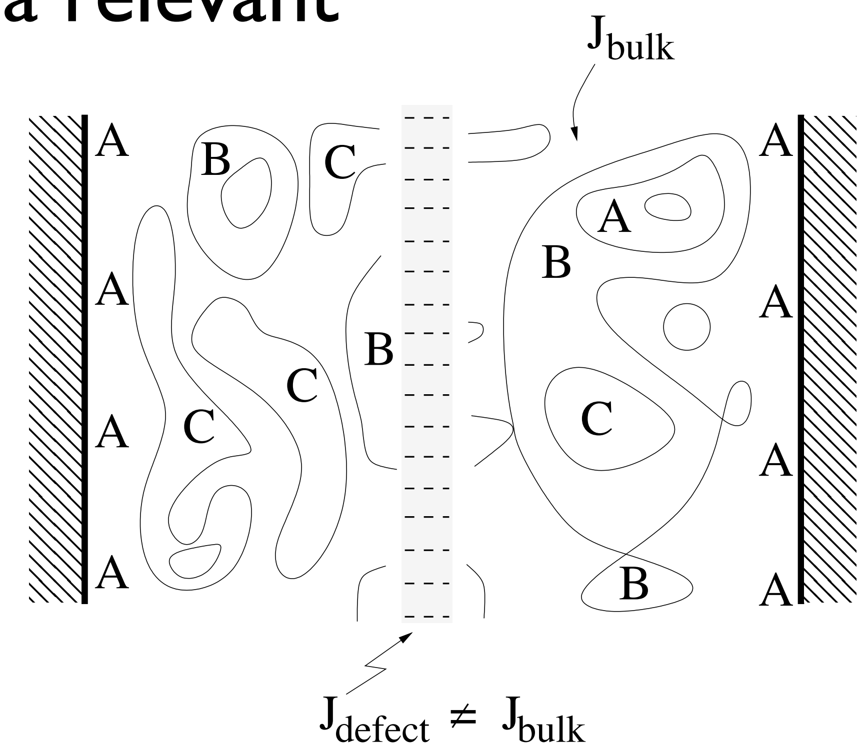
Bishara and Nayak, in prep.

see also: Rosenow et al. '08
Overbosch and Wen '08

Free b.c. in the 3-State Potts model

- Weakening a line of bonds is a relevant perturbation, $\Delta = 4/5$





- Flow to (Free, Free) b.c.

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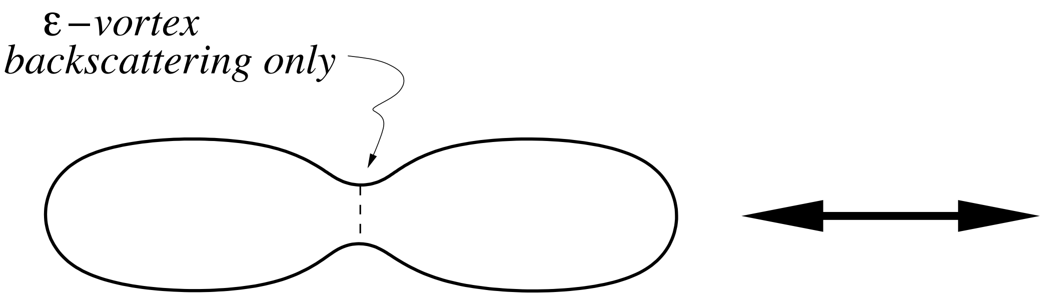
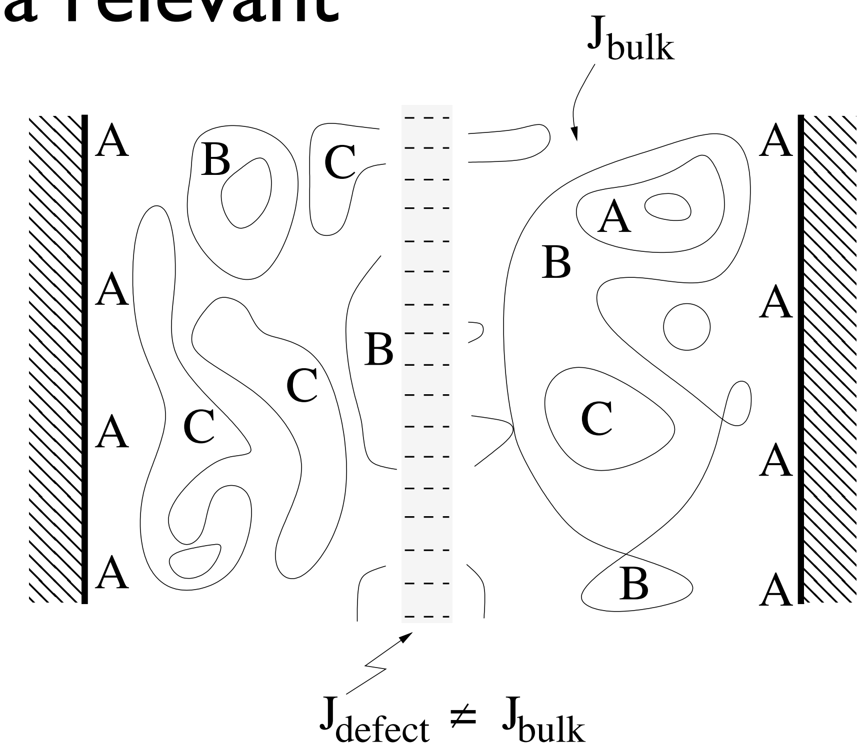
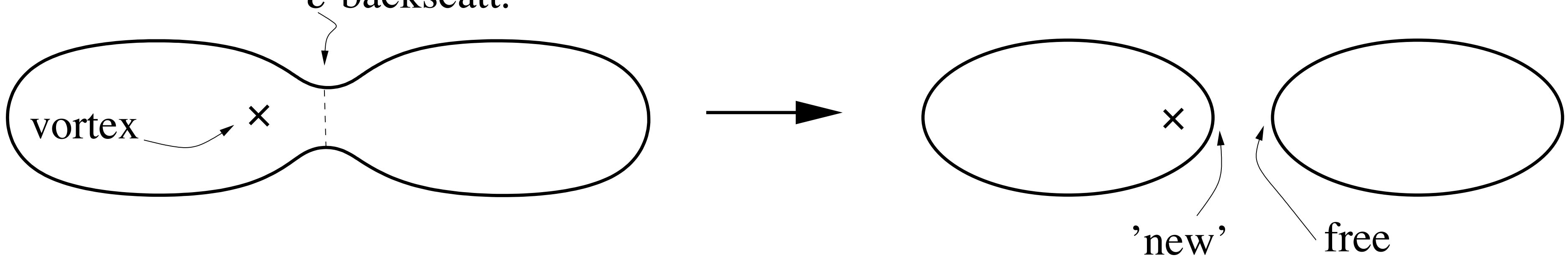




- Flow to (Free, Free) b.c.

- There is an 8th conformally-inv. b.c.:



Possible Relevance to Experiment

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- Recently, Willett et al. have measured the current through a 2-pt. contact interferometer.
- There are regions in which the oscillation period as a function of side gate voltage (a proxy for area) corresp. to e/4 qps and regions in which they corresp. to e/2 qps.
- These regions may correspond to even/odd qp. numbers in the interference loop.
- If so, the tunneling current should have temp., voltage dependence det'd by the CFT discussed above.

The long. resistance should scale with temp. differently in different possible states.

$$R_{xx} \sim T^{2g-2}$$

$\nu = \frac{5}{2}$	e^*	nA?	θ	g_c	g_n	g
MR:	$e/4$	yes	$e^{i\pi/4}$	$1/8$	$1/8$	$1/4$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
$\overline{\text{Pf}}$:	$e/4$	yes	$e^{-i\pi/4}$	$1/8$	$3/8$	$1/2$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
$\text{SU}(2)_2$:	$e/4$	yes	$e^{i\pi/2}$	$1/8$	$3/8$	$1/2$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
K=8:	$e/4$	no	$e^{i\pi/8}$	$1/8$	0	$1/8$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
(3,3,1):	$e/4$	no	$e^{i3\pi/8}$	$1/8$	$1/4$	$3/8$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$

Different states, qps. - different coherence lengths

$$L_\phi(T) = \frac{1}{2\pi T}\left(\frac{g_c}{v_c} + \frac{g_n}{v_n}\right)^{-1}$$

$e/4$	MR	$\overline{\text{Pf}}/\text{SU}(2)_2$	K=8	(3,3,1)	e/2
L^* in μm	1.4	0.5	19	0.7	4.8
T^* in mK	36	13	484	19	121

$\nu = \frac{12}{5}$	e^*	nA?	θ	g_c	g_n	g
$\text{HH}_{2/5}$:	$e/5$	no	$e^{i3\pi/5}$	$1/5$	$2/5$	$3/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$
$\text{BS}_{2/5}$:	$e/5$	yes	$e^{i9\pi/40}$	$1/10$	$1/8$	$9/40$
	$e/5$	no	$e^{-i2\pi/5}$	$1/10$	$1/2$	$3/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$
$\overline{\text{BS}}^{\psi}_{3/5}$:	$e/5$	yes	$e^{-i11\pi/40}$	$1/10$	$3/8$	$19/40$
	$e/5$	no	$e^{-i2\pi/5}$	$1/10$	$1/2$	$3/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$
$\overline{\text{RR}}_{k=3}$:	$e/5$	yes	$e^{-i\pi/5}$	$1/10$	$3/10$	$2/5$
	$2e/5$	no	$e^{i2\pi/5}$	$2/5$	0	$2/5$

see also, X. Wan et al. '07
K. LeHur, '02

Summary

- Partition functions of quantum Hall droplets are given by critical 2D stat. mech. models with boundary conditions det'd by quasiparticles in the bulk.

- Inter-edge quasiparticle tunneling causes flows from one conformally-invariant b.c. to another.

- Even/odd effect = CD vs. CN b.c. for Ising defect

- Simple interp. for 8 conf. inv. b.c. of 3-State Potts, esp. free and 'new'.