

# Angular Oscillations in a Tilted Magnetic Field in Layered Metals and Bilayers: Q2D, Q1D, Graphite/Graphene, etc.

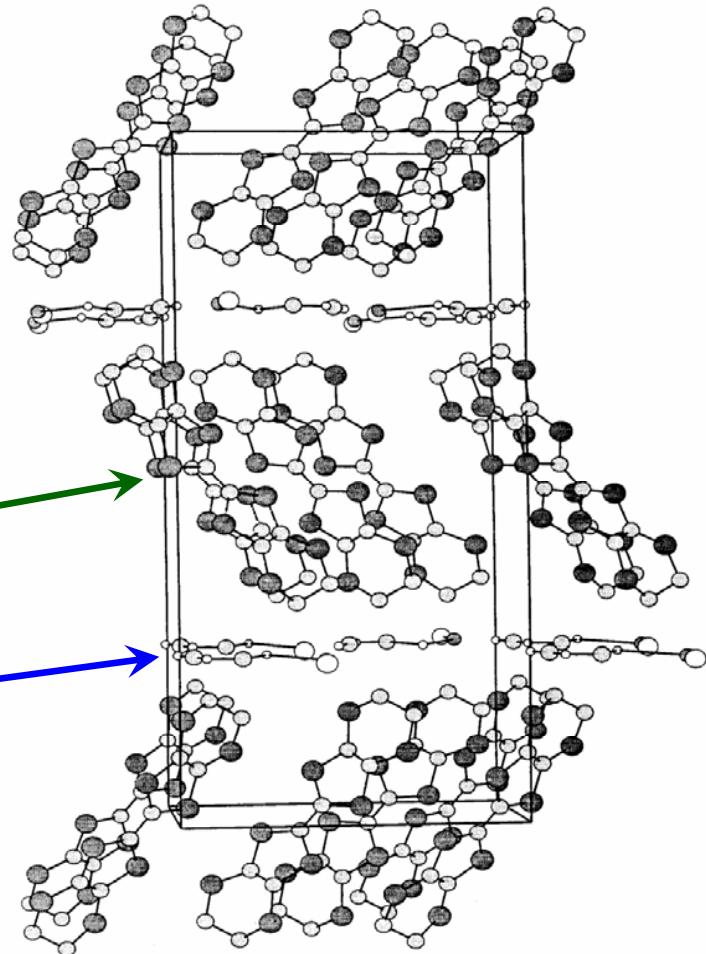
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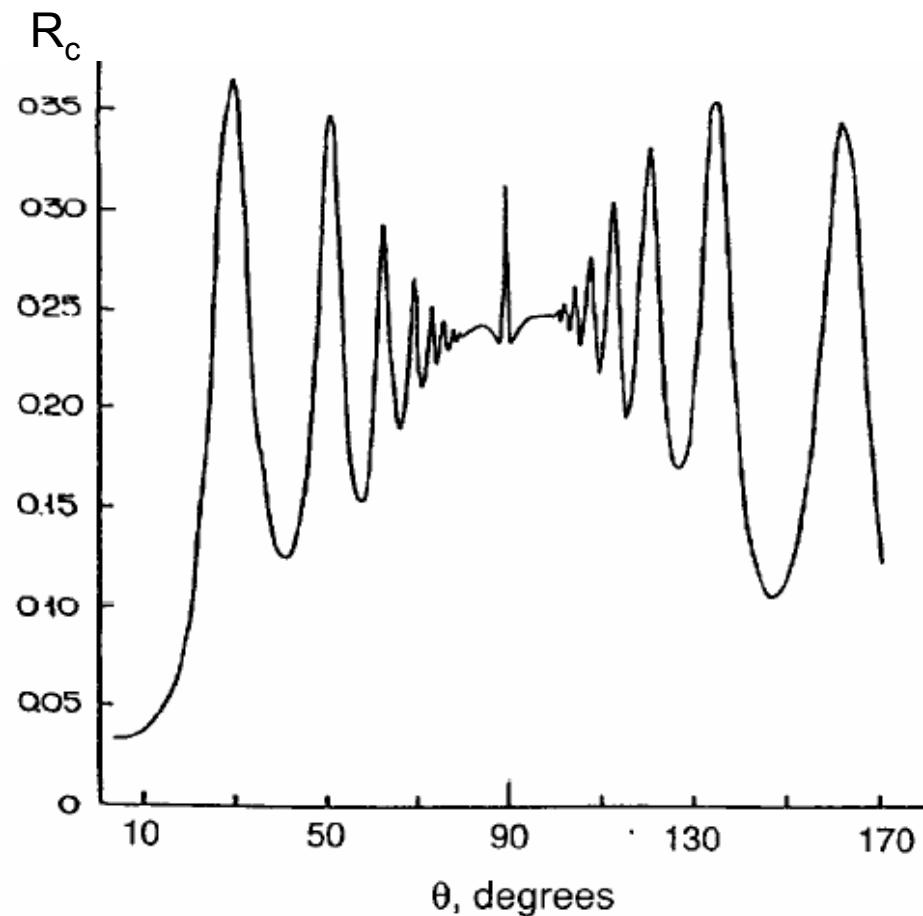
- Angular magnetoresistance oscillations (AMRO) in Q2D metals: 20 years since discovery in 1988
- AMRO in **semiconducting bilayers**: theory and experiment  
*Physica E* **34**, 128 (2006); *PRB* **78**, 155320 (2008)
- Proposal for AMRO in **graphene bilayers** and **multi-layers**
- Two-parameter pattern of AMRO in **Q1D** materials, arrays of **quantum wires**, driven **superconducting qubits**, ...  
*PRL* **96**, 037001 (2006); *PRB* **78**, 125404 (2008)

The  $(\text{BEDT-TTF})_2\text{X}$  organic conductors are **layered**,  
quasi-2D metals.

They consist of **BEDT-TTF** layers intercalated with layers of anions  $\text{X}=\text{I}_3^-$ ,  $\text{IBr}_2^-$ , etc.



# Angular magnetoresistance oscillations



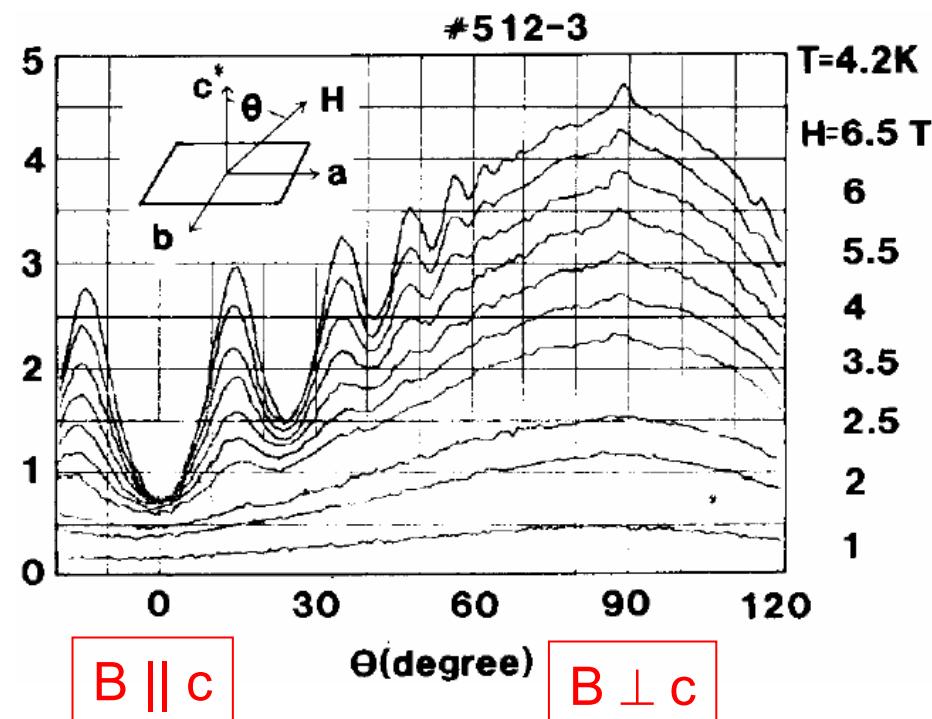
$B \parallel c$

$\theta$ , degrees

$B \perp c$

$B \parallel c$

$\beta$ -(BEDT-TTF)<sub>2</sub>IBr<sub>2</sub>,  $B = 14$  T,  $T=1.4$  K  
Kartsovnik *et al.*, *JETP Lett.* **48**, 541 (1988)



$B \parallel c$

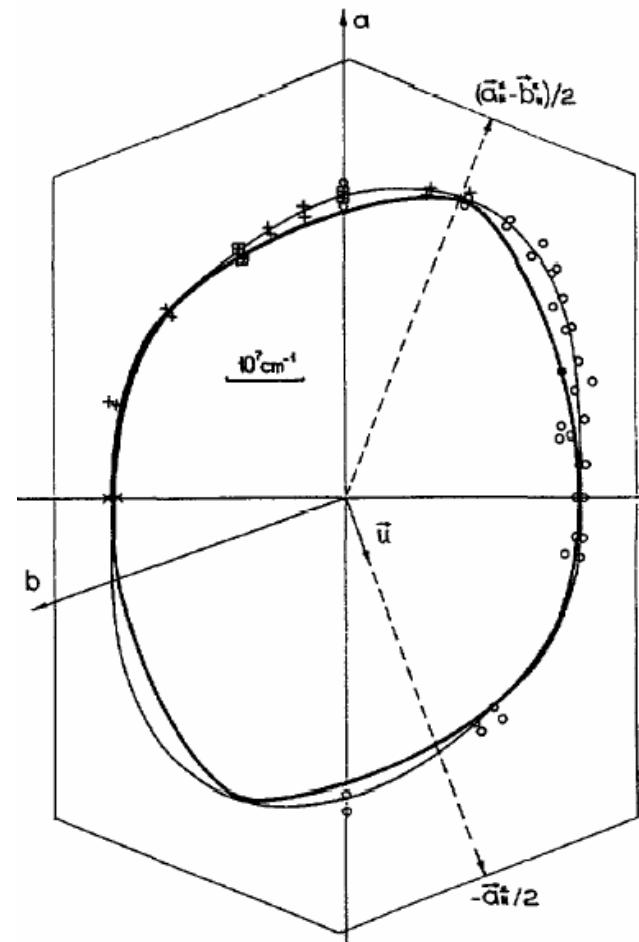
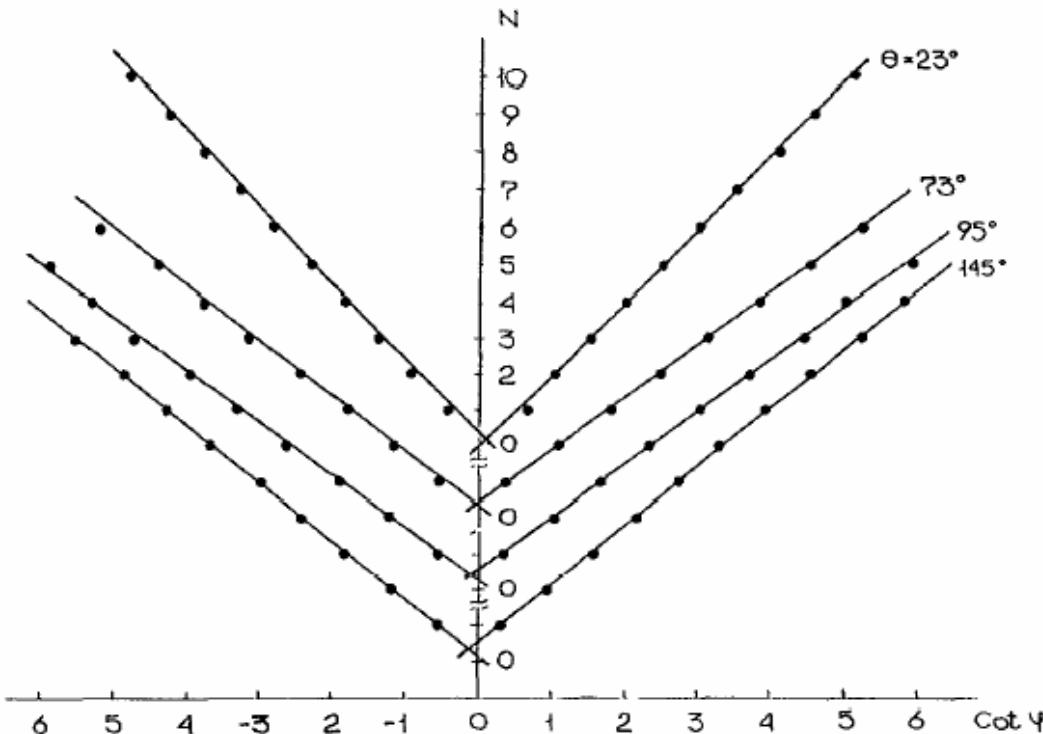
$\theta$ (degree)

$B \perp c$

$\theta$ -(BEDT-TTF)<sub>2</sub>I<sub>3</sub>, Kajita *et al.*,  
*Sol. St. Comm.* **70**, 1079 (1989)

Peaks of  $R_{zz}$  (and  $R_{xx}$ ) at  
the “magic angles”  $\Theta_n$

# Angular magnetoresistance oscillations



Resistance maxima:  $\tan \theta_n = \frac{\pi(n - 1/4)}{k_F d}$

*n* is an integer,

*d* is the distance between layers,

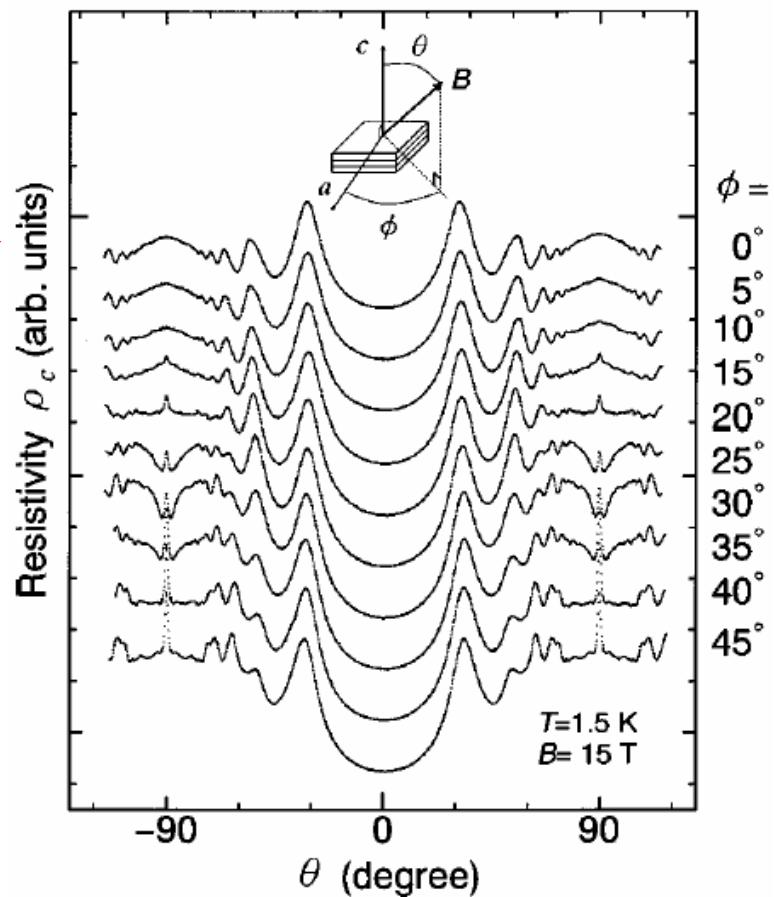
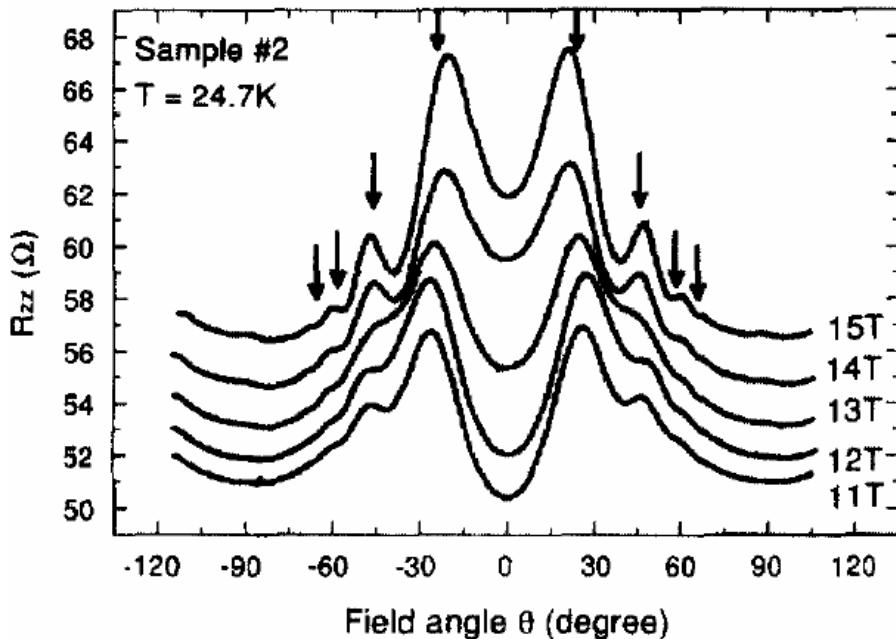
$k_F$  is the in-plane Fermi momentum.

Reconstruction of the  
Fermi surface from AMRO

$\beta$ -(BEDT-TTF)<sub>2</sub>IBr<sub>2</sub>, Kartsovnik *et al.*, J. Phys. I (France) **2**, 89 (1992)

# AMRO in other layered materials

- Intercalated graphite, Iye *et al.*,  
J. Phys. Soc. Jpn. **63**, 1643 (1994)
- $\text{Sr}_2\text{RuO}_4$ , Ohmichi *et al.*,  
Phys. Rev. B **59**, 7263 (1999)
- High- $T_c$  superconductor  $\text{Tl}_2\text{Ba}_2\text{CuO}_6$ ,  
Hussey *et al.*, Nature **425**, 814 (2003)



- GaAs/AlGaAs superlattices,  
Kawamura *et al.*, Physica B **249-251**, 882 (1998)  
Notice high temperature 25 K

# AMRO in intercalated graphite

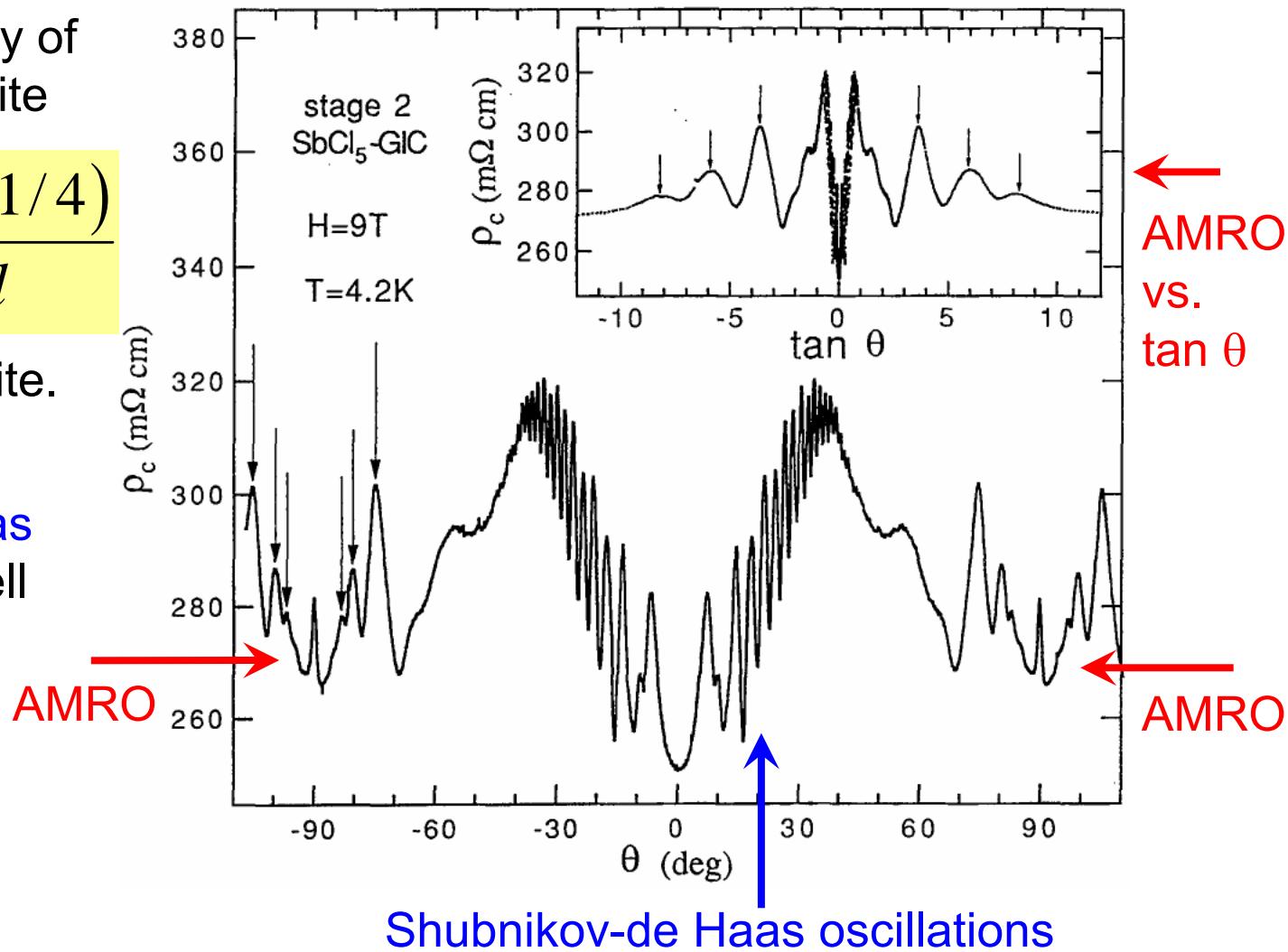
Iye, Baxendale, Mordkovich, *J. Phys. Soc. Jpn.* **63**, 1643 (1994)

Interlayer resistivity of  
intercalated graphite

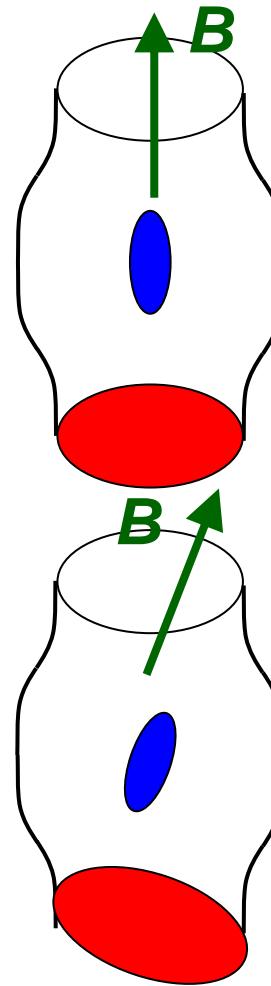
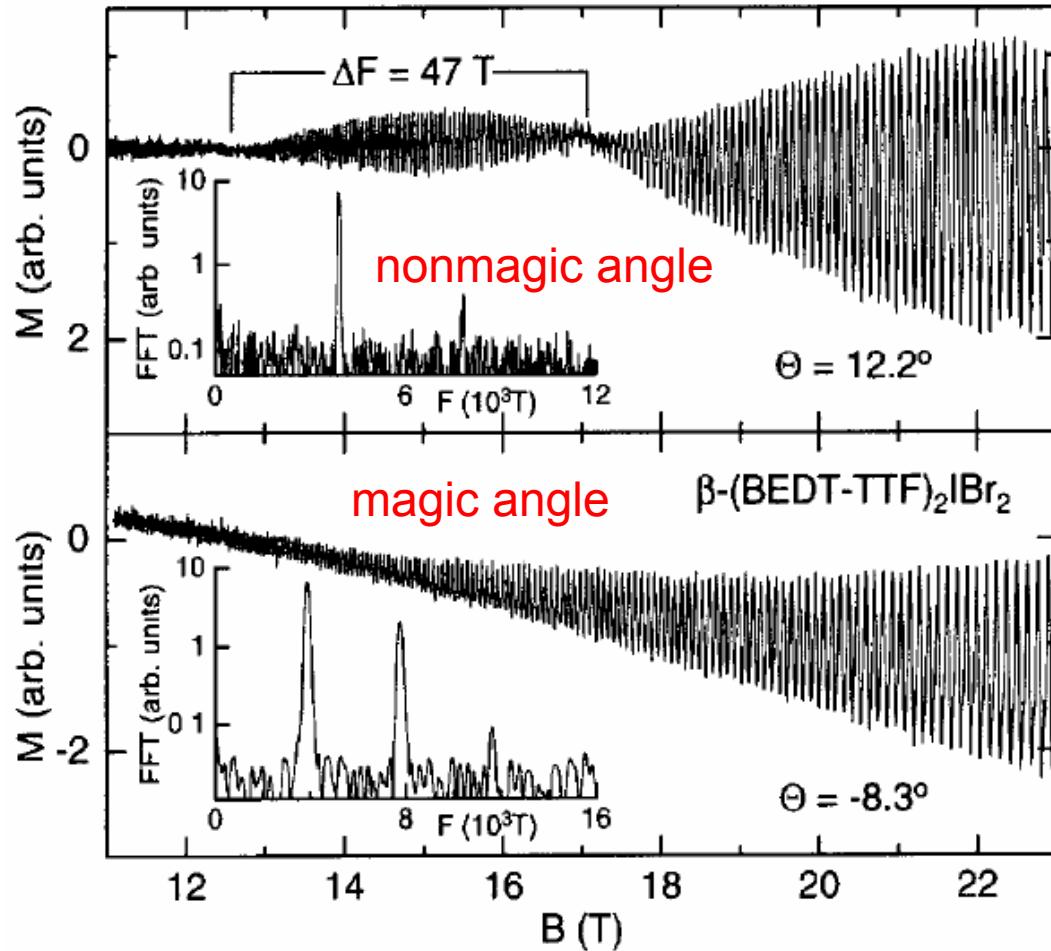
$$\tan \theta_N = \frac{\pi(N - 1/4)}{k_F d}$$

$k_F d \ll 1$  for graphite.

So, AMRO and  
Shubnikov-de Haas  
oscillations are well  
separated.



# Shubnikov-de Haas and de Hass-van Alphen oscillations beatings

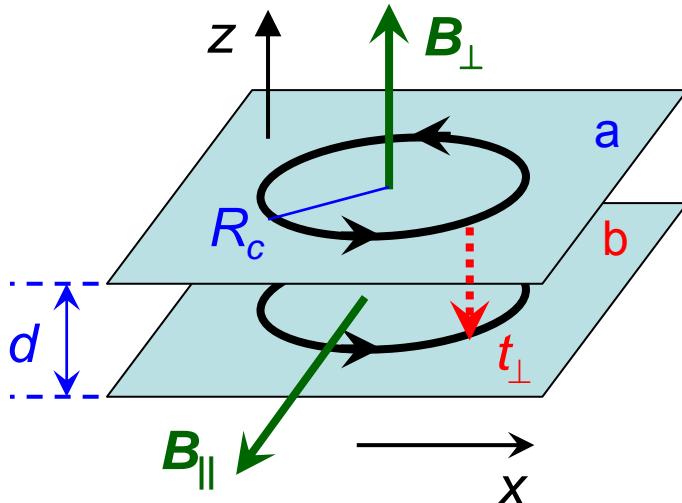


Yamaji JPSJ 1989:  
 $S_{\max} = S_{\min}$  at the  
 magic angles  
 $\Rightarrow$  no beatings.  
 The average  
 velocity  $\langle v_z \rangle \rightarrow 0$ ,  
 so  $\sigma_z \rightarrow 0$ ,  $R_z \rightarrow \infty$ .

$$\begin{aligned}\langle v_z \rangle &= \left\langle \frac{\partial \epsilon}{\partial k_z} \right\rangle \\ &= - \frac{\partial S / \partial P_z}{\partial S / \partial \epsilon} \\ &= - \frac{\partial S / \partial P_z}{2\pi m}\end{aligned}$$

$\beta\text{-}(\text{BEDT-TTF})_2\text{IBr}_2$ ,  $T=0.4 \text{ K}$ , Wosnitza *et al.*, J. Phys. I (France) **6**, 1597 (1996)

# Theory of AMRO in bilayers



Interlayer tunneling, gauge  $A_z = B_{\parallel}x$

$$\hat{H}_{\perp} = t_{\perp} \hat{\psi}_a^{\dagger}(\mathbf{r}) \hat{\psi}_b(\mathbf{r}) e^{ieA_z(\mathbf{r})d/\hbar c} + \text{h.c.}$$

For quasiclassical cyclotron motion in the plane, the effective amplitude of inter-plane tunneling is obtained by averaging of the phase over time  $t$ :

$$\tilde{t}_{\perp} = t_{\perp} \left\langle e^{ieB_{\perp}x(t)d/\hbar c} \right\rangle_t, \quad x(t) = R_c \cos(\omega_c t), \quad R_c = k_F \hbar c / B_{\perp} e$$

$$\tilde{t}_{\perp} = t_{\perp} J_0 \left( \frac{B_{\perp}}{B_{\parallel}} k_F d \right) \propto t_{\perp} \cos \left( \frac{B_{\perp}}{B_{\parallel}} k_F d - \frac{\pi}{4} \right) \quad - \text{Bessel function}$$

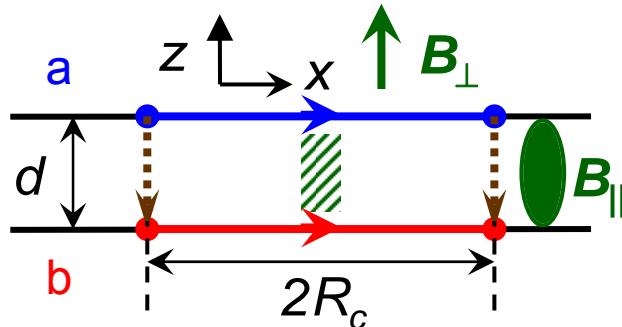
The effective amplitude  $\tilde{t}_{\perp} \rightarrow 0$  when

$$\frac{B_{\perp}}{B_{\parallel}} = \tan \theta_n = \frac{\pi(n-1/4)}{k_F d}$$

The layers decouple, so  $\sigma_z \rightarrow 0$  ( $R_z \rightarrow \infty$ ), and beatings disappear.

# AMRO as Aharonov-Bohm interference

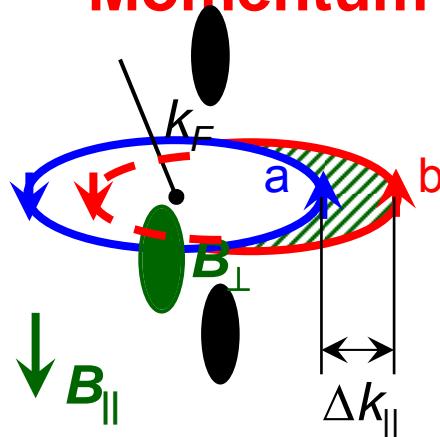
**Real-space picture:** Periodicity in the magnetic flux of  $B_{\parallel}$



$$\Phi = B_{\square}^{(n)} 2R_c d = \phi_0(n + C), \quad R_c = \frac{k_F \phi_0}{2\pi B_{\perp}}$$

$$\frac{B_{\square}^{(n)}}{B_{\perp}} = \tan \theta_n = \frac{\pi(n + C)}{k_F d}$$

**Momentum-space picture:** Interlayer tunneling  $t_{\perp} e^{ixeB_{\square}d/c\hbar}$  injects the in-plane momentum  $\Delta k_{\parallel} = eB_{\parallel}d/c$ .



Interference between trajectories a and b:

$$(n + C)B_{\perp}^{(n)} = \frac{c}{2\pi e\hbar} 2k_F \Delta k_{\square} \Rightarrow B_{\perp}^{(n)} = \frac{B_{\square} k_F d}{\pi(n + C)}$$

**Interlayer matrix element between Landau wave functions:**

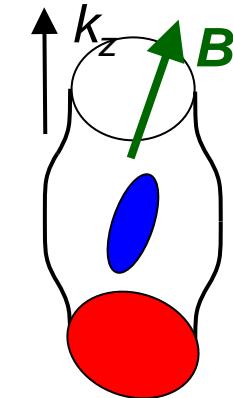
Hu & MacDonald, *PRB* **46**, 12554 (1992)  $\langle n', a | \hat{H}_{\perp} | n, b \rangle \propto L_n^{n'-n} \rightarrow J_{n'-n} \rightarrow J_0$

Kurihara, *J. Phys. Soc. Jpn.* **61**, 975 (1992): Laguerre  $\rightarrow$  Bessel for  $n \gg 1$

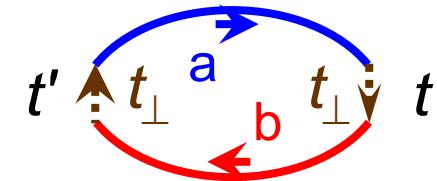
# Interlayer conductance $\sigma_z$ with finite $\tau$

Shockley tube integral: Yagi *et al.*, *JPSJ* **59**, 3069 (1990)

$$\sigma_z \propto \int_t^\infty dt' \left\langle v_z(t)v_z(t') e^{-(t'-t)/\tau} \right\rangle_t, \quad v_z(t) = 2t_\perp \sin[k_z(t)d]$$



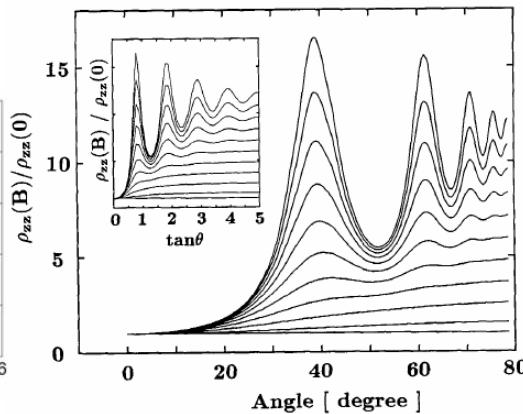
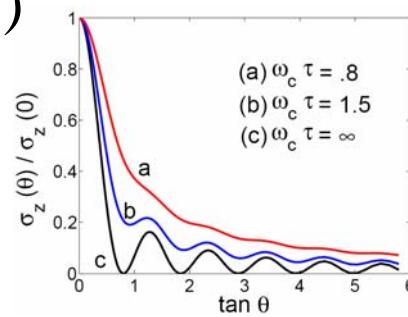
Interlayer tunneling: McKenzie & Moses, *PRL* **81**, 4492 (1998), *PRB* **60**, 7998 (1999)



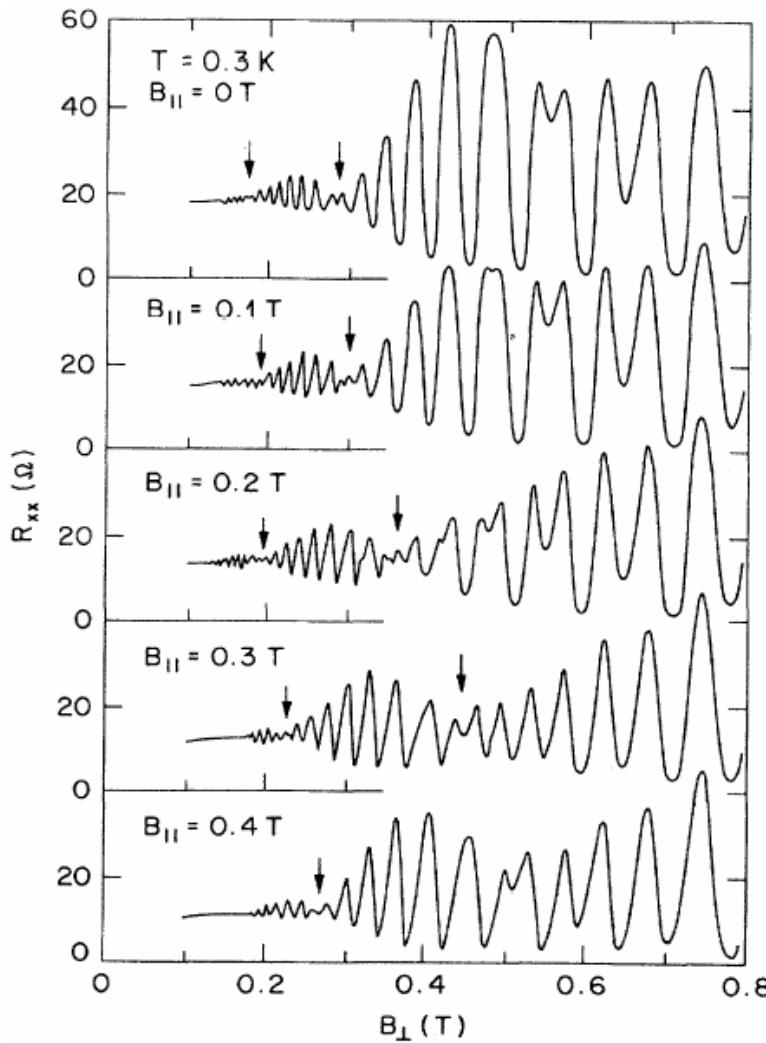
$$\sigma_z \propto t_\perp^2 \int_t^\infty dt' \left\langle e^{\frac{ieB_0d}{\hbar c}[x(t)-x(t')] - \frac{t'-t}{\tau}} \right\rangle_t \propto \begin{cases} \tilde{t}_\perp^2 \tau & \text{for } \omega_c \tau \gg 1 \\ t_\perp^2 \tau & \text{for } \omega_c \tau \ll 1 \end{cases}$$

$$\frac{\sigma_z}{\sigma_z^0} = J_0^2(k_F d \tan \theta) + 2 \sum_{m=1}^{\infty} \frac{J_m^2(k_F d \tan \theta)}{1 + (m \omega_c \tau)^2}$$

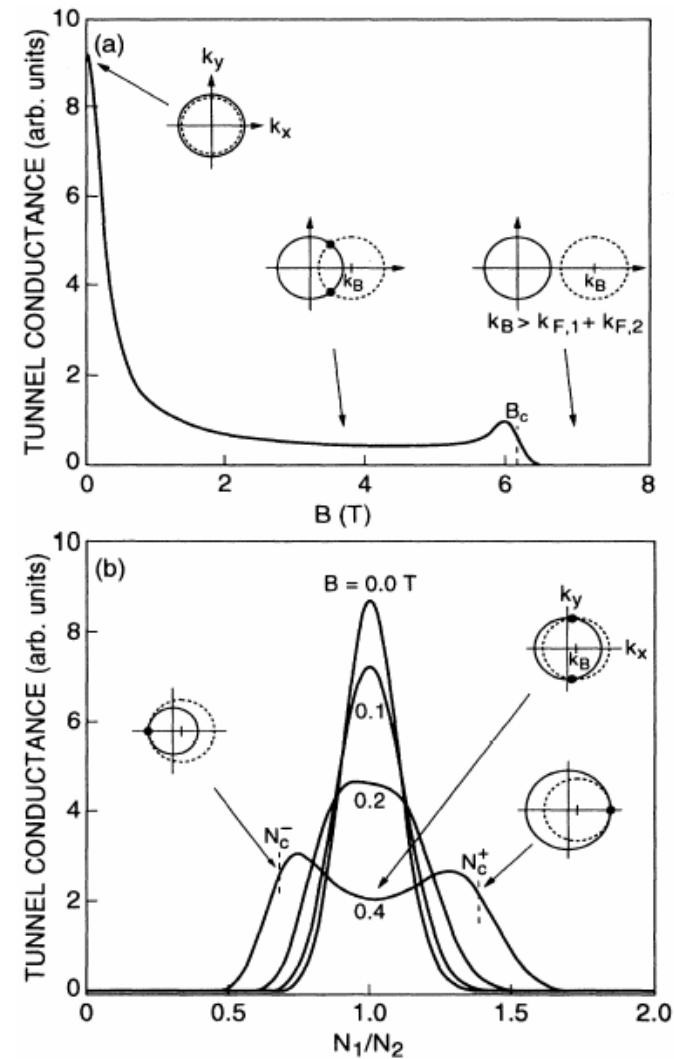
Interpolation between strong AMRO for  $\omega_c \tau \gg 1$  and no AMRO for  $\omega_c \tau \ll 1$



# Experiments in bilayers



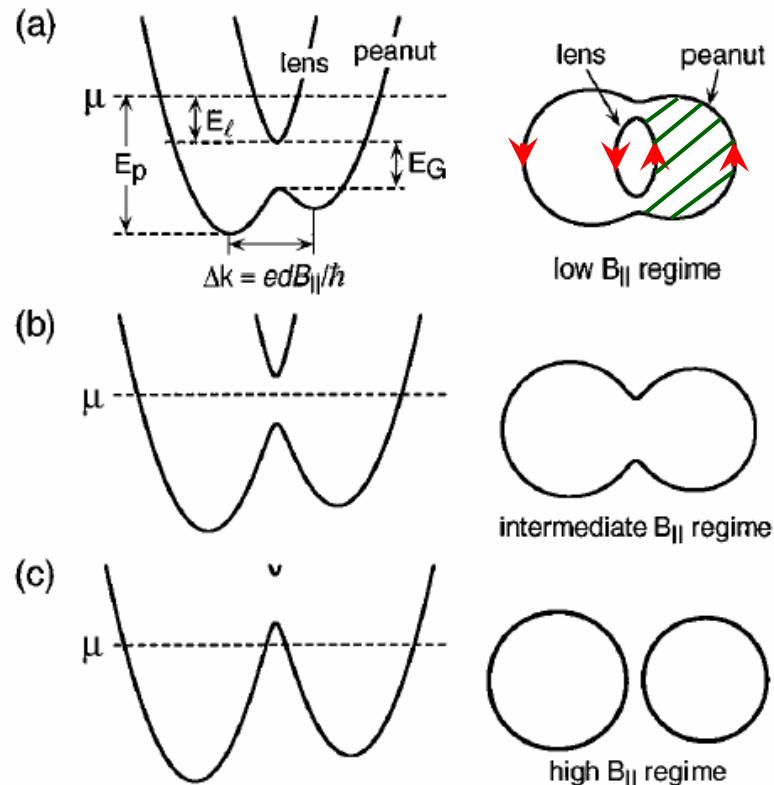
Beating period in  $B_\perp$  increases with  $B_{\parallel}$   
 Boebinger et al., PRB 43, 12673 (1991)



Interlayer conductance vs.  $B_{\parallel}$   
 Eisenstein et al., PRB 44, 6511 (1991)

# Experiments in bilayers

Harff, Simmons et al., PRB 55, 13405 (1997)

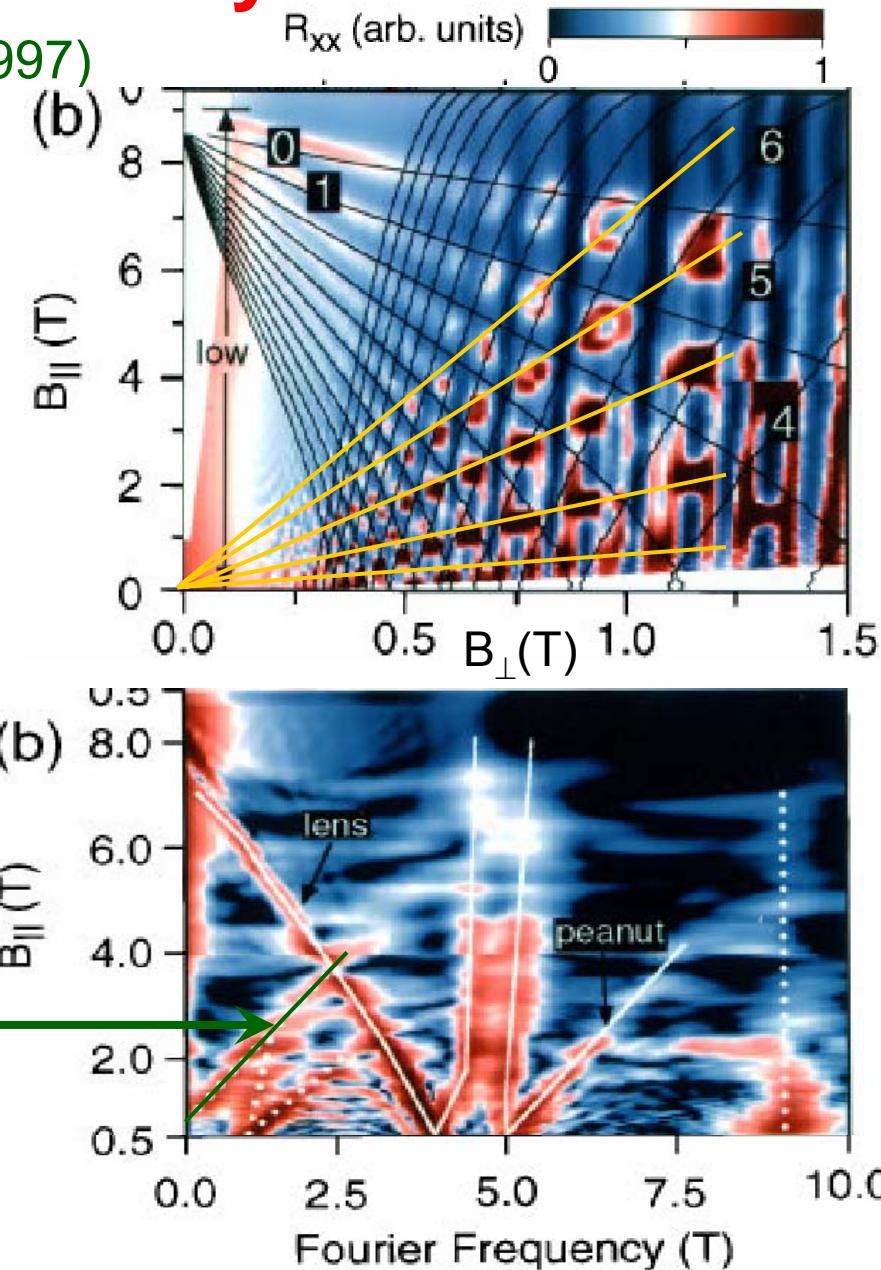


AMRO in  
bilayers?

Interference  
oscillations?

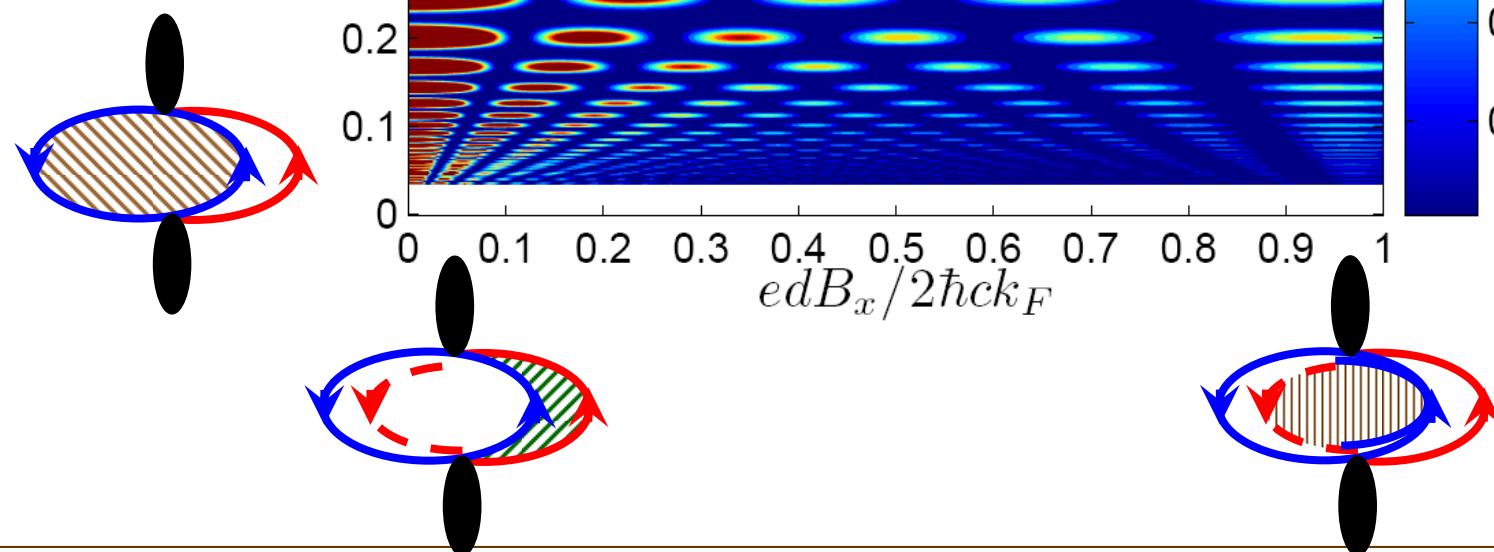
$$B_{\perp}^{(n)} \approx \frac{B_{\parallel} k_F d}{\pi(n + C)}$$

$R_{zz}$  is needed



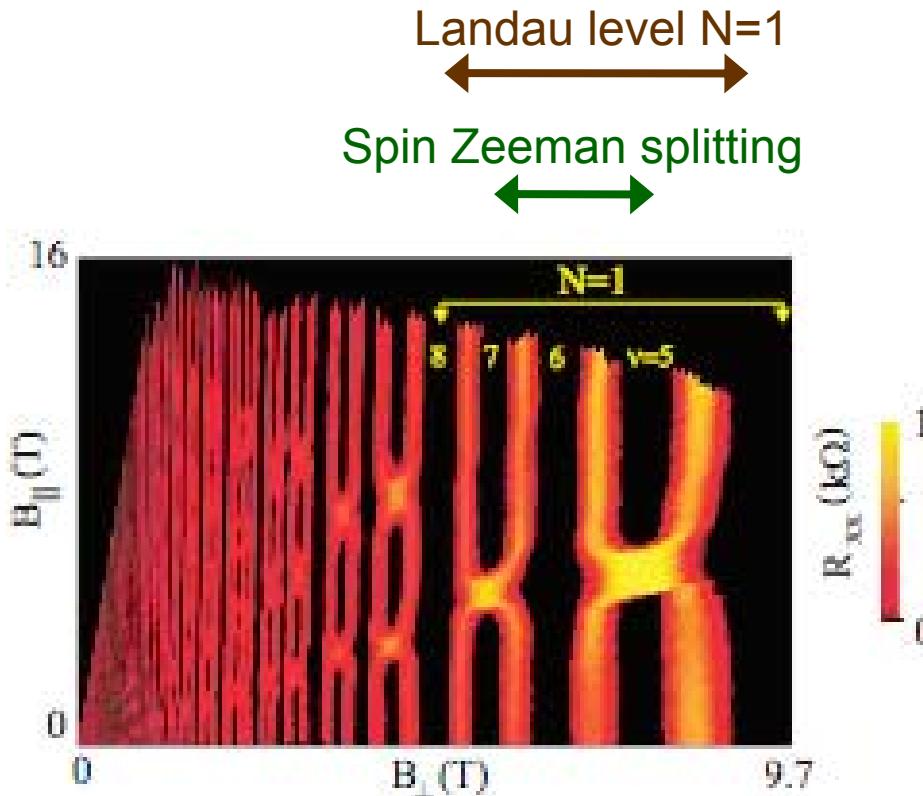
# SdH and AMRO in bilayer with parabolic dispersion

Contour plot of the calculated  $\sigma_{zz}$  vs.  $B_x$  and  $B_z$



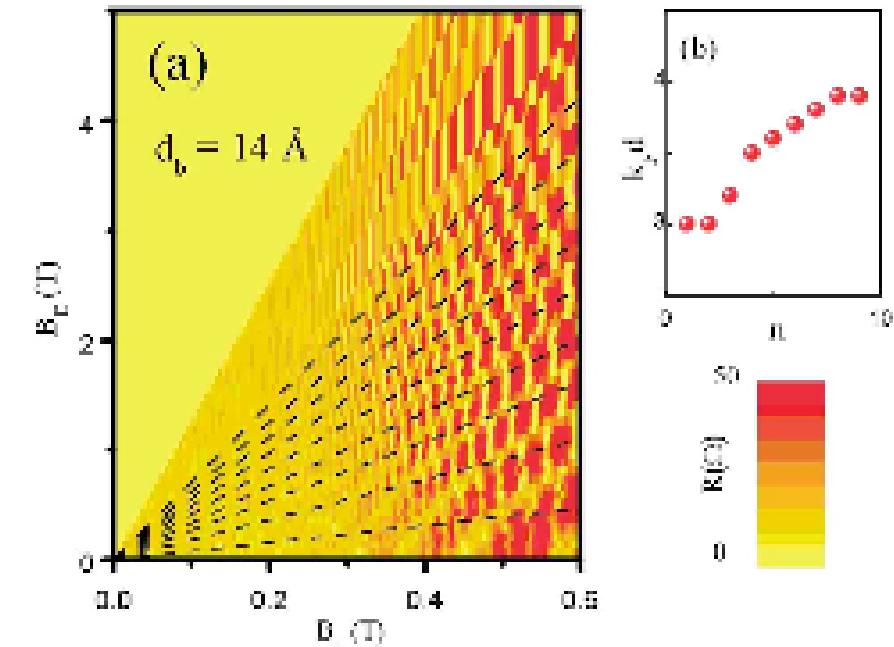
# Experimental observation of AMRO in bilayers

Gusev *et al.*, PRB 78, 155320 (2008)



Bilayer splitting  $\leftrightarrow \Delta_{SAS}=2t_{\perp}$

Bilayer splitting vanishes  
for certain values of  $B_{\parallel}$

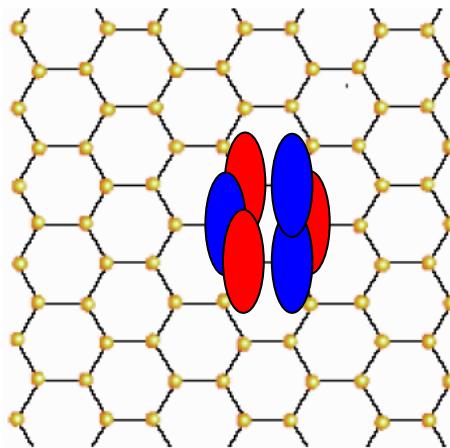


Observation of AMRO for  
high Landau levels,  
described by semiclassical  
cyclotron orbits

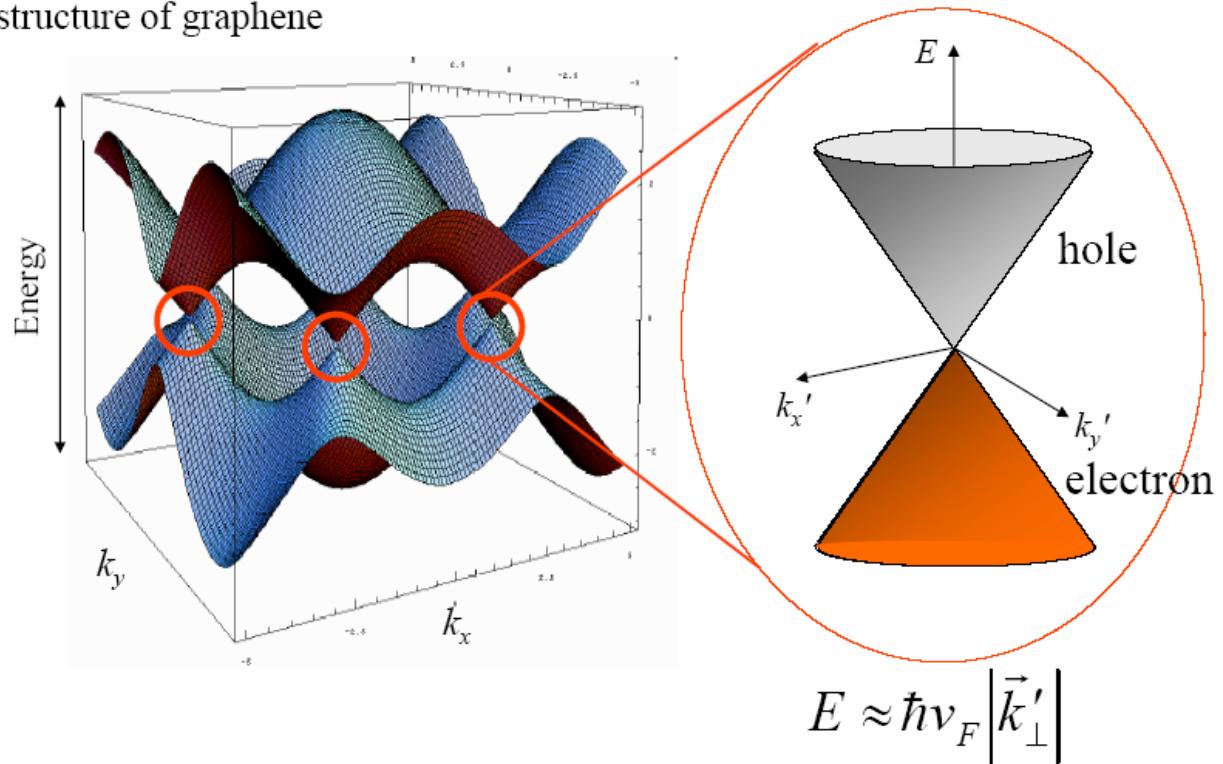
# Graphene – the single-layer graphite

## Graphene : Dirac Particles in 2D Box

Band structure of graphene



Real-space structure:  
hexagonal lattice,  
two sublattices



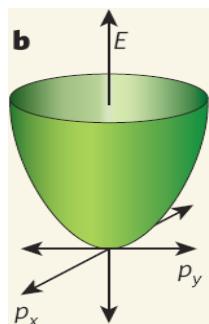
Massless Dirac Particles with effective speed of light  $v_F$

Energy dispersion: two  
bands touching at points

Linear dispersion  
near the Dirac point

Graphics courtesy of Philip Kim

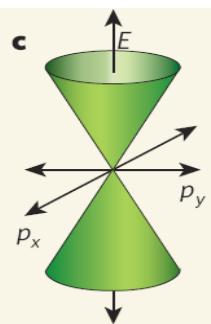
# Parabolic dispersion (semiconductors)



$\psi(\mathbf{p})$  – scalar

$$\epsilon(\mathbf{p}) = \frac{\mathbf{p}^2}{2m}$$

# Linear dispersion (graphene)



$\psi(\mathbf{p})$  – spinor

$$\epsilon(\mathbf{p}) = v_F p$$

**In a perpendicular magnetic field**

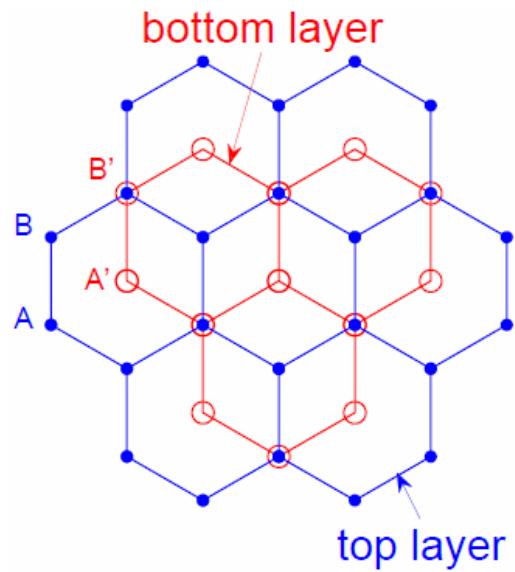
$$E_n = \frac{\hbar e B_z}{m} \left( n + 1/2 \right)$$

$$\psi_n(x, y) = |n\rangle_{h.o}$$

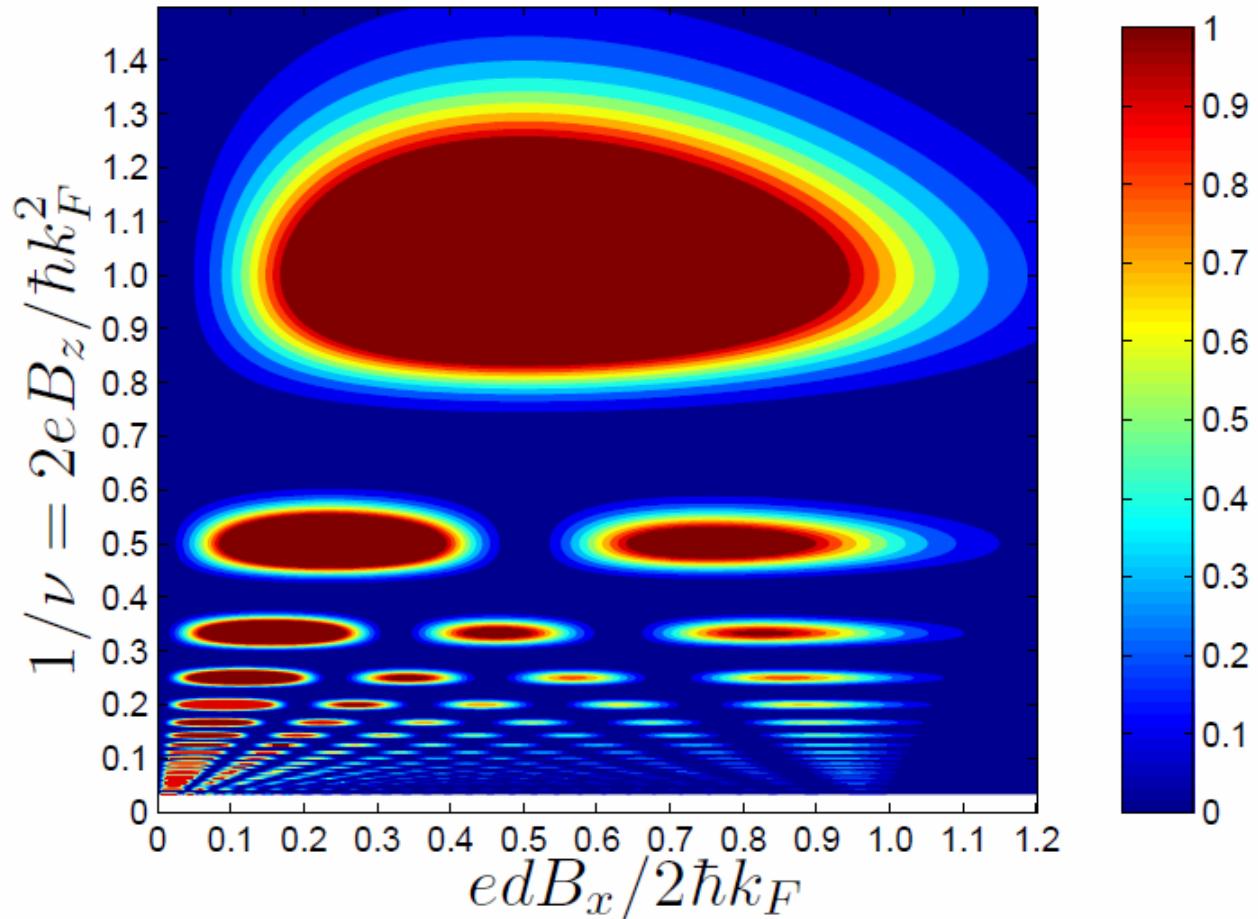
$$E_n = \pm \sqrt{2nB_z e \hbar v_F^2}$$

$$\psi_n(x, y) = \begin{pmatrix} |n\rangle_{h.o} \\ |n-1\rangle_{h.o} \end{pmatrix}$$

# Shubnikov-de Hass oscillations and AMRO in graphene bilayers with ABAB stacking

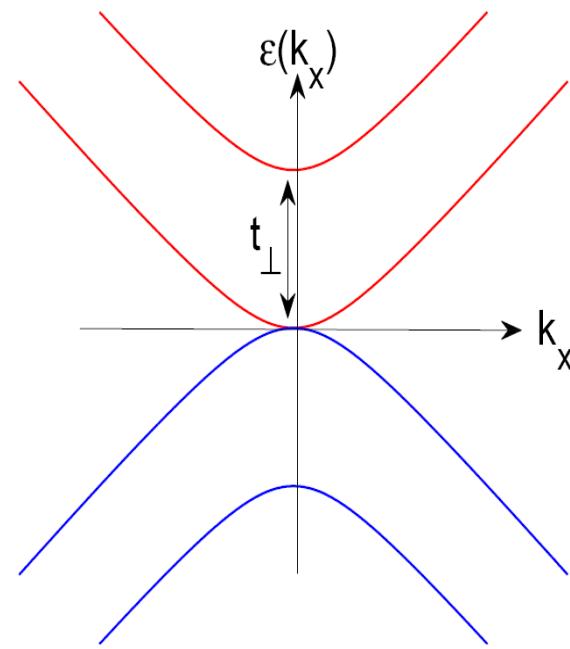
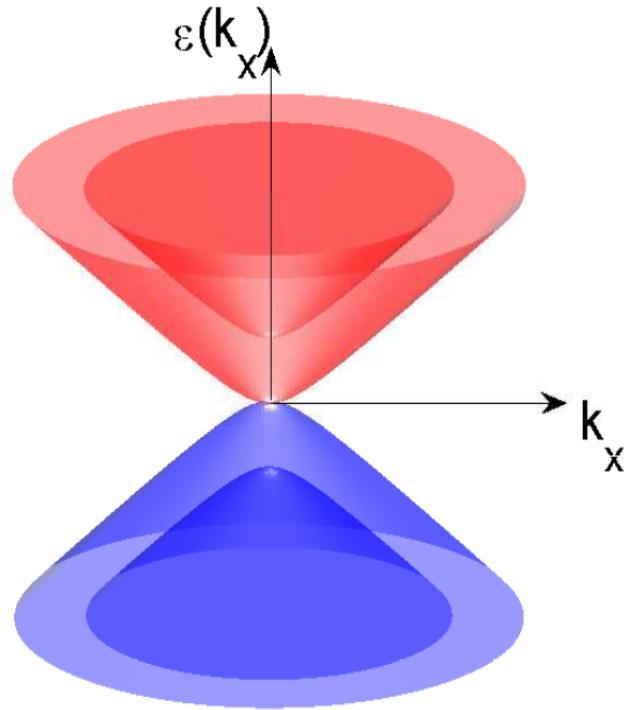


Sublattice A of one layer  
couples to sublattice B' of another layer



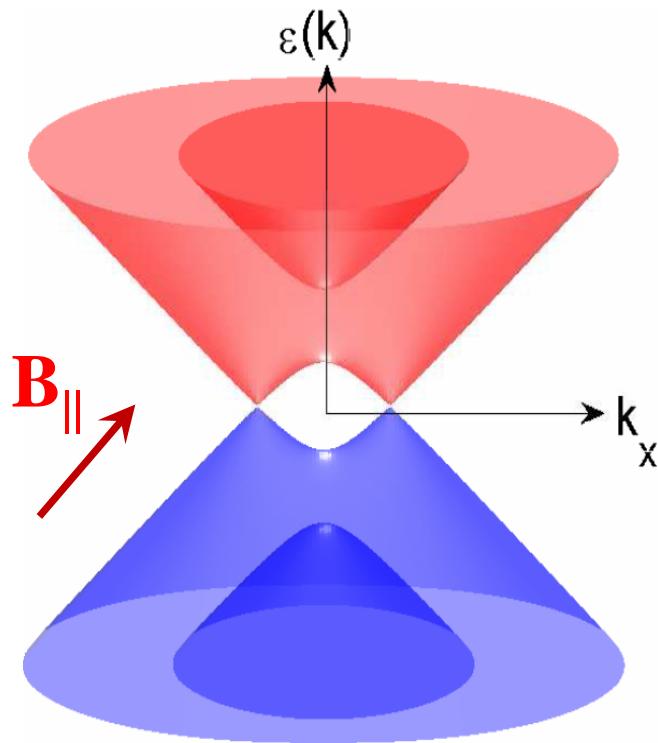
## Contour plot of $\sigma_{zz}$ vs. $B_x$ and $B_z$

# Nonperturbative treatment of $t_{\perp}$ in graphene bilayer

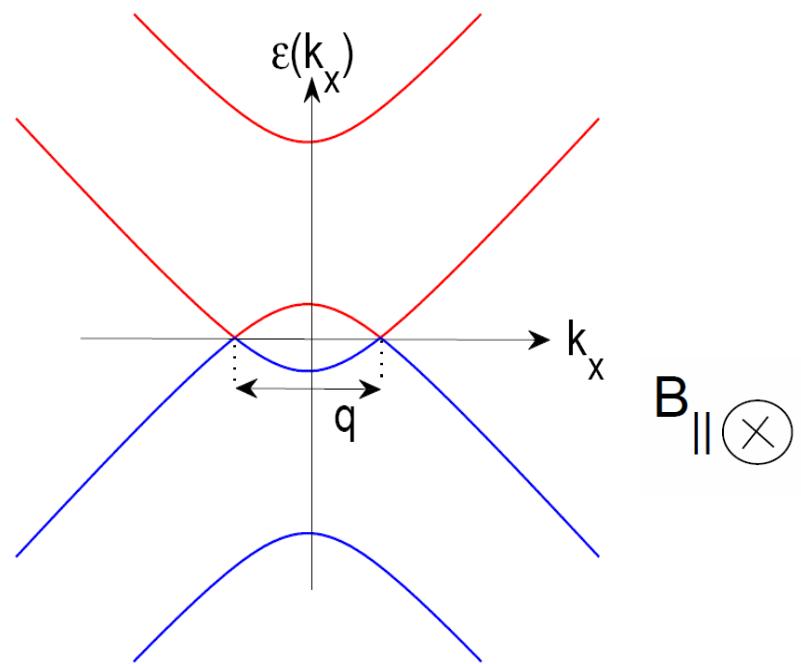


Electron dispersion is parabolic; Dirac points are lost.

# Dispersion in a parallel magnetic field



In a parallel magnetic field,  
the Dirac cones reappear.

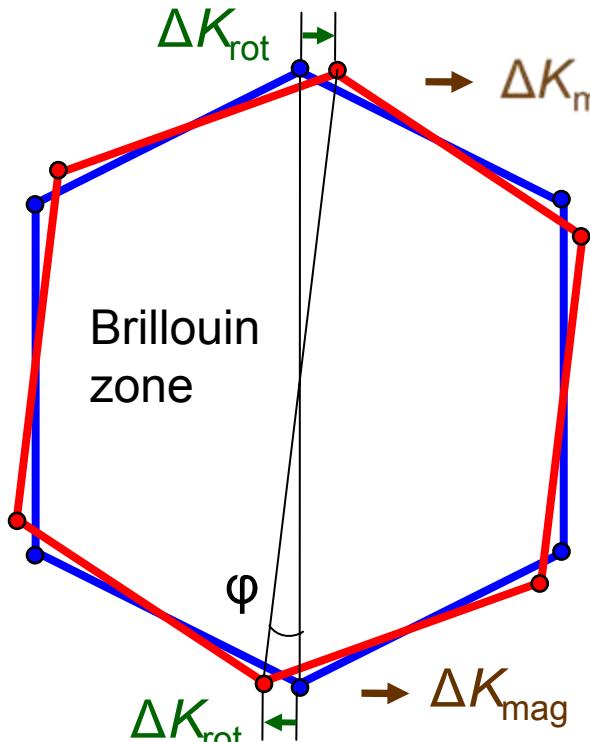


The parallel magnetic field shifts the in-plane momentum of electrons by  $q$  when they tunnel between layers.

Electronic properties of a graphene bilayer can be controlled by the orbital effect of a parallel magnetic field.

# Graphene bilayer with a twist

Theory: Lopes dos Santos, Peres, Castro Neto, *PRL* **99**, 256802 (2007)



$$\Delta K_{\text{rot}} = K\varphi = (4\pi/3a)\varphi \\ = 6 \times 10^8 \text{ m}^{-1} \text{ for } \varphi = 2^\circ$$

$$\Delta K_{\text{mag}} = eB_{\parallel}d/\hbar = 4 \times 10^6 \text{ m}^{-1} \text{ for } B_{\parallel} = 50 \text{ T}$$

The magnetic shift of the in-plane momentum is much smaller than the rotational shift:

$$\Delta K_{\text{mag}}/\Delta K_{\text{rot}} = 0.7 \times 10^{-2}$$

In general,

$$\frac{\Delta K_{\text{mag}}}{\Delta K_{\text{rot}}} = \frac{3a}{4\pi\varphi} \frac{eB_{\parallel}d}{\hbar} = \frac{3}{4\pi} \left( \frac{\Phi_{ad}}{\Phi_0} \right) \frac{1}{\varphi},$$

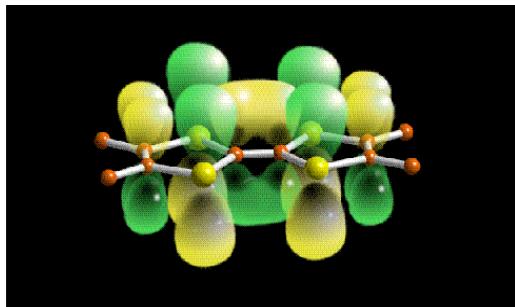
where  $\Phi_{ad} = B_{\parallel}ad$  and  $\Phi_0 = \hbar/e$ .

$$\Delta E_{\text{mag}} = \hbar v_F \Delta K_{\text{mag}} = v_F e B_{\parallel} d = 17 \text{ meV for } B_{\parallel} = 50 \text{ T}$$

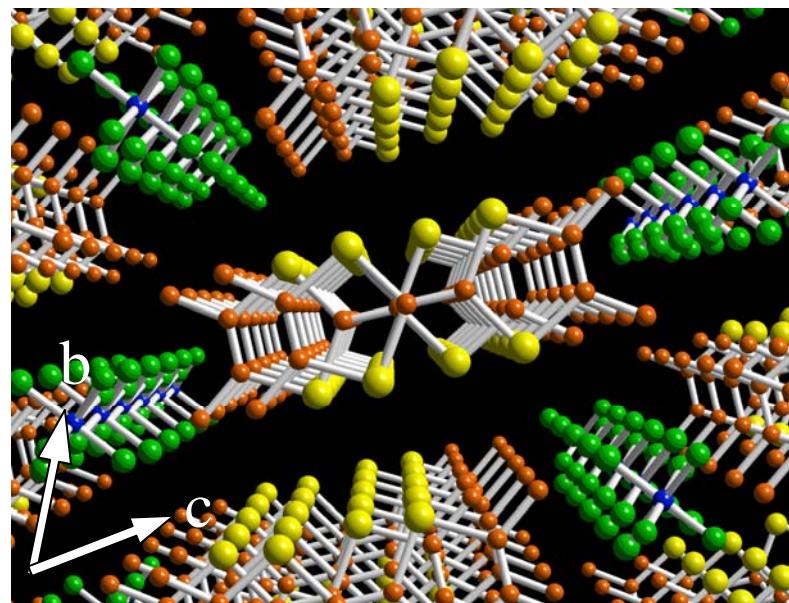
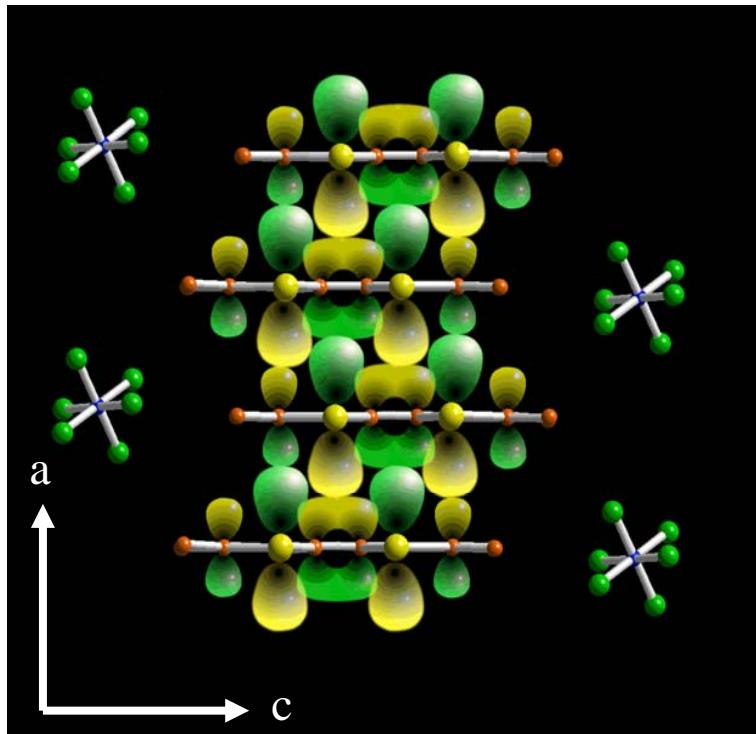
The van Hove singularity in DOS from the saddle point in the electron dispersion due to  $\Delta K_{\text{rot}}$  for a twisted bilayer was observed with STM by Eva Andrei.

# Q1D conductors $(\text{TMTSF})_2\text{X}$ (Bechgaard salts)

The **TMTSF** molecule



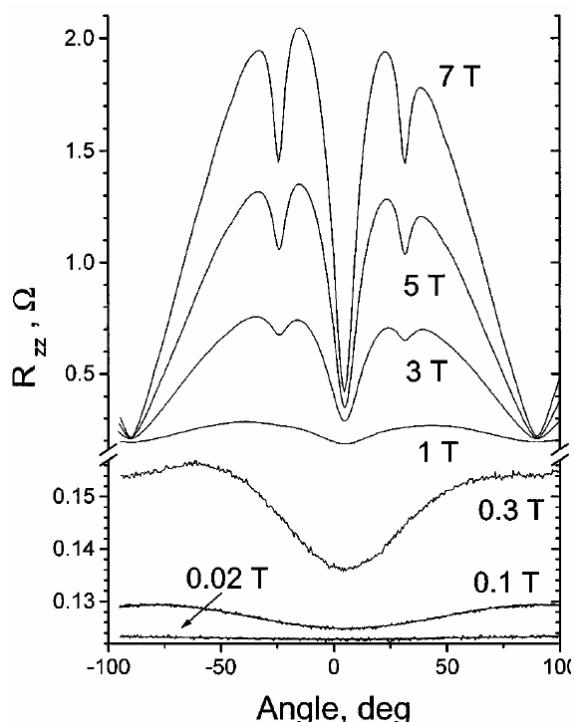
A stack of **TMTSF** molecules,  
with anions **X=PF<sub>6</sub>, ClO<sub>4</sub>**



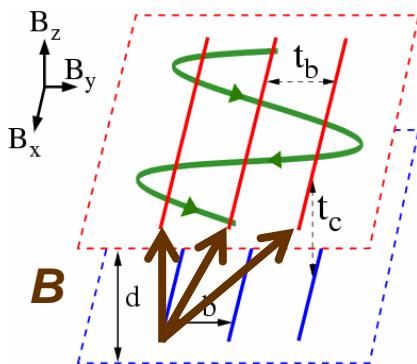
View along the **TMTSF** chains

# Different types of AMRO in Q1D conductors

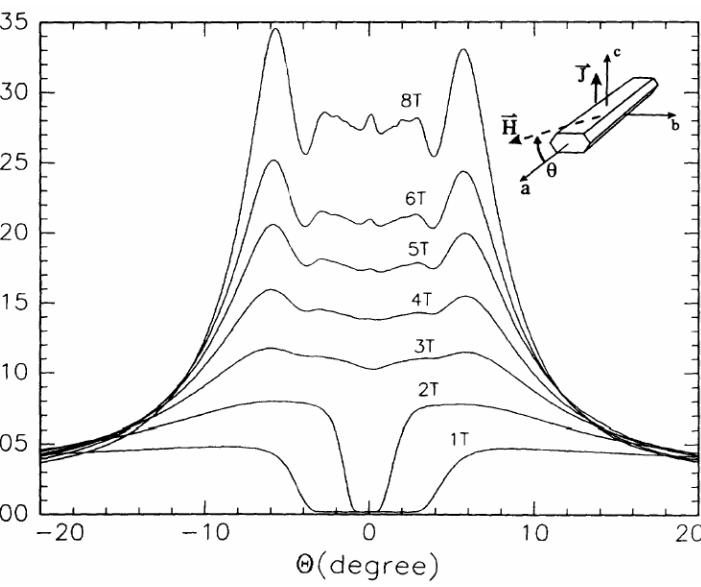
Lebed magic angles:  
 $(z,y)$  rotation, magnetic  
 field points from one  
 chain to another



Chashechkina, Chaikin,  
*PRL* **80**, 2181 (1998)

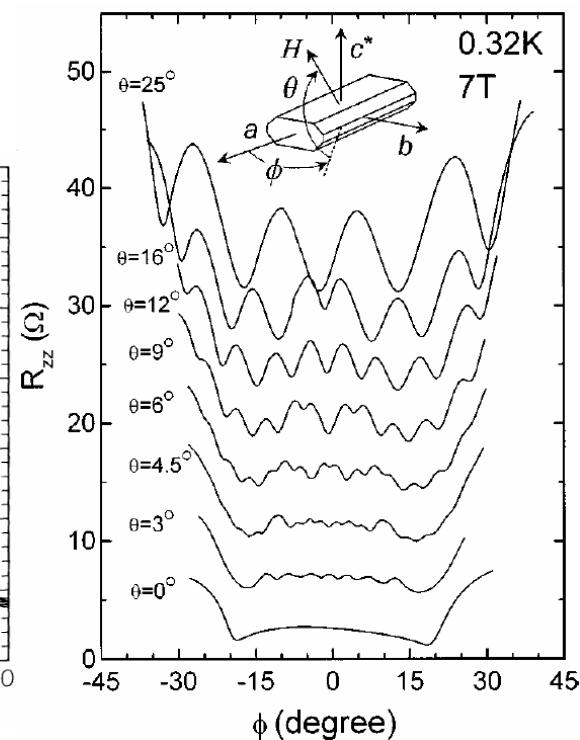


DKC effect:  $(x,z)$  rotation



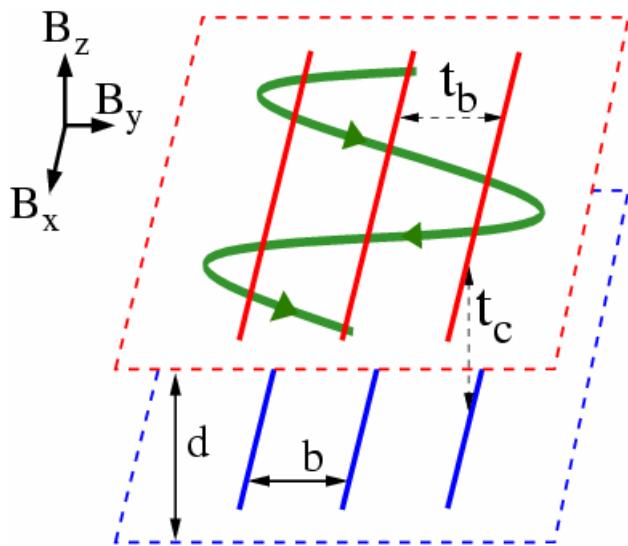
Danner, Kang, Chaikin,  
*PRL* **72**, 3714 (1994)

Third angular effect:  
 $(x,y)$  rotation.  
 Generic rotation –  
 all effects together.



Lee, Naughton, *PRB*  
**57**, 7423 (1998)

# Tunneling between two Q1D layers



Effective amplitude of inter-plane tunneling is obtained by phase averaging:

Interlayer tunneling:

$$\hat{H}_\perp = t_c \int d^2r \hat{\psi}_1^\dagger(\mathbf{r}) \hat{\psi}_2(\mathbf{r}) e^{i\phi(\mathbf{r})} + \text{H.c.}$$

$$\phi = \frac{ed}{\hbar c} A_z, \quad A_z = B_x y - B_y x$$

Quasiclassical in-plane motion:

$$x(t) = x_0 \pm v_F t, \quad y(t) = y_0 \mp \frac{2t_b c}{e v_F B_z} \sin(\omega_c t)$$

$$\omega_c = \frac{e b v_F B_z}{\hbar c}, \quad B_x' = \frac{B_x}{B_z} \frac{2t_b d}{\hbar v_F}, \quad B_y' = \frac{B_y}{B_z} \frac{d}{b}$$

$$\tilde{t}_c = t_c \left\langle e^{i\phi(t)} \right\rangle_t = t_c J_n \left( B_x' \right) \quad \text{for} \quad B_y' = n$$

- The condition  $n=B_y'$  corresponds to the Lebed magic angles
- Oscillations of  $J_n(B_x')$  represent the Danner-Kang-Chaikin effect

# Interlayer conductivity $\sigma_c$

Shockley tube integral (solution of the Boltzmann equation):

Yagi *et al.* (1990), Danner, Kang, Chaikin (1994)

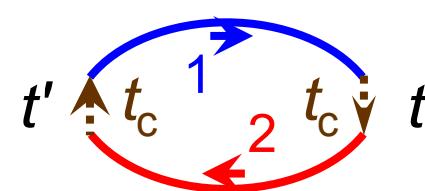
$$\sigma_c \propto \int_t^\infty dt' \left\langle v_z(t)v_z(t') e^{-(t'-t)/\tau} \right\rangle_t, \quad v_z(t) = 2t_c \sin[k_z(t)d]$$

Kubo formula: Osada, Kagoshima, Miura (1992),  
Lebed', Naughton (2003)

Interlayer tunneling conductance: McKenzie, Moses, Lundin  
(1998-2004), Osada (2002-2003)

$$\sigma_c \propto t_c^2 \int_t^\infty dt' \left\langle e^{i\phi(t) - i\phi(t') - (t'-t)/\tau} \right\rangle_t \propto \begin{cases} \tilde{t}_c^2 \tau & \text{for } \omega_c \tau \gg 1 \\ t_c^2 \tau & \text{for } \omega_c \tau \ll 1 \end{cases}$$

$$\frac{\sigma_c(\mathbf{B})}{\sigma_c(0)} = \sum_{n=-\infty}^{\infty} \frac{J_n^2(B_x')}{1 + (\omega_c \tau)^2 (n - B_y')^2}$$



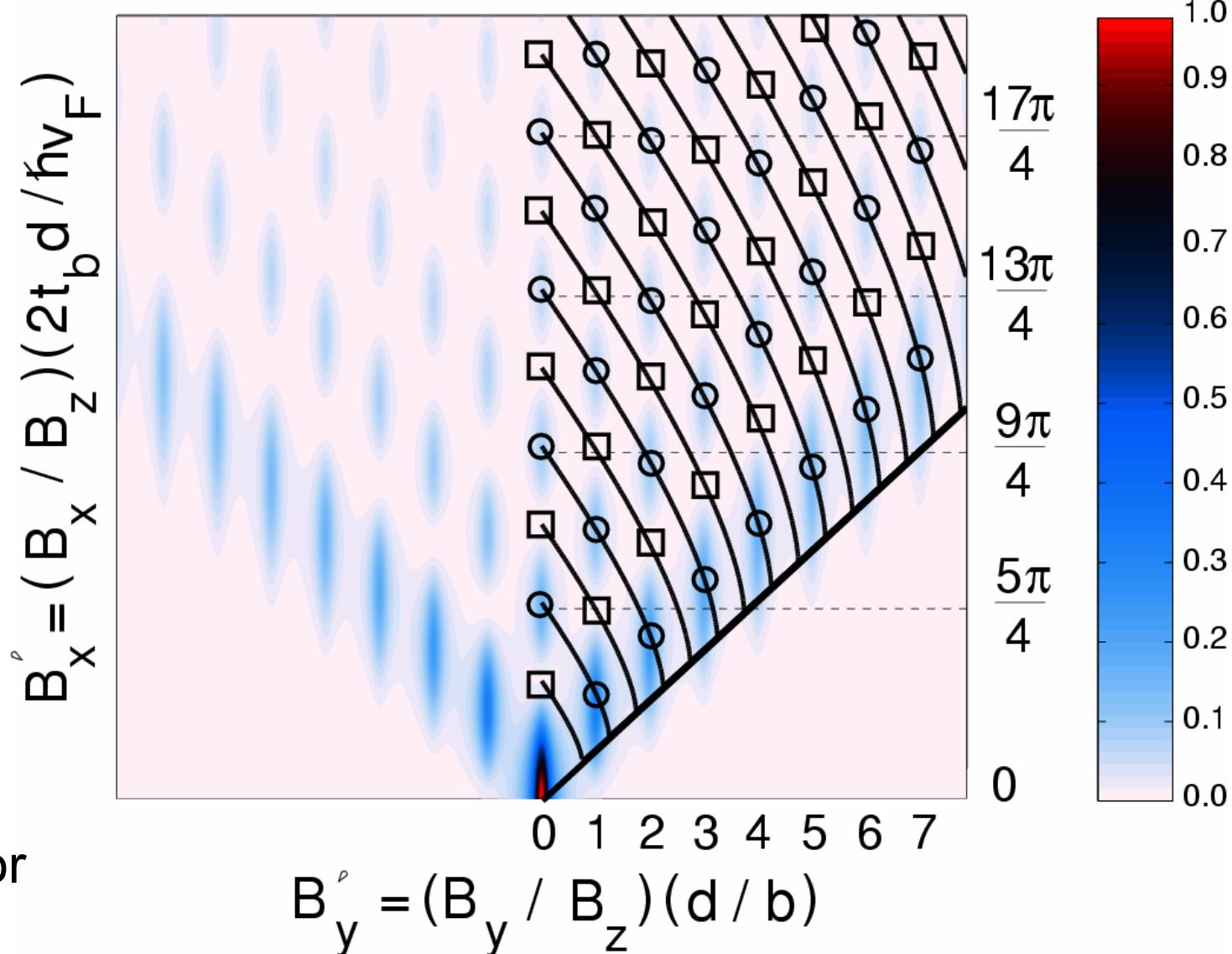
# The two-parameter pattern of AMRO in $\sigma_c$

Circles –  
maxima

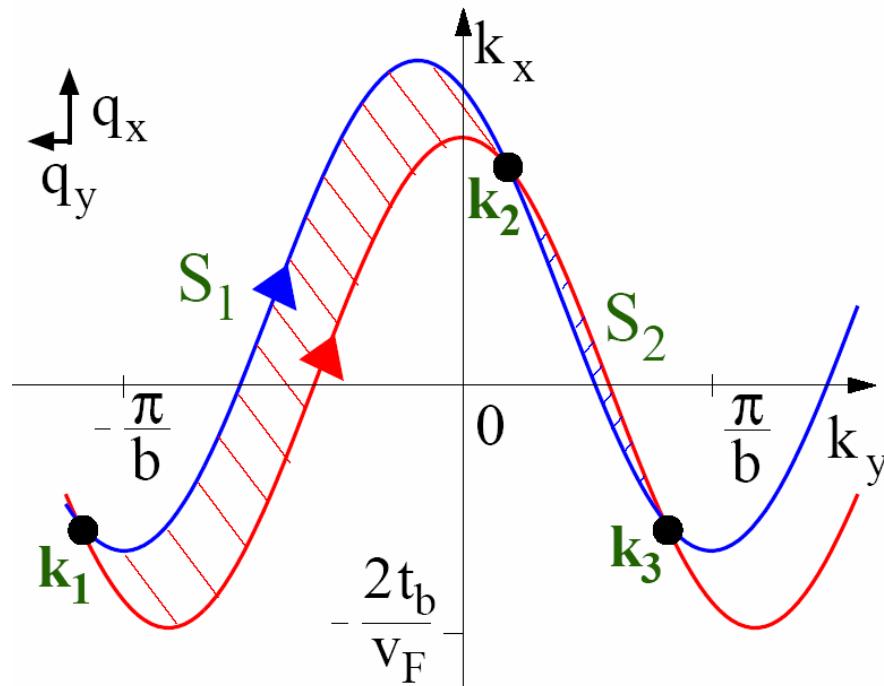
Squares –  
minima

Diagonal  
line –  
the third  
angular  
effect

Calculated for  
 $(\omega_c \tau)^2 = 50$



# Interlayer interference in momentum space



Interlayer tunneling changes the in-plane momentum by  $\mathbf{q}$ :

$$\mathbf{q} = \frac{ed}{c} (B_y, -B_x)$$

The Fermi surfaces of the two layers are shifted by the vector  $\mathbf{q}$ . Tunneling is possible only at  $\mathbf{k}_1$ ,  $\mathbf{k}_2$ ,  $\mathbf{k}_3$ , etc.

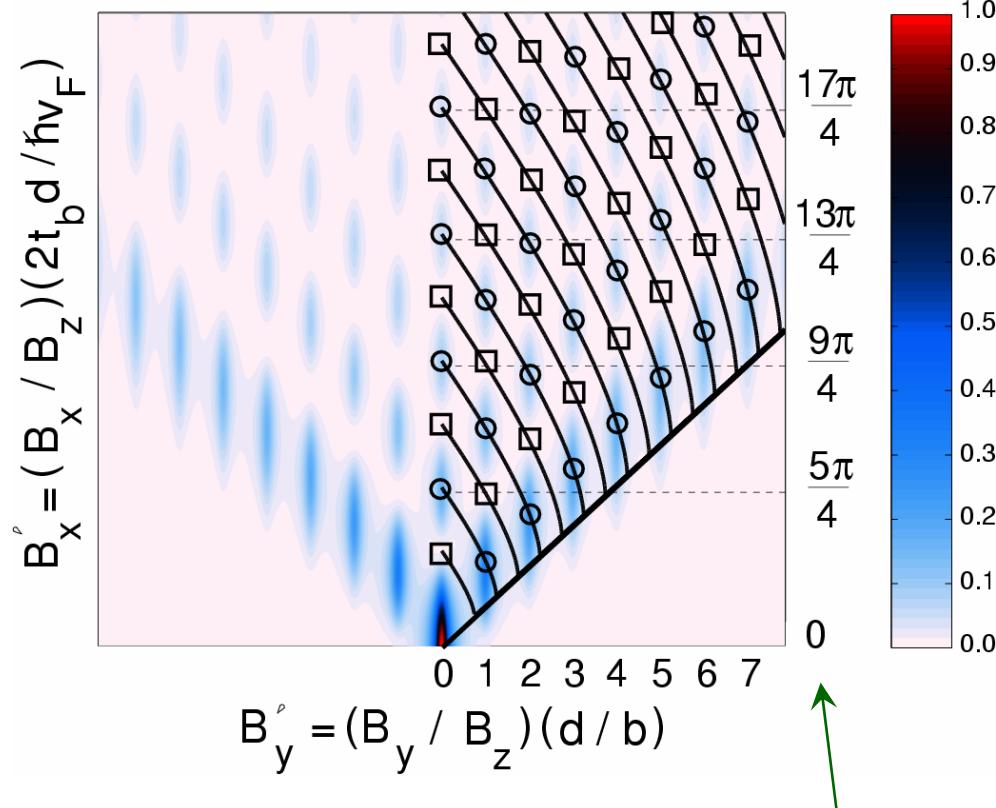
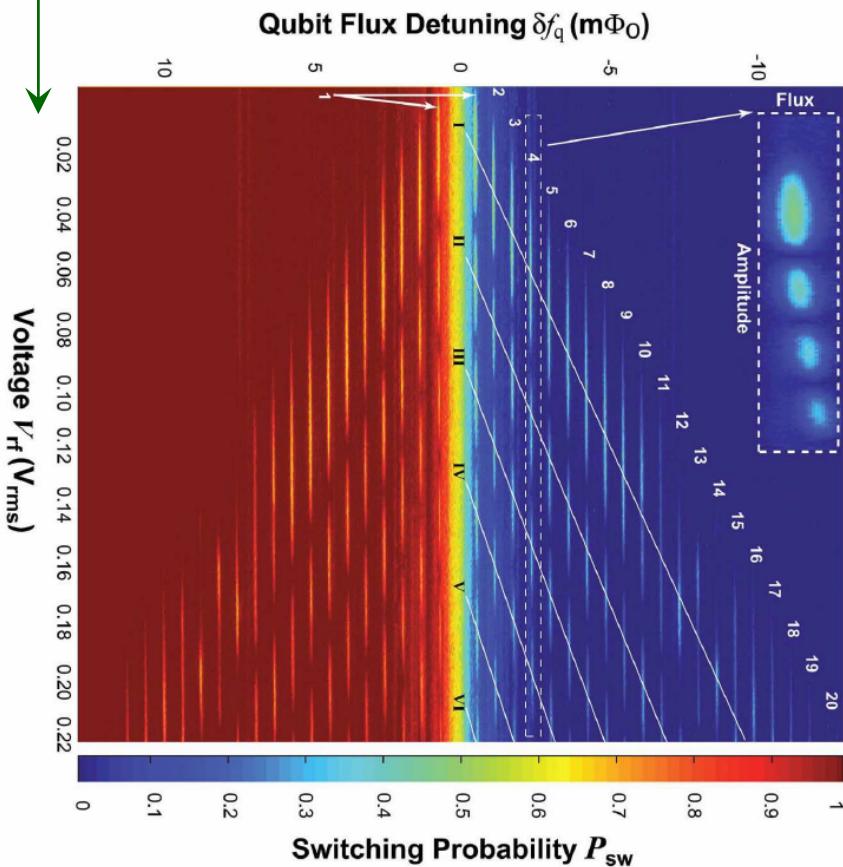
Constructive interference between  $\mathbf{k}_1$  and  $\mathbf{k}_3$  due to  $B_z$  requires  $(S_1 + S_2)c/\hbar e B_z = (2\pi)^2 n$ , equivalent to the Lebed condition  $B_y' = n$ .

Constructive (destructive) interference between  $\mathbf{k}_1$  and  $\mathbf{k}_2$  requires  $S_1 c / \hbar e B_z = (2\pi)^2 (j + 1/4)$  with  $j$  integer (half-integer) – the DKC oscillations.

If the displaced Fermi surfaces do not cross, there are no interference and no AMRO – the third angular effect.

# Coherent quantum interference in a driven superconducting qubit and Q1D organic conductors

Oliver, Yu, Lee, Berggren, Levitov, Orlando, *Science* **310**, 1653 (2005) – experiment and theory for a driven superconducting qubit



Cooper, Yakovenko, *PRL* **96**, 037001 (2006) – theory of AMRO in Q1D conductors

# Conclusions

- AMRO result from periodic modulation of the effective interlayer tunneling amplitude due to the quantum Aharonov-Bohm interference between electron trajectories in the real or momentum space.
- AMRO have been observed in many layered metals and recently in semiconducting bilayers.
- An observation of AMRO in graphene bilayers and multilayers is desirable.
- In Q1D conductors, there are two Aharonov-Bohm interference areas, so AMRO are two-parameter oscillations described by  $\sigma_c(B_x', B_y')$ . These results can be also applied to arrays of quantum wires.
- The formula  $\sigma_c(B_x', B_y')$  derived for Q1D conductors can be applied to a driven superconducting qubit and to the Shapiro steps in Josephson junctions with a finite decoherence time.
- The quantum interference effects are limited by decoherence time  $\tau$  due to loss of phase memory.