

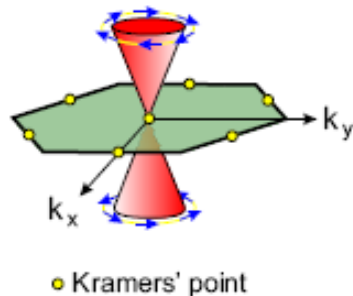
First observation of spin-textured helical Dirac cone in Topological Insulators

M Zahid Hasan

Department of Physics, Princeton University

Discovery of Strong Topological Insulator : [Nature\(2008\)](#), [Science\(2009\)](#)
(work done in 2007)

“Hydrogen Atom” of Topological Insulator Bi_2Se_3 (2008)
Observed (also Theoretically predicted) by Xia, Lin, MZH et.al.,
[Nature Physics '09](#), N&V: [Nat.Phys.'09](#)

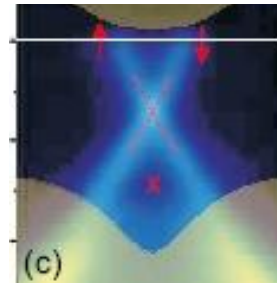
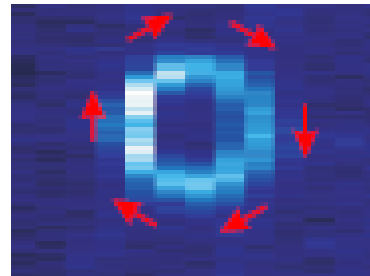


spin-Helical Dirac fermions in Topological Insulators

Hsieh, Xia, MZH et.al., arXiv:0904.1260v1

Direct signature of Topological Order

Bi_2Te_3

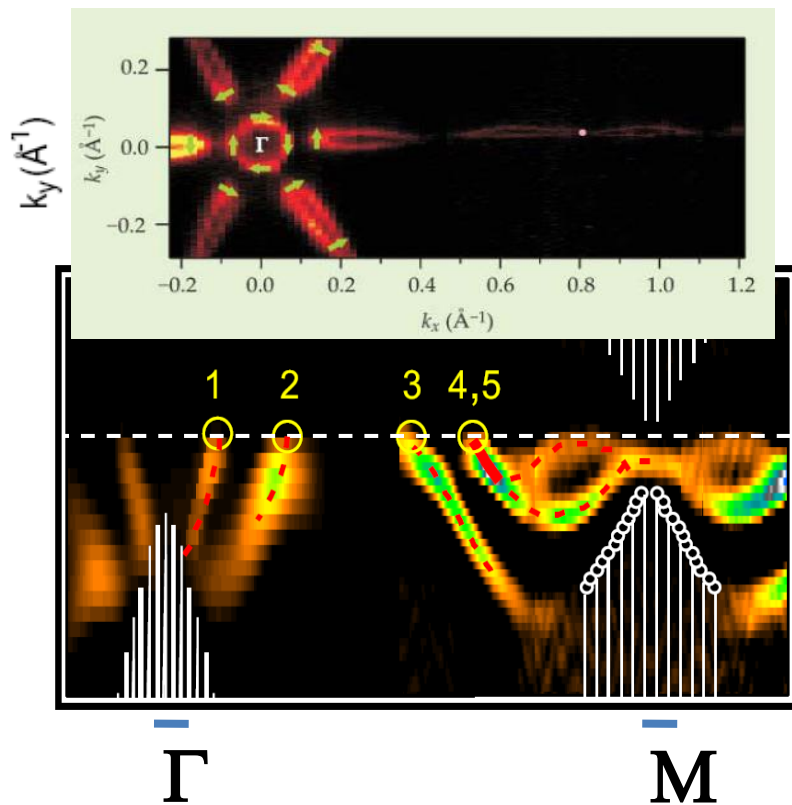


Bi_2Se_3

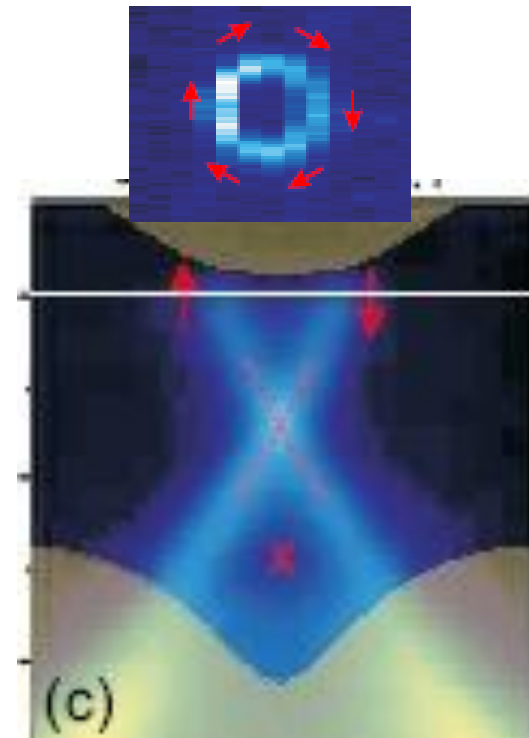


Experimental realizations of Strong Topo Insulators (Theta-Vacuum states) in nature :

Nature 08, Science 09



NatPhys 09, Nature 09



These two are actually 2 distinct types of strong topo insulators !



Physics Team

David Hsieh, Dong Qian, Andrew Wray, Matt Xia, MZH



Undergrads/JPs:

Cathy Kunkel
Matt Severson
Ram Shankar

High-purity Materials/Crystal Growth/Characterization

Y.S. Hor, R.J. Cava (Chem), J. Chechelsky, N.P. Ong (Physics),
D. Hsieh, Y. Xia, MZH (Physics), N.L. Wang (Beijing), H. Eisaki (Tokyo)



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Physics Experimental team : D. Hsieh, Y. Xia, D. Qian, L. Wray, A. Pal

Beamline/Detector/Instrumentation :

A. Fedorov, J. Osterwalder, L. Patthey, H. Dil, Y. Chuang

Quantum Hall effect/Topological Field Theory :

C.L. Kane, D. Haldane, J.E. Moore, F. Wilczek, A. Bernevig, X.-G. Wen, D. Tsui
Liang Fu, L. Balents, S.C. Zhang, D. Huse, N.P. Ong, B. Halperin, C. Callan

U.S. DOE, NSF, A.P.Sloan Foundation, R.H.Dicke Fdn (Princeton), MRSEC

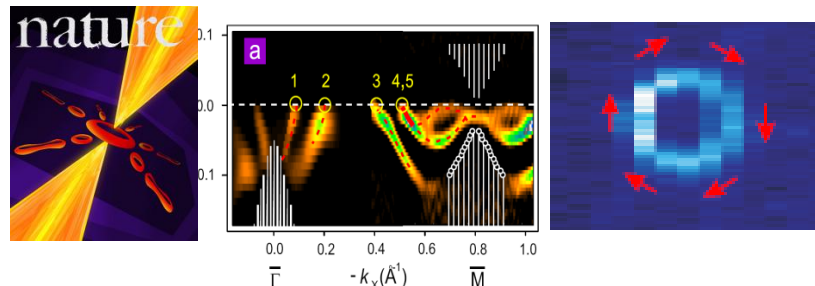
Topological Insulators *Quantum Spin Hall effect, Topological-Order, Spintronics*

Angle-, Spin-Resolved Spectroscopy

Nature (2008), N&V: Nature Phys (2008)

Science (2009), Nature Phys (2009)

Nature (acpt, 2009), Nature (in rev, 2009)



Exotic Superconductors *Fermi surface, Spontaneous SB, SC gap, Self-energy*

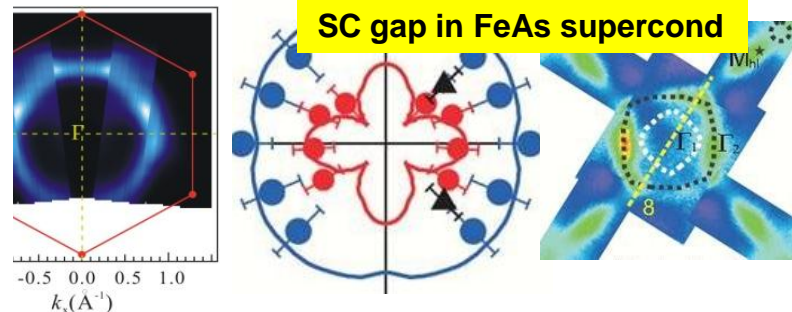
Photoemission, Resonant X-ray Scattering

Nature (in rev) (2009), Phys.Rev.Lett.(2009)

Phys.Rev.Lett.s (2007a,b,c), PRL/B (2008)

Phys.Rev.Lett.s (2006 a,b,c)

Science (2000), Phys.Rev.Lett.(2004)

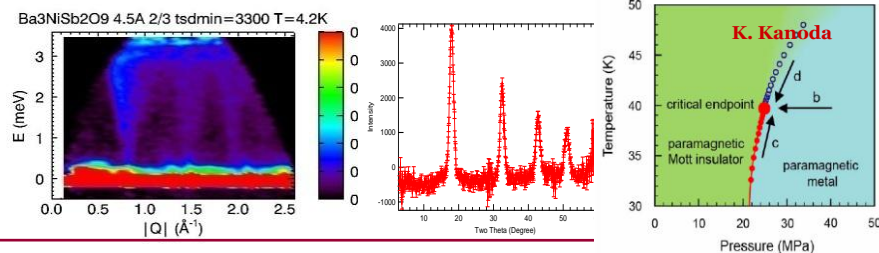


Frustrated Magnetism *Spinon dynamics, Quantum Criticality* Quantum Spin-liquid TL Organics (2006)

Polarized Neutron Scattering

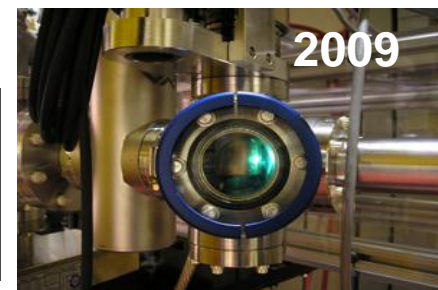
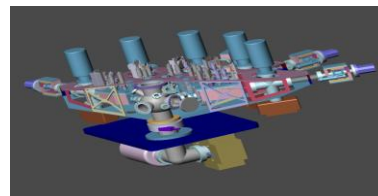
Phys. Rev.Lett. (subm.) (2009)

Z. Tan Senior-Thesis, P.U. (2005)



Spectrometer development

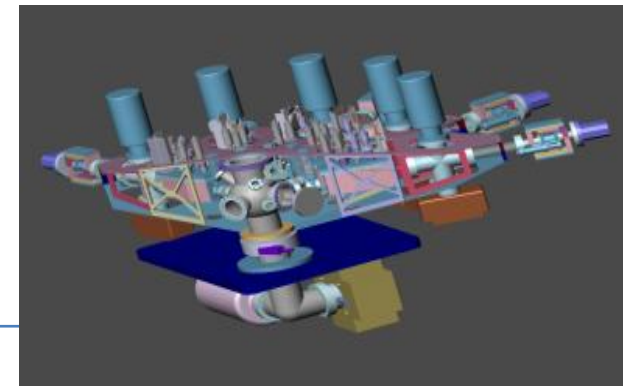
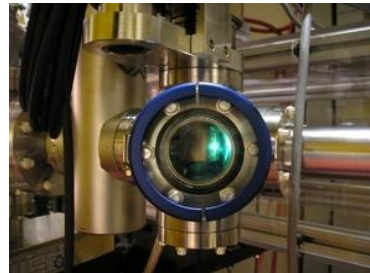
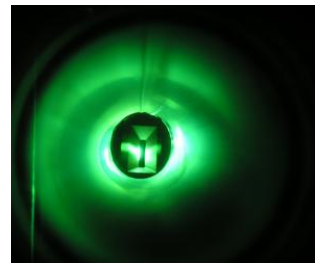
MERLIN Project
Spin detector Project
Projects at LBNL & SLAC



Accelerator Facilities



Beamline/Spectrometer/Detector development :



Collaboration with LBNL(ALS)

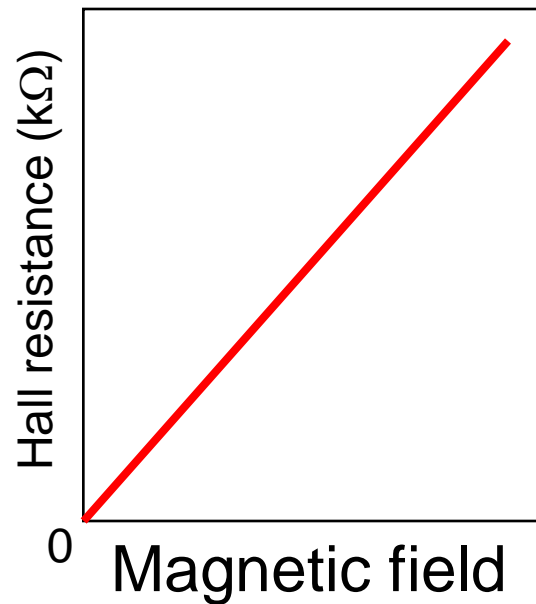
~~ Outline ~~

- **QHE & Topological Invariants**
- **How to measure Topo Quantum numbers?**
- **Spin-resolved-photoemission (spin-ARPES)**
- **Observation of Topo Insulators**
- **Determination of non-trivial Berry's phase**
- **Future Directions (Topo. QC, TEC etc.)**

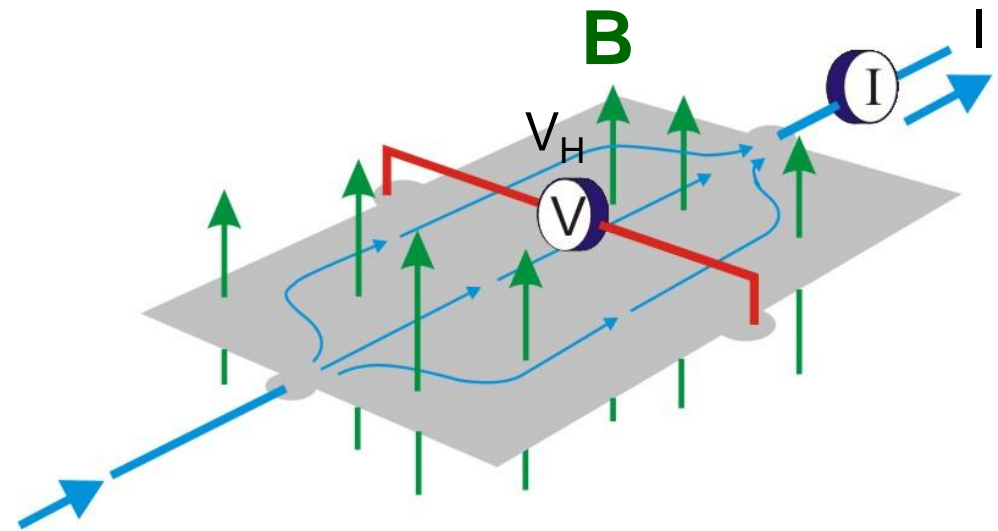
Metals feature no gap: current can flow (no energy gap):
How many carriers ? what sign ?

Hall Effect

Edwin H. Hall (1879)



B



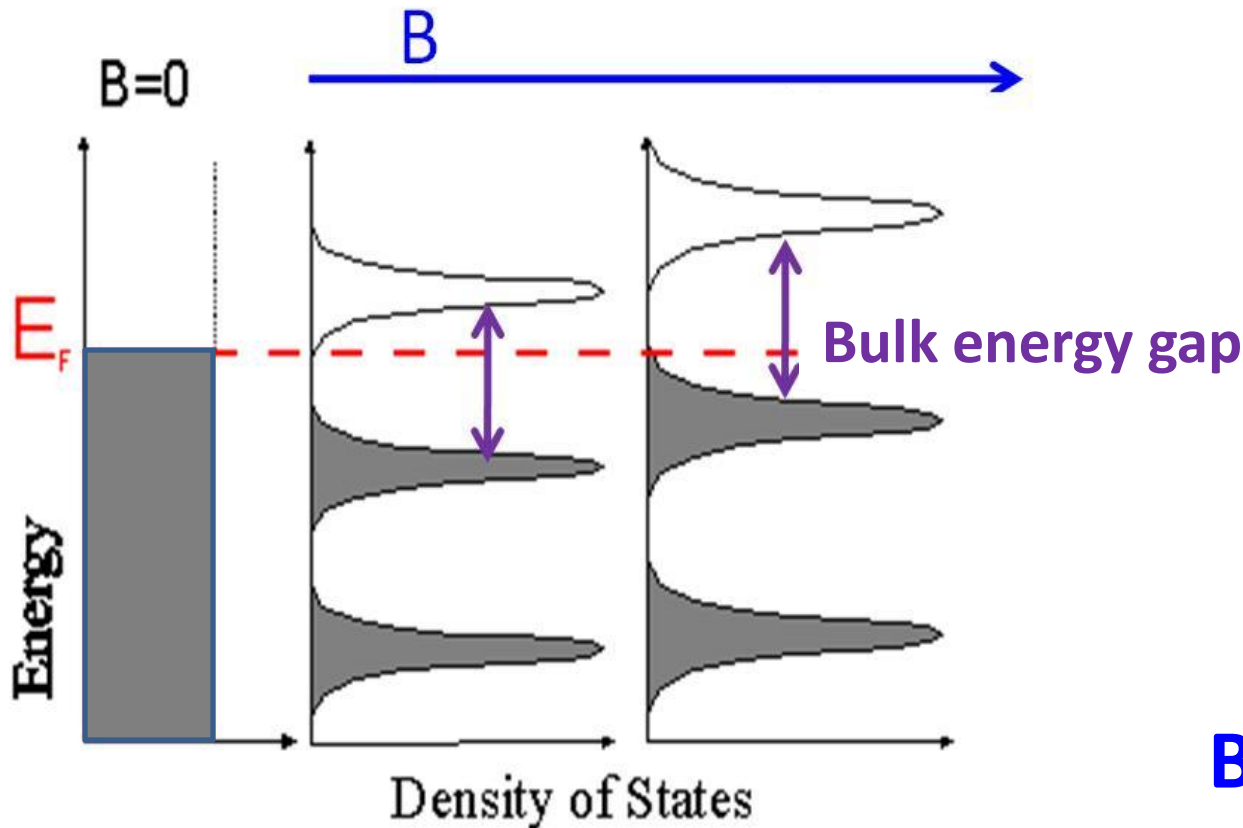
Hall resistance $R_H = V_H / I$

$$R_H = V_H / I$$

Hall Effect at high magnetic fields

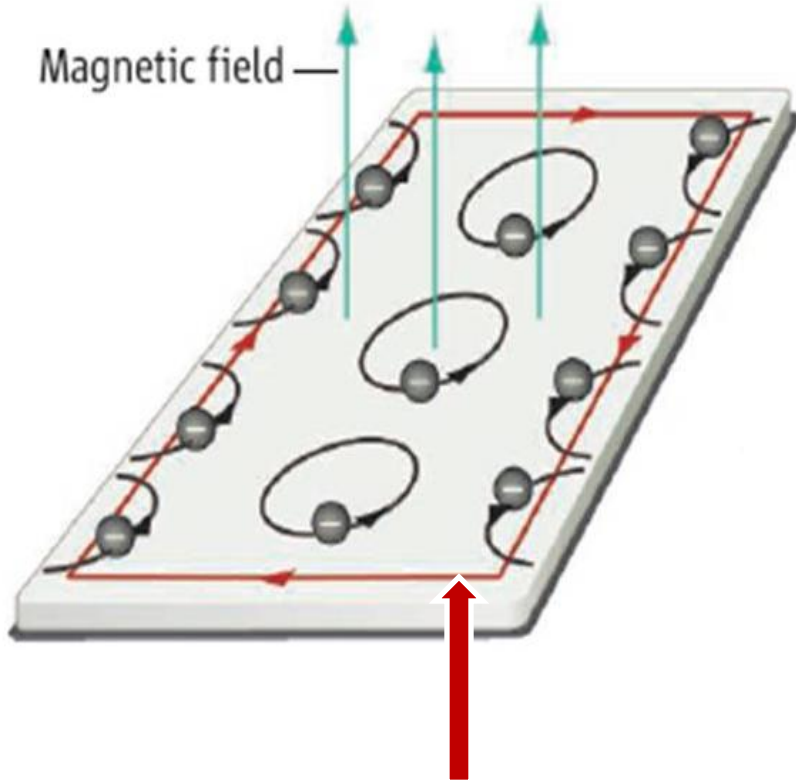
Electrons + strong magnetic field

→ Discrete Energy Levels



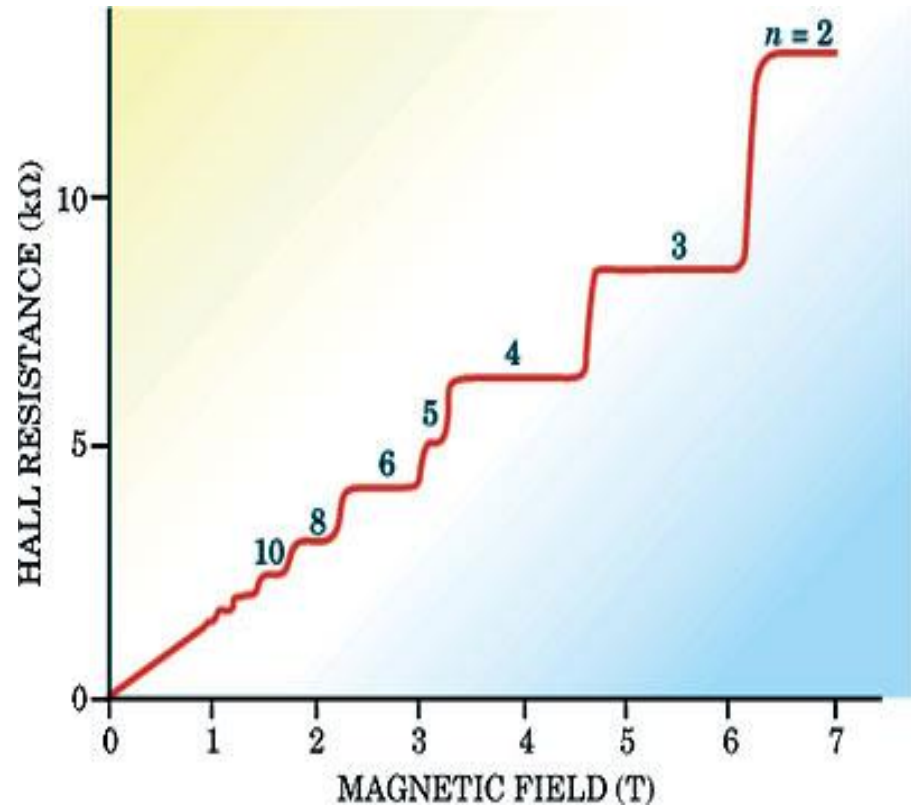
Band Insulator?

Bulk Insulator but Conduction through the Edge



**Conduction through
the boundary**

Finite Hall Conductivity

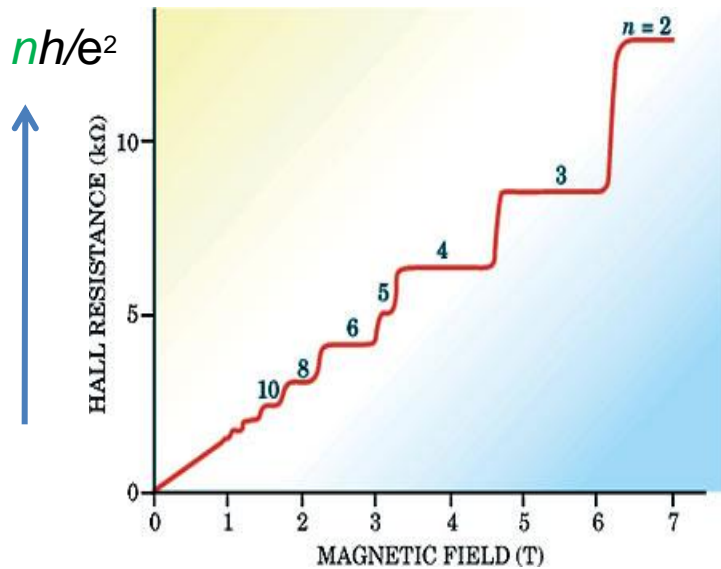


Quantum Hall Effect

K. von Klitzing et.al., (Nobel Prize, '85)

D.C. Tsui et.al., (Nobel Prize, '98)

Quantum Hall State : Topological, TR-breaking



Hall conductance:

$$\sigma_{xy} = ne^2/h$$

n = Chern no. (**Edge** states)

S. Chern : Quantum version (Hilbert space) of Gauss-Bonnet formula

$$n = \frac{1}{2\pi} \int_{BZ} [\nabla_{\mathbf{k}} \times \mathbf{A}(k_x, k_y)]_z d^2\mathbf{k}$$

$$\mathbf{A} = -i \langle u_{\mathbf{k}} | \nabla_{\mathbf{k}} | u_{\mathbf{k}} \rangle$$

Electron-occupied **Bulk** bands

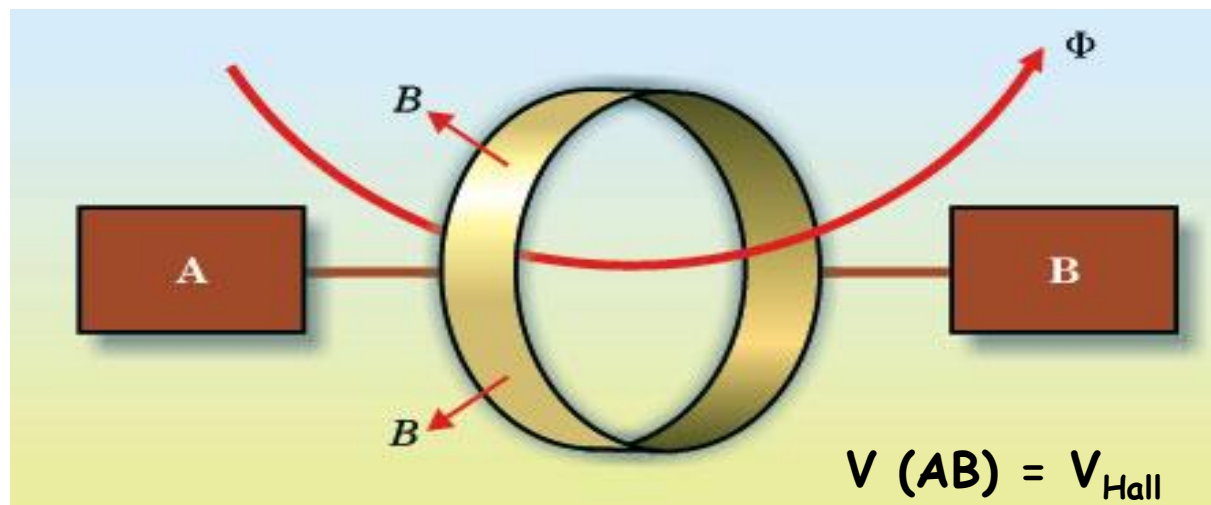
Topological Property

TKNN invariant



Finite $n \rightarrow$ topologically “protected” edge-states

Laughlin cylinder : Quantum Hall State (IQH)



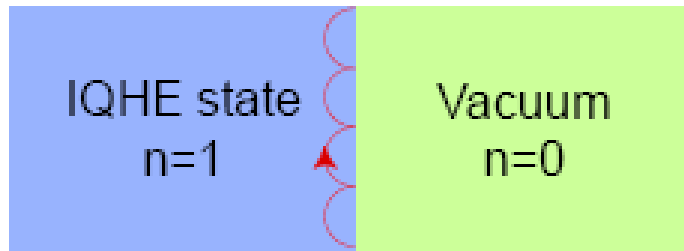
$\phi = h/e$ inserted through the cylinder (at filling N) results in a transfer of N electrons from one end of the cylinder to the other (quantum charge pump). **Change in charge polarization.**

A generalization of Gauss-Bonnet formula to the geometry of the eigenstates parameterized on a surface (geometry \leftrightarrow topology/#handles). A topological invariant the Chern integer characterizes the IQHE state precisely. It characterizes this quantized change in charge polarization. **Weak sensitivity to disorder.**

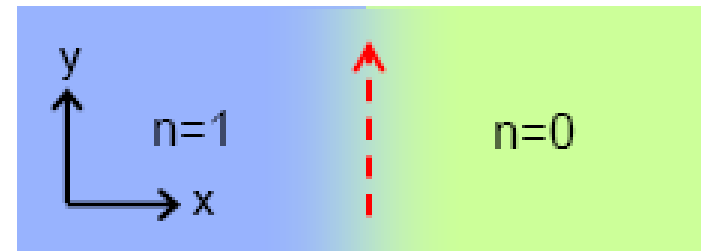
No SBS, no local OP.

Edge States

Gapless states **must** exist at the interface between topologically distinct phases



Edge states ~ skipping orbits



Smooth Interpolation

2 Band Model : Dirac Eq.

$$H = -iv(\sigma_x \partial_x + \sigma_y \partial_y) + M(x)\sigma_z$$

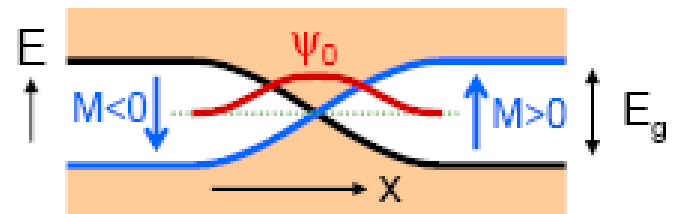
$M(x) = M_0$: Gap $E_g = 2M_0$

$$E(k_x, k_y) = \pm \sqrt{v^2 |k|^2 + M_0^2}$$

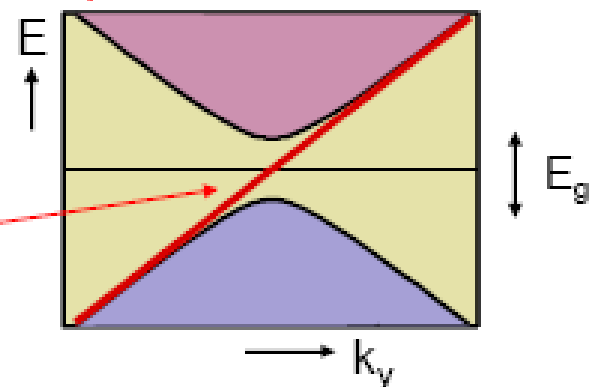
$M(x) \sim M_0 \text{sgn}(x)$: "Domain Wall bound state"

$$\psi_0(k_y, x) \propto e^{ik_y y} e^{-M_0|x|/v} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$E_0(k_y) = vk_y$$



Gapless Chiral Fermions

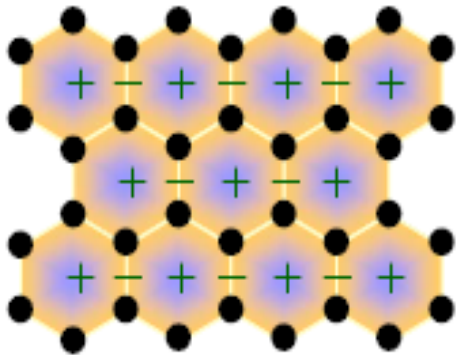


“Quantum Hall Effect” without Magnetic Field?

Haldane Model (1988)

Alternating field but net magnetic field is zero

Bulk band-structure



$B(r) = 0$
Zero gap,
Dirac point

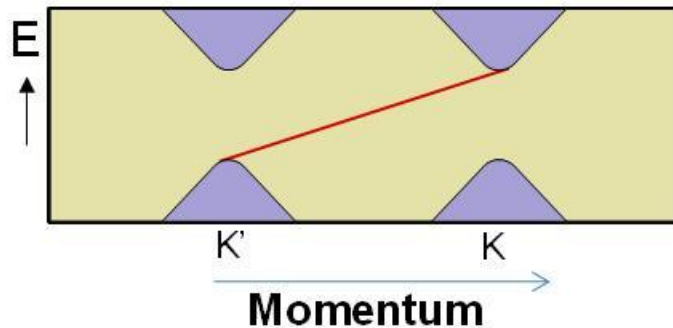


Dirac point

$B(r) \neq 0$
Energy gap
 $\sigma_{xy} = e^2/h$



Massive Dirac particles



Edge structure

Chiral fermions

*Hall Transport
is determined
by band-structure
Edge \leftrightarrow Bulk*

Do QHE-like topological phases exist in nature that are time-reversal invariant?

Topological state in 2D (**spin Hall state**)

Kane & Mele (2005), Bernevig & Zhang (06), Murakami (06)
Sheng, Haldane et.al.(2006) also others

Distinct new Topo state in 3D (**Theta-vac, Axionic state**)

Moore & Balents (07), Fu, Kane & Mele (07), Roy (09)
Qi, Hughes, Zhang (08), Essin, Franz, Moore, Vanderbilt, X.G. Wen (08)
Ryu, Ludwig, Schnyder (08), Kitaev (08) and others
Earlier work by Wilczek, Callan, Witten and others (70-80s)

Experimental realization : Nature 08, Science 09, NatPhys 09

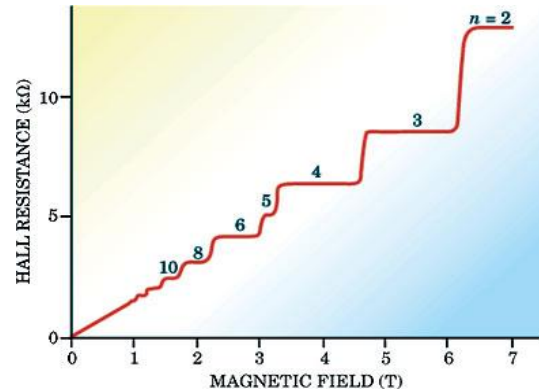
Topological Theta vacuum (3D) is distinct from spin Hall state (2D).
This is not the case for quantum Hall state. 2D and 3D QH states are the same!
Theta vacuum (3D) or the 3D Topological Insulator is the most exotic state!

QHE phases

$$\sigma_{xy} = n e^2/h$$



Topological quantum number



Transport

Topo Insulators

How to experimentally “measure” the topological quantum numbers (ν_i) associated with the quantum spin Hall (QSH) or Strong Topological Insulator (STI) phases?

No quantized transport!

$$\{\nu_i\}$$

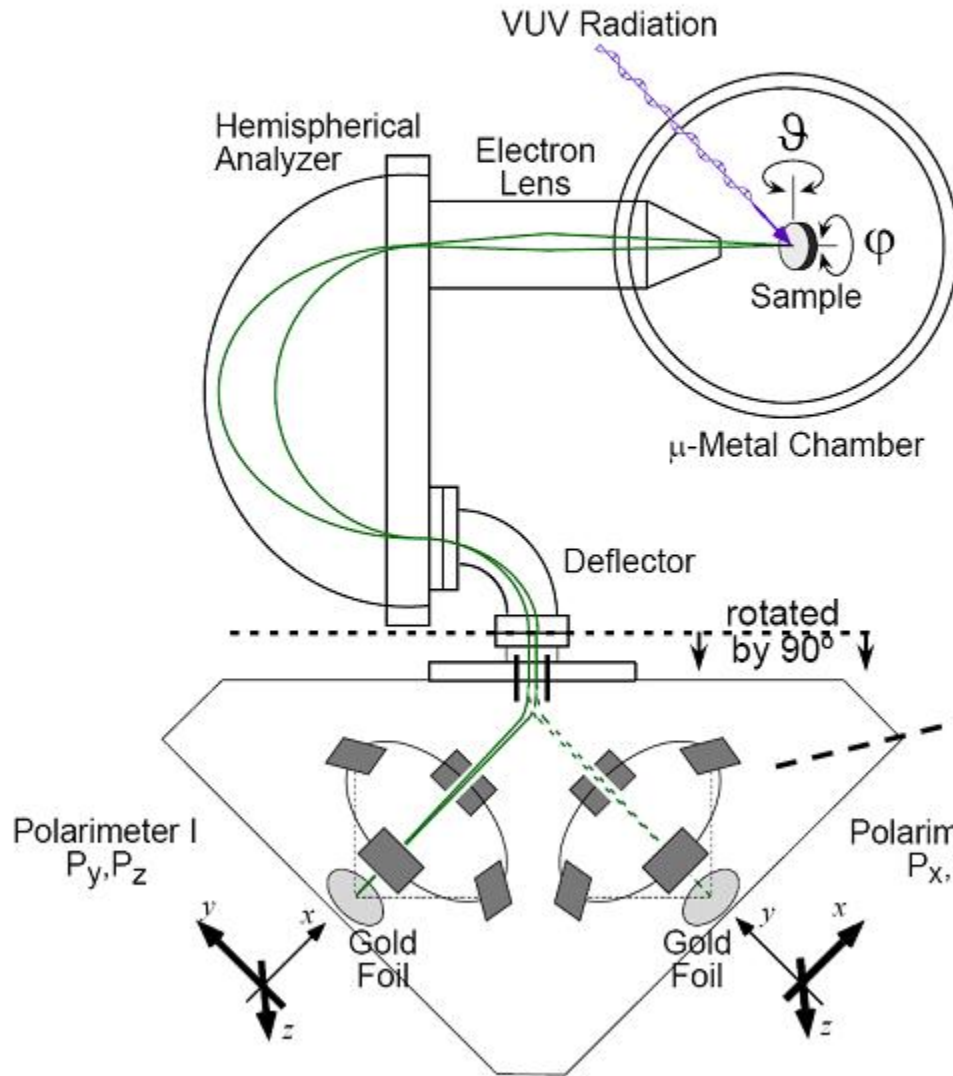


Topological quantum number

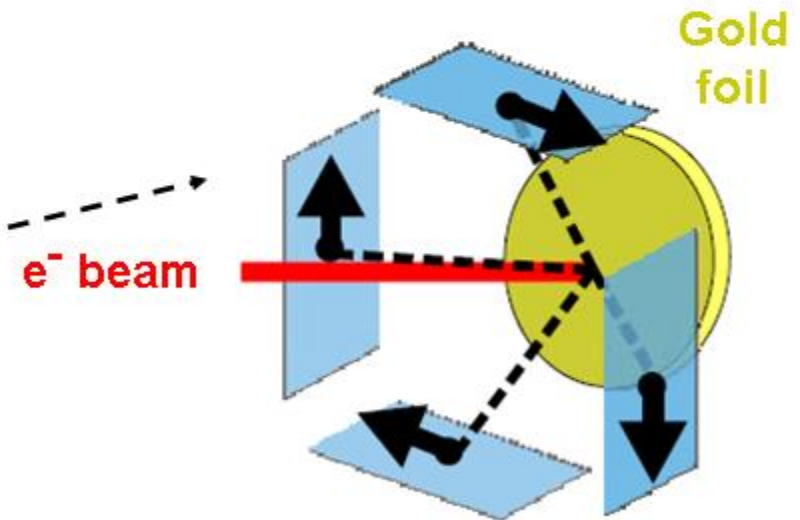
?

Spin-sensitive
Momentum-resolved
Measurements!

Double Mott detector configuration



- Each Mott detector measures 2 orthogonal spin components
- Double Mott detector captures 4 (3 indep.) spin components



First generation materials:

Gap too small *Graphene*

No SS in *Pb(Sn)Te*

2D

(spin Hall state)

Hg(Cd)Te

Charge Transport

measurements of
edge-states

Konig et.al., (Molenkamp).

SCIENCE (2007)

3D

(Theta-vac state)

Bi_{1-x}Sb_x

spin-ARPES (spin-sensitive)

measurements of edge-states

Hsieh et.al., (Hasan).

NATURE (2008)

SCIENCE (2009)

Second generation : “hydrogen atom” materials



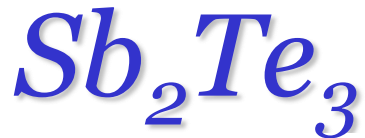
“Hydrogen Atom” of Topological Insulator

Xia et.al., arXiv:0812.2078v1

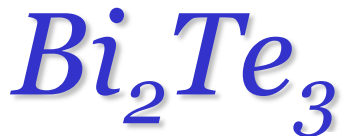
NATURE PHYSICS (2009)



SCIENCE (2009)



see my **KITP talk (2008)**

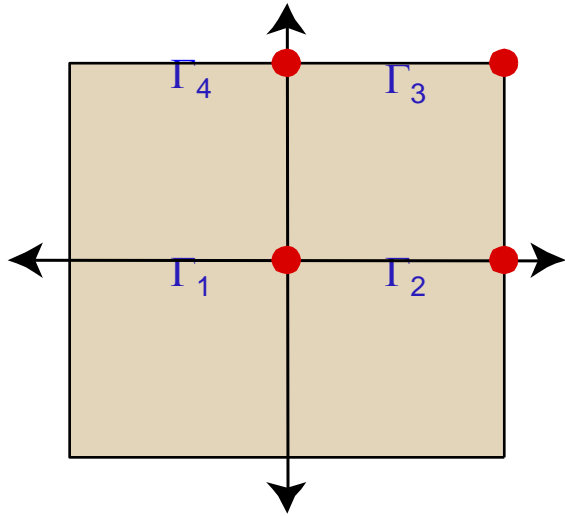


Hsieh et.al., arXiv:0904.1260



Topological (Z_2) invariants :

Inversion Symmetry : Parity of occupied Bloch states at $\Gamma_{1,2,3,4}$



Bulk Brillouin Zone

$$P|\psi_n(\Gamma_i)\rangle = \xi_n(\Gamma_i)|\psi_n(\Gamma_i)\rangle$$
$$\xi_n(\Gamma_i) = \pm 1$$

$$(-1)^{\nu} = \prod_{i=1}^4 \prod_n \xi_{2n}(\Gamma_i)$$

$$\nu = \mathbf{0} \text{ or } \mathbf{1}$$

In 3D (8 bulk KPs) there are **4 Z_2 invariants**: $(\nu_0; \nu_1\nu_2\nu_3)$
Characterizing the bulk. These determine how edge states
Connect \rightarrow 16 topological phases!!

Direct/unique Detection of Topological Order
via

Direct Imaging of Edge-structure



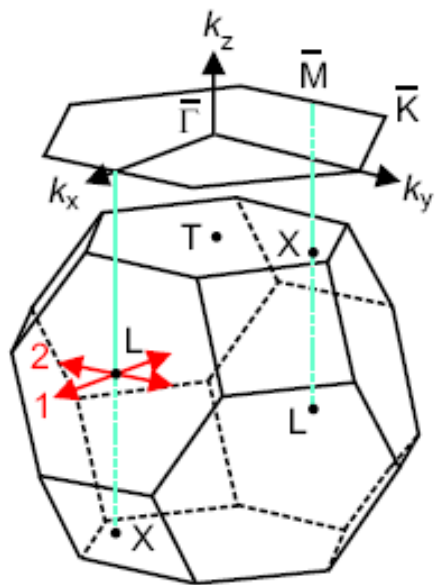
Surface/Interface Dirac Fermions

Spin Texture of surface/edge electrons

Bulk band-structure, parity set

Insulating bismuth-Antimony

Band inversions at 3 L points
T point is not inverted



Bulk Low-lying states(Bi)

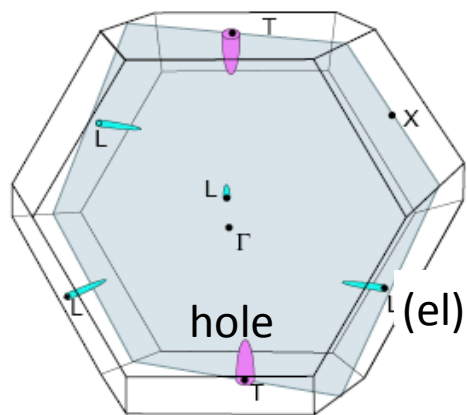
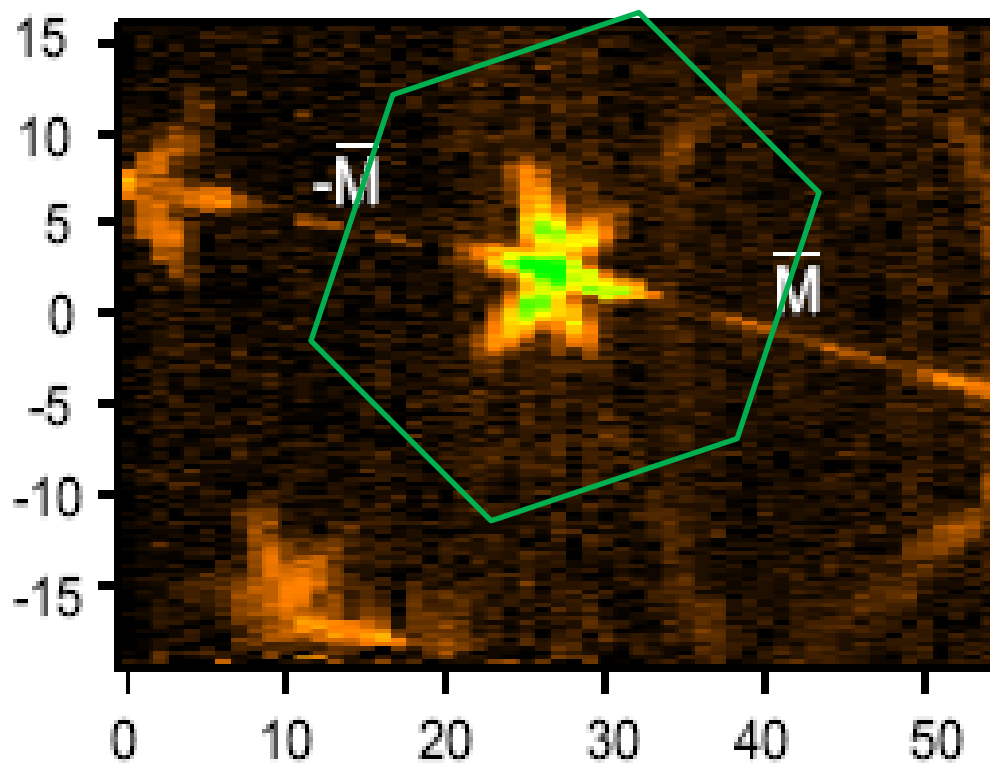


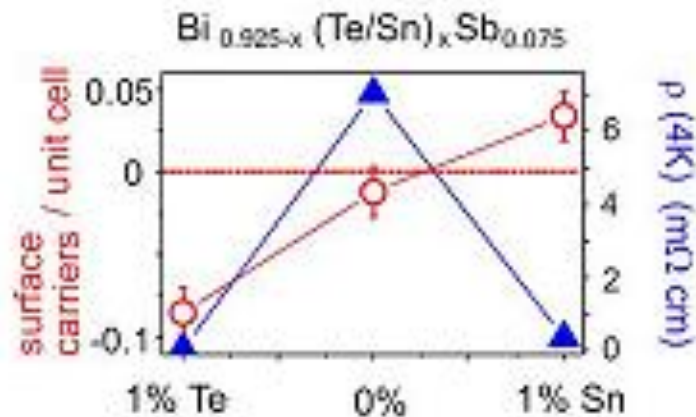
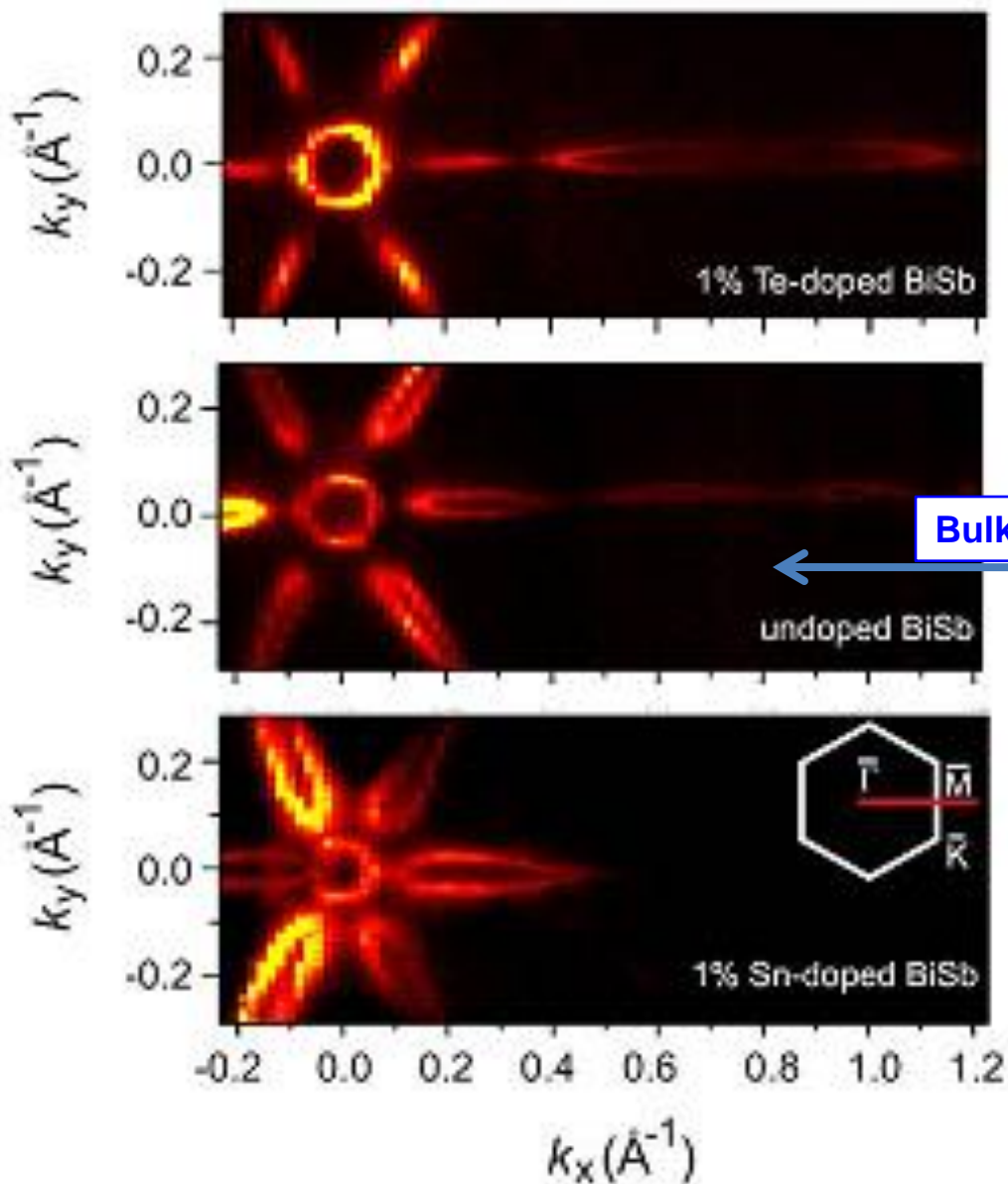
Photo-Emission intensity at Fermi level

emission angle (deg)

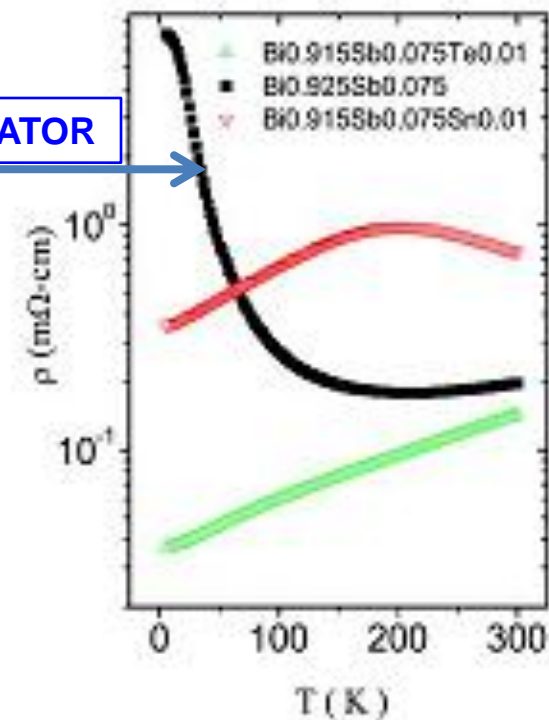


emission angle (deg)

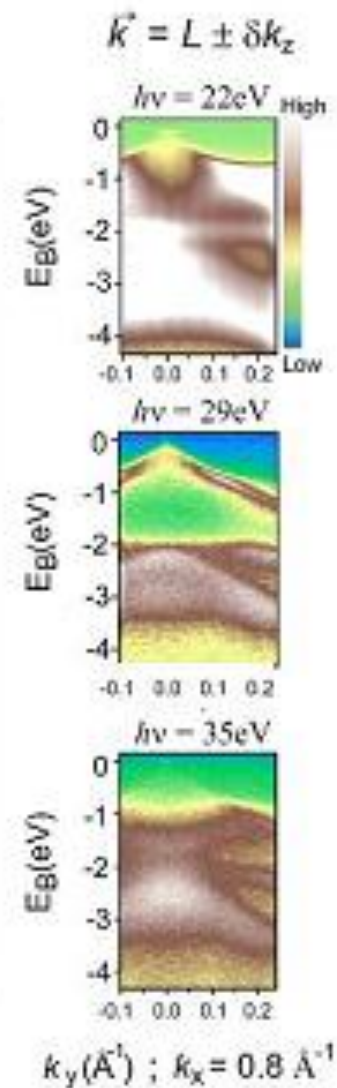
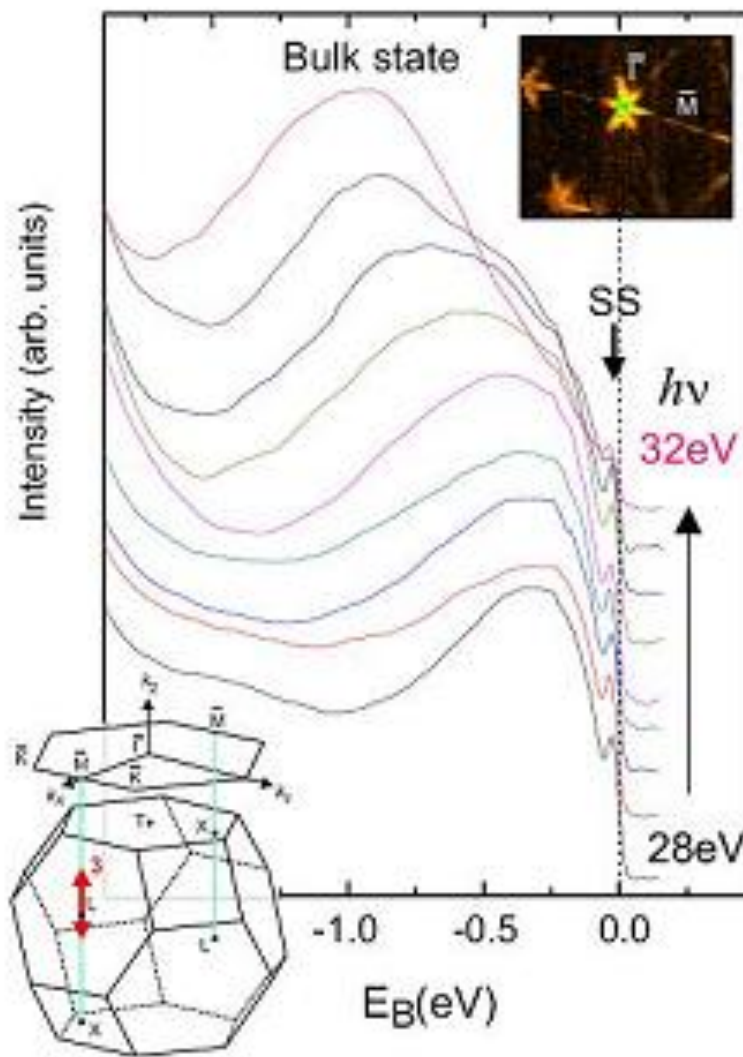
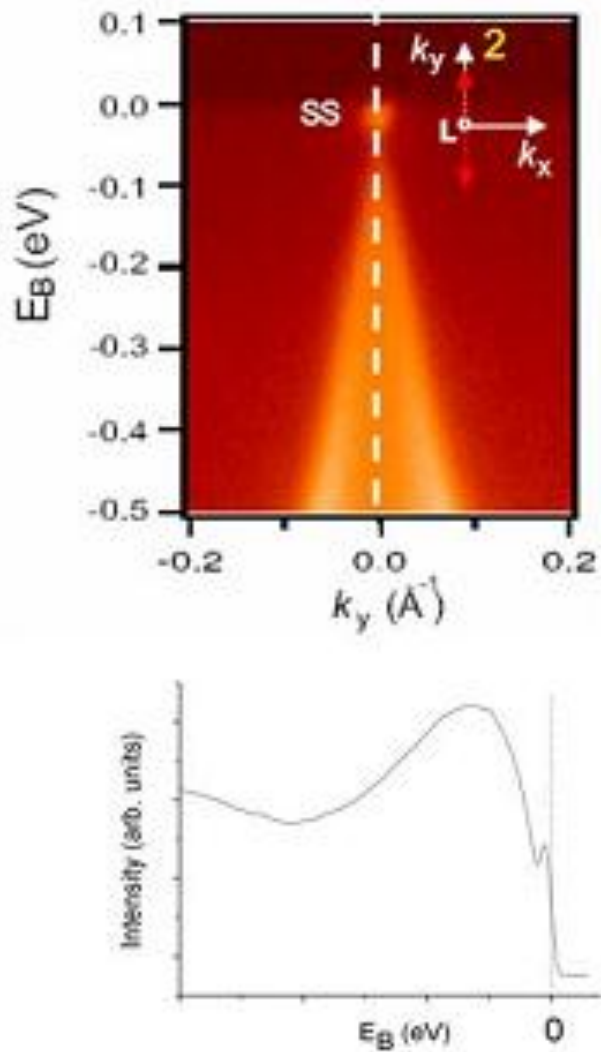
Tuning to the topological insulator regime in $\text{Bi}_{1-x}\text{Sb}_x$

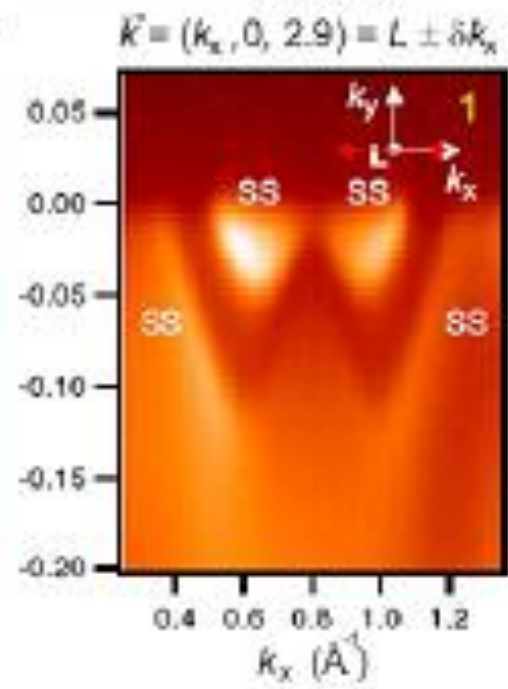
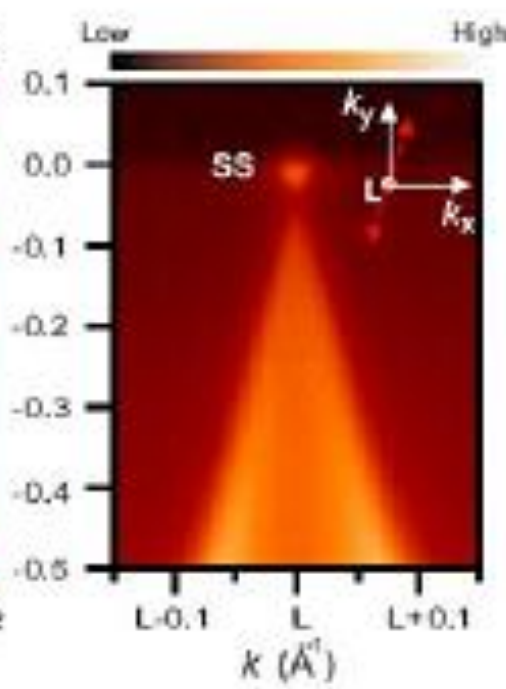
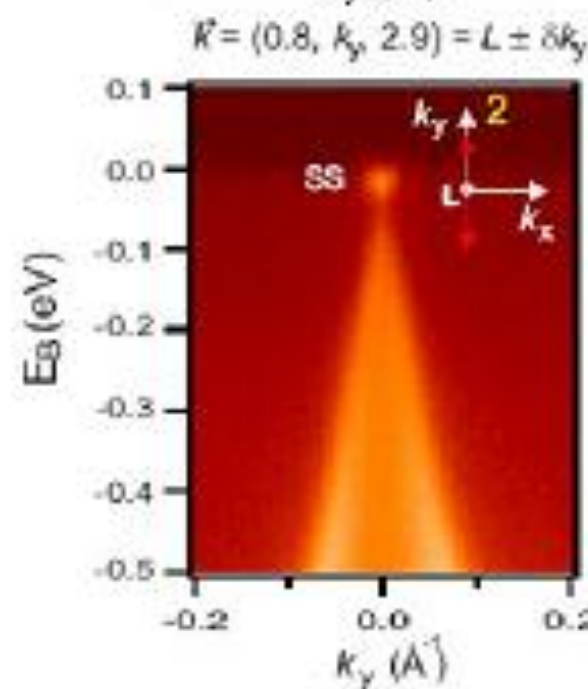
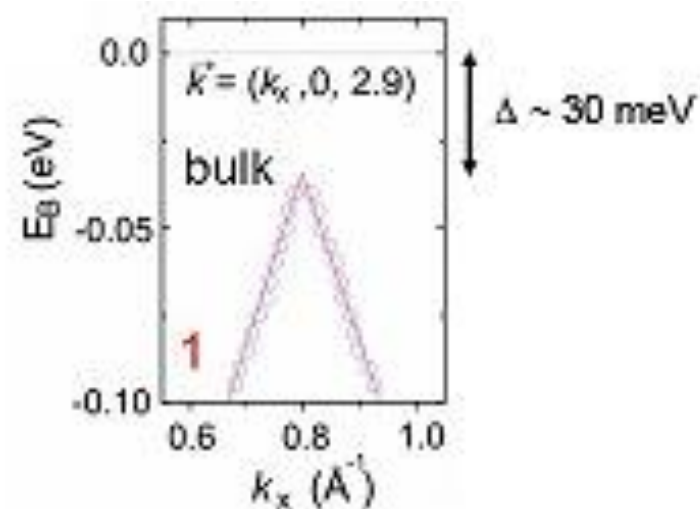
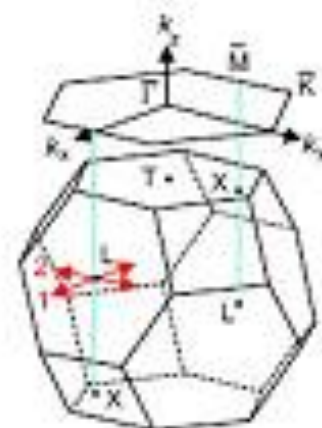
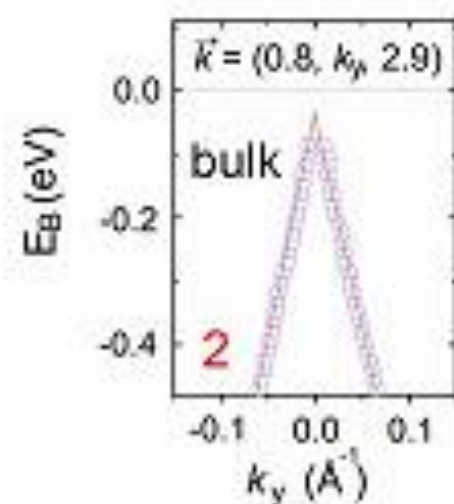


Bulk INSULATOR

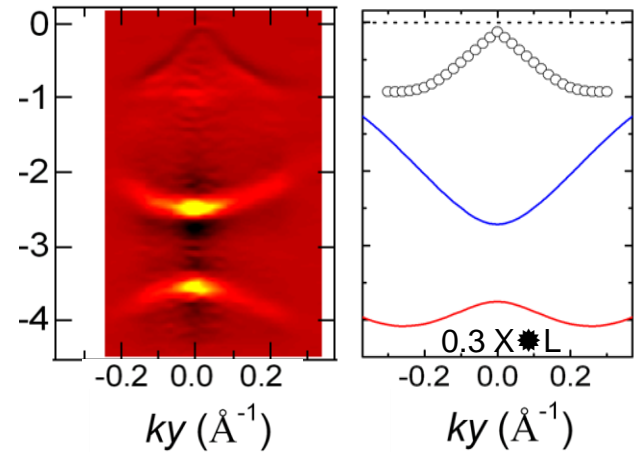
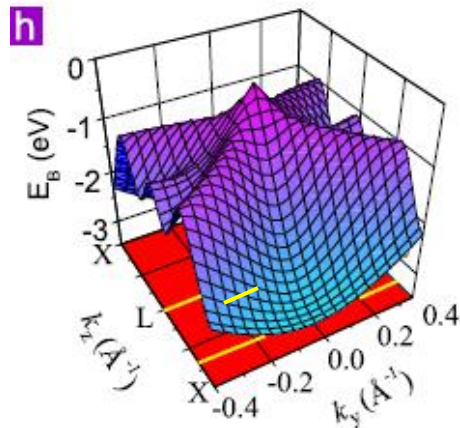
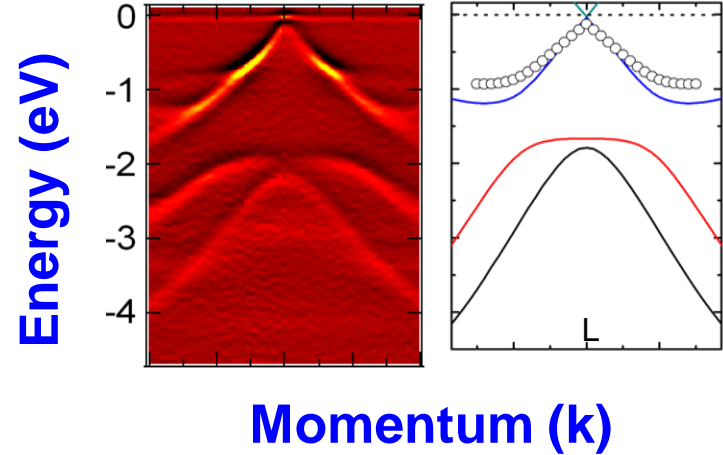
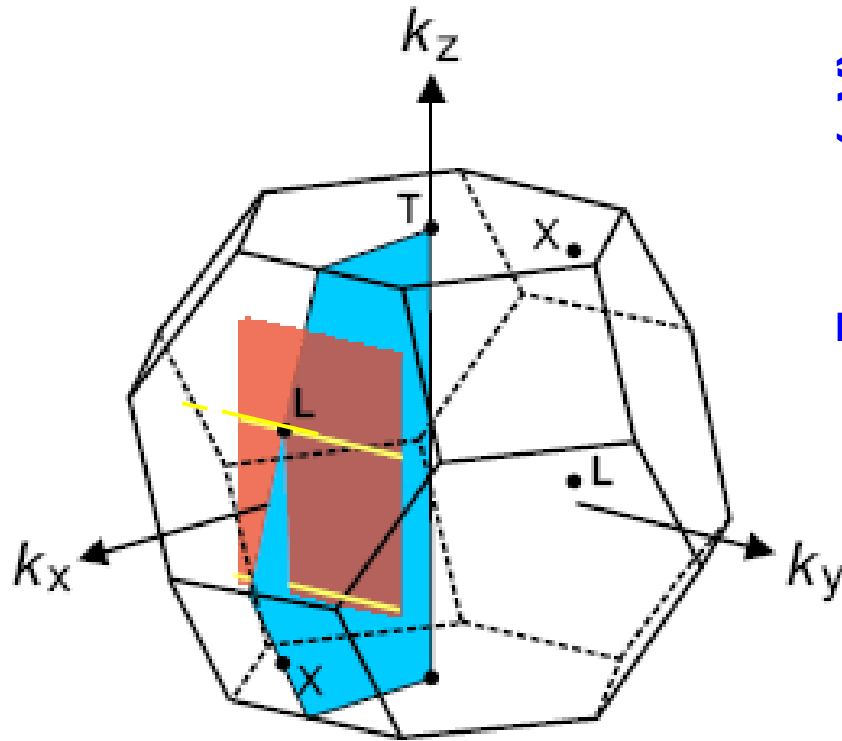


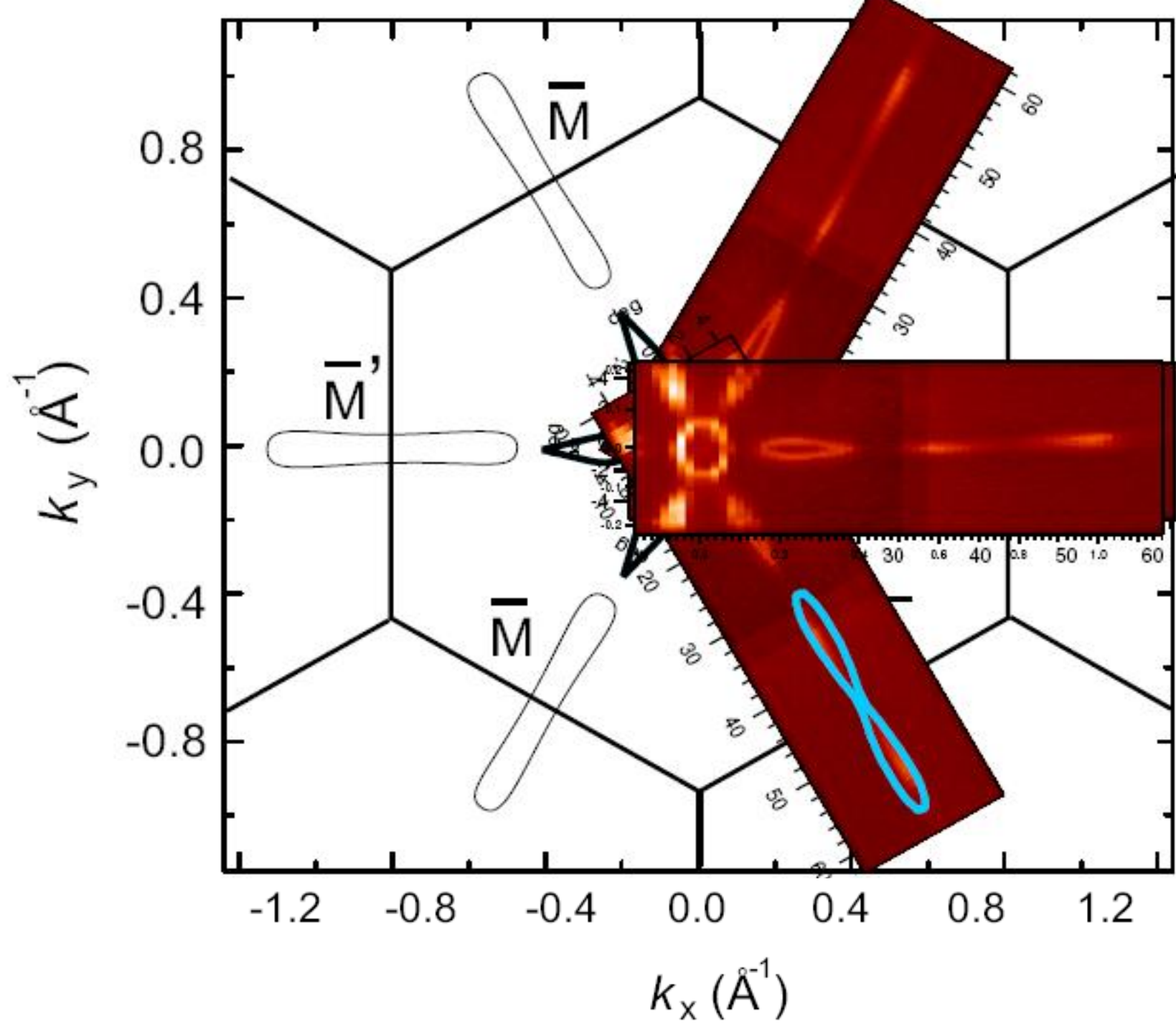
How to isolate Bulk vs. Surface states ?





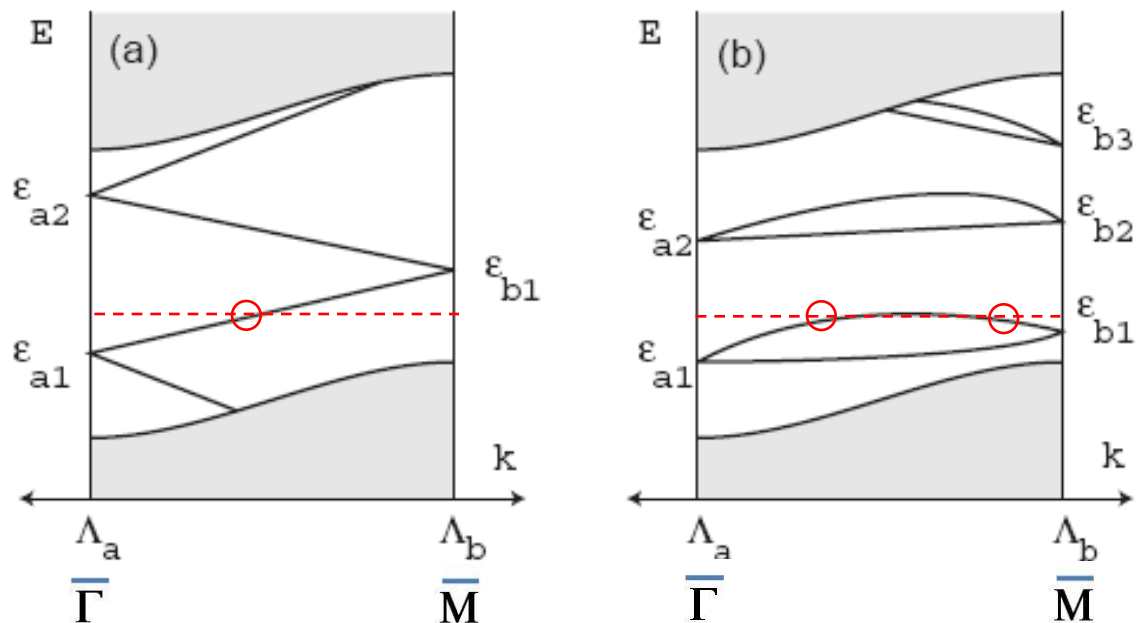
Expt. data vs. bulk band calculation





Signatures of the Topological edge states

Z_2 invariant characterizes the presence or absence of Kramers degeneracy associated with a boundary



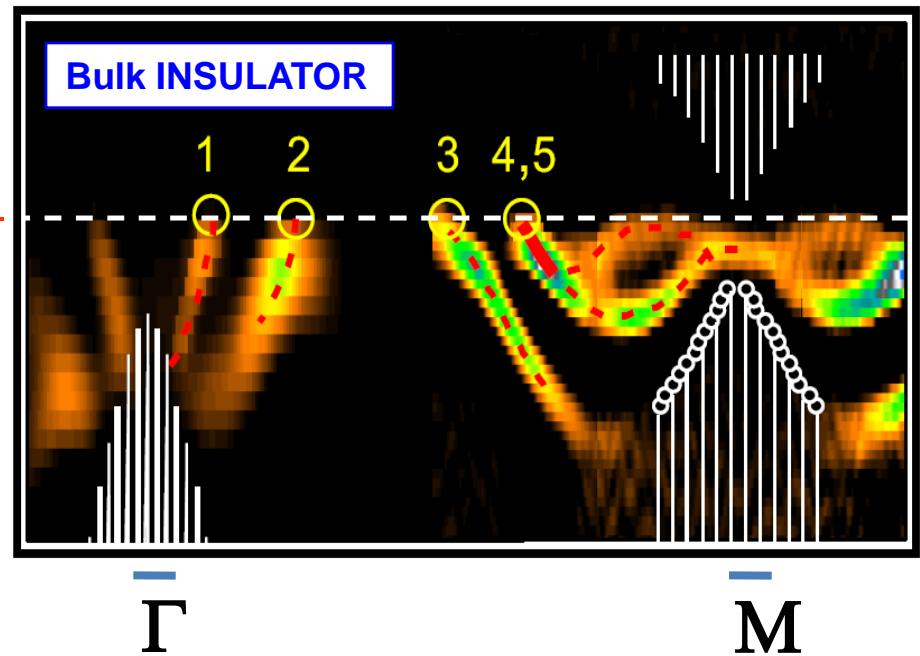
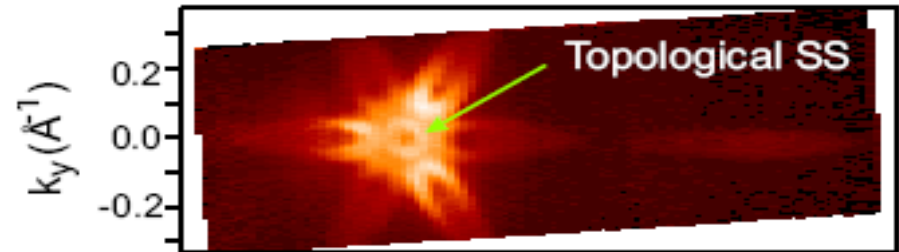
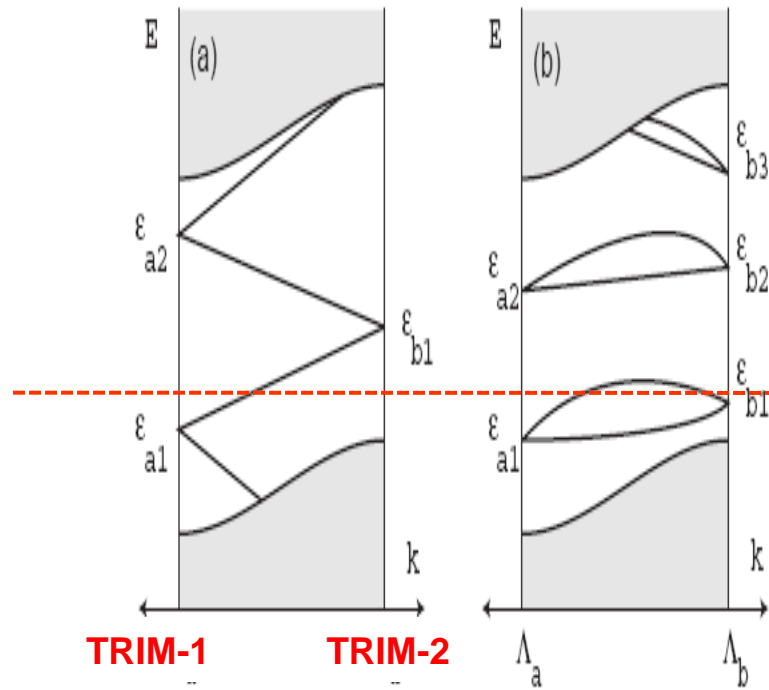
# crossing between TRIM	odd	even
Z_2 class	-1	+1
Protected bulk crossing	yes	no

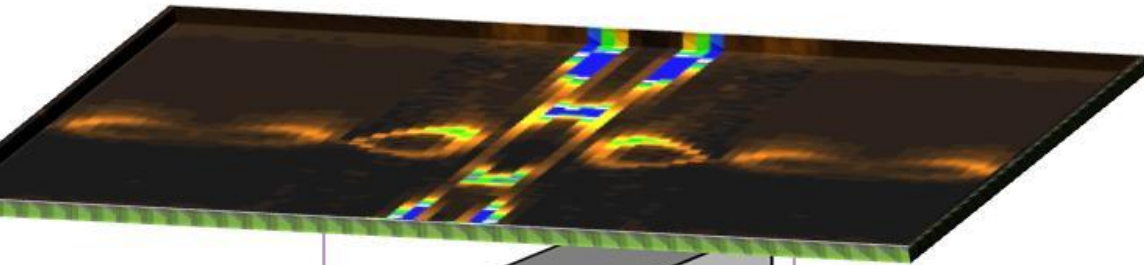
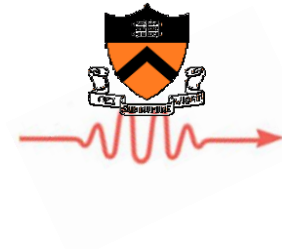
Gapless odd number modes at the boundary

Bi-Sb

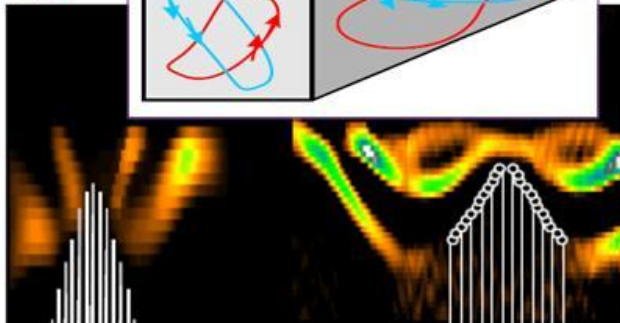
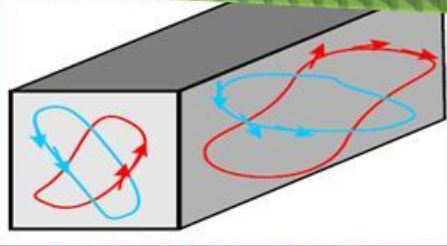
Topological

Trivial

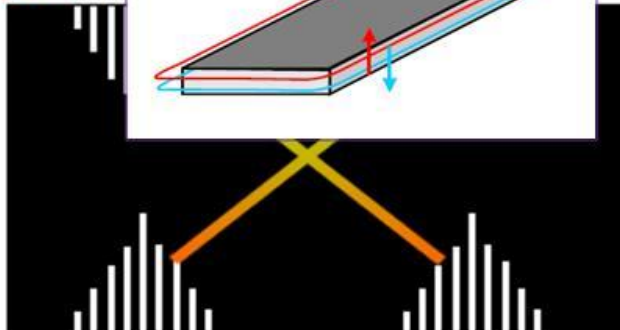
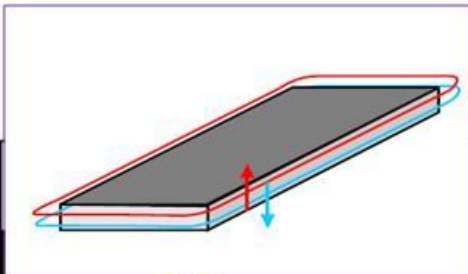




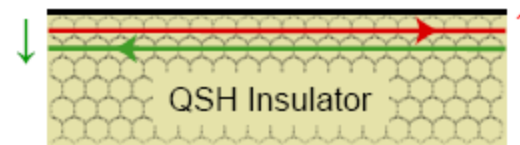
3D



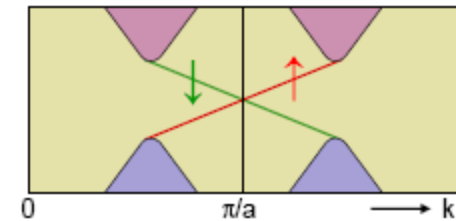
2D



Spin Filtered edge states
vacuum



Edge band structure

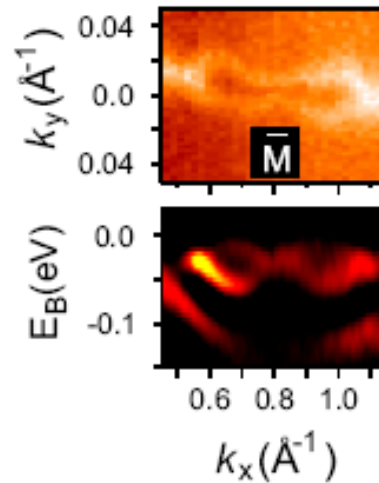
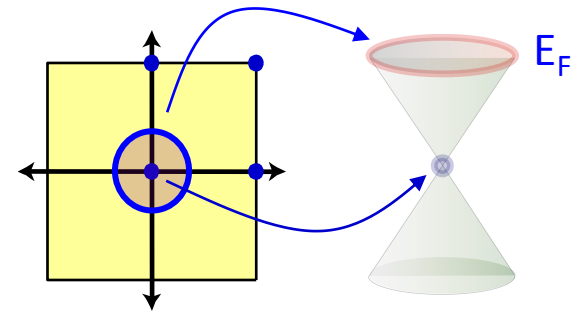
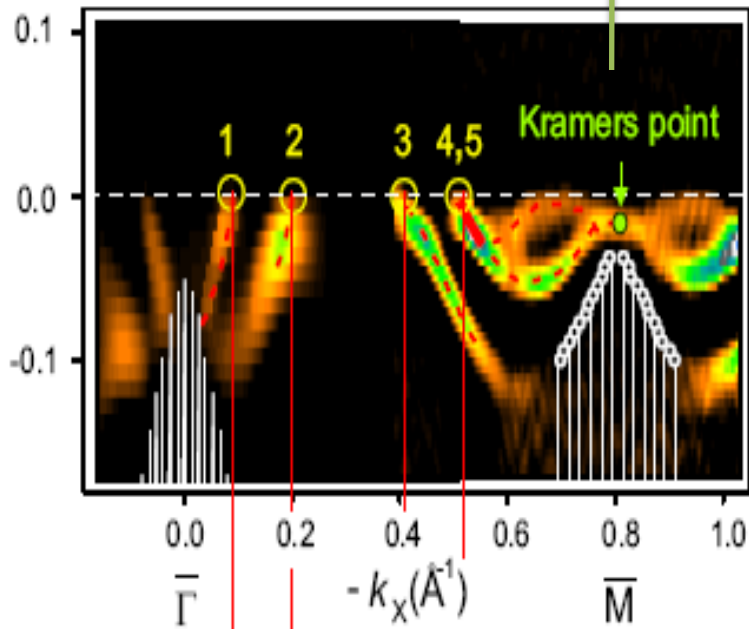
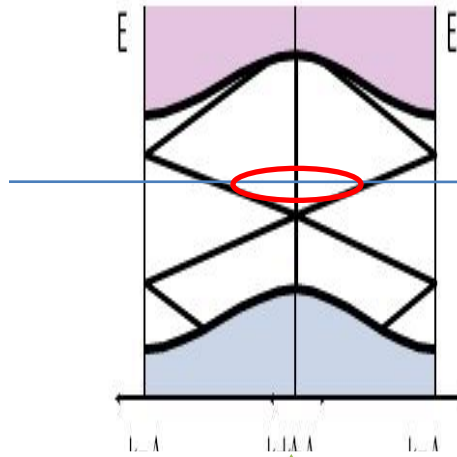


Edge states form a unique 1D electronic conductor

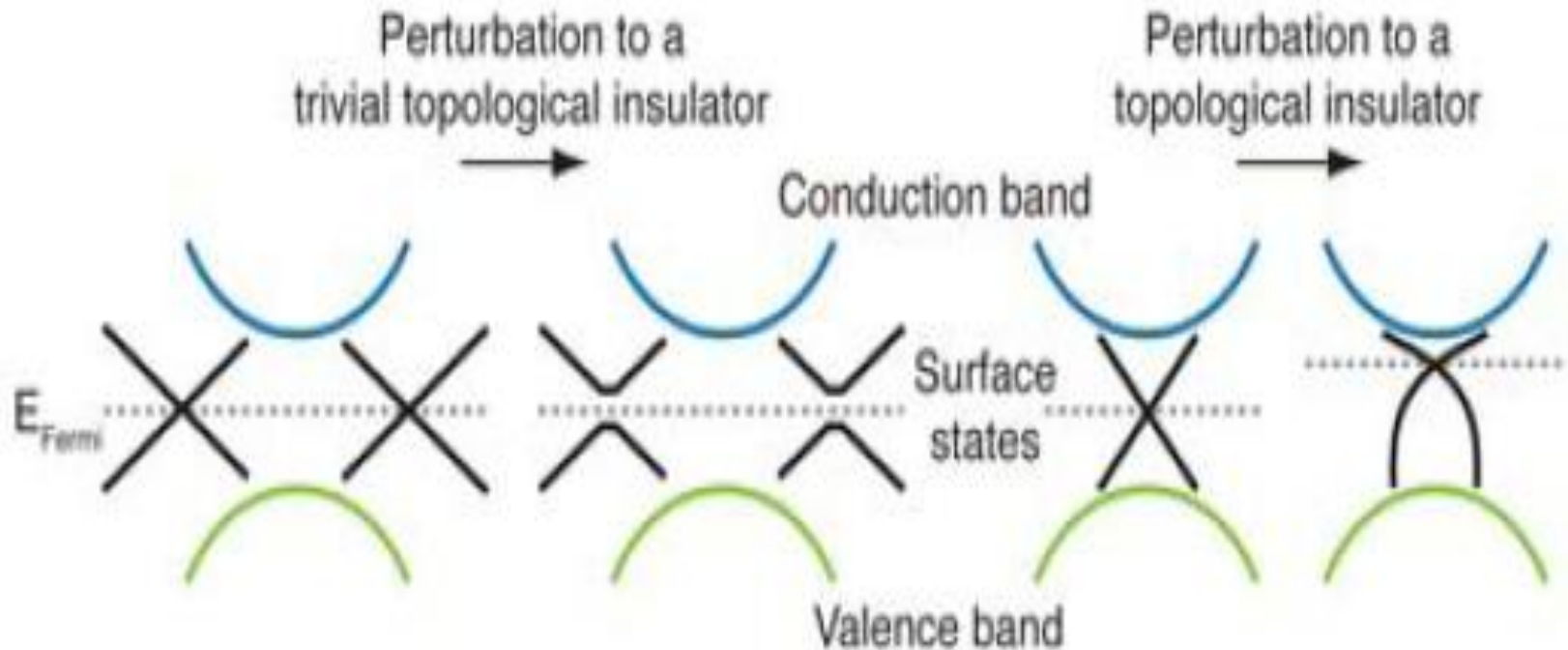
- HALF an ordinary 1D electron gas
- Protected by Time Reversal Symmetry (conservation of S_z is NOT essential)
- Elastic Backscattering is forbidden. No 1D Anderson localization

Topological Insulators

Surface Fermi pockets must enclose ODD number of Kramers points (Dirac points)

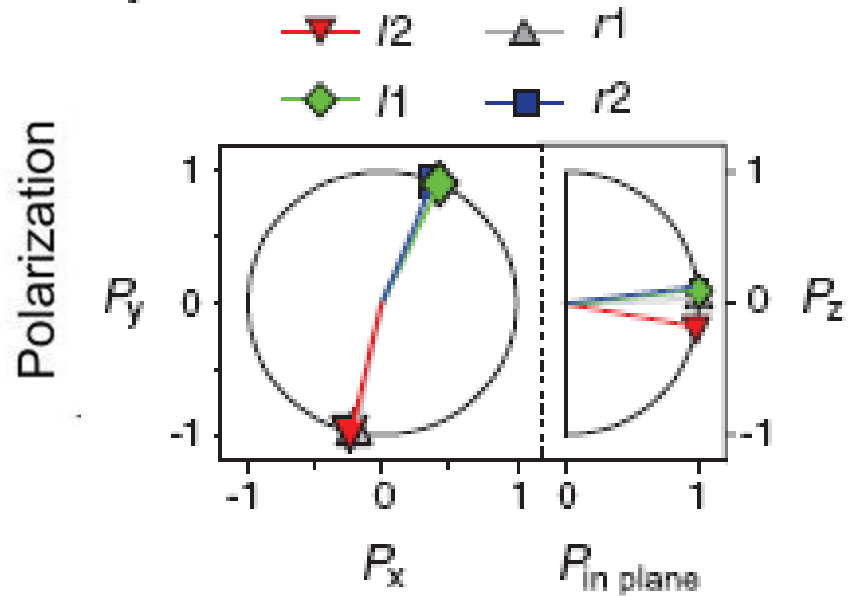
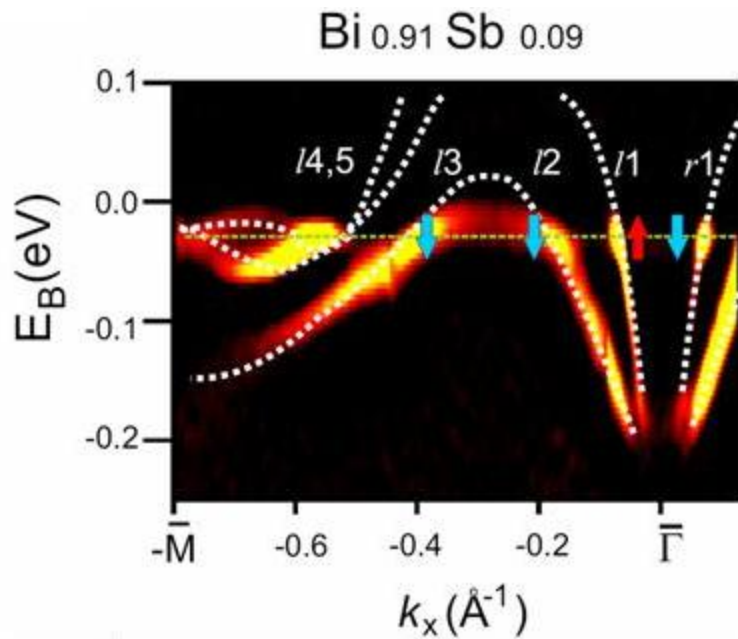
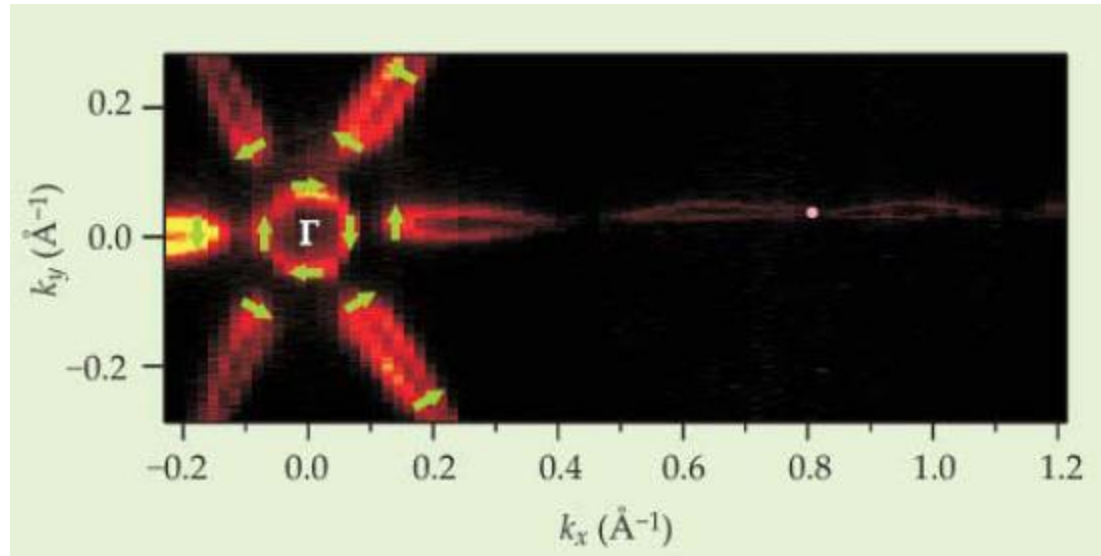


Why Dirac cone is distorted ?

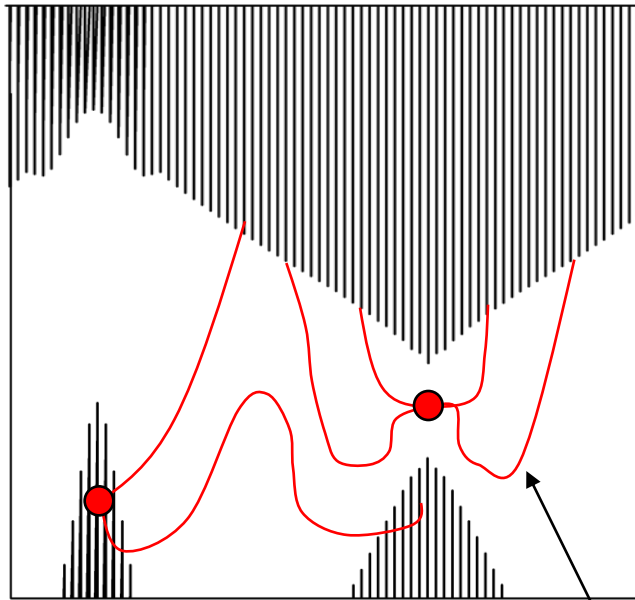


S.C Zhang (on BiSb) : Physics [APS news&views] (2008)

Surface state spin-textures



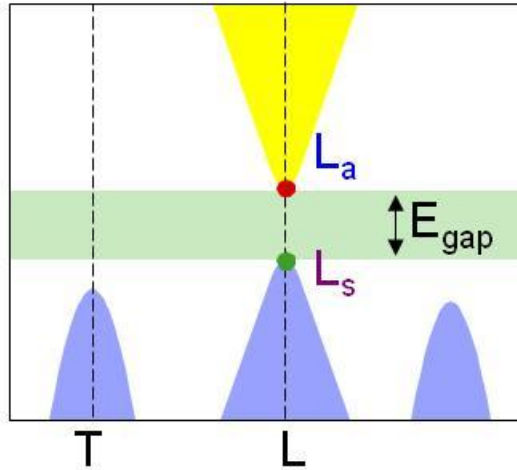
Which one is the "Knot" band?



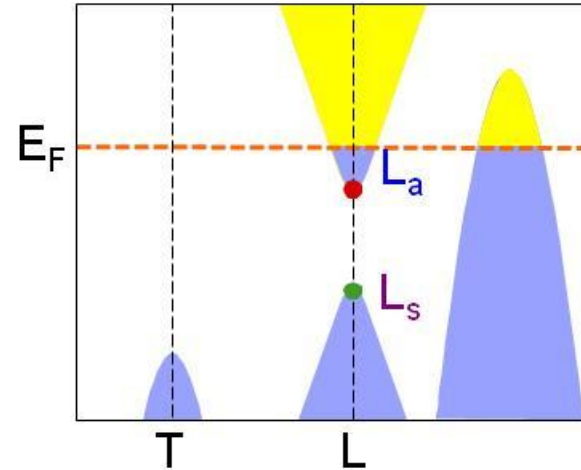
Originates from
conduction band

Parity Table for $\text{Bi}_{1-x}\text{Sb}_x$

Alloy : $.09 < x < .18$
 Band gap $\sim 30\text{-}50\text{ meV}$



Pure Antimony
 semimetal



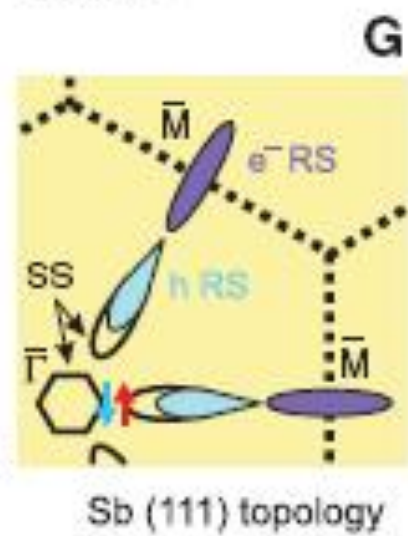
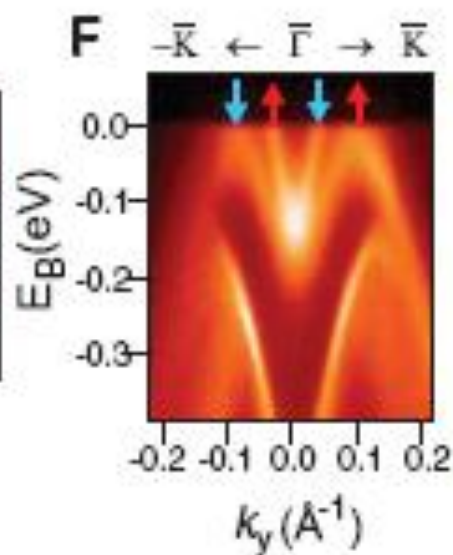
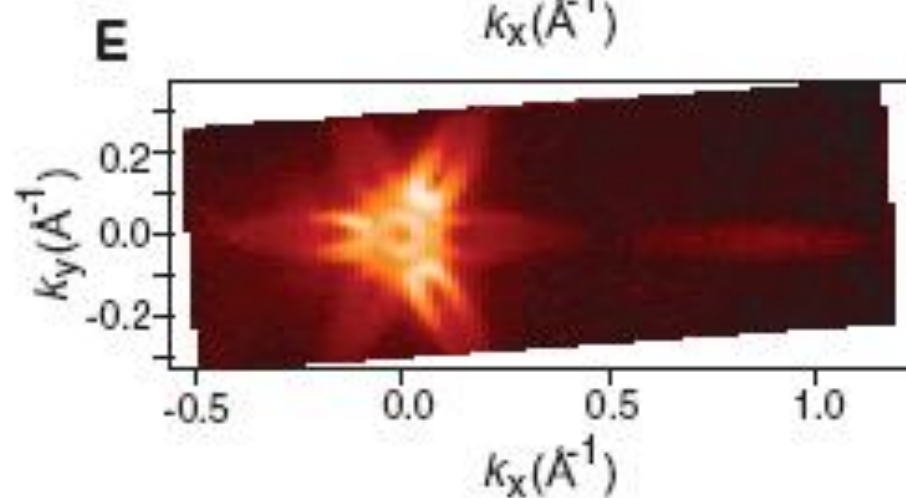
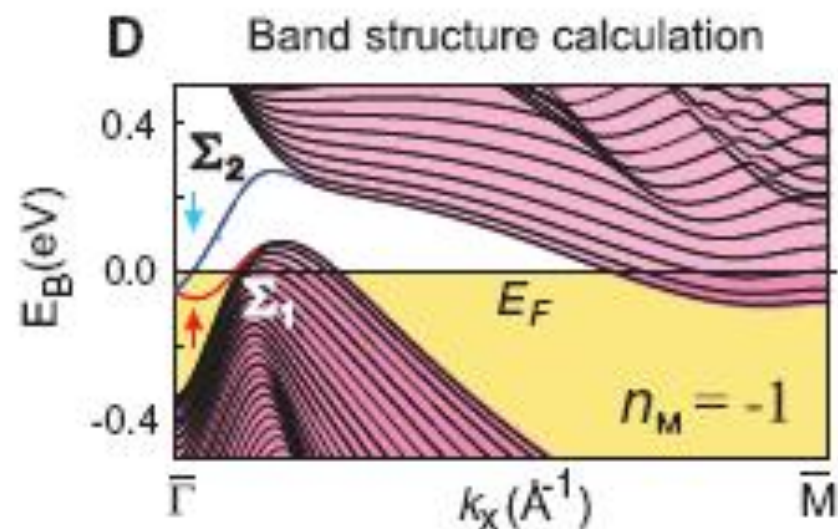
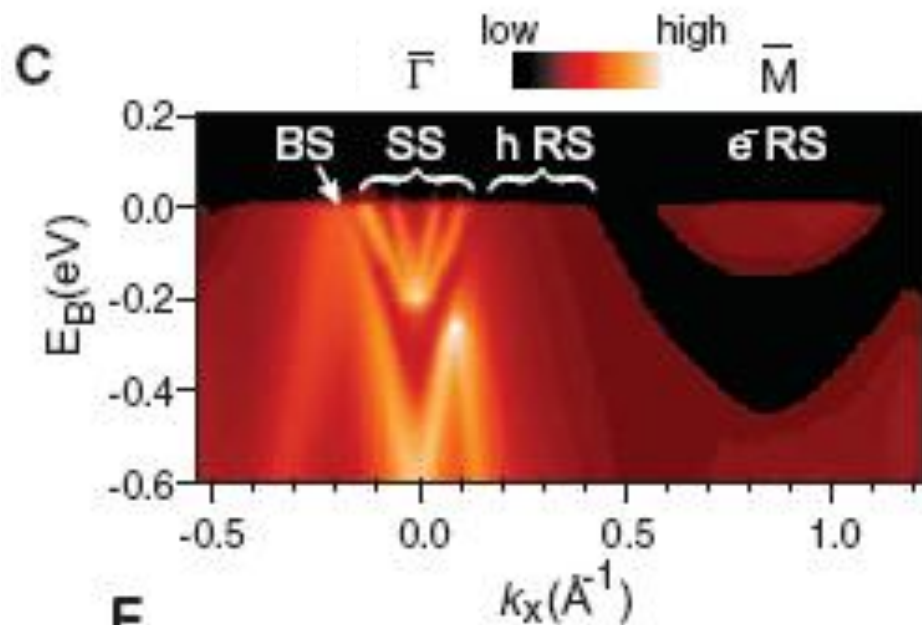
E
 k

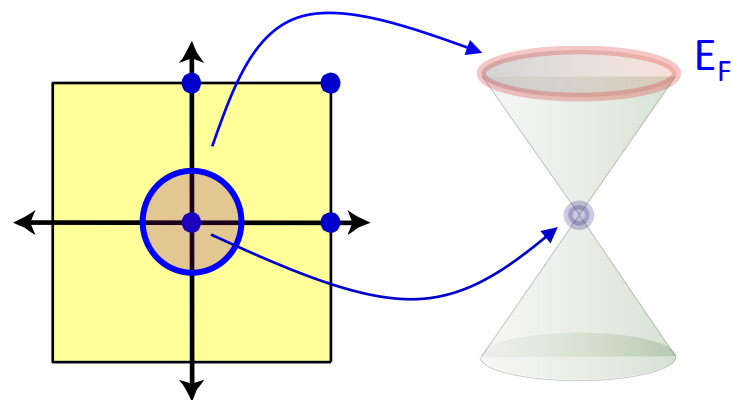
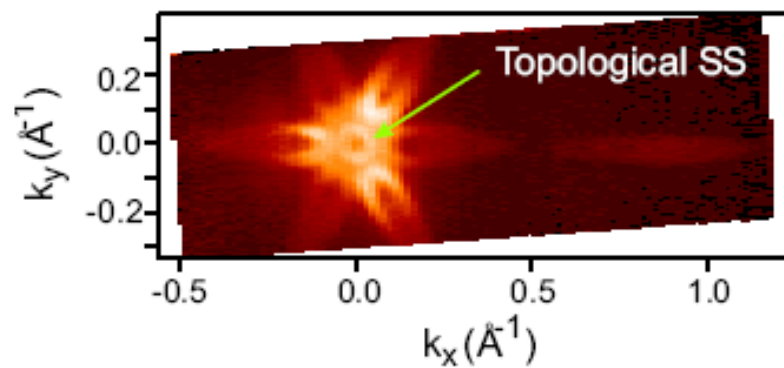
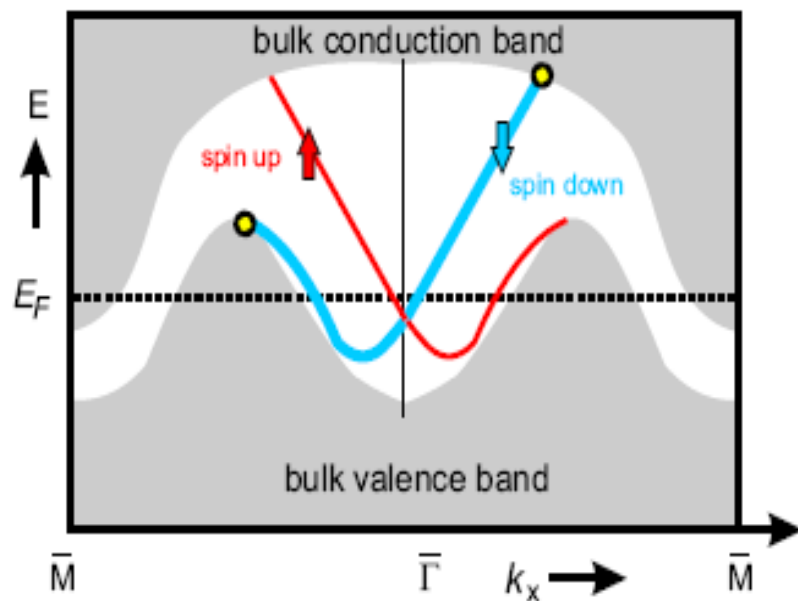
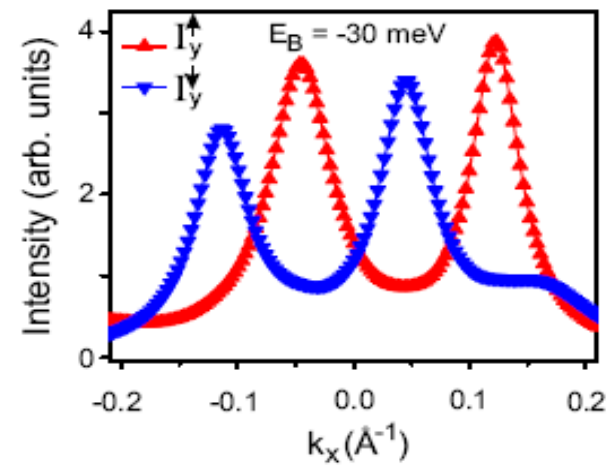
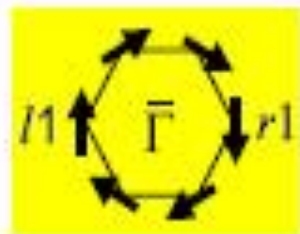
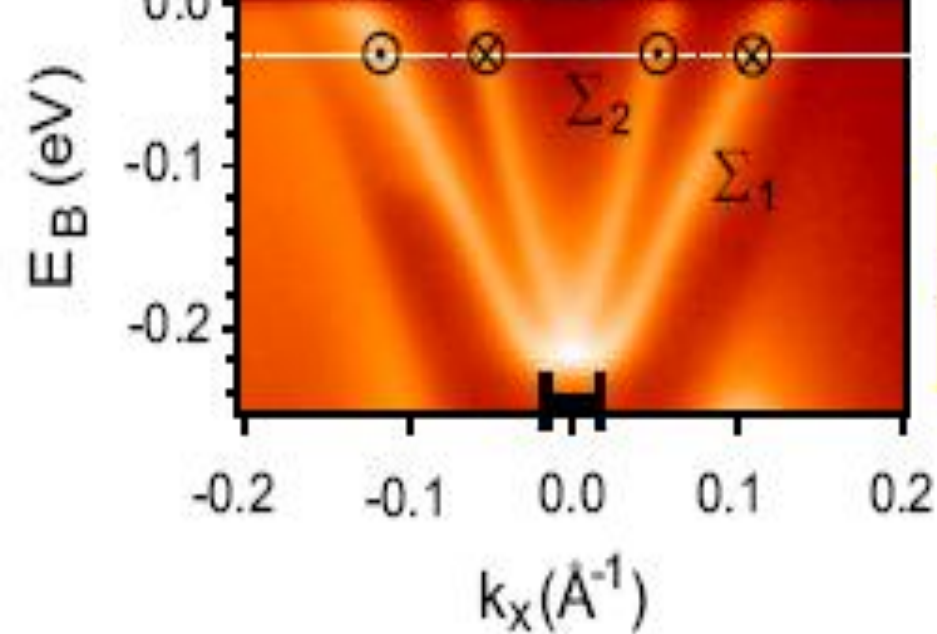
Inversion symmetry \Rightarrow

$$(-1)^{\nu_0} = \prod_{i=1}^8 \prod_n \xi_{2n}(\Gamma_i)$$

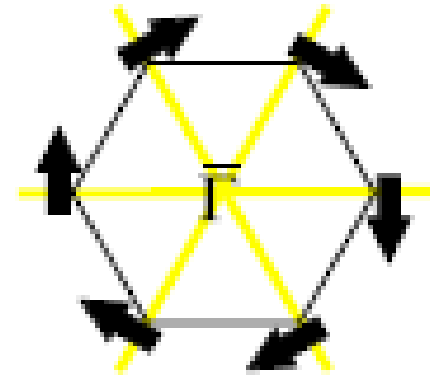
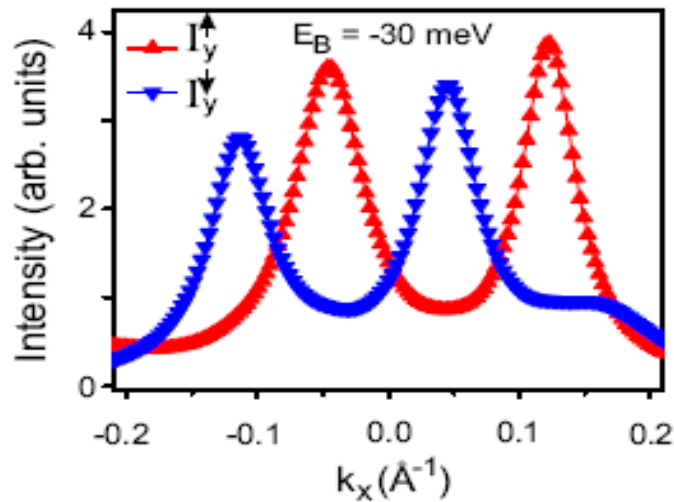
Antimony

1Γ	Γ_6^+	Γ_6^-	Γ_6^+	Γ_6^+	Γ_{45}^+	-
$3L$	L_s	L_a	L_s	L_a	L_s	+
$3X$	X_a	X_s	X_s	X_a	X_a	-
$1T$	T_6^-	T_6^+	T_6^-	T_6^+	T_{45}^-	-
	Z ₂ class					(1; 111)





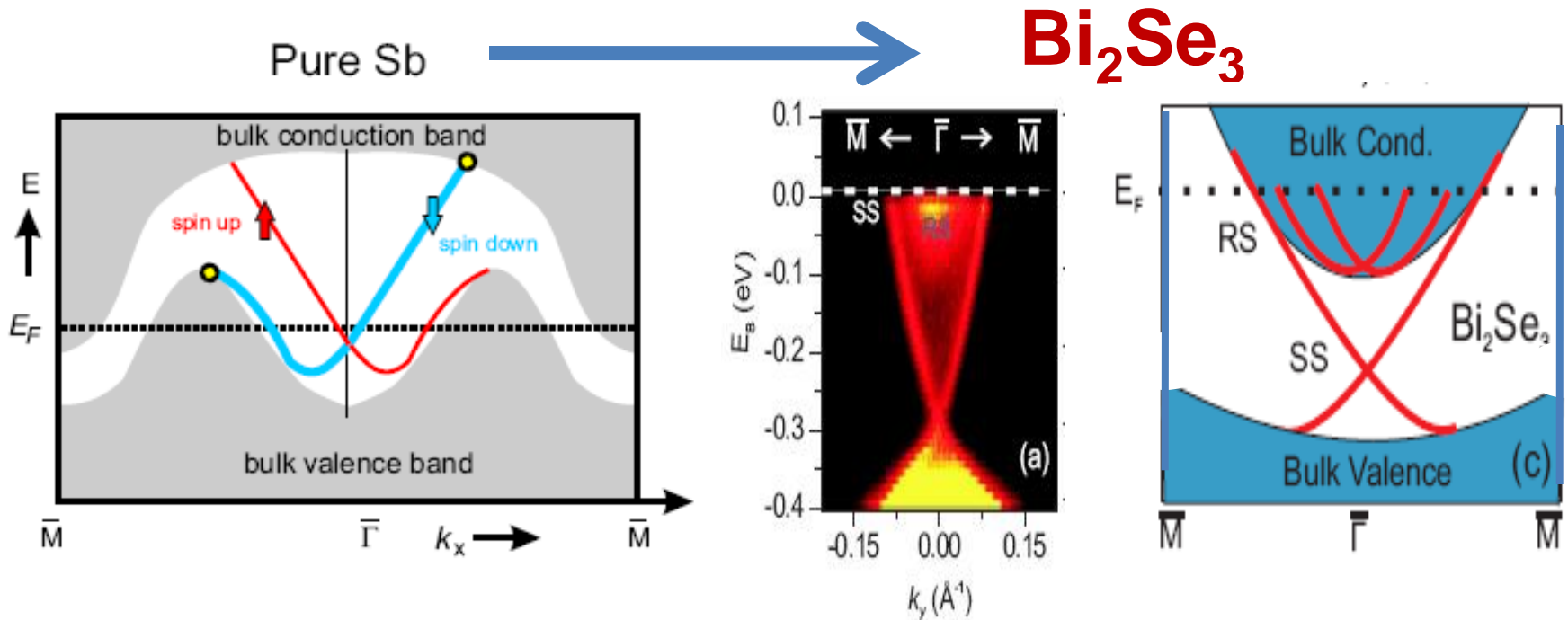
Spin Chirality & Topological Berry's Phase



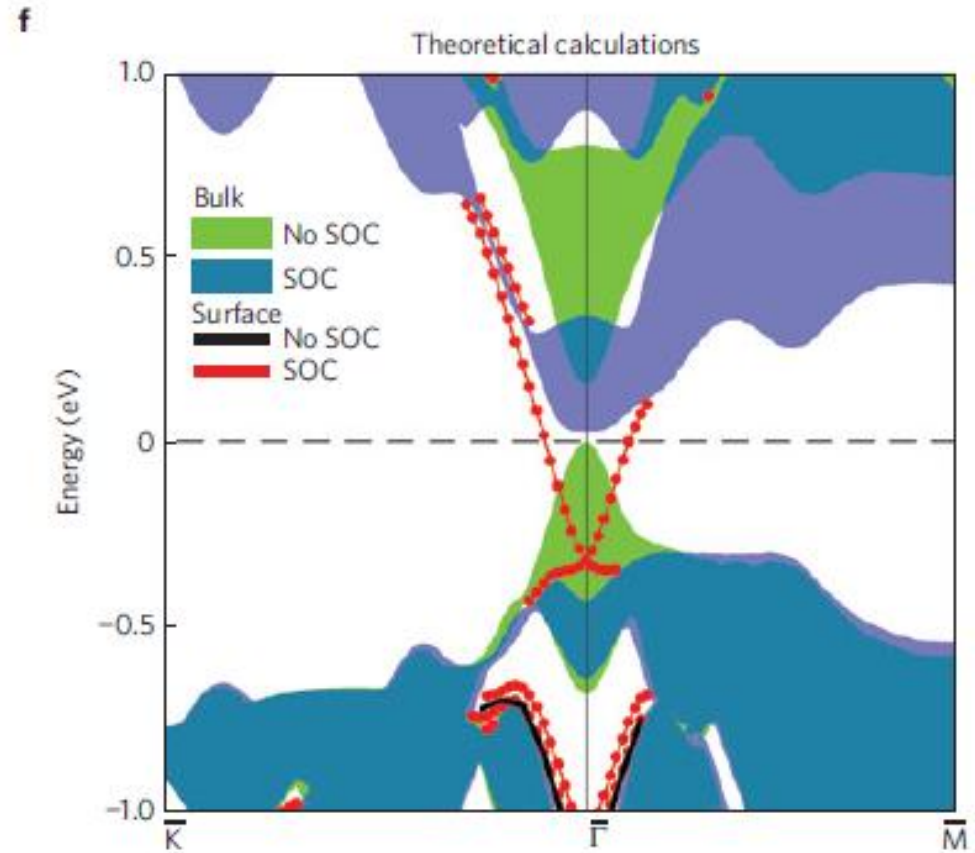
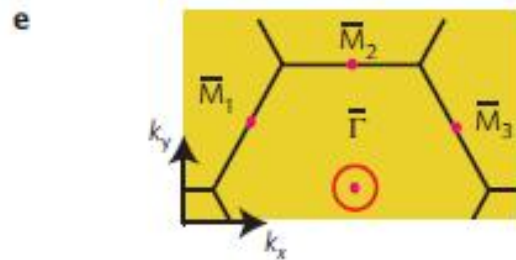
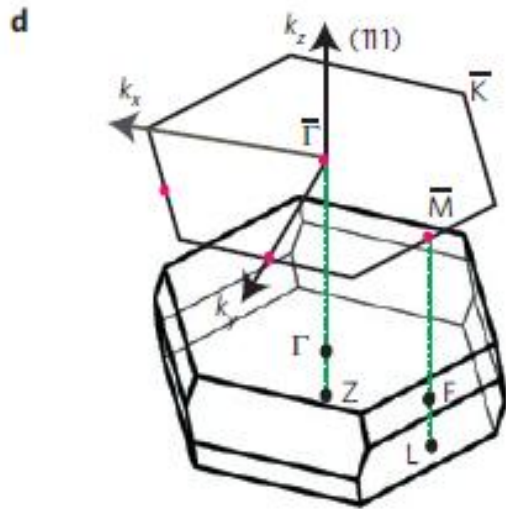
$$e^{i\phi(\Gamma)} = \exp i \oint_{\Gamma} \mathcal{A}_\mu dg^\mu = -1$$

$$\oint_{\Gamma} \mathcal{A}_\mu(\mathbf{g}) = -i \langle \Psi(\mathbf{g}) | \partial_\mu \Psi(\mathbf{g}) \rangle \sim \pi$$

Can we make an insulating version of Sb?

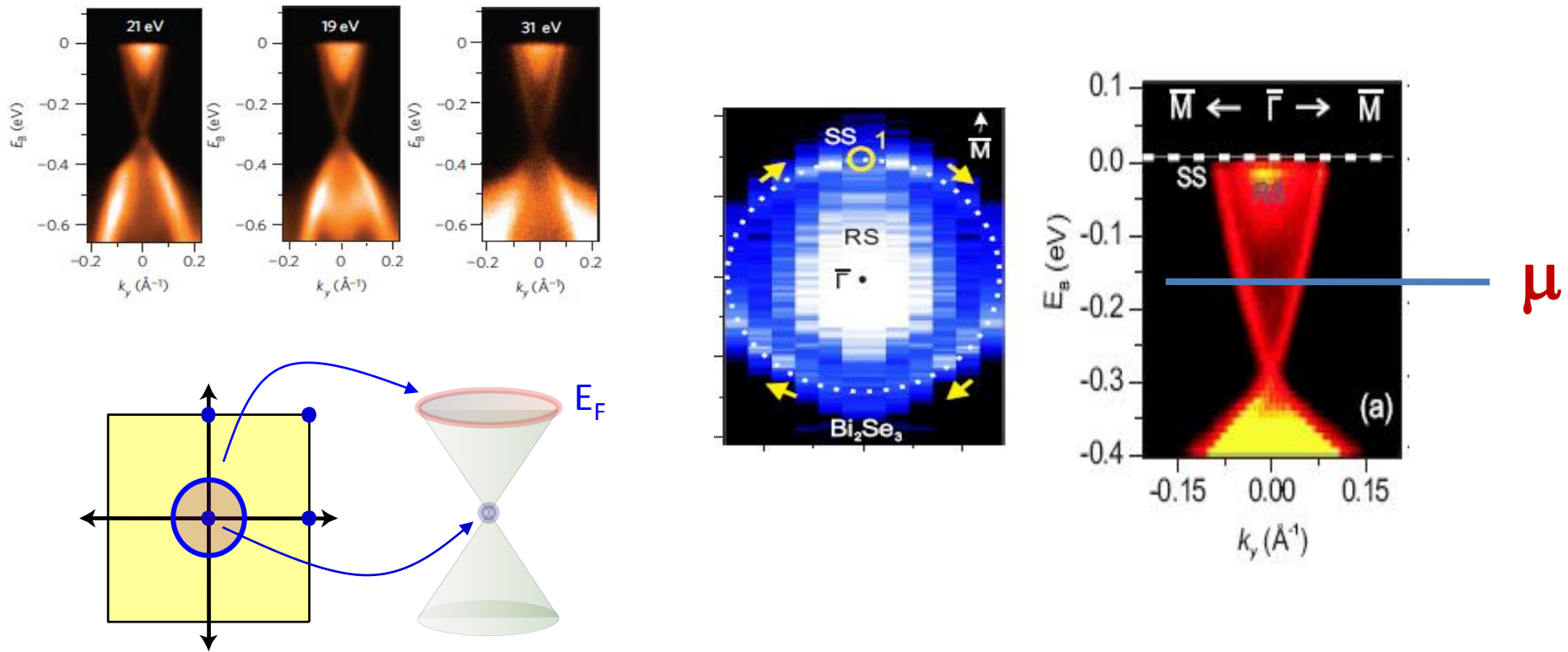


Our theoretical prediction of Topological Dirac fermions in Bi_2X_3 class



Xia et.al., **Nature Physics** 5, 398 - 402 (2009)
Published online: 10 May 2009 | doi:[10.1038/nphys1274](https://doi.org/10.1038/nphys1274)

Experimental Observation of “Hydrogen atom” of Strong Topological Insulator

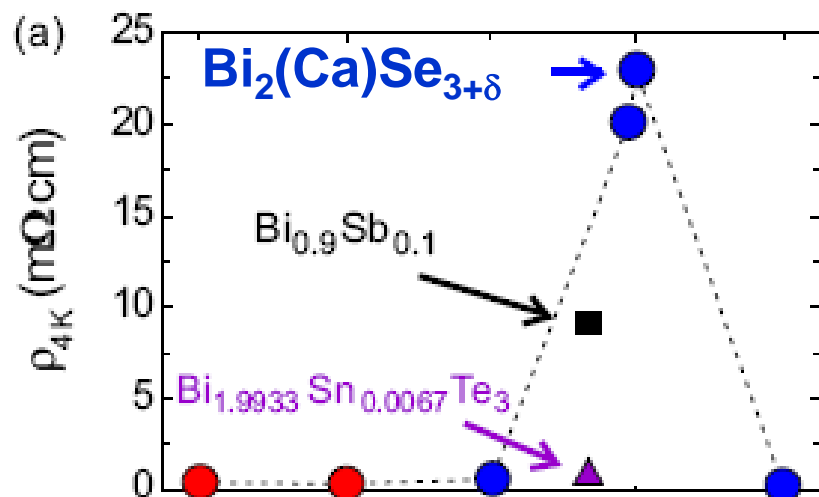


Xia et.al., **Nature Physics** 5, 398 - 402 (2009)

Published online: 10 May 2009 | doi:[10.1038/nphys1274](https://doi.org/10.1038/nphys1274)

News & Views : <http://www.nature.com/nphys/journal/v5/n6/full/nphys1294.html>

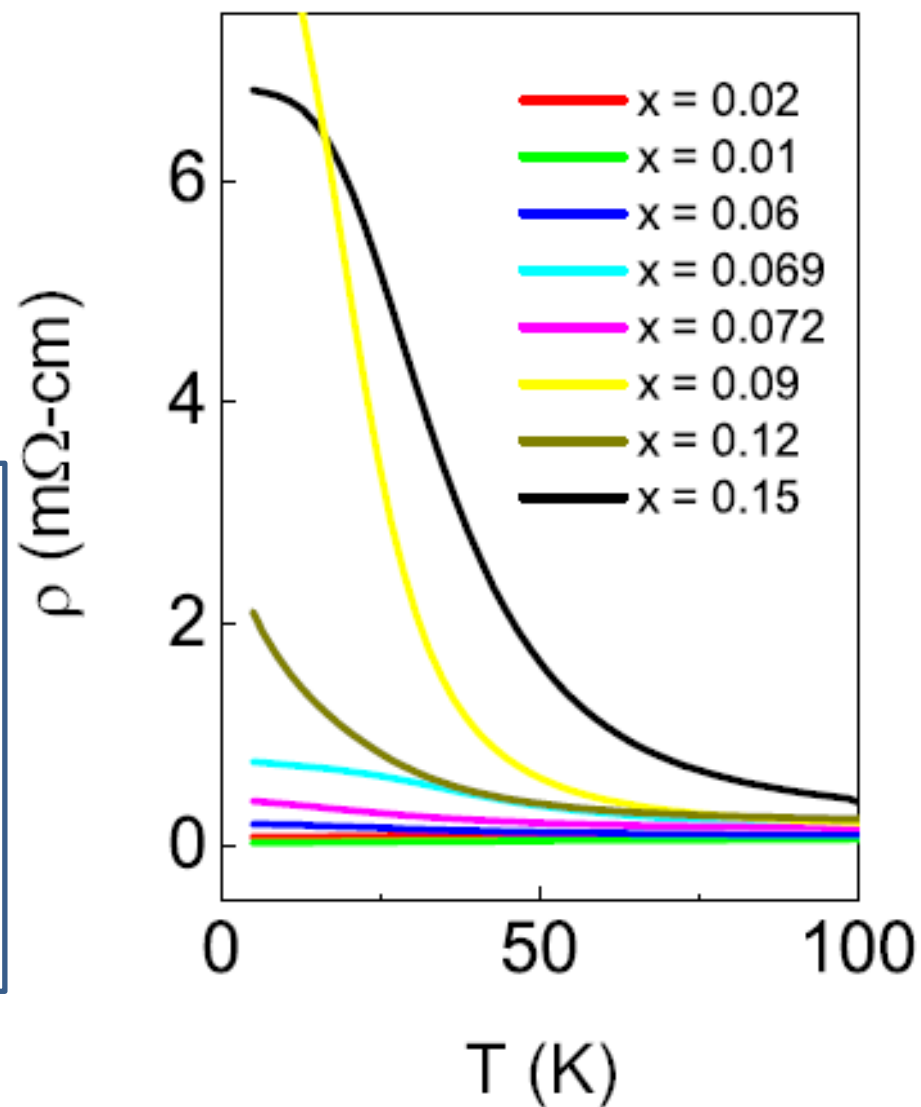
Bulk Insulator



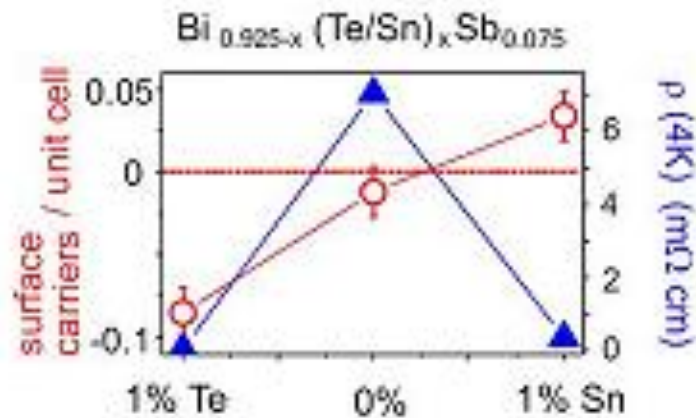
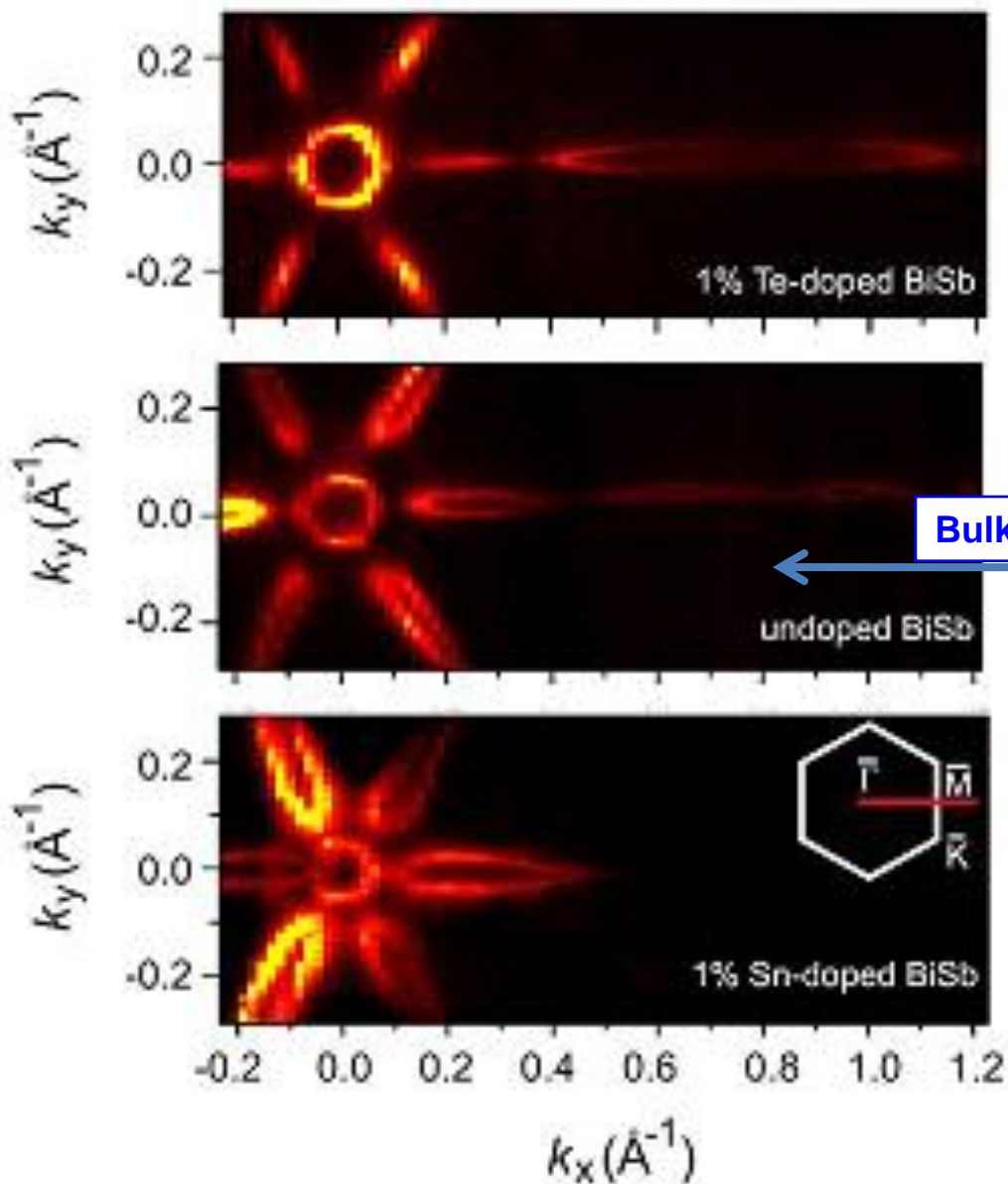
Data by Checkelsky & Ong (2009)

Charge compensation

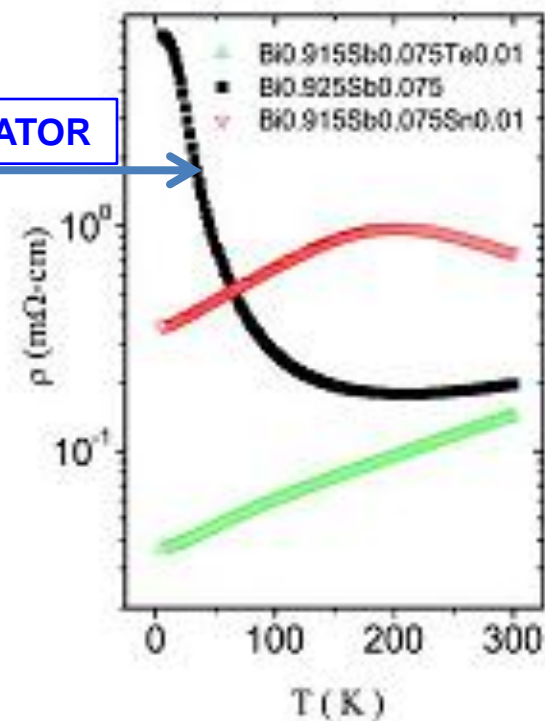
$\text{Bi}_{1-x}\text{Sb}_x$



Tuning to the topological insulator regime in $\text{Bi}_{1-x}\text{Sb}_x$

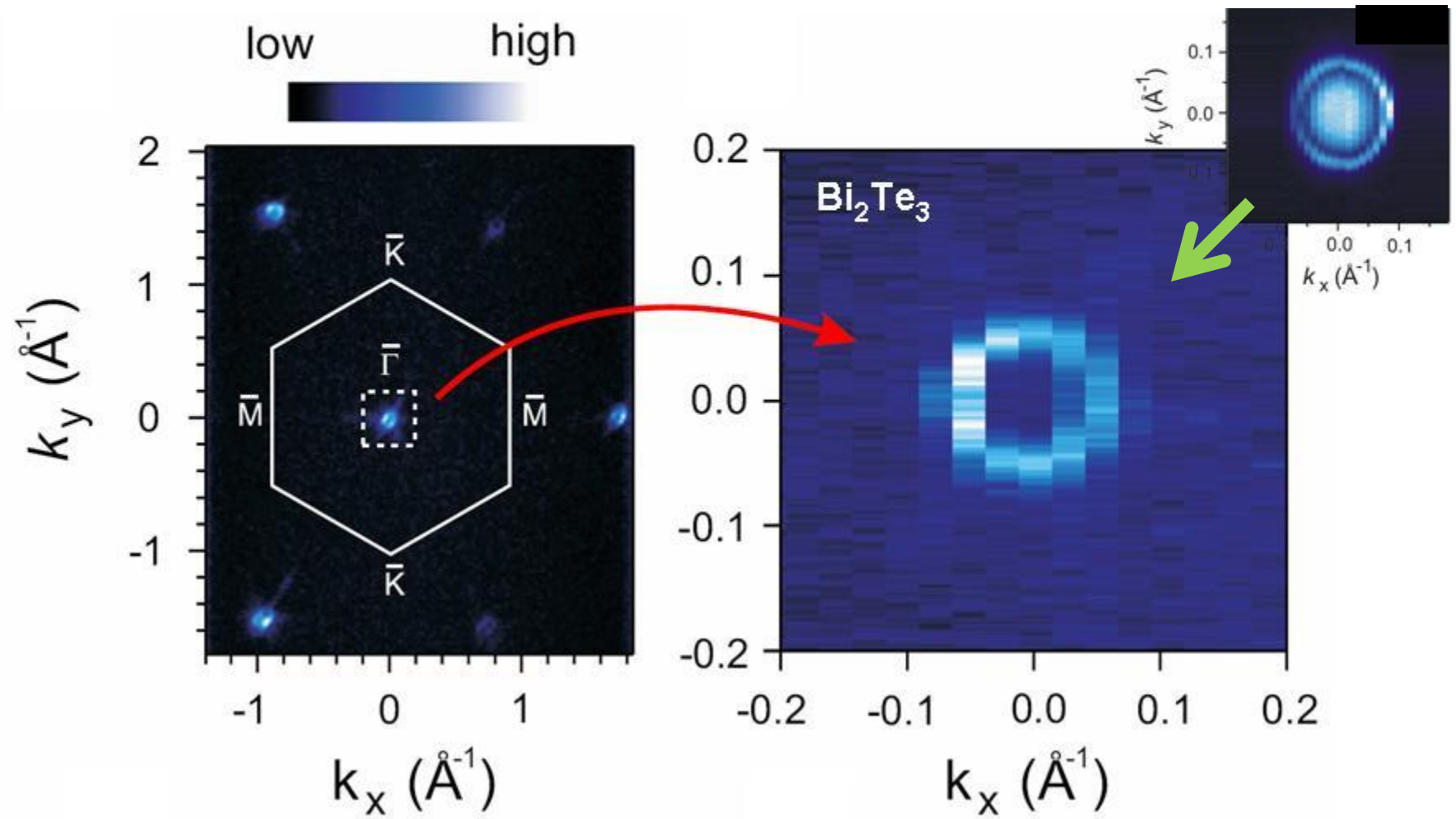


Bulk INSULATOR



First observation of **spin-Dirac** fermions and topological insulator phase in Bi_2Te_3 probed by spin-ARPES (previous work, Noh et.,al. was not spin polarized detection):

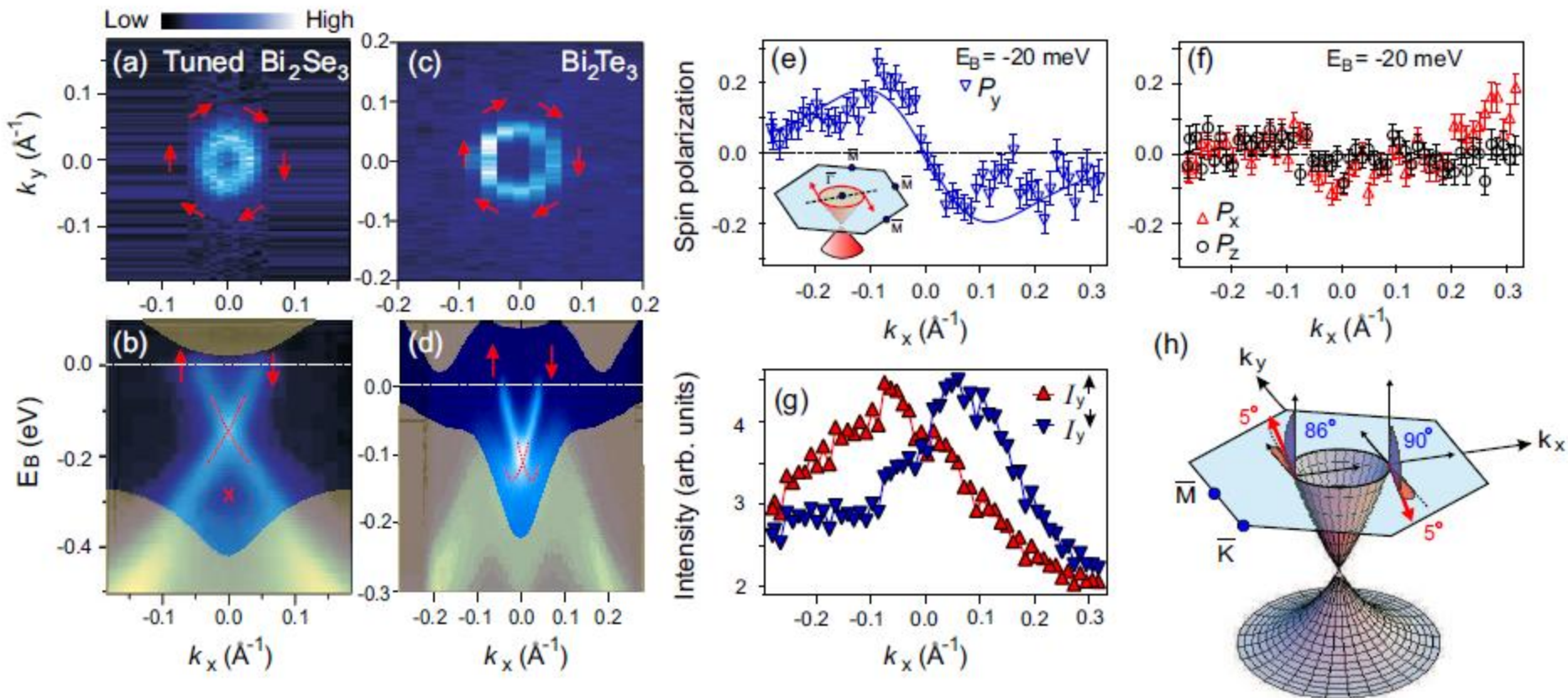
Hsieh et.al., <http://aps.arxiv.org/abs/0904.1260>





Helical Dirac fermions

Spin-momentum locking



Hsieh et.al.,

<http://aps.arxiv.org/abs/0904.1260>

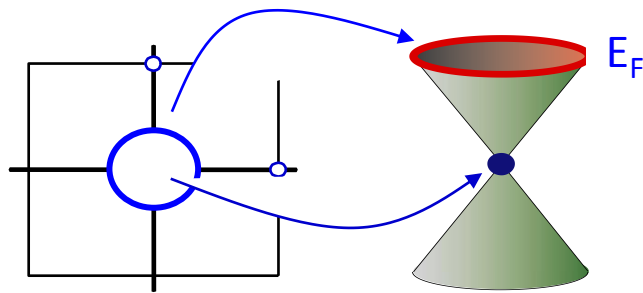
Strong Topological Insulator ($\nu_0 = 1$)

Boundary Fermi pocket encloses **odd** number of Dirac points

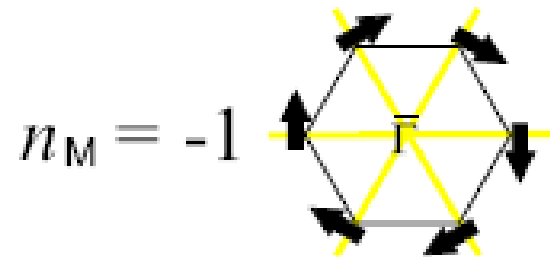
Surface states form a “Topological Metal” phase :

This topological state is fundamentally different from conventional 2D matter (2DEG) and the integer QHE state.

- Berry's phase π around the Fermi pocket
- Half-integer charge QHE
- Opposite to Anderson Localization (weak “anti-localization”)

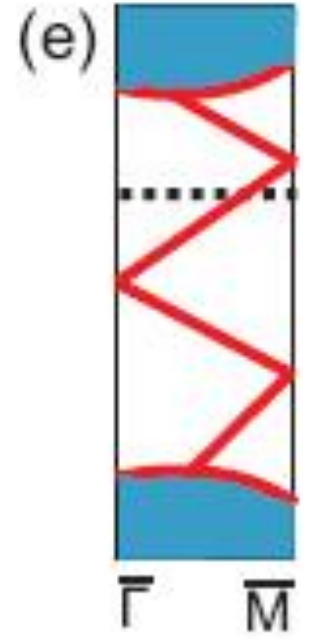
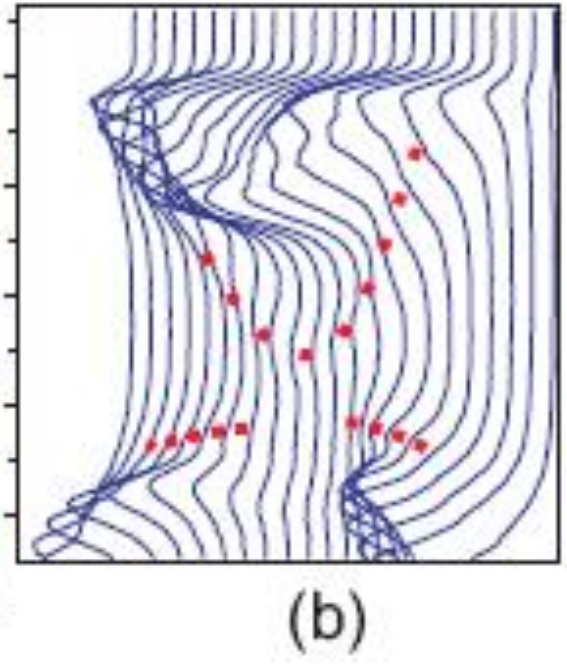
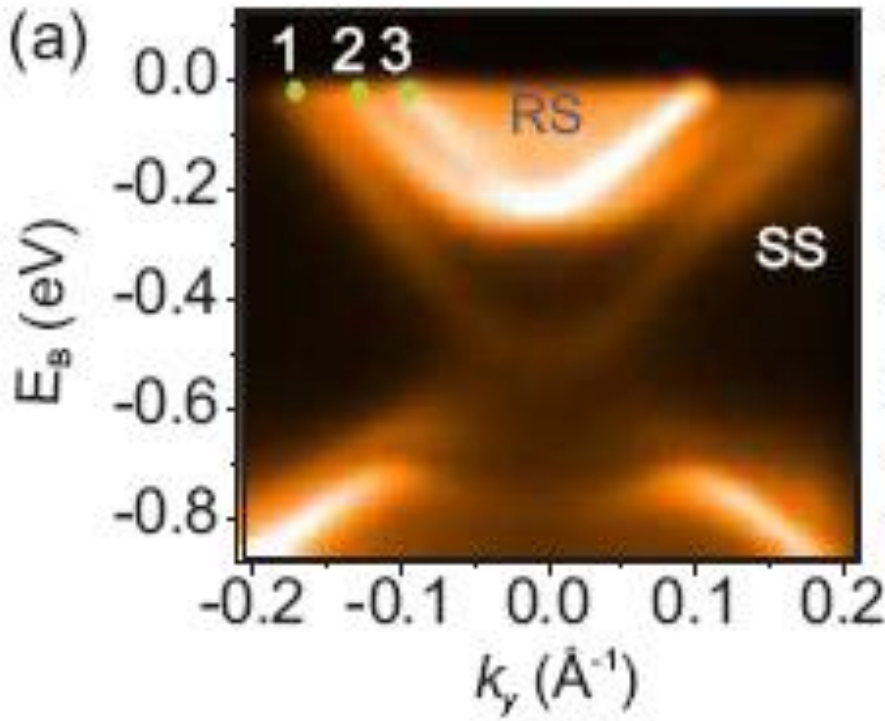


Spin-textured Dirac cone



Mirror Chern number

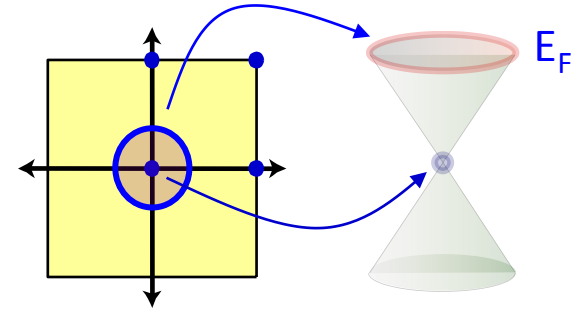
Deposit a magnetic submonolayer on STI



See our preprint for details : Xia, Hasan et.al., [arXiv:0812.2078v1](https://arxiv.org/abs/0812.2078v1) (2008)



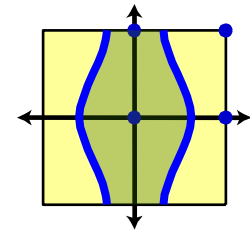
Strong Topological Insulator ($\nu_0 = 1$)



Fermi arc encloses odd number of Dirac points
Dirac structure has unique spin-texture
Leading to π Berry's phase

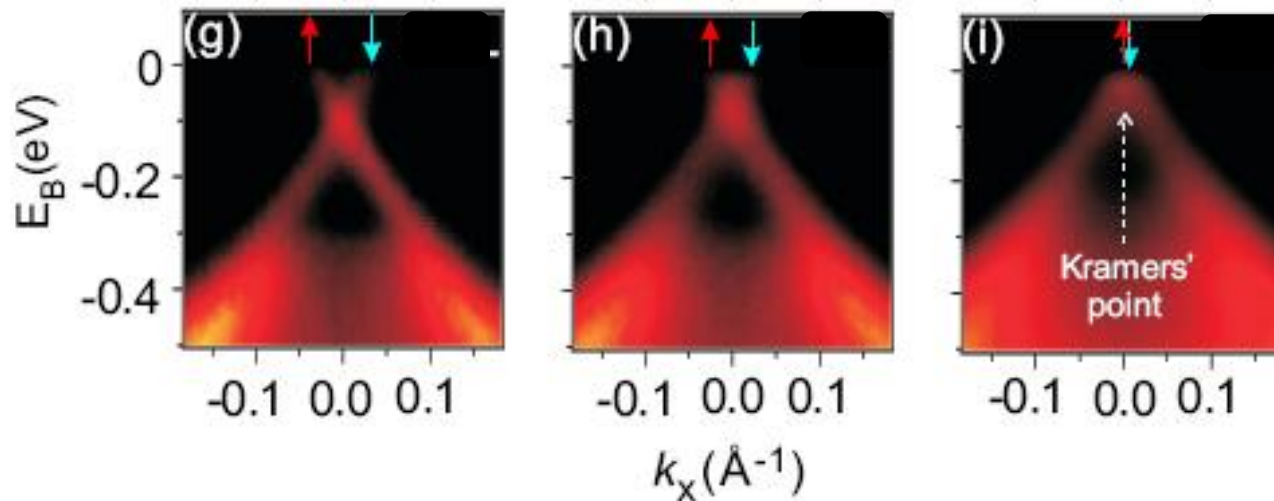
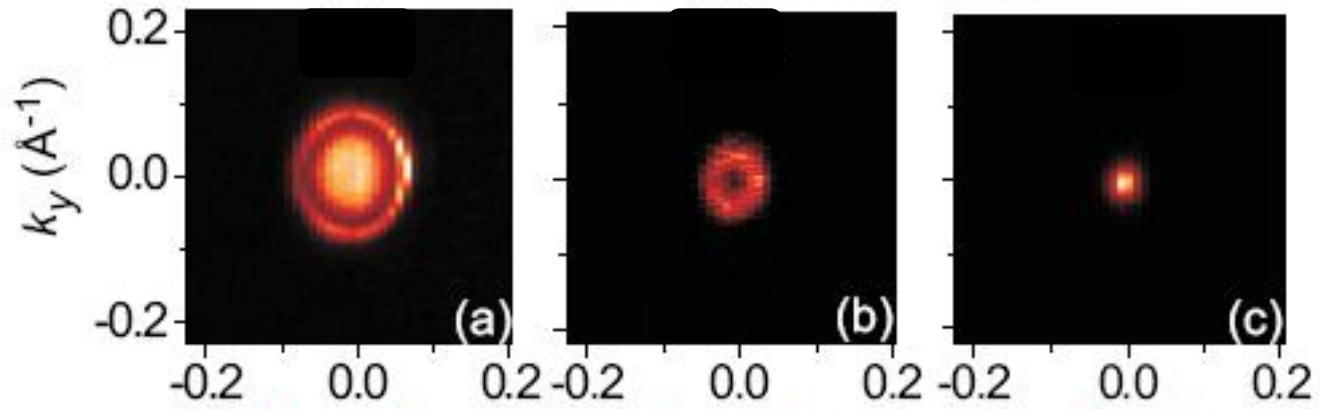
Weak Topological Insulator ($\nu_0 = 0$)

Fermi arc encloses even number of Dirac points
Dirac structure has spin-texture but trivial phase

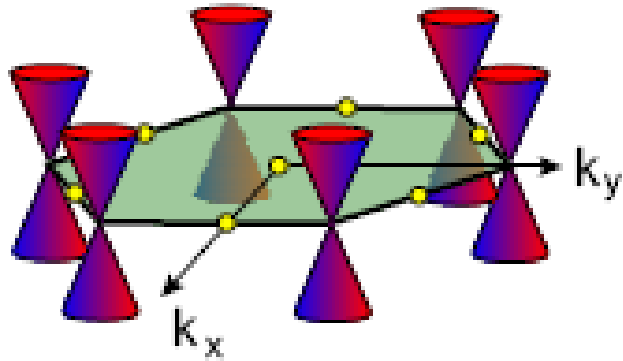


Trivial 3D
Generalization
of 2D QSH phase

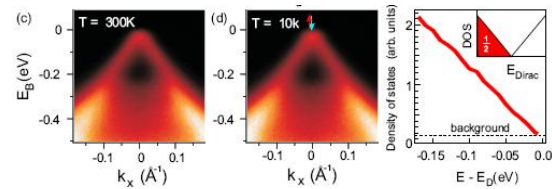
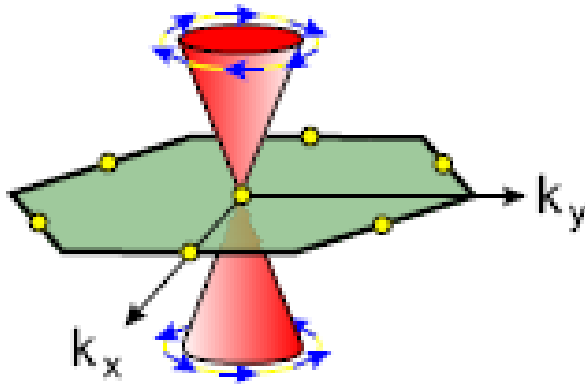
TOPOLOGICAL "Graphene"



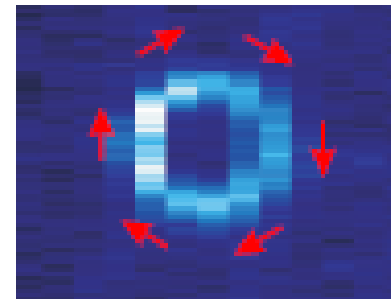
Chiral Dirac ground state



Graphene



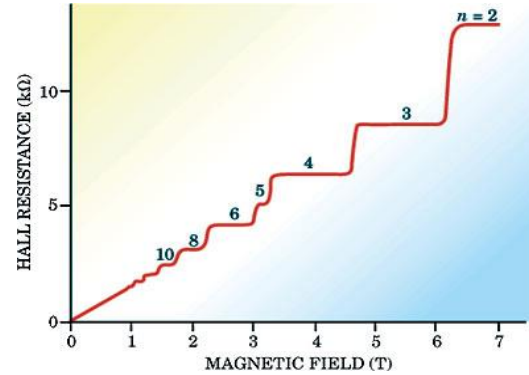
Topo Insulator



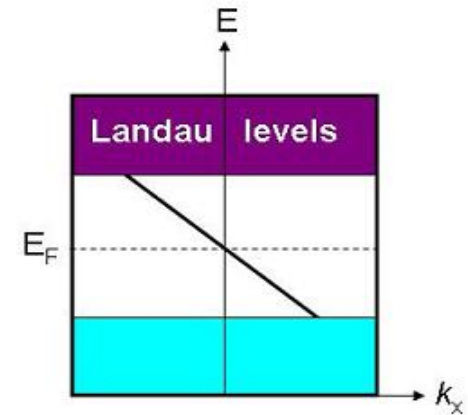
Quantum Hall effect

$$\sigma_{xy} = n e^2/h$$

Topological quantum number



Chiral degenerate fermions



Topological Insulators

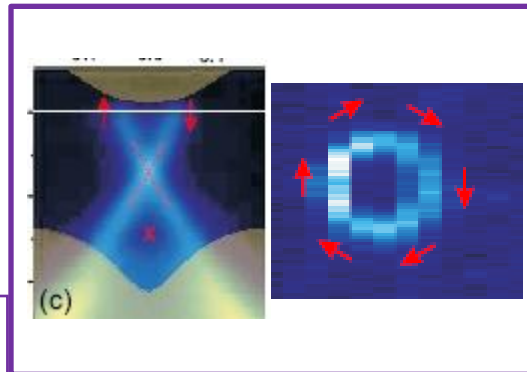
Helical spin-Dirac fermions

How to experimentally “measure” the topological quantum numbers (ν_i) associated with the quantum spin Hall or Strong Topological Insulator (STI) phases?

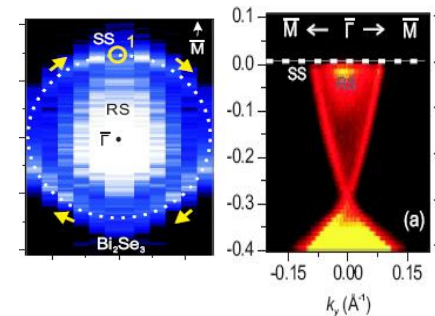
No quantized transport!

$$\{\nu_i\}$$

Topological quantum number



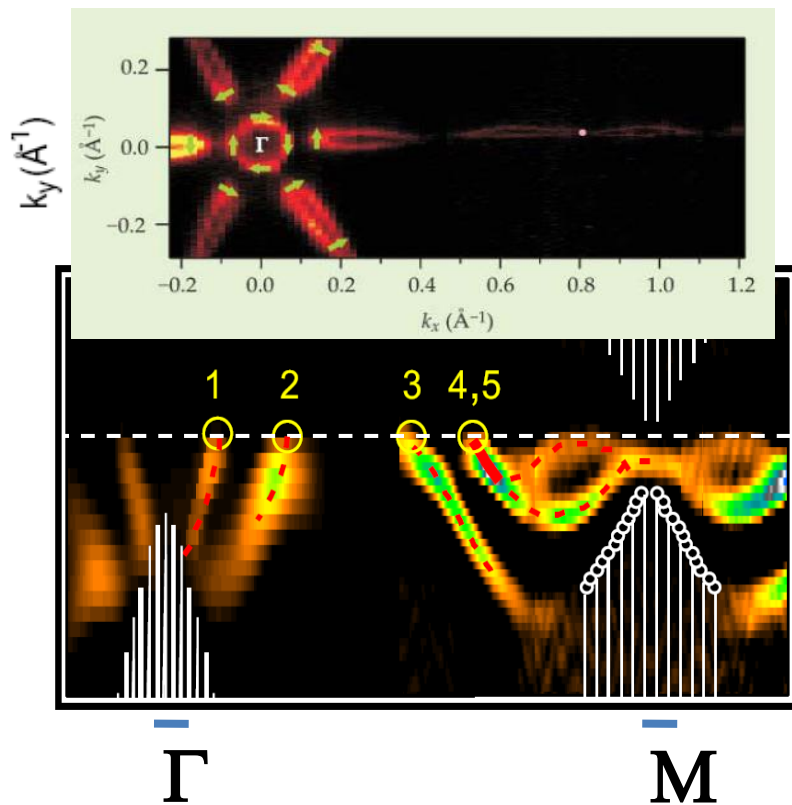
Spin-textured Dirac



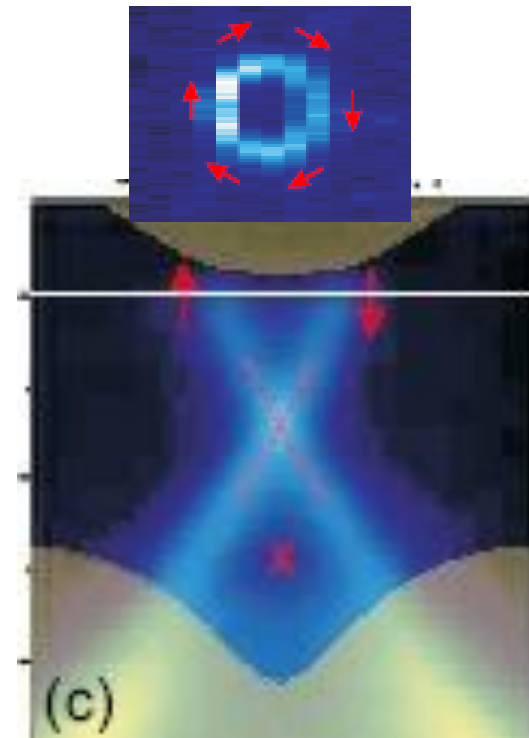


Experimental realizations of Strong Topo Insulators (Theta-Vacuum states) in nature :

Nature 08, Science 09



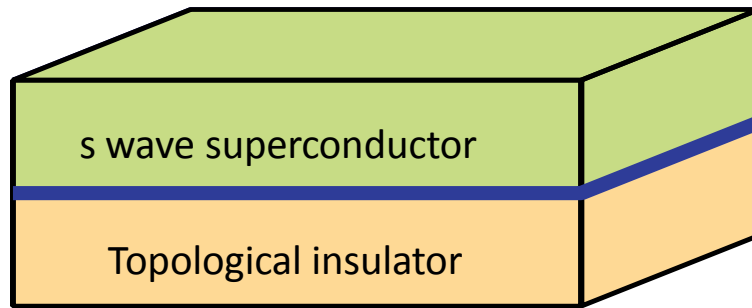
NatPhys 09, Nature 09



These two are actually 2 distinct types of strong topo insulators !

STI/Superconduct interface

Kane-Fu proposal



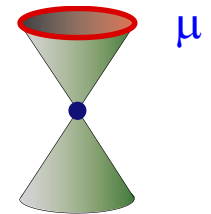
2D interface state with energy gap and exotic topological order

Resembles 2D spinless p_x+ip_y superconductor but does not violate time reversal symmetry

$$H = \psi^\dagger (-iv\vec{\sigma}\cdot\vec{\nabla} - \mu)\psi + \Delta\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \Delta^*\psi_\downarrow\psi_\uparrow$$

Dirac surface states
(no spin degeneracy)

proximity induced
superconductivity

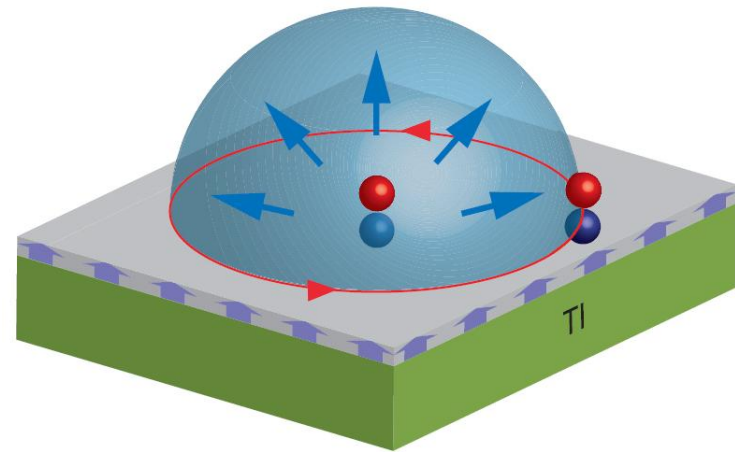
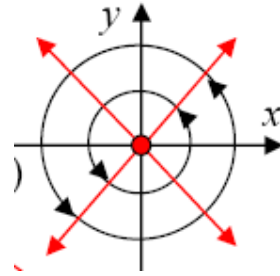
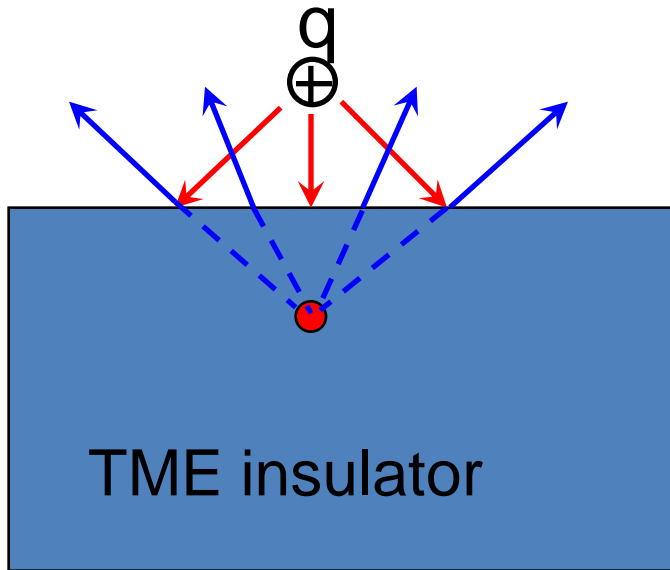


Majorana bound state at a vortex :

$$\Delta = \Delta(r)e^{i\theta}$$

- bound state solution to BdG equation at exactly zero energy
- $c_0 = c_0^\dagger$ (electron=hole) Majorana fermion = "1/2 a state"
- Also predicted in $\nu=5/2$ FQHE, Sr_2RuO_4 , cold atoms, etc

Seeing the magnetic monopole thru the mirror of a TME insulator, (Qi, Zhang et al, Science 323, 1184, 2009)

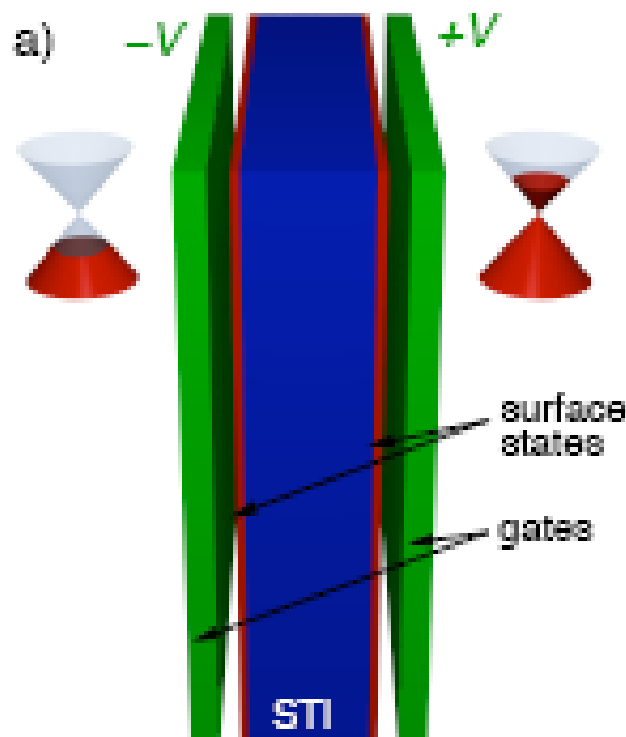


Magnitude of B:

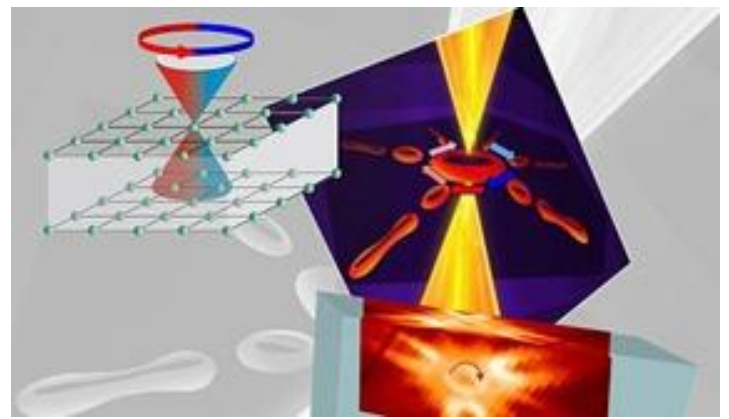
$$10^6 \text{ V/m} \rightarrow 0.25 \text{ G}$$

Topological Exciton Condensation:

B. Seradjeh,¹ J.E. Moore,^{2,3} and M. Franz⁴



An odd number of gapless Dirac fermions is guaranteed to exist at a surface of a strong topological insulator. We show that in a thin-film geometry and under external bias, electron-hole pairs that reside in these surface states can condense to form a coherent exciton condensate, similar in general terms to the exciton condensate recently argued to exist in a biased graphene bilayer, but with different topological properties. Such a ‘topological’ exciton condensate (TEC) exhibits a host of unusual properties; the most interesting among them is the fractional charge $\pm e/2$ carried by a singly quantized vortex in the TEC order parameter.



Future Directions :

Strong Topological Insulators are expected to exhibit :

- Half-integer charge quantum Hall effect
- Opposite to Anderson localization (weak anti-localization)
- Superconducting proximity: Majorana Fermions :
Fu, Kane et.al., Beenakker et.al., Kitaev
- Magnetoelectric/ Axion effects: Moore, Franz et.al,
- Topo exciton (BEC) condensates: Moore, Franz et.al,
- Magnetic proximity effect, Monopole: Qi, Zhang et.al,
- Helical-metal in Dislocations: Ran, Vishwanath et.al.,
- Topological (fault-tolerant) quant.computing Kitaev, Kane, Beenakker

Our Expts on Topological Insulator

(Theta-vacuum or Axionic state)

Spin-sensitive direct edge-state imaging methods

“A topological Dirac insulator in a Quantum Spin Hall Phase”

NATURE (2008)

“Spin-textures & Berry’s phase in a Topo Insulator” [Bi-Sb series]

SCIENCE (2009)

“Charge compensation in a Topo Insulator” [Bi₂(Ca)Se₃]

Phys. Review B (2009)

“A large-gap Topological with single Dirac cone” [Bi₂Se₃]

NATURE PHYS (2009)

“A new Topological Insulator”

NATURE (in review) (2009)

Conclusion :

A fundamentally new quantum phase of matter ($z_2 = -1$, $\nu_0 = 1$ θ -vacuum state) has been observed (Topological Insulator) which is distinct from the quantum Hall effect phase yet topological in nature.

NATURE (2008)

News&Views : Nature (2008)

Phys.Rev. B (09)

Editor's Suggestion

SCIENCE (2009)

News&Views : Science (2009)

NATURE PHYS (2009)

News&Views : Nature Phys. (2009)

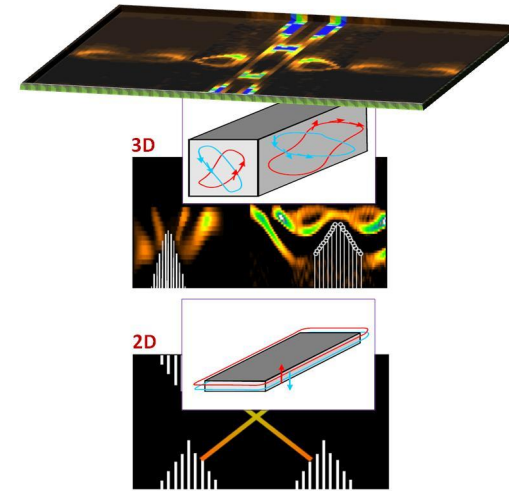
PHYSICS TODAY "Search&Discovery" April (2009)

See commentary by F. Wilczek in NATURE (2009)

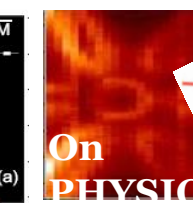
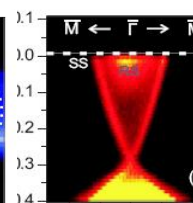
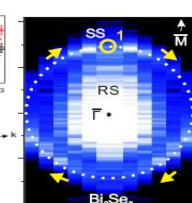
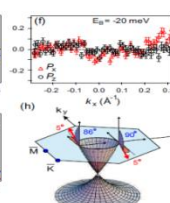
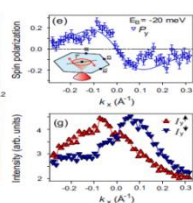
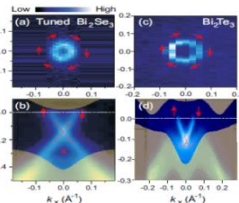
also by B.I. Halperin at CMP Journal Club (2008)

NATURE (in rev.) (09)

A tuneable Topological Insulator



Topo Insulator is the realization of long-sought θ -vacuum/Axions (next talk by Moore).



Thanks !