# Experiments on pinning modes in quantum Hall systems

L. W. Engel

*National High Magnetic Field Laboratory,*
*Florida State University*

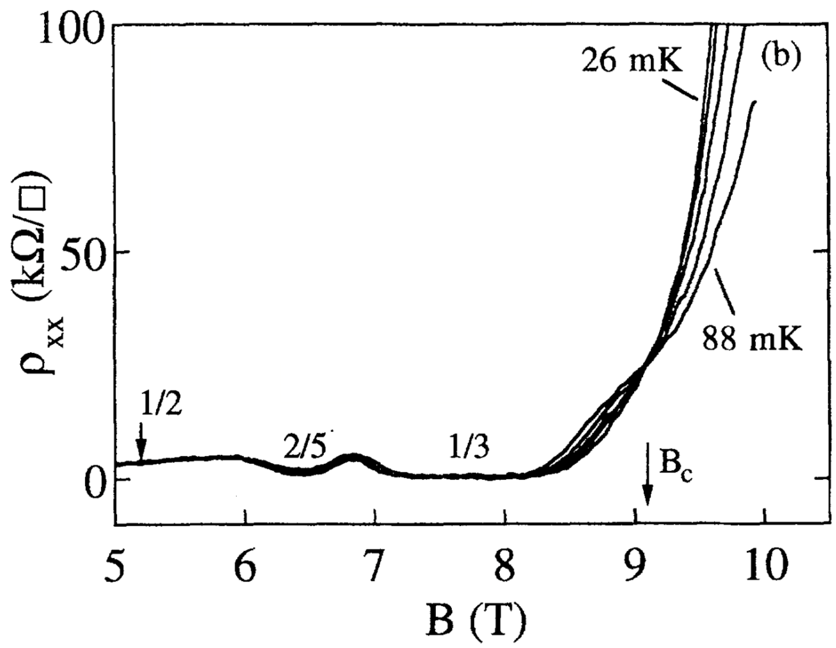
Collaborators:
Yong Chen,
R. M. Lewis,
G. Sambandamurthy,
Zhihai Wang
P. D. Ye,
Han Zhu

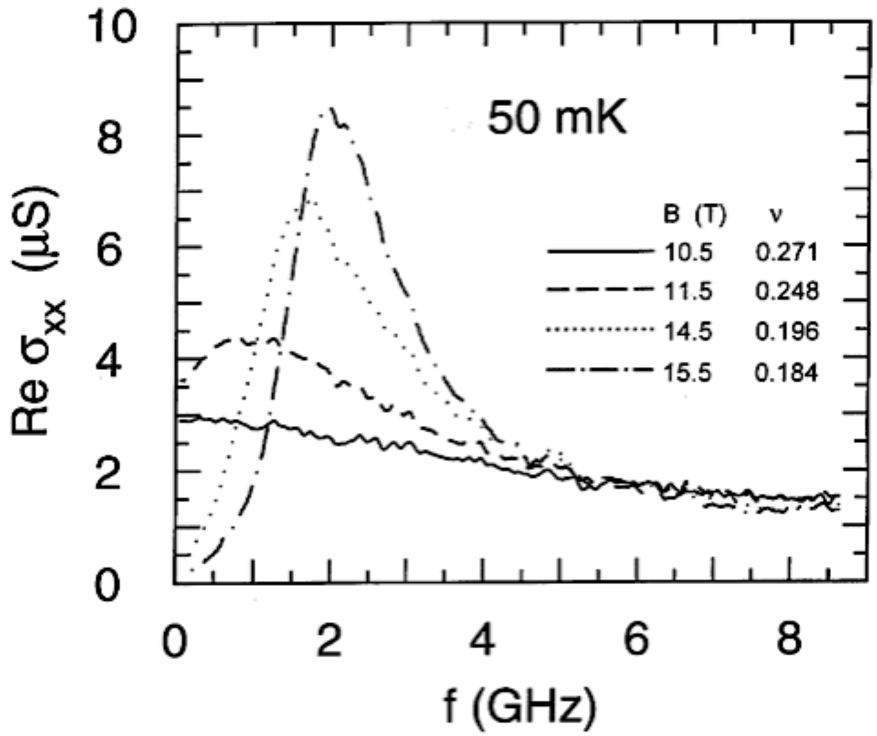
D. C. Tsui,

L. N. Pfeiffer , K. W. West

M. Shayegan

# Phenomenon of 2DES in high B :
# Insulating phase accompanied by microwave resonance





One example: insulator at termination of FQH series
Other examples to be presented…

Broadband microwave spectroscopy works where dc transport is problematic, resonance makes it a powerful tool for study of insulators

# High B ground state: Wigner crystal

- Low $B$ : Coulomb energy – kinetic energy

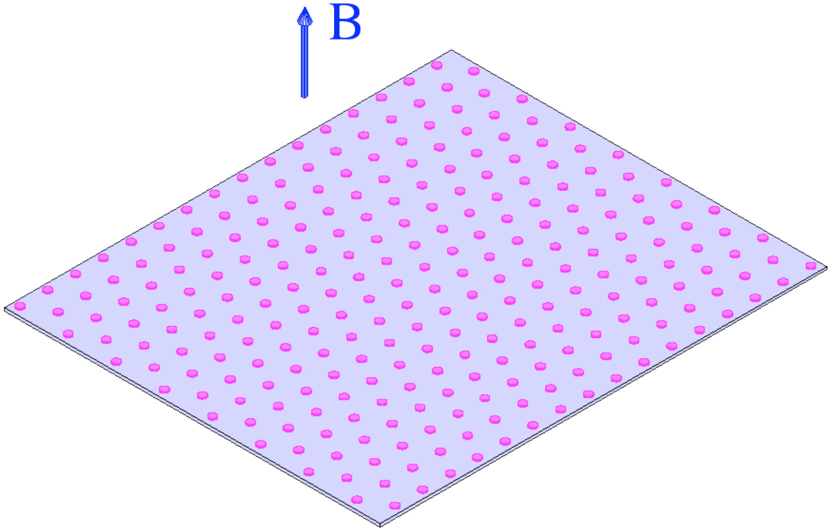
  $E_C - E_F$

- High $B$ : Kinetic energy frozen out

  Landau level filling $\nu < 1/7$ to $1/6$

  (high enough $n$ )

  Lam and Girvin; Levesque, Weis, MacDonald `84,
  Lozovik and Yudson `75; Yang and Rezayi, ‘02

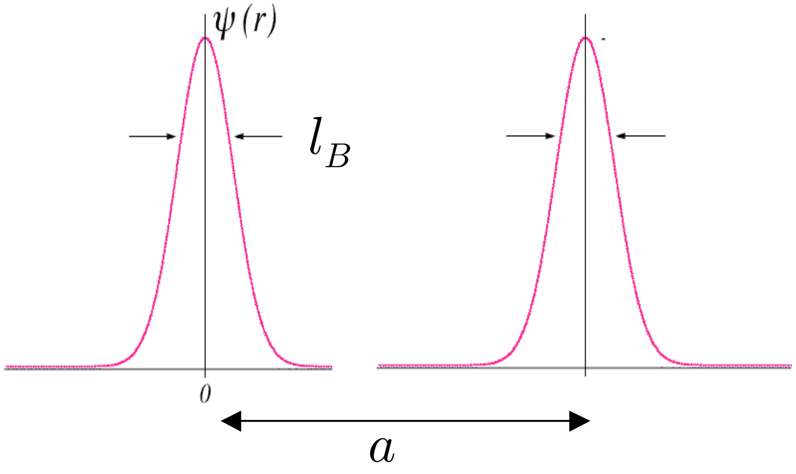


$l_B$ : size of electron wavefunction in lowest Landau level

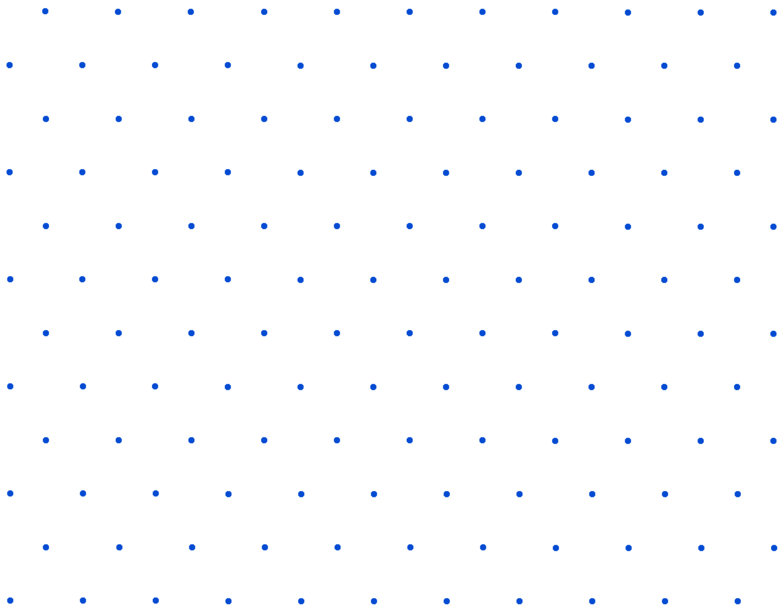
$a$ : crystal lattice constant

$$\nu = nh/eB = (l_B/a)^2 \; (4\pi/\; 3^{\;1/2}\;)$$

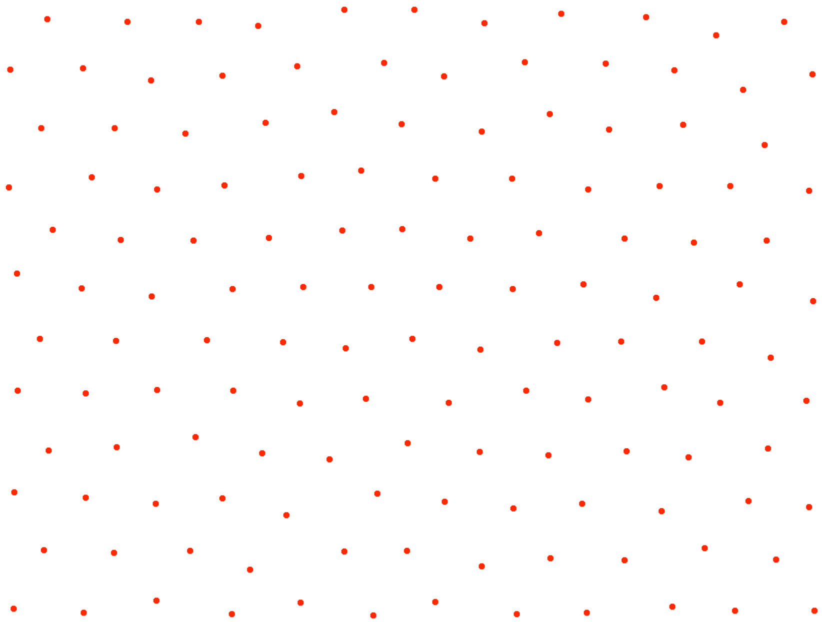
$$l_B = (\hbar/eB)^{1/2}$$



# Disorder pins Wigner crystal, makes insulator



disorder off



(small) disorder on

Equilibrium positions minimizes total energy:
(electron-electron, stiffness) + (electron-disorder, pinning)
Disorder results in finite correlation length of crystalline order, *L*

Disorder: Impurities, interface roughness, donor potential

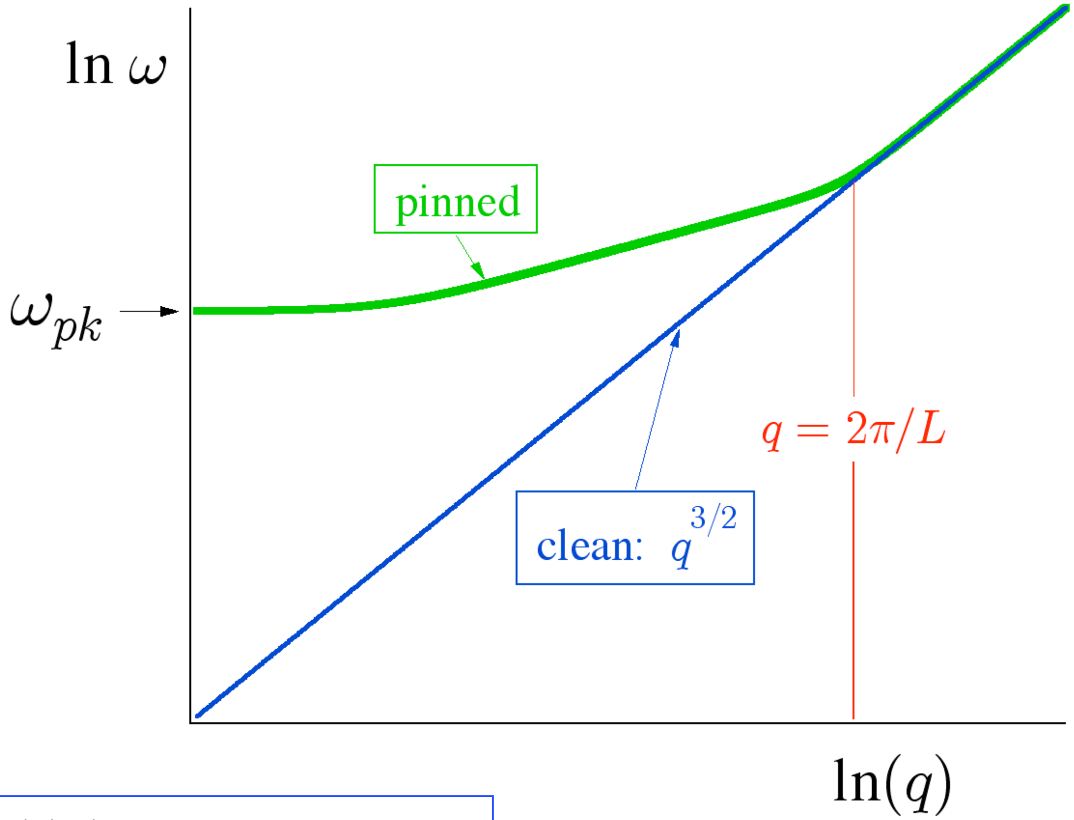
# Resonance: pinning mode interpretation

*Disorder* induces "Pinning" Mode: small oscillation about pinned positions

Disorder

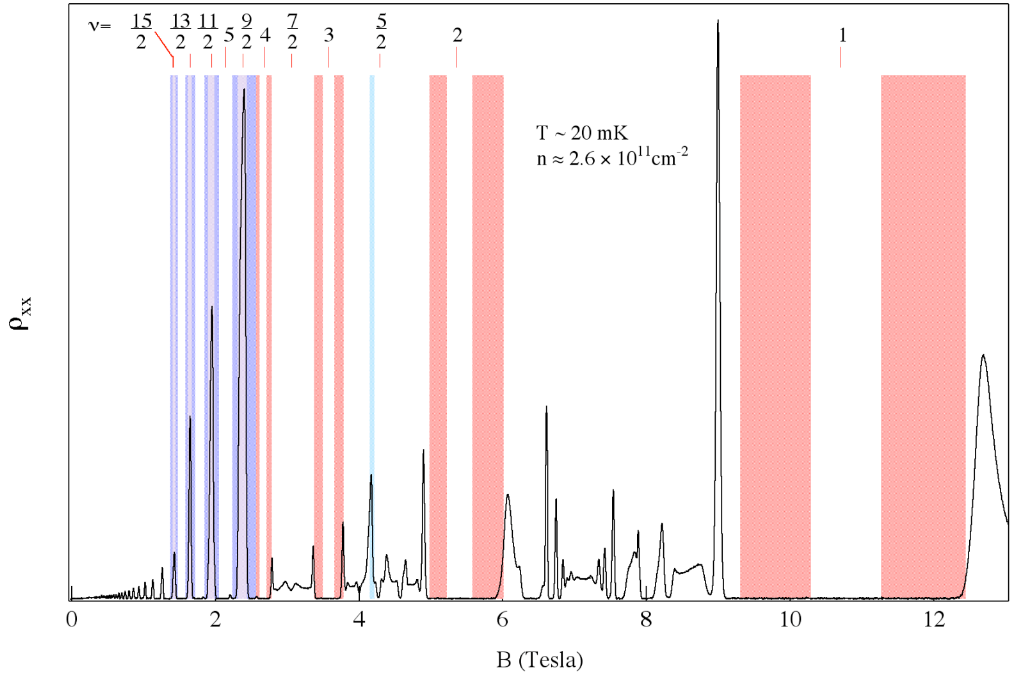
$\Rightarrow L$

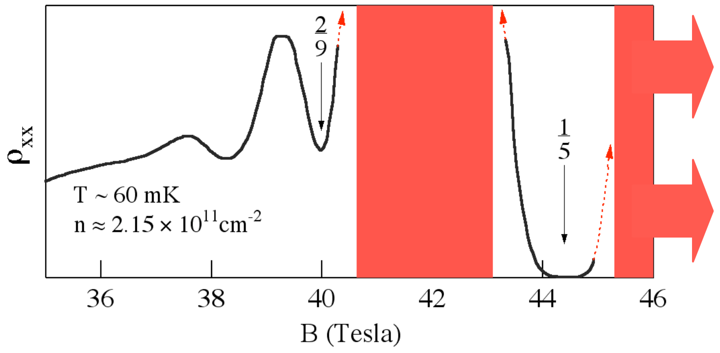
$\Rightarrow \omega_{pk} \sim \omega(q \sim 2\pi/L)$



Classical Wigner crystal in high B:
Without disorder: $\omega \sim q^{3/2}\ B$
With disorder: saturates at low $q \sim 2\pi/L$

# ν ranges of electron solids





🟥 Integer Quantum Hall Effect Wigner Crystal

🟦 Bubble phase

🟪 Stripe phase (anisotropic)

🟦 N=1 Landau level bubble phase

🟥 High B Wigner Crystal

dc transport data courtesy G. Gervais

# Outline

- Microwave measuring technique

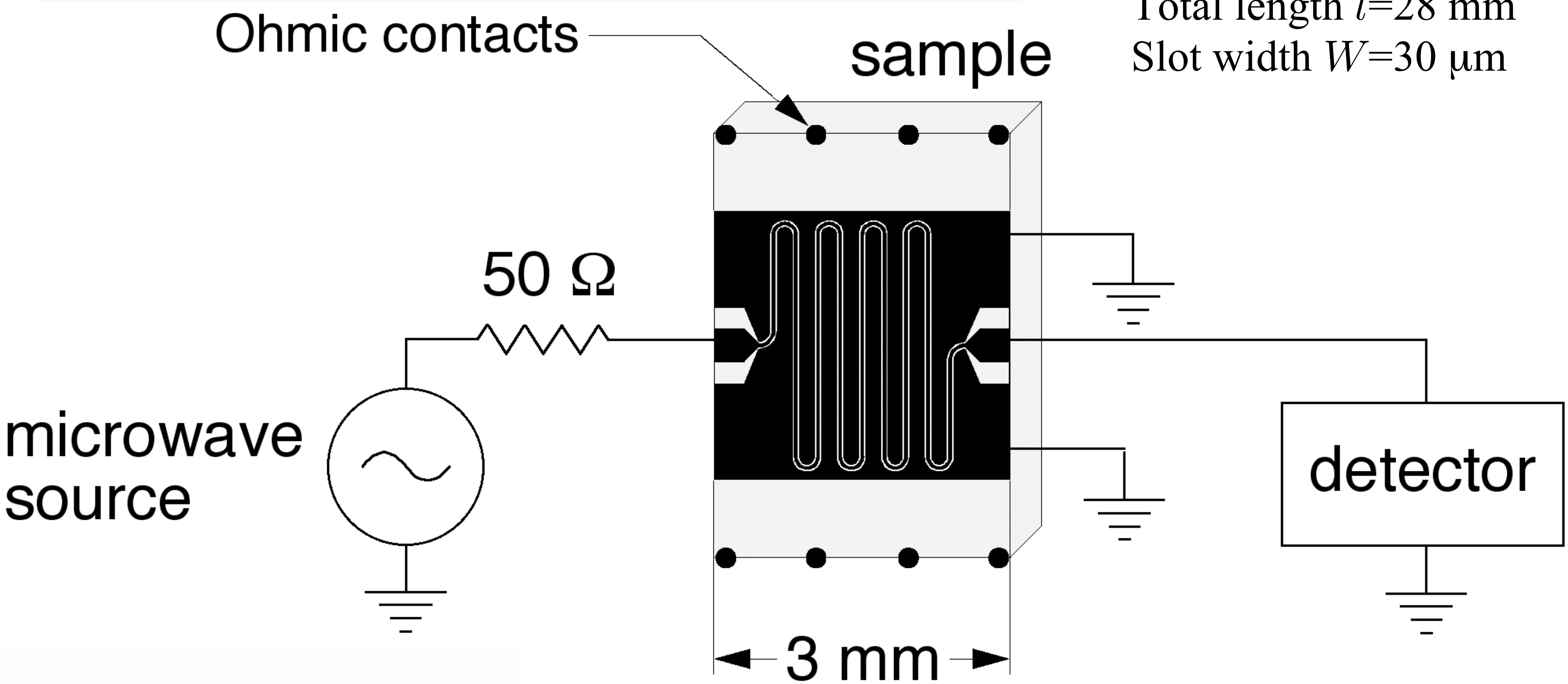
- pinning mode "basics"
  - Sample (confinement) dependence
  - Evidence of collective localization
  - Oscillator model and sum rule
  - $f_{\mathrm{pk}}$ vs $n$ , $B$
  - Correlation lengths of crystalline order

*Examples of "applications":*

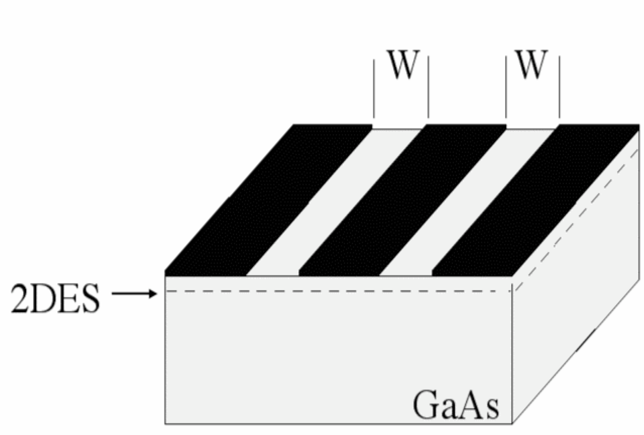
- resonance in IQHE plateau regions:
  - Wigner crystallization in IQHE + skyrmion effects

- bilayers: evolution with effective separation d/a
  - Evidence for interlayer correlation

# Microwave measuring technique

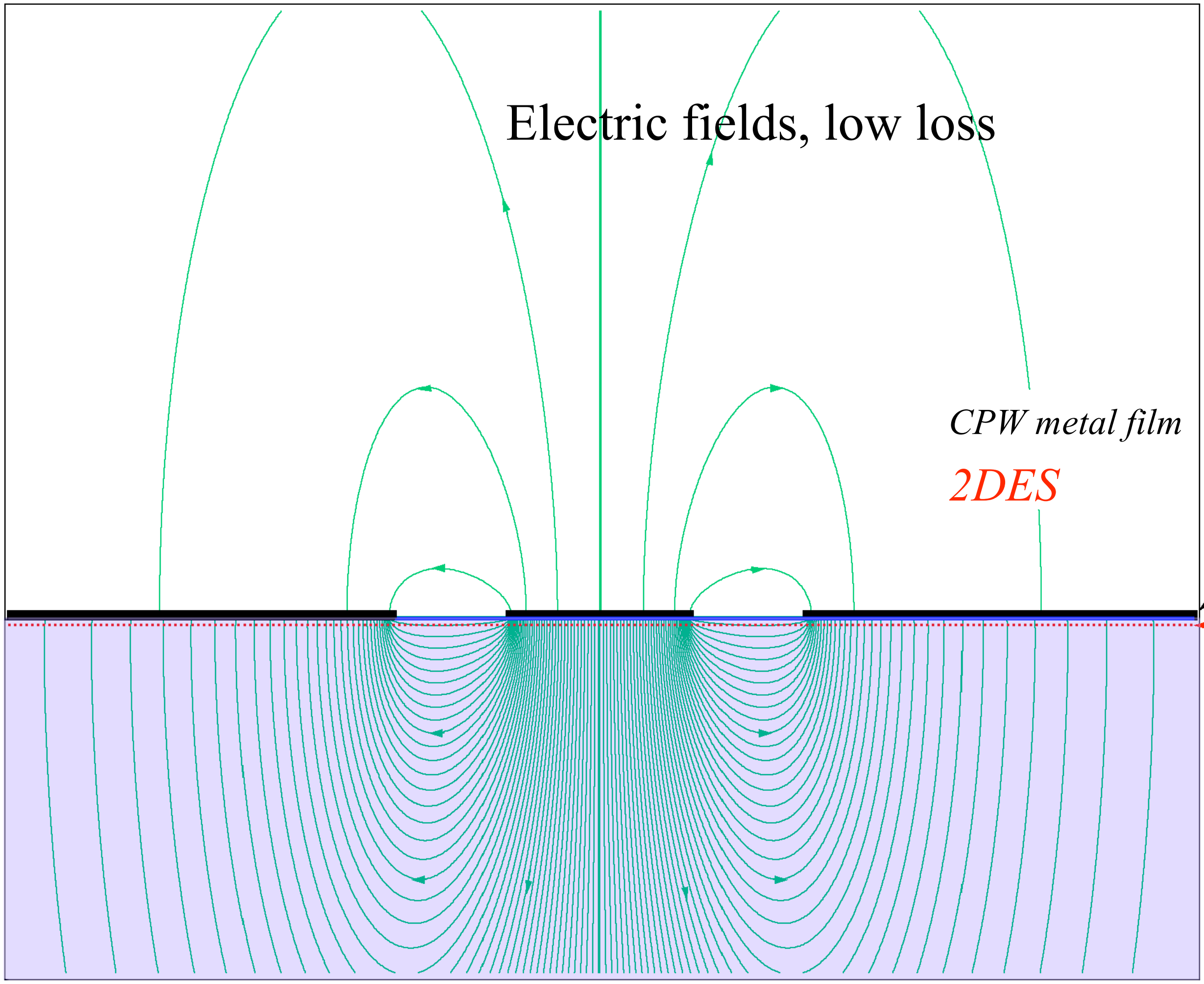


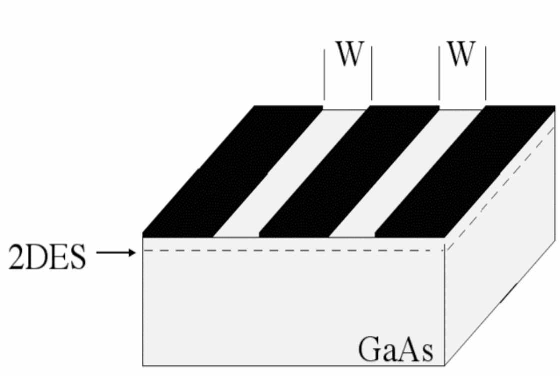
Total length $l$=28 mm
Slot width $W$=30 μm



- Metal film makes coplanar waveguide (CPW) transmission line
- Center conductor driven, side planes grounded "like coax"
- Re($\sigma_{xx}$) from 2DES effect on signal

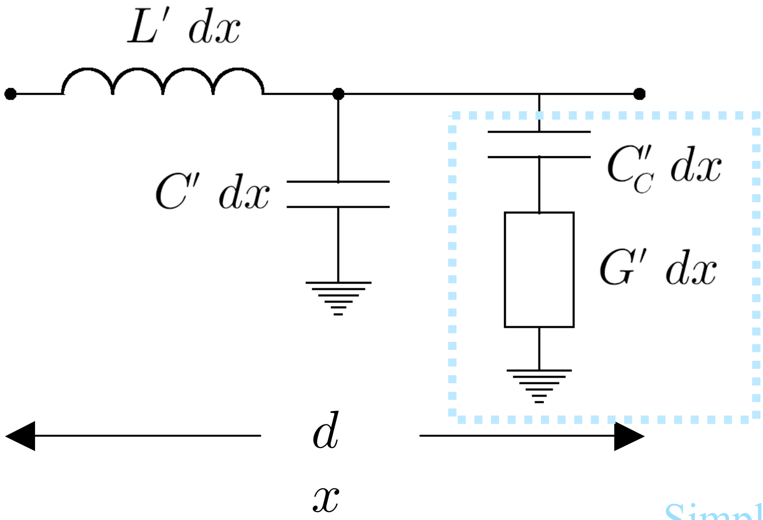
# Microwave measurement, Electric fields





- Relevant 2DES is under the slots
- CPW - 2DES coupling is capacitive

# Re($\sigma_{xx}$) from transmitted power: Circuit model



Simplified 2DES and coupling

$$G' = 2\sigma_{xx}/W$$

$$\omega C'_C \gg C'$$

$$Z_0 = \sqrt{\frac{L'}{C'}} = 50 \ \Omega$$

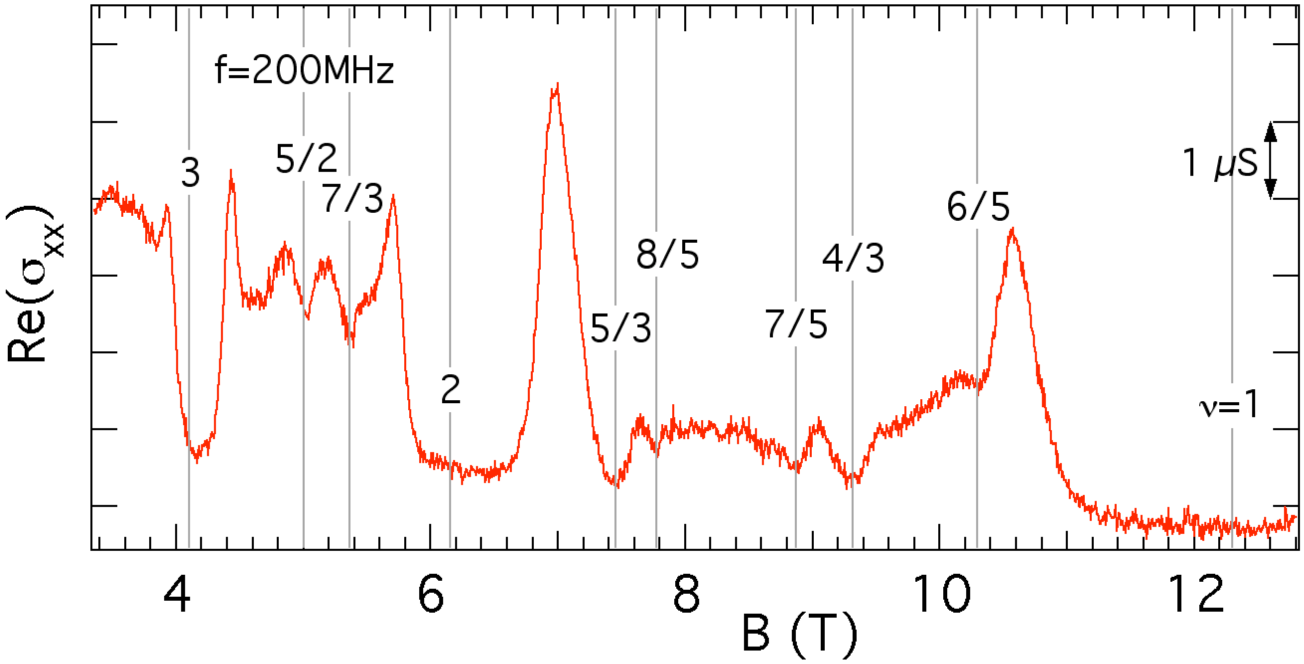
Characteristic impedance

Relative transmitted power (in low loss limit, $|\sigma_{xx}| \ll \omega C'W$):

$$P = \exp(-lZ_0 \mathrm{Re}\ \sigma_{xx}/2W)$$

- *f*-independent extinction ($l$ is length along line)
- Corrections are considered for reflections, larger loss, distributed capacitive coupling
- Assumes no $q$-dependence of $\sigma_{xx}$ accessed by transmission line

# Quantum Hall effect: finite frequency transport

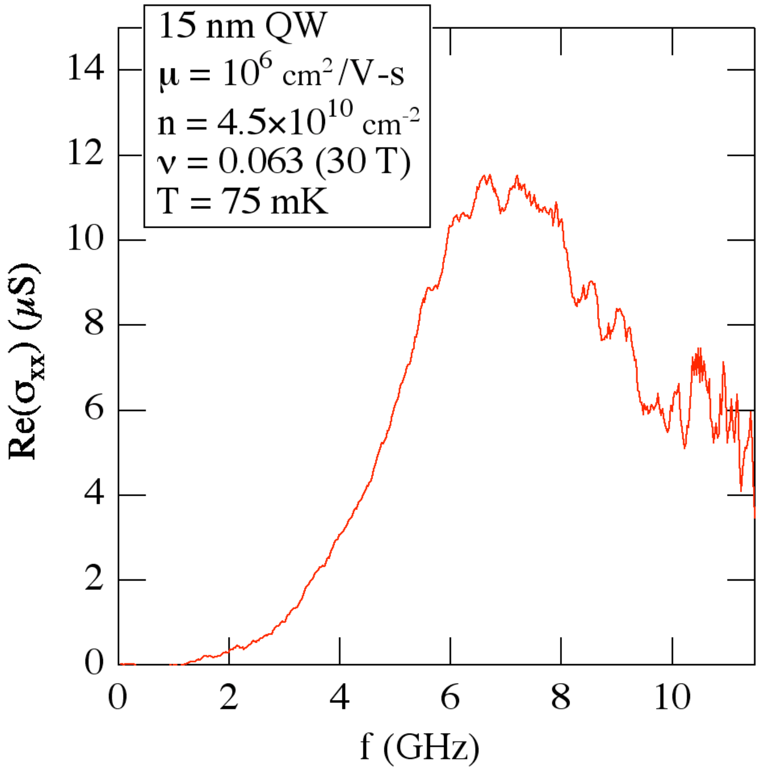


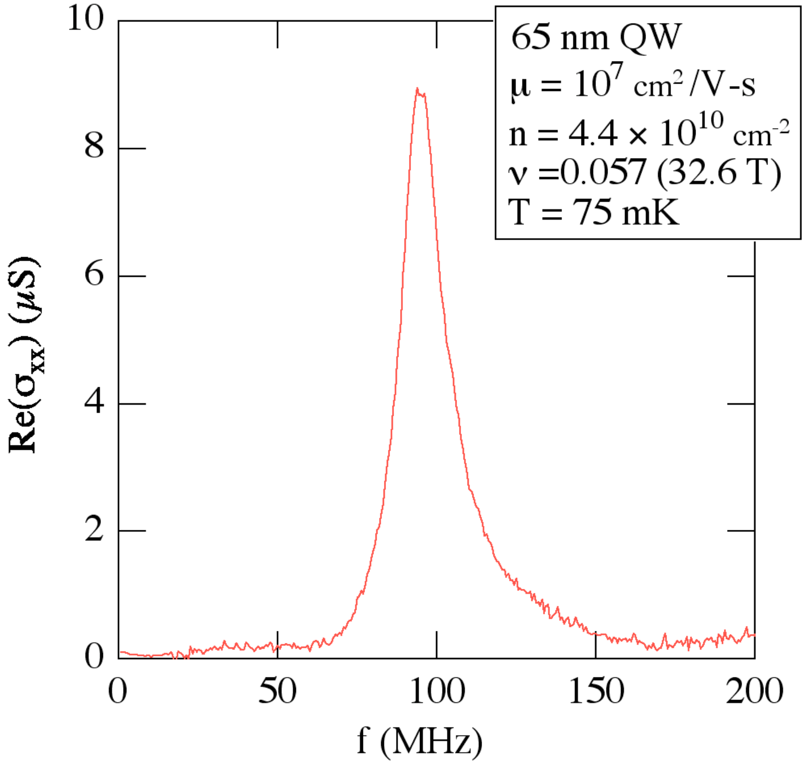
Data presented as $\mathrm{Re}(\sigma_{xx})$

$\sigma_{xx}=\rho_{yy}/(\rho_{xx}\,\rho_{yy}+\rho_{xy}{}^2)$

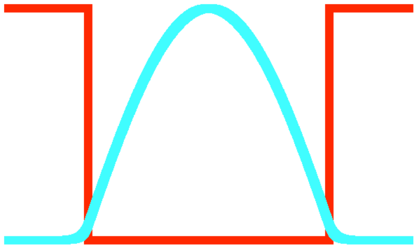
low wavevector ( *q* ) limit (most of talk)

# $f_{pk}$ is larger for larger disorder: Quantum Wells



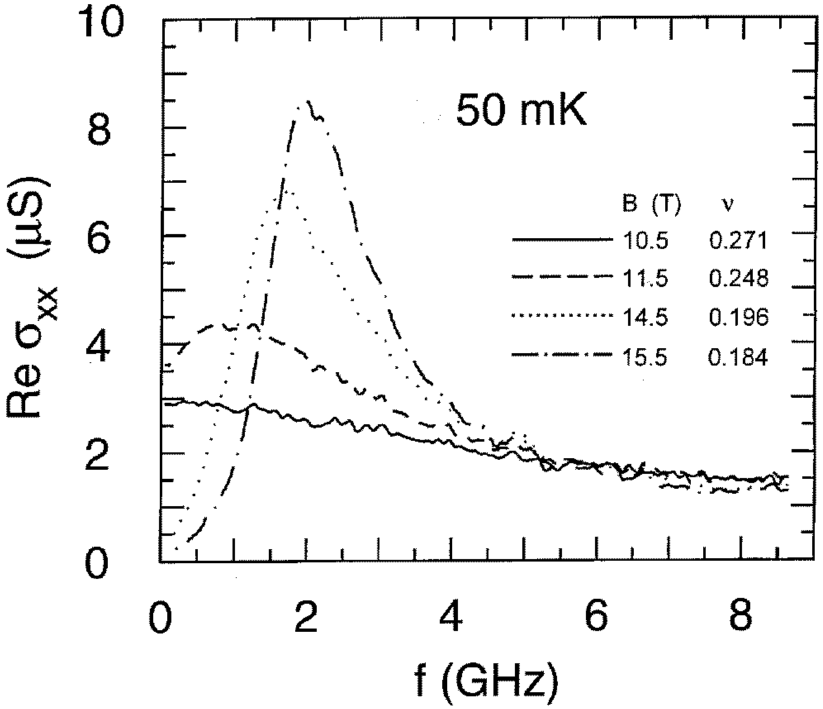


- Enormous range of $f_{pk}$
- Interface roughness (Fertig '99) as relevant disorder (vs impurities, remote ionized donor potential):
- Wide QW
  ⇒ Reduced influence of interfaces on confinement
  ⇒ reduced disorder



Quantum Well

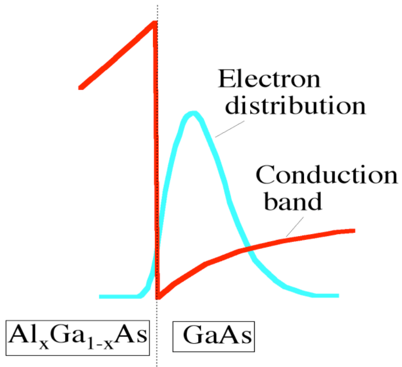
# $f_{pk}$ is larger for larger disorder: heterojunctions

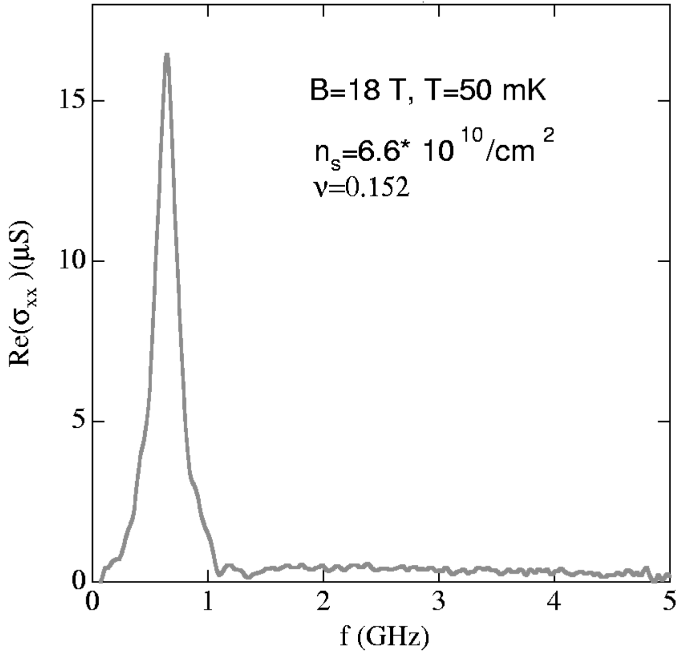


$n \approx 6.9\times10^{10}$ cm$^{-2}$

$\mu \approx 5.0 \times 10^{5}$ cm$^2$/Vs

$f_{pk} \approx$ 2 GHz, at 15.5 T





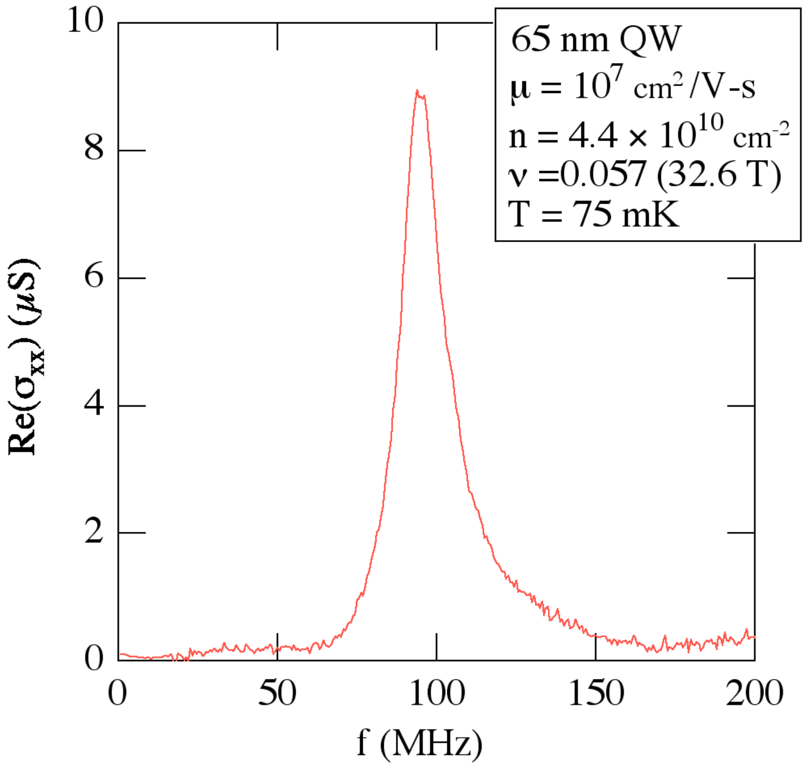
$n \approx 6.6\times10^{10}$ cm$^{-2}$

$\mu \approx 5.0 \times 10^{6}$ cm$^2$/Vs

$f_{pk} \approx$ 0.6 GHz

(sample 1)

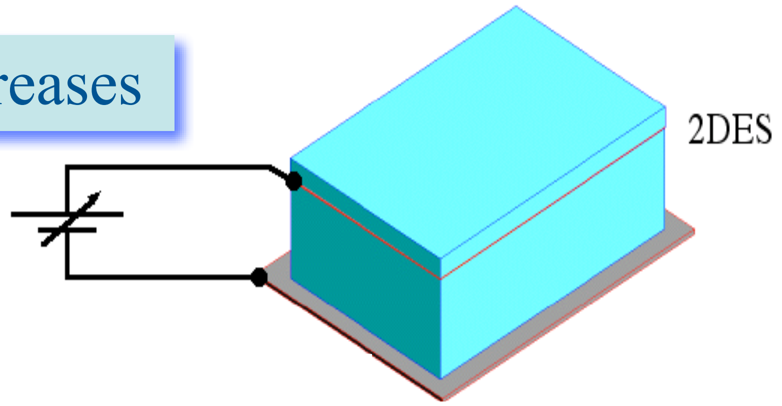
# Correlated Electron Solid

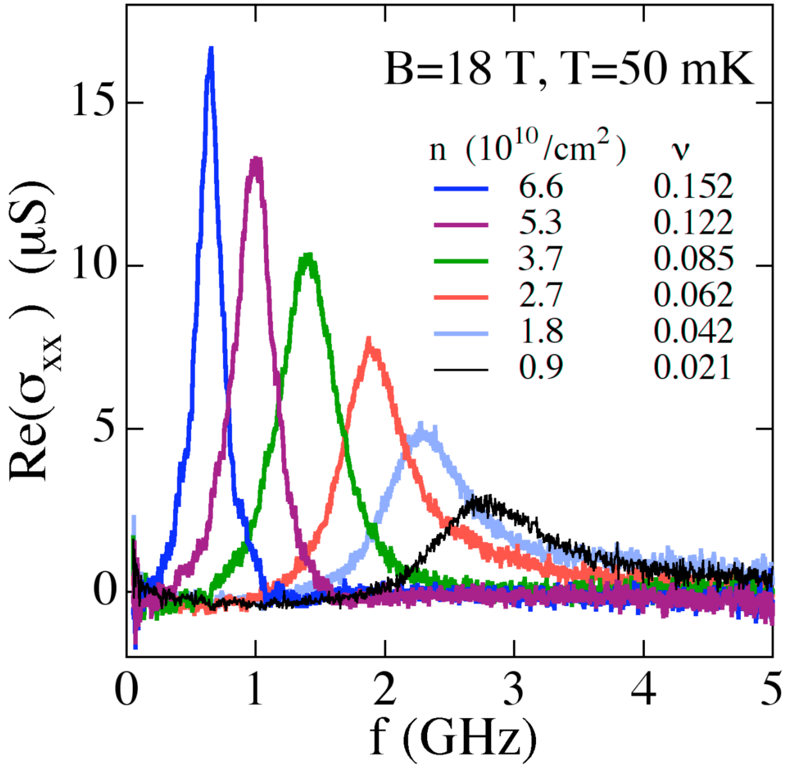


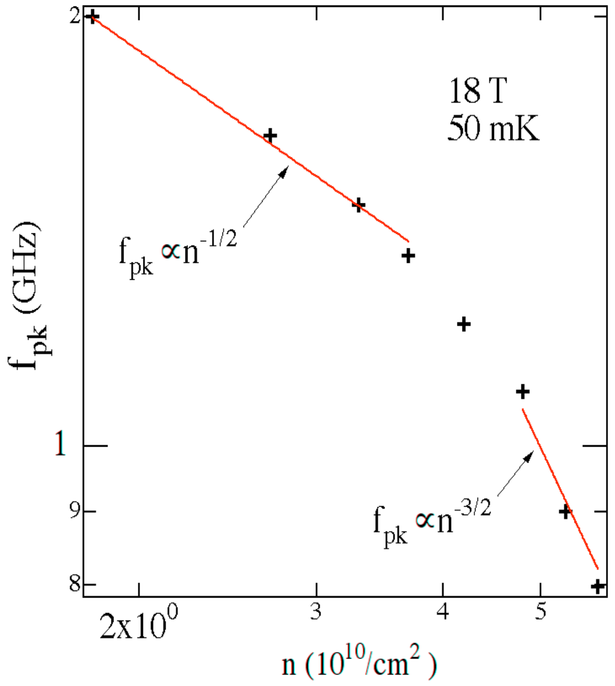
- Lowest $f_{pk}$ and $\Delta f$ 95 MHz 10 MHz
  4.5 mK 0.5 mK
- $hf_{pk}/k_B \ll T$ ⇒ pinning, collective localization, electron solid
- Rules out individual electrons in traps
- General for cleaner samples at higher $n$, includes other insulators

# Decrease *density* with backgate: $f_{pk}$ increases

Single layer 2DES sample







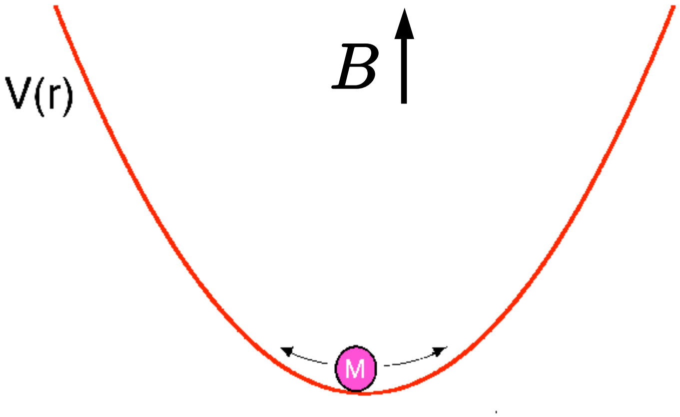
Weak pinning: minimize (impurity + deformation) energy ⇒ $L$

Reducing $n$ ⇒ weaker carrier-carrier interaction

⇒ Carriers "fall further into impurity potential"

⇒ Average pinning, so $f_{pk}$ increases

# Harmonic oscillator model of pinning mode



Charge in “pinning” potential

$$V(x,y)=\frac{M\omega_0^2(x^2+y^2)}{2}$$

“pinning” frequency $\omega_0$,
cyclotron frequency $\omega_c = eB/m^*$

Two modes: $\omega_+ \geq \omega_c$

$$\omega_- = \omega_0^2/\omega_c \;(\text{ for } \omega_0 \ll \omega_c)$$

Microwave resonance

Observed resonance frequency: $f_{pk}=\omega_0^2/2\pi\omega_c$

$\omega_-$ sum rule: $$S_- = \int_0^\infty \mathrm{Re}[\sigma_{xx}(f)]df = \frac{n_s^* e\pi f_{pk}}{2B}$$

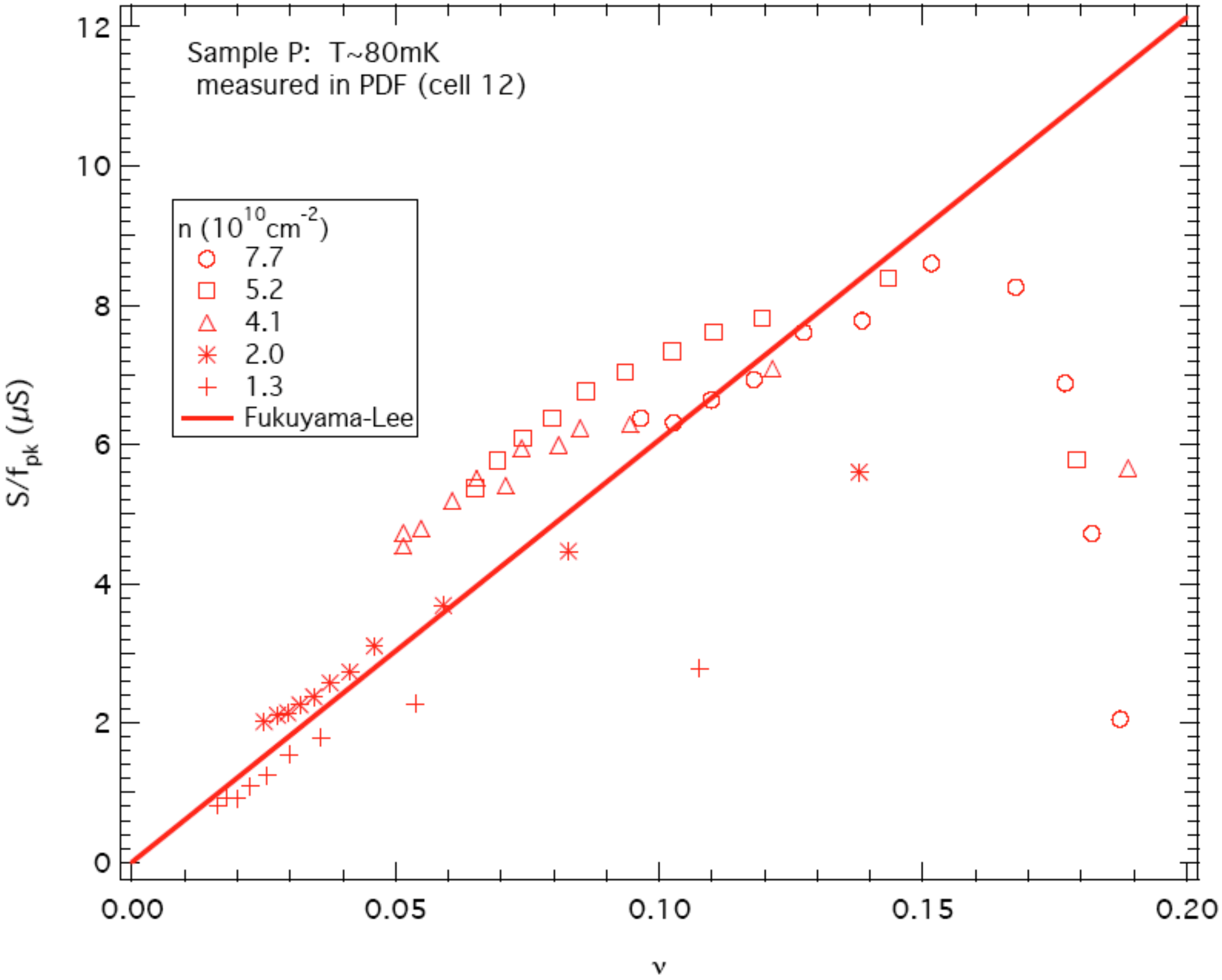
$$\boxed{\frac{S_-}{f_{pk}} = \frac{n_s^* e\pi}{2B}}$$

$n_s$* : participating
carrier density

$$S_- \propto \frac{\omega_0^2}{\omega_c^2} \qquad (S_- \ll S_+)$$

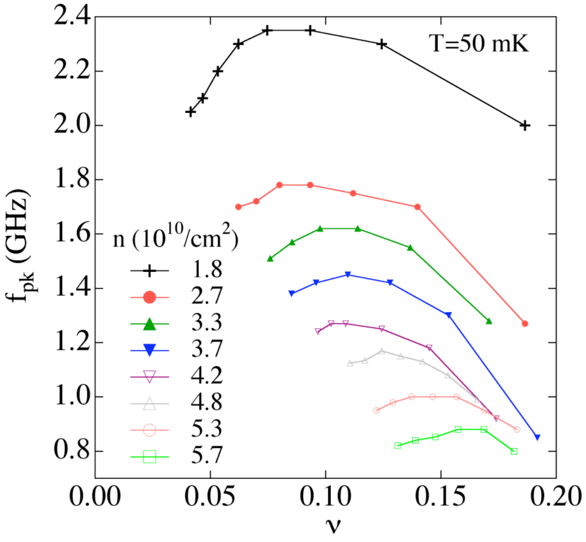
Fukuyama and Lee; Millis, Normand, Littlewood

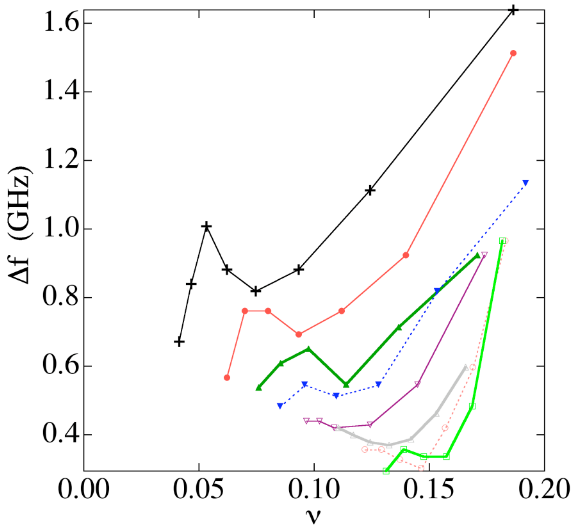
# $S/f_{pk}$ vs $n$: linear, slope close to oscillator model value



Full participation: not isolated traps

# $f_{pk,}$ $\Delta f$ vs $\nu$





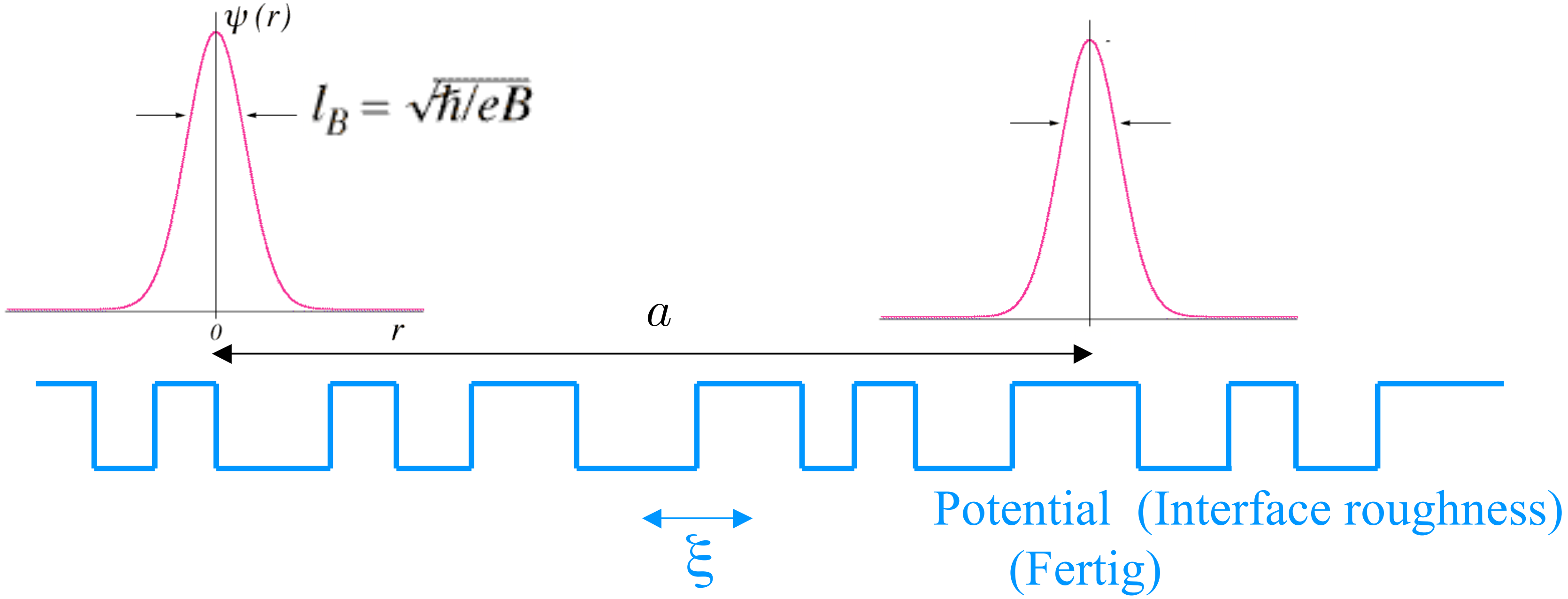
- Maxima in $f_{pk}$ vs $\nu$
- Oscillator model: $f_{pk} \propto \nu$ (for fixed $n$)--not achieved
- Existing theories for weak pinning limit,

  based on interplay of $l_B$ and disorder lengths:

  $f_{pk} \propto 1/\nu$ (Fertig '99)

  or $1/\nu^2$ (Chitra'98, Fogler & Huse '01)
- $\Delta f$ : complicated…

# B dependence of $f_{pk}$, discussion



$l_B$ : size of electron wavefunction in lowest Landau level

$a$ : lattice constant

$\xi$ : disorder correlation length(s)

B dependence of $f_{pk}$ from electron-impurity interaction:
interplay between $l_B$ and $\xi$

# How can a pinning mode be so sharp?

domain size (coherence length) $L$

Possibilities:

1. Dilute identical impurities as oscillators:
   but oscillator strength too large

2. Motion coherent over large length scale
   $L_B - L$, averaging disorder
   Due e-e interaction + high B

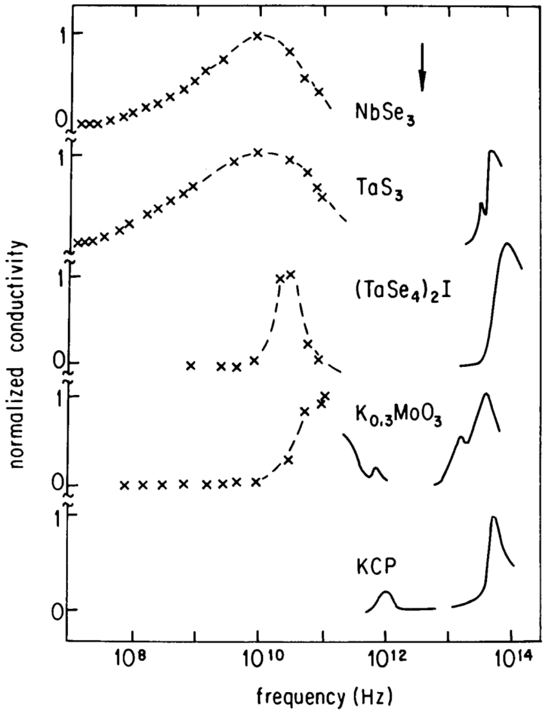


**Figure 9.4.** Frequency dependent conductivity measured in several compounds in their charge density wave state (Grüner, 1988). The arrow indicates the gap measured by tunneling for NbSe$_3$.

Theories: Fertig, Fogler and Huse, Chitra and Giamarchi

# Correlation lengths of Wigner crystalline order (from $f_{pk}$)

$L$ is *Larkin* length over which (deviations from lattice) ~ (disorder correlation length $\xi$)
*or* (deformation energy) ~ (pinning energy)

$L_a$ is length over which (deviations from lattice) ~ ( lattice constant $a$)

For interface roughness, $\xi \ll a$

$L_a/L \sim (a/\xi)^{\beta}$, $\beta \sim 3$ (Fogler and Huse, '00)

Find $L$ from
$f_{pk}$ and elastic properties of classical WC:

$$\mu_{t,cl} = \frac{0.245 e^2 n^{3/2}}{4\pi\epsilon_0\epsilon}; \quad \epsilon = 12.8$$

$$\omega_0 = \sqrt{2\pi f_{pk}\omega_c}$$

$$= \frac{2\pi c_t}{L}$$

$c_t = (\mu_t/nm^*)^{1/2}$, where
$c_t$ is $B = 0$ transverse phonon
$\mu_{t,cl}$ is shear modulus (Bonsall and Maradudin '77)

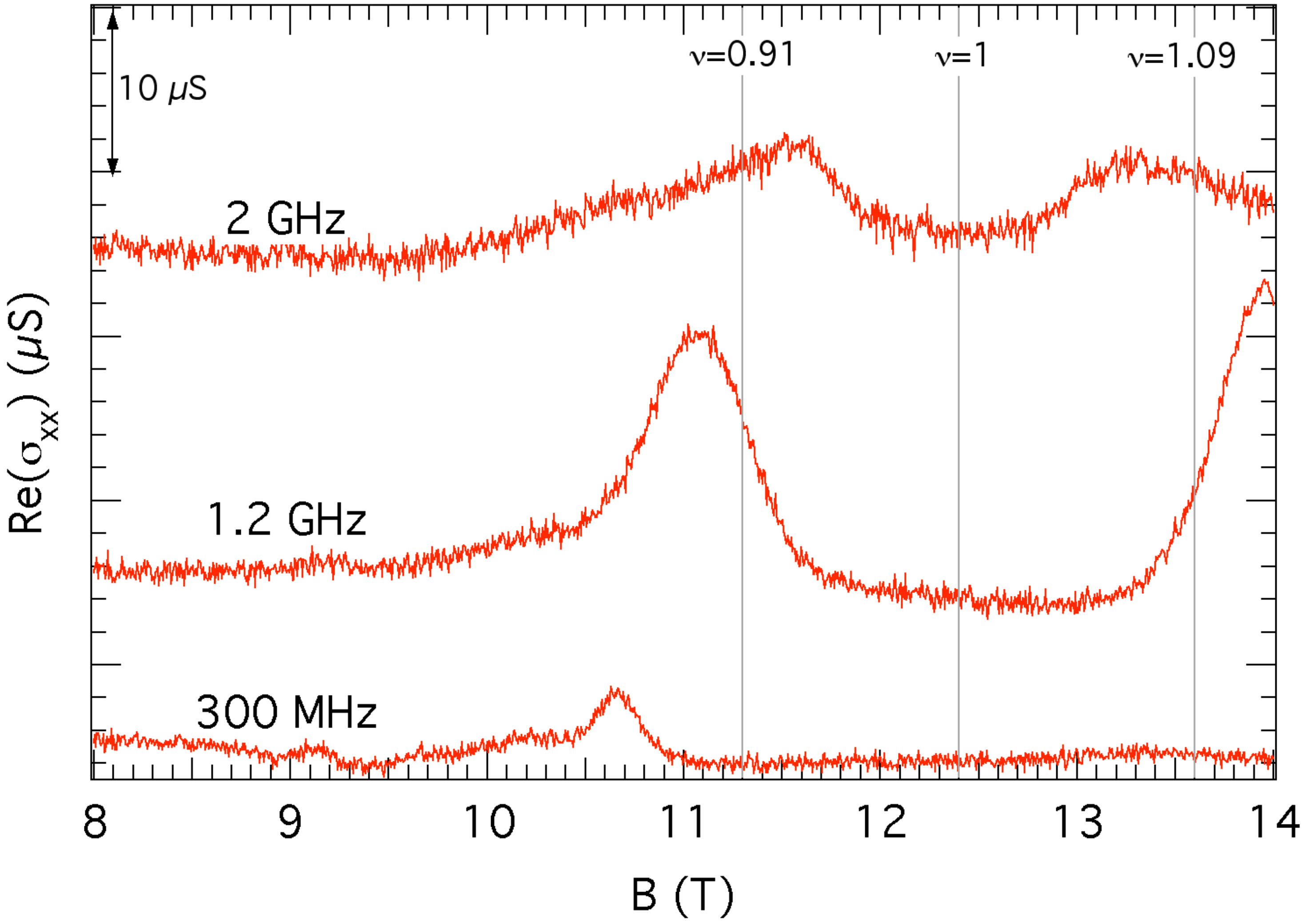
$$L = \left(\frac{2\pi\mu_{t,cl}}{nm^* f_{pk}\omega_c}\right)^{1/2} = \left(\frac{2\pi\mu_{t,cl}}{neB f_{pk}}\right)^{1/2}$$

For $n \approx 4.5 \times 10^{10}/\mathrm{cm}^2$, $a \approx 0.051$ μm:

| $f_{pk}$ | $L/a$ |
|---|---|
| 95 MHz | 21 |
| 7 GHz | 2.7 |

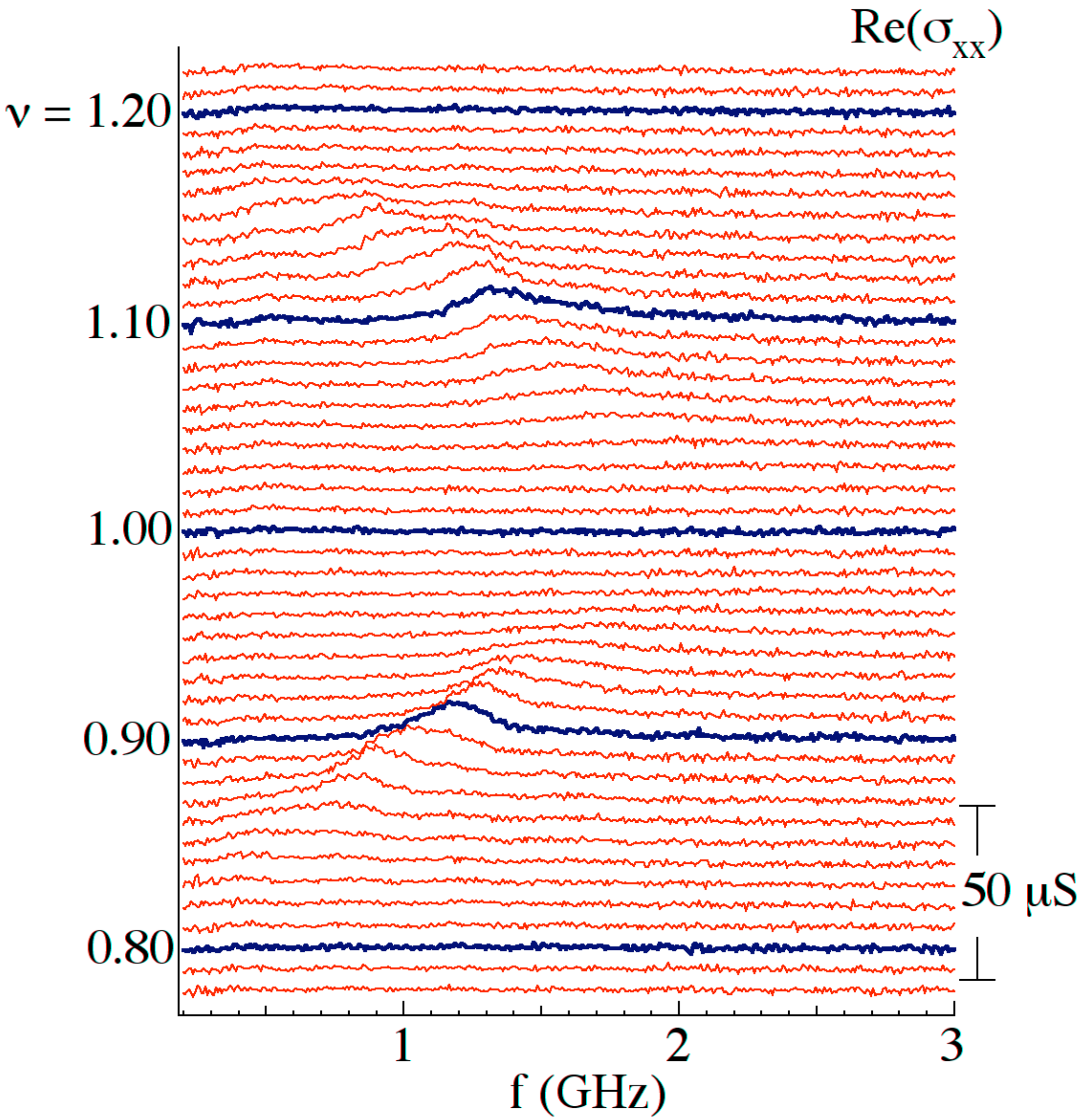
Crystal may be quite ordered, $L_a \gg a$, even when $L \sim a$

# Higher ν: evidence for Wigner crystallization in IQHE



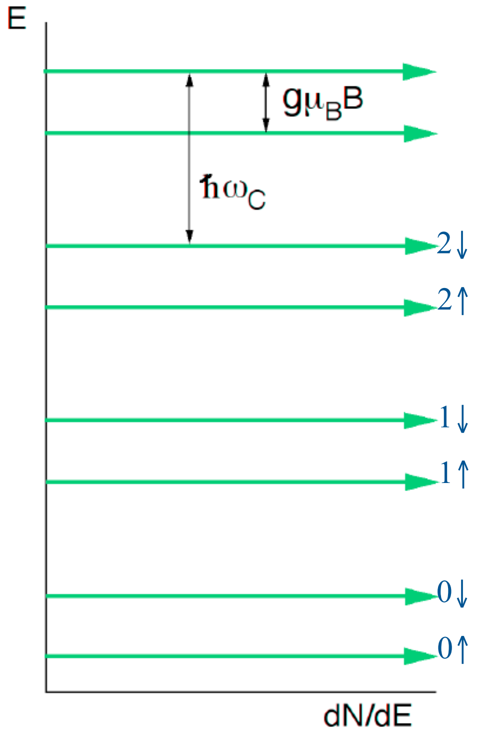
Narrowing of IQHE minimum with $f$, $\mathrm{Re}[\sigma_{xx}(f)]$ not monotonic

# Resonances on either side of ν=1, vanishing just on ν=1



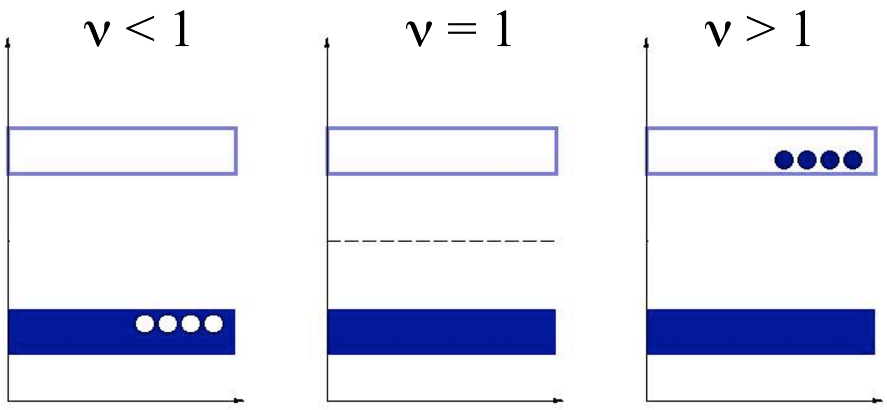
$f_{pk}$ increases as ν→1

# Interpretation: "Integer Quantum Hall Wigner Crystal" (IQHWC)



… made up of top Landau level (LL) electrons or holes

IQHE when ( top LL contribution to dc $\sigma_{xx}$) → 0



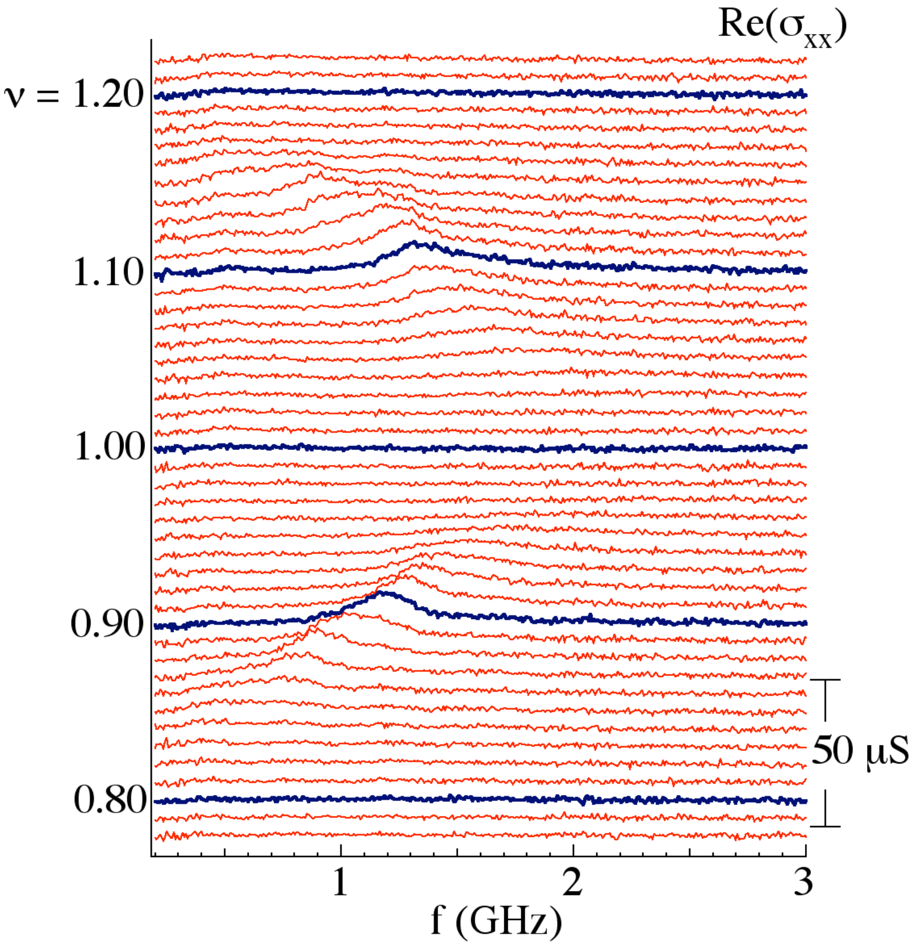
$\nu = K + \nu^*$, $K$ an integer

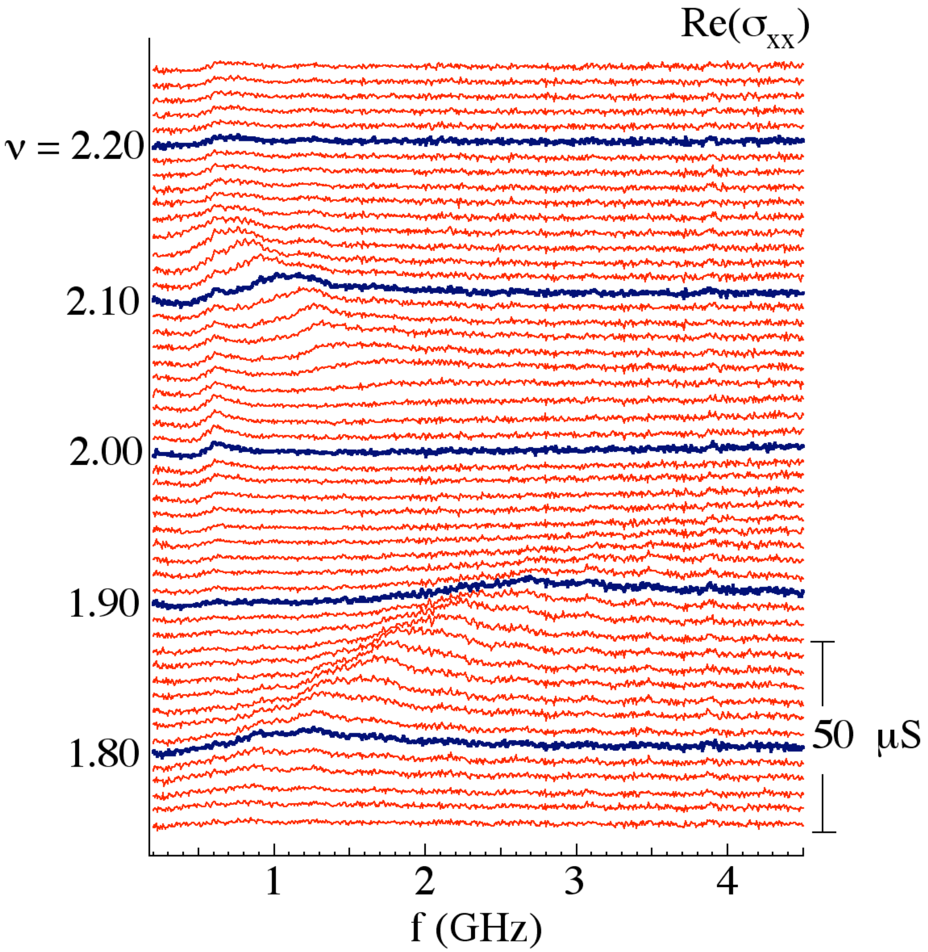
Density of top LL $\begin{matrix}\text{electrons}\\ \text{holes}\end{matrix} = \pm n^* = \nu^* n / \nu - \nu^* n / K$

$n^* \to 0$ for exact integer ν, T=0

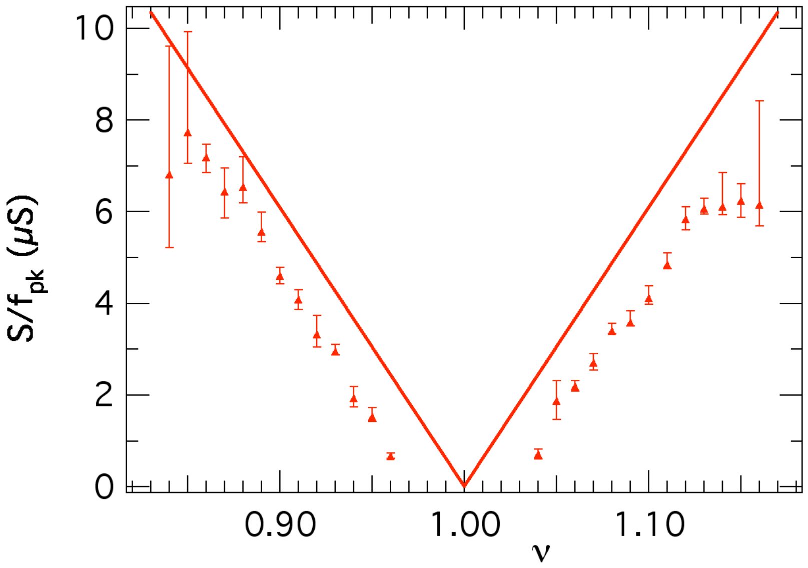
Interpret analogous to high B lowest LL except varying $B$ varies $n^*$

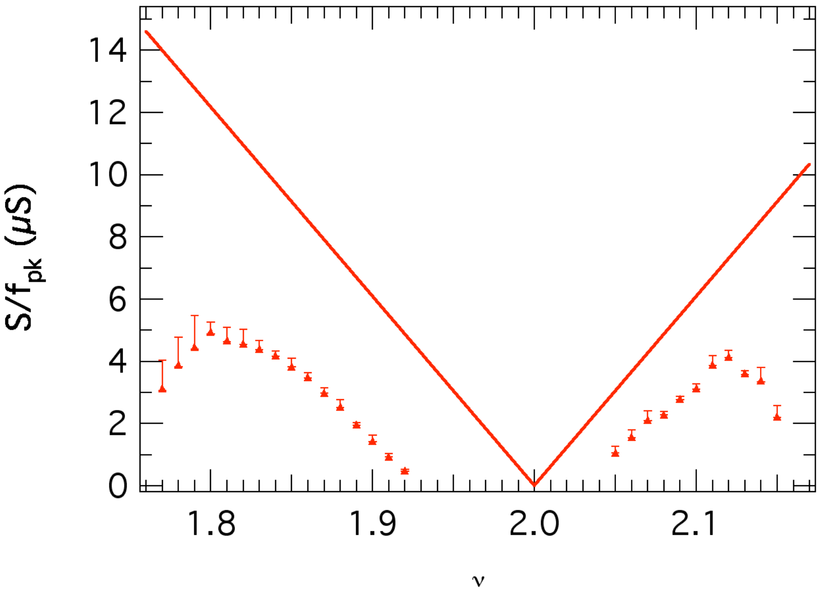
# Resonance around ν=1 and ν=2





# “IQHE-WC” resonance oscillator strengths around ν=1 and 2

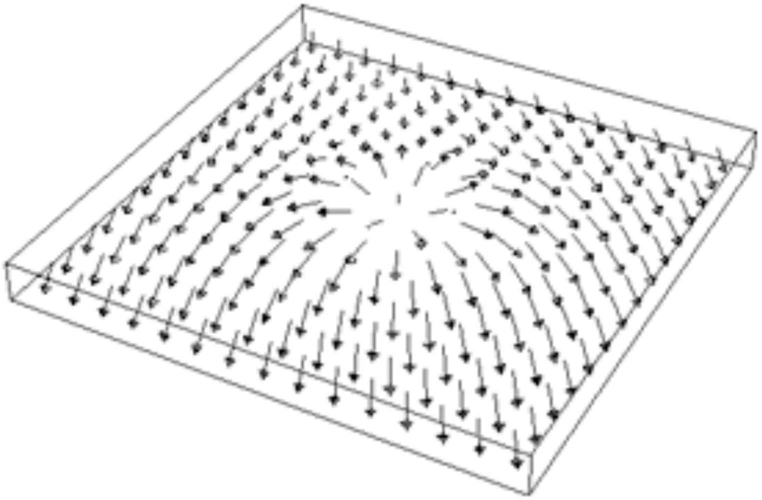




$S/f_{pk} \propto$ participating density

# Skyrmions: charged excitations near $\nu = 1$

Problem:
resonances around $\nu = 1$ and 2
(and 3 and 4) are similar, but the
particles near $\nu$=1 are
expected to be skyrmions.



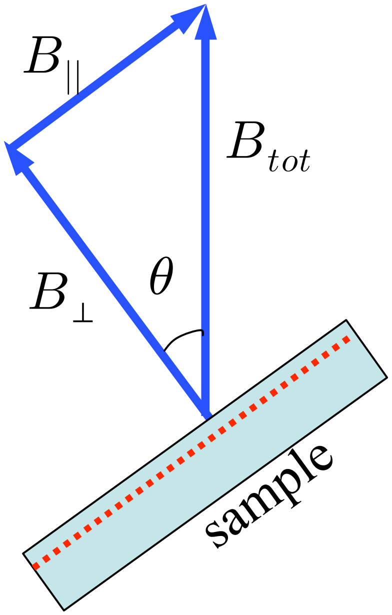
- Exchange energy gained at expense of Zeeman energy
- $\tilde{g} = E_Z/E_C = g\mu_B B/(e^2/4\pi\varepsilon\varepsilon_0 l_B) \approx .012$ for $n$=$10^{11}$ cm$^{-2}$ in perp. B
- Larger $\tilde{g}$ decreases spin, "Size of skyrmion", $S$
- $S$ decreases to $1$ for $\tilde{g} > 0.054$ (Sondhi et al 93)

Experimental evidence:
Barrett et al 95 (Knight shift) ; Bayot et al. 96, heat capacity
Schmeller et al 95 (T dependent transport); Aifer, Goldberg,Broido (optics)

# Tilted field

For given $\nu$, $l_B$  $(B_\perp)$
Vary Zeeman energy $(B_{tot})$

$$\tilde{g} = E_Z/E_C = g\mu_B B/(e^2/4\pi\varepsilon\varepsilon_0 l_B)$$



Also affects vertical wavefunction

Schmeller et al 95  (T dependent transport)

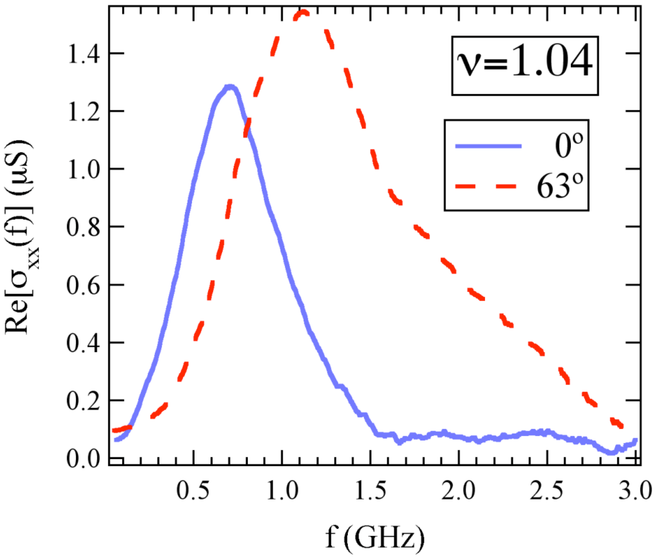
# Compare near $\nu = 1$ and 2

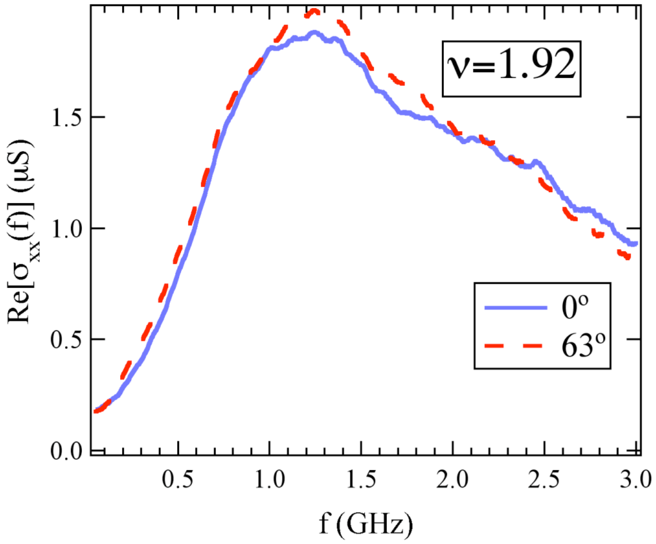
$n$=1.15 ×$10^{11}$ cm$^{-2}$, $\mu$~$10^7$ cm$^2$/Vs 50nm QW

B=4.6 T for $\nu = 1$, $\theta = 0^\circ$

$n^*$=0.46×$10^{10}$ cm$^{-2}$

$f_{\mathrm{pk}}$ (GHz)





$\theta = 0^\circ$ : $\tilde{g}$= 0.013

$\theta = 63^\circ$: $\tilde{g}$= 0.028

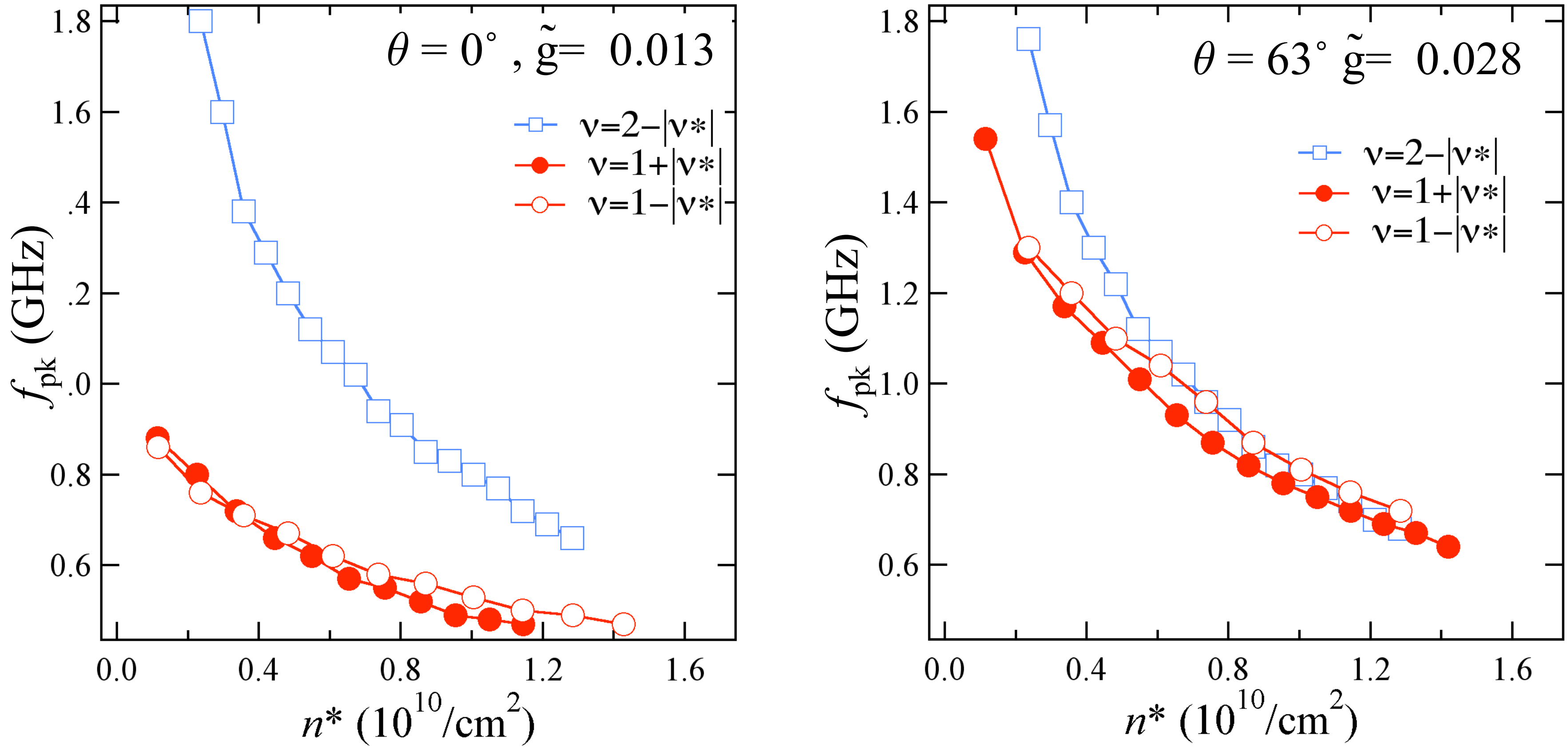
Same n*, same Landau level index, N=0

Absence of any effect at $\nu$=1.92 indicates little effect of confinement due to $B_{||}$

# $f_{pk}$ vs $n^*$

$n$=1.15 $\times 10^{11}$ cm$^{-2}$, $\mu \sim 10^7$ cm$^2$/Vs

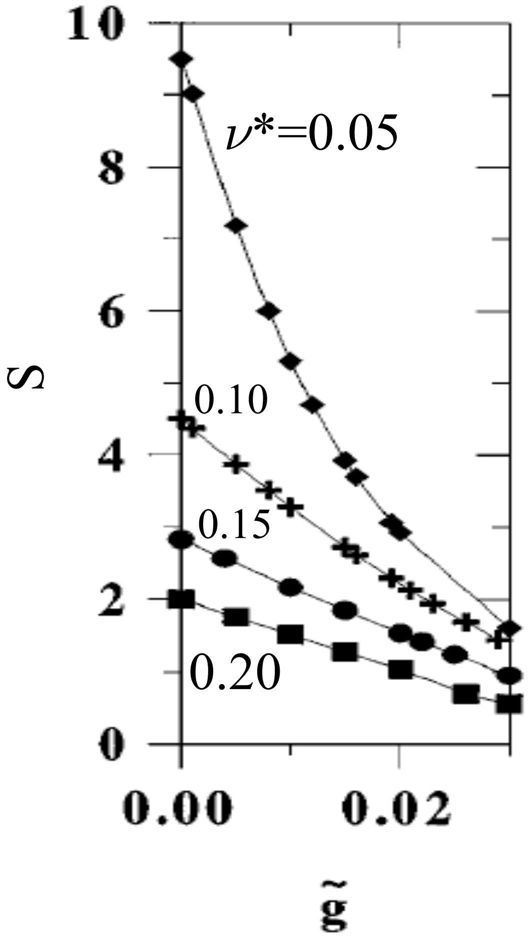
B=4.6 T for $\nu = 1$, $\theta = 0^\circ$

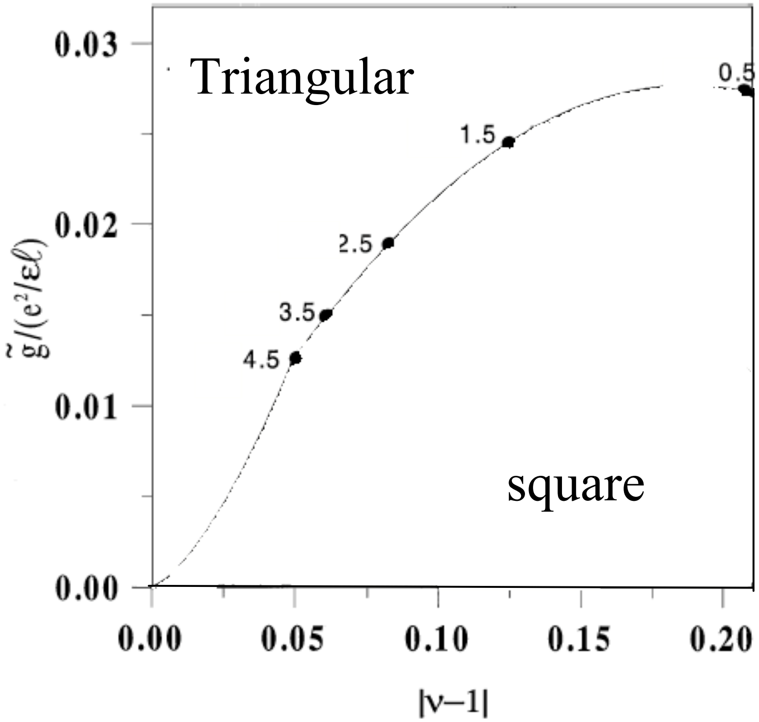


Curves for 1 ± $\nu^*$ approach those for 2 - $\nu^*$ as $\theta$ increases, so they lie together at largest n*

# Skyrmion crystals

Côté, MacDonald,Brey,Fertig,Girvin,Stoof, PRL '97



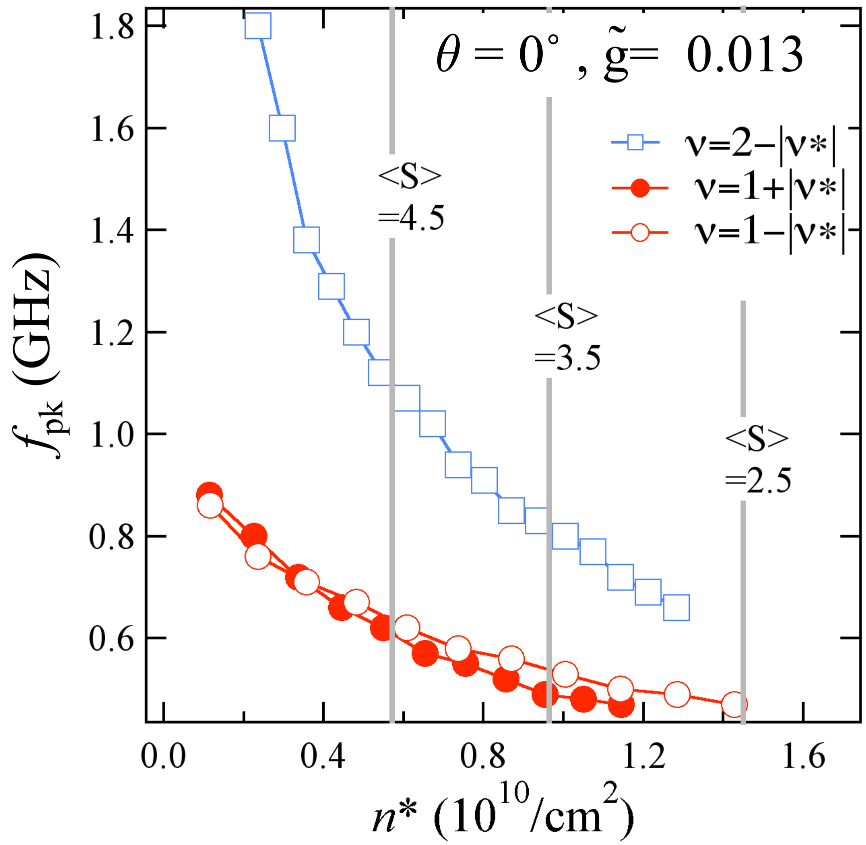


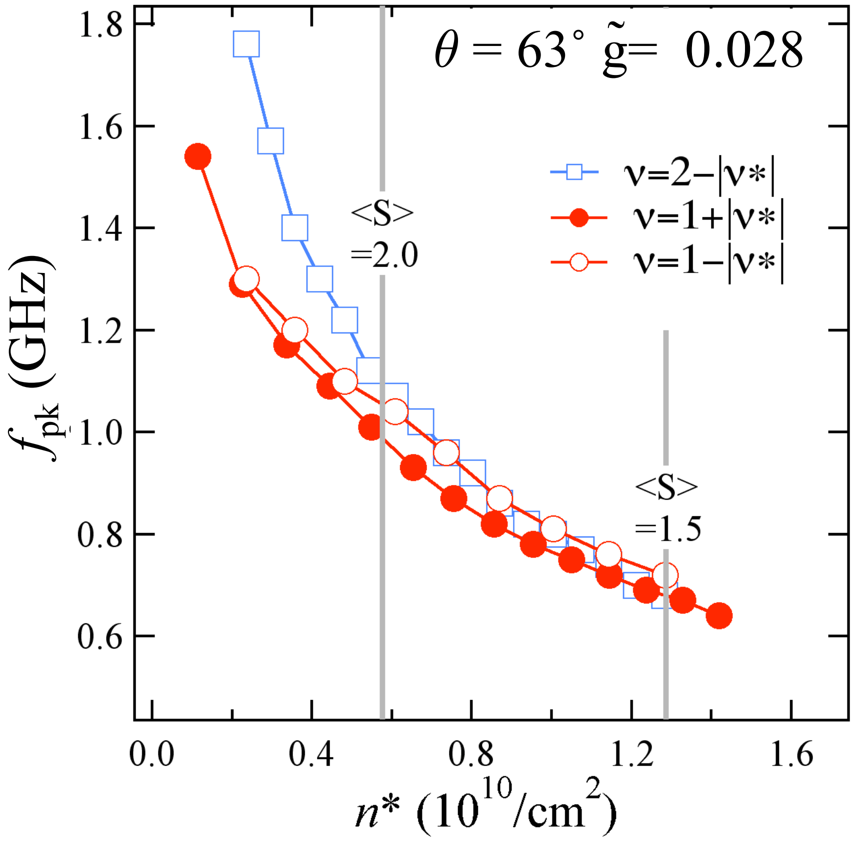
1) Larger skyrmions are less favored when crystal is denser
2) Square to triangular transition

# $f_{pk}$ vs $n^*$

$n$=1.15 ×$10^{11}$ cm$^{-2}$, $\mu$~$10^7$ cm$^2$/Vs

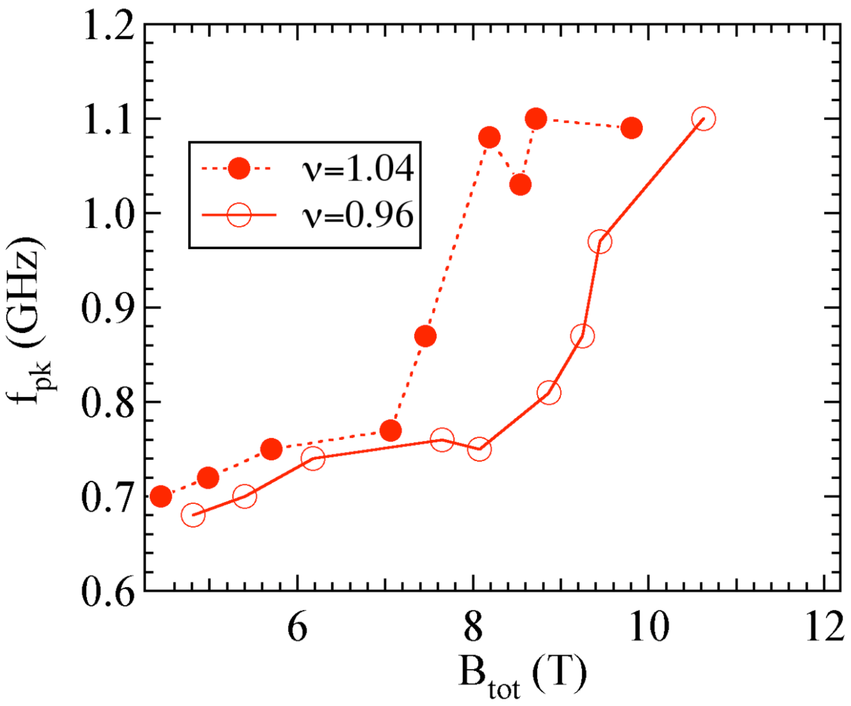
B=4.6 T for $\nu$=1, $\theta = 0^\circ$





$f_{pk}$ difference visible for predicted S>~2

# Particle vs hole: different threshold $B_{tot}$



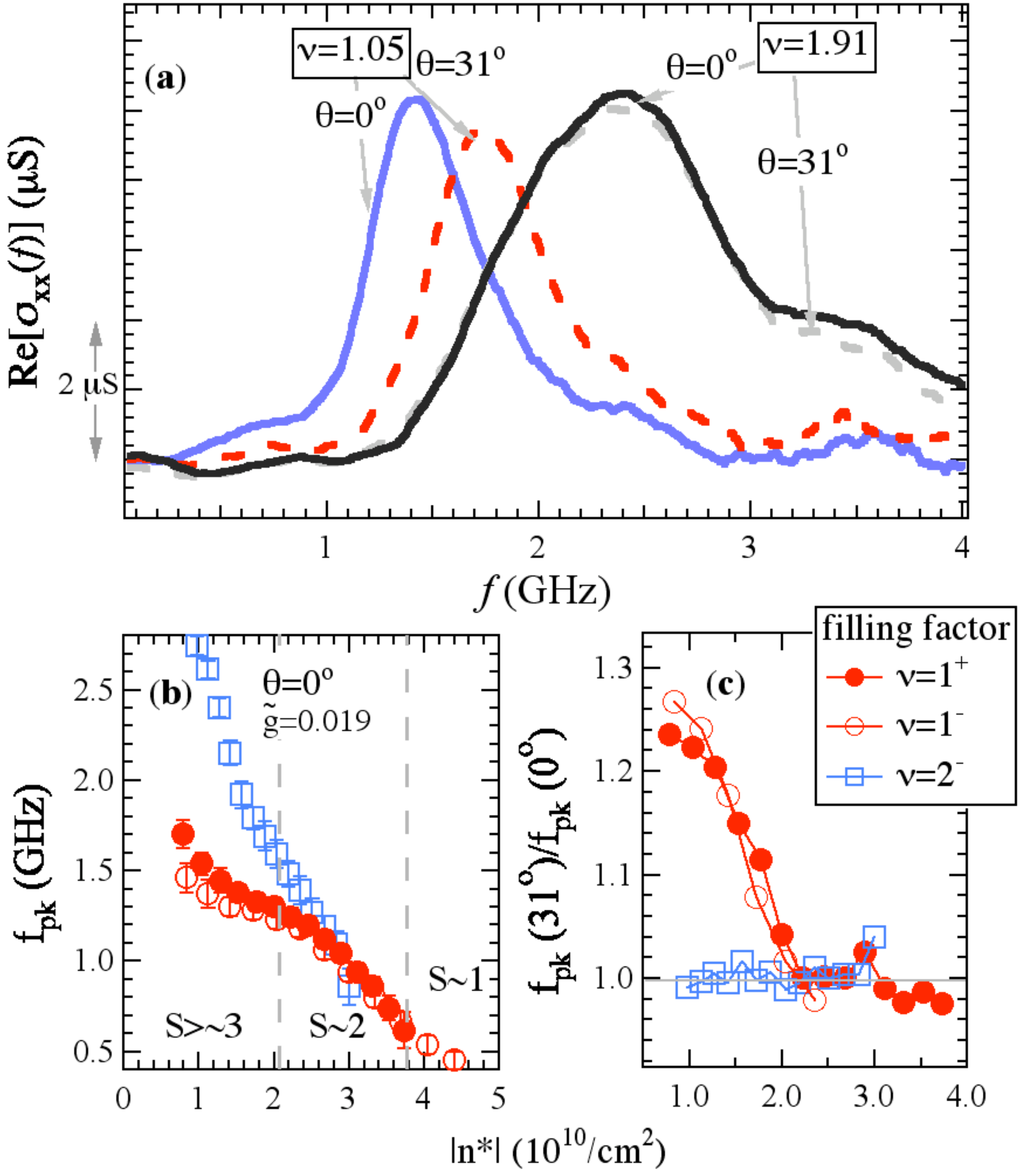
Different charge distribution in skyrmion vs antiskyrmion?

# High n sample

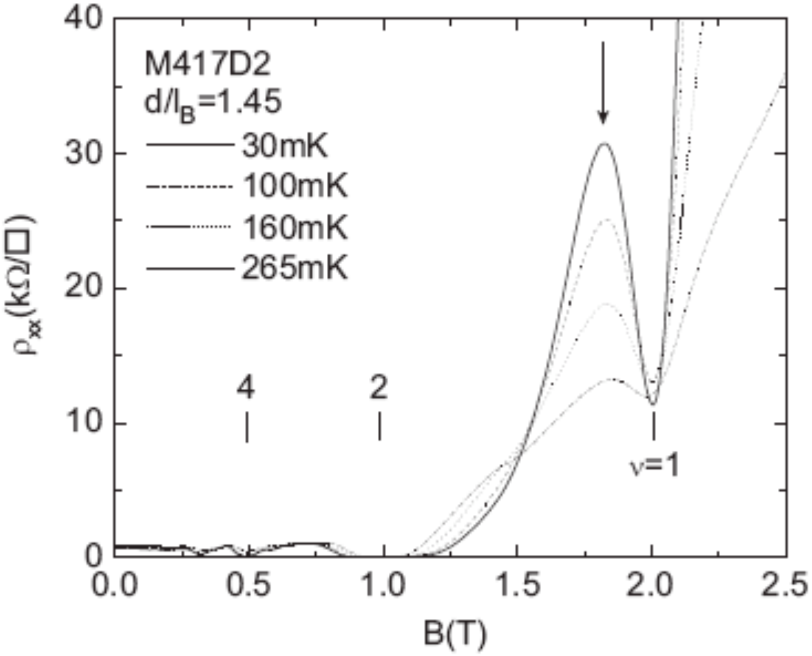
$n = 2.7 \times 10^{11}/\mathrm{cm}^2$
$\mu \sim 25 \times 10^{6}\ \mathrm{cm}^2/\mathrm{Vs}$
30 nm QW

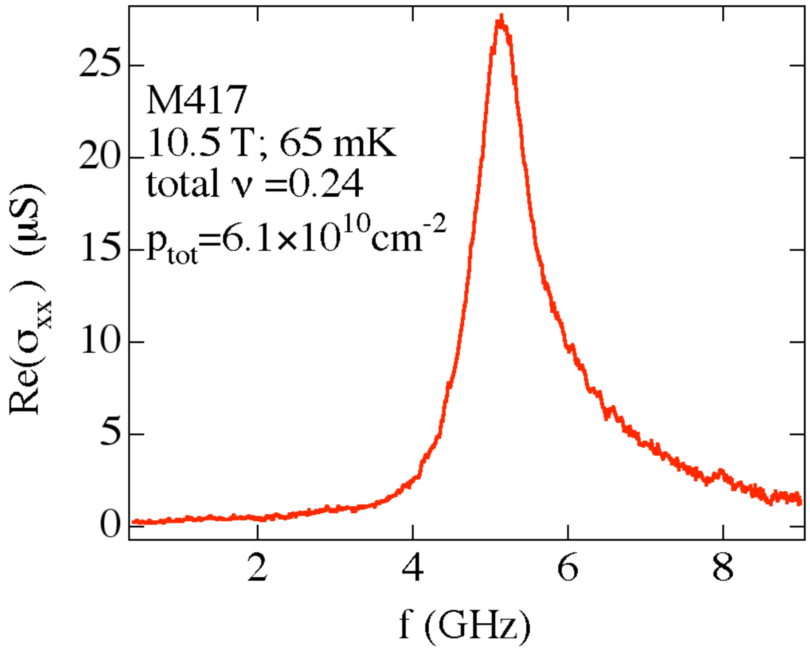
Smaller range of $\theta$

Larger $\tilde{g}$ : 0.019 at 0º



# High B Termination of QH series (bilayer)





Insulating phase accompanied by microwave resonance

Bilayer p-type, per layer p=$3.05\times10^{10}$/cm$^2$; well-center separation d=26 nm

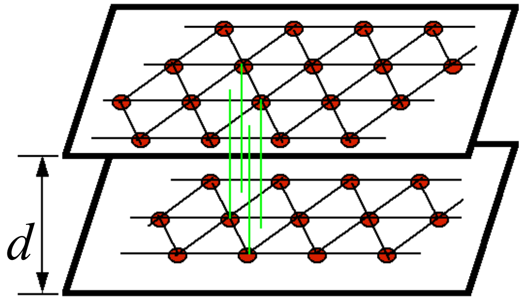
d/(avg. spacing in a layer)~d$(\pi p)^{1/2}$=0.8

(Earliest observation of pinning mode in bilayer: Doveston et al. 2002))

# Bilayer Wigner Crystals (BWC) (balanced)

Type of BWC depends on separation/ in-plane spacing, *measured by* $\tilde{d} = 2d(\pi p)^{1/2}$ $p$ is density/layer (holes), $d$ is well separation

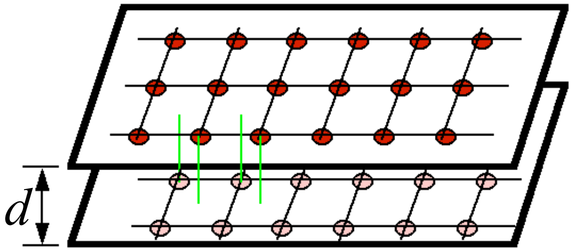
Pseudospin

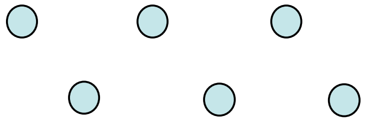


Staggered triangular

Antiferromagnet
AFMBWC

(No interlayer tunneling)

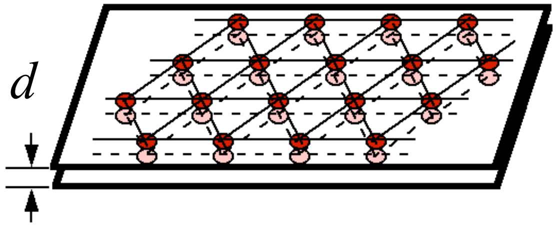


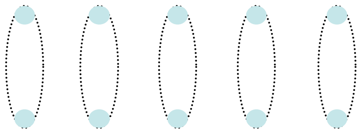


Staggered square

Antiferromagnet
AFMBWC

(No interlayer tunneling)





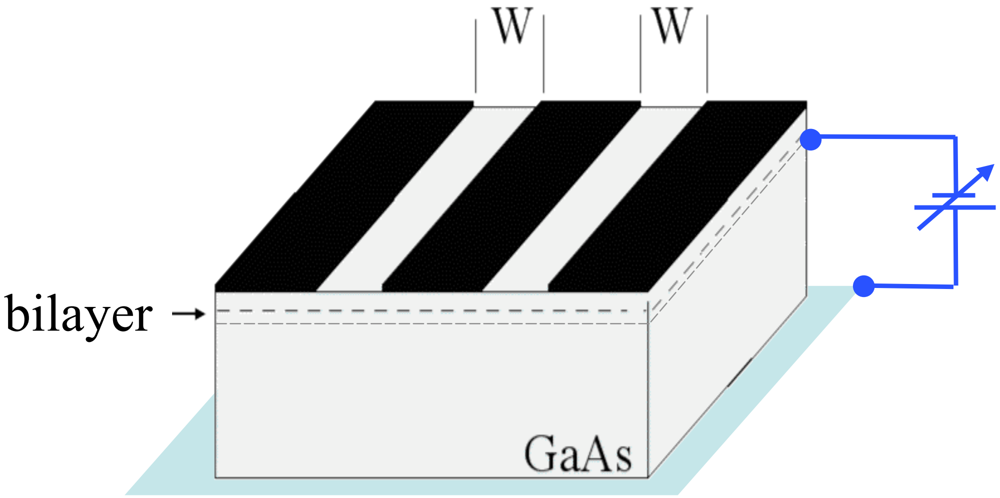
One-component triangular

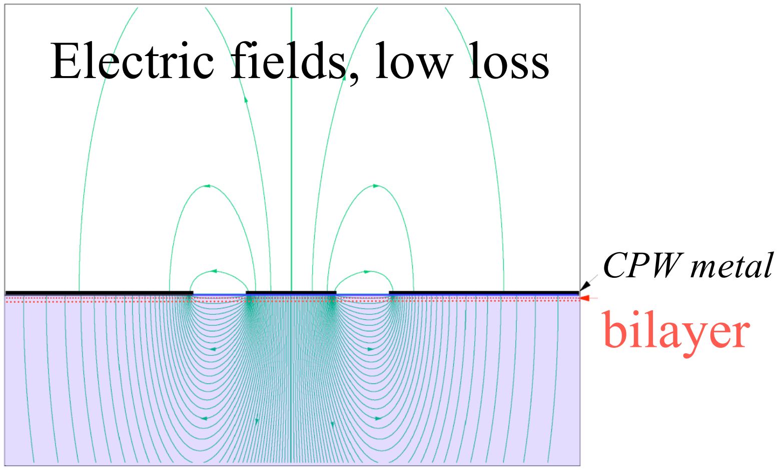
easy plane ferromagnet
FMBWC

Theories: Narasimhan and Ho, Zheng and Fertig. PRB ‘95

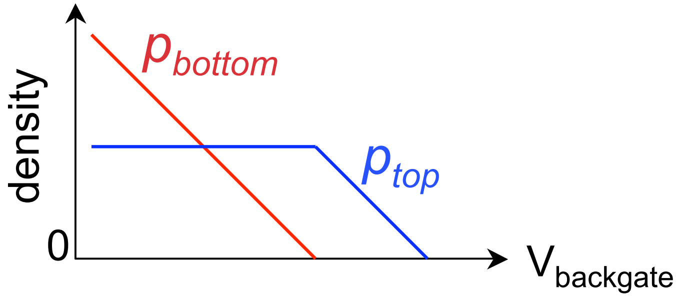
Evidence for phases: BWC pinning modes behavior $\Leftarrow \tilde{d}$

# Microwave measurement, coupling to bilayer





backgate only (no front gate)
one balanced density per cooldown



backgate only (no front gate) one balanced density per cooldown

As cooled, $p_B > p_T$, + backgate bias decreases $p_B$ first, $p_T$ following $p_B$ depletion

- Microwave electric field penetrates both layers with small perturbation

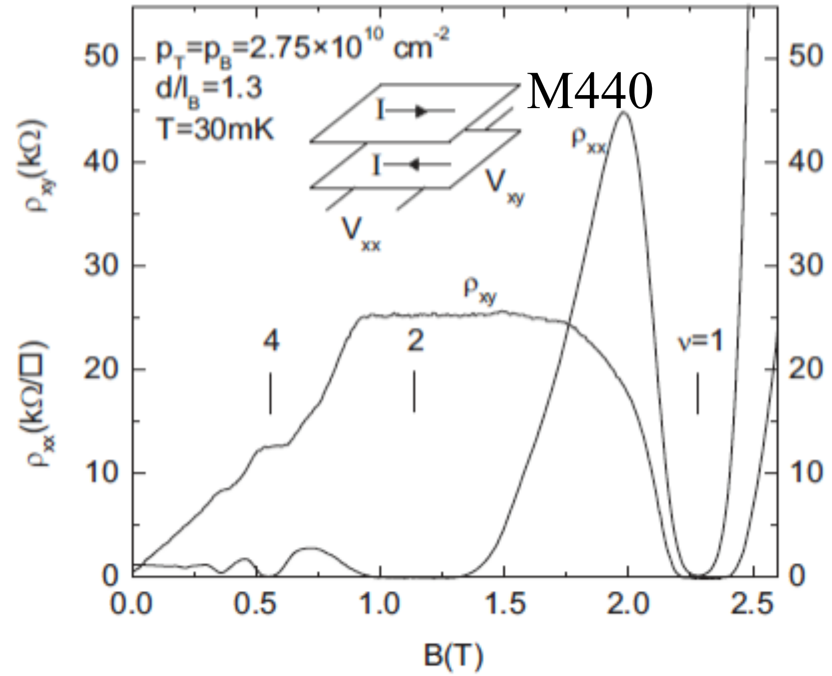
Independent layers with $\sigma_1, \sigma_2$ would give $(\sigma_1 + \sigma_2)$

# Samples

<311A> grown bilayers of holes
Negligible interlayer tunneling

Series of samples, 1.4 <d̃ < 18

deep in insulating phases, $\nu \lesssim 1/2$



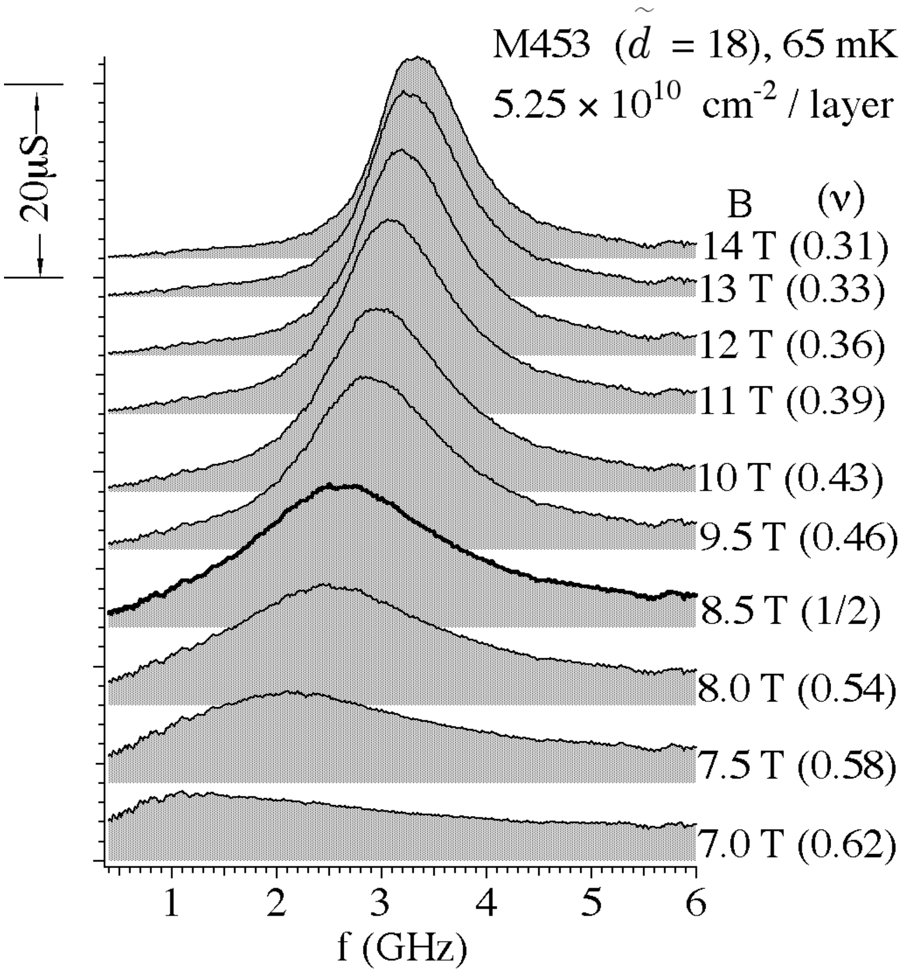
For small d̃, total ν=1:
excitonic Bose-Einstein Condensate,
counterflow superfluid

(pseudospin easy plane ferromagnet)

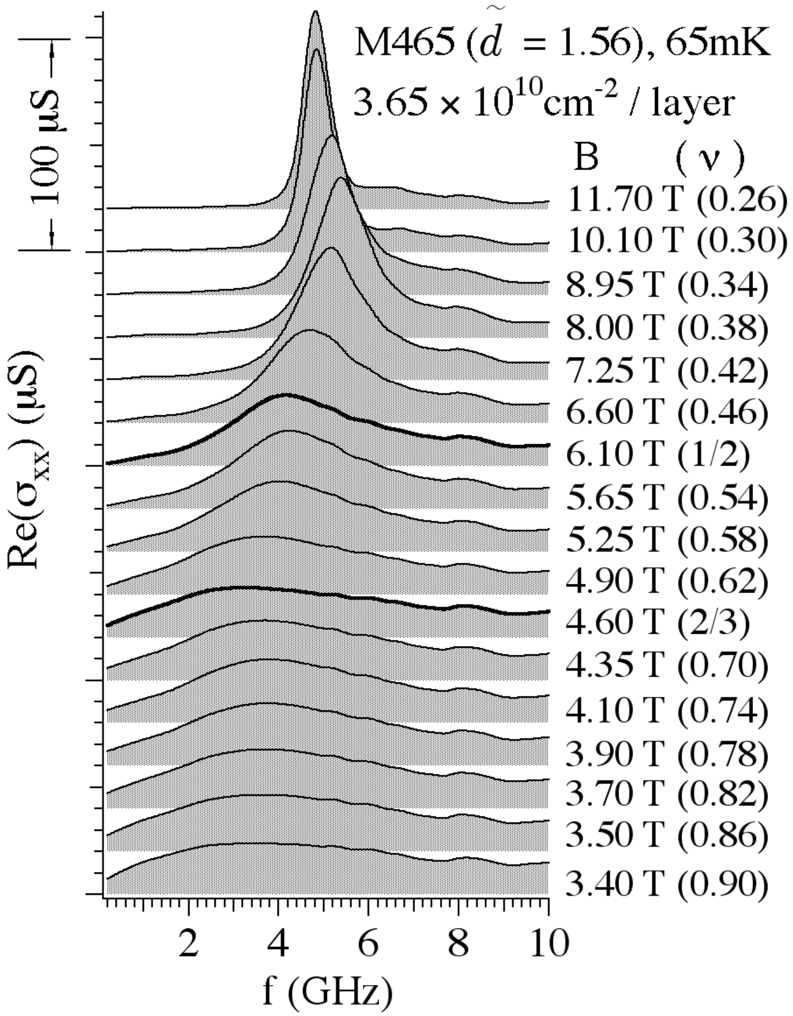
Tutuc, Shayegan, and Huse, PRL’04
n type: Kellogg *et al.*, PRL ‘04

*($\tilde{d} = d/l_{B1}$, where $l_{B1}$ is magnetic length when total filling $\nu = 1$)*

# Balanced BWC: Microwave Spectra, evolution with ν

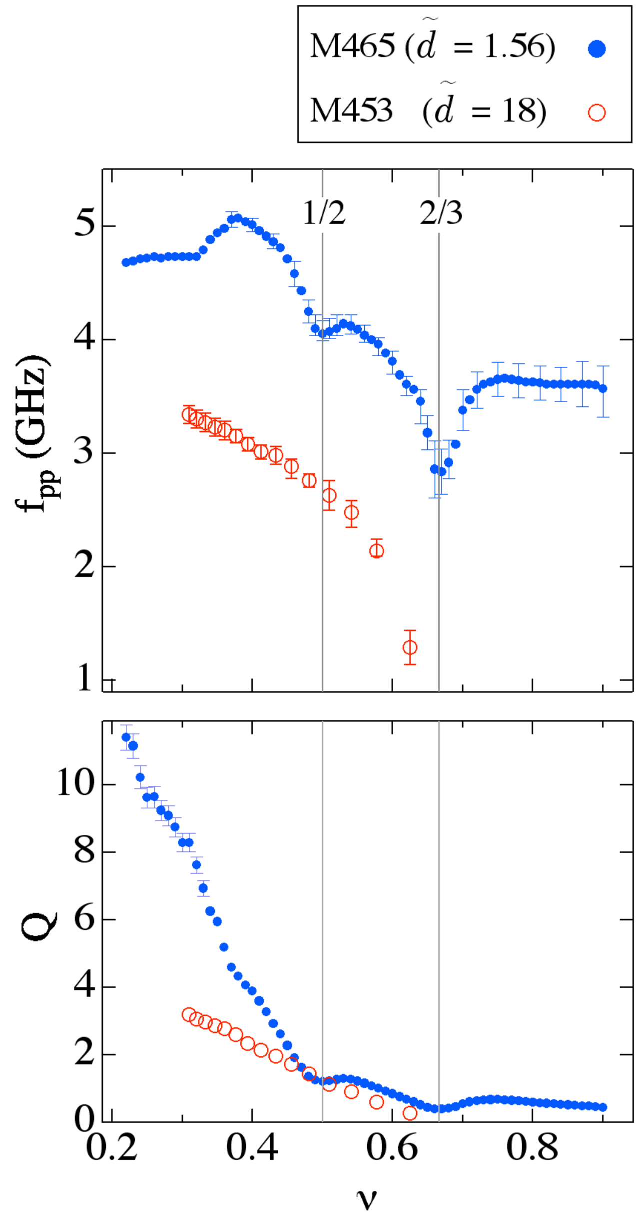


M453: Spectra similar to single-layer case.



M465: $f_{pk}$ drop at ν=2/3 &1/2, possibly indicating some FQH correlation in pinned bilayer WC.

# Balanced BWC: Resonance $f_{pk}$ and Q vs ν



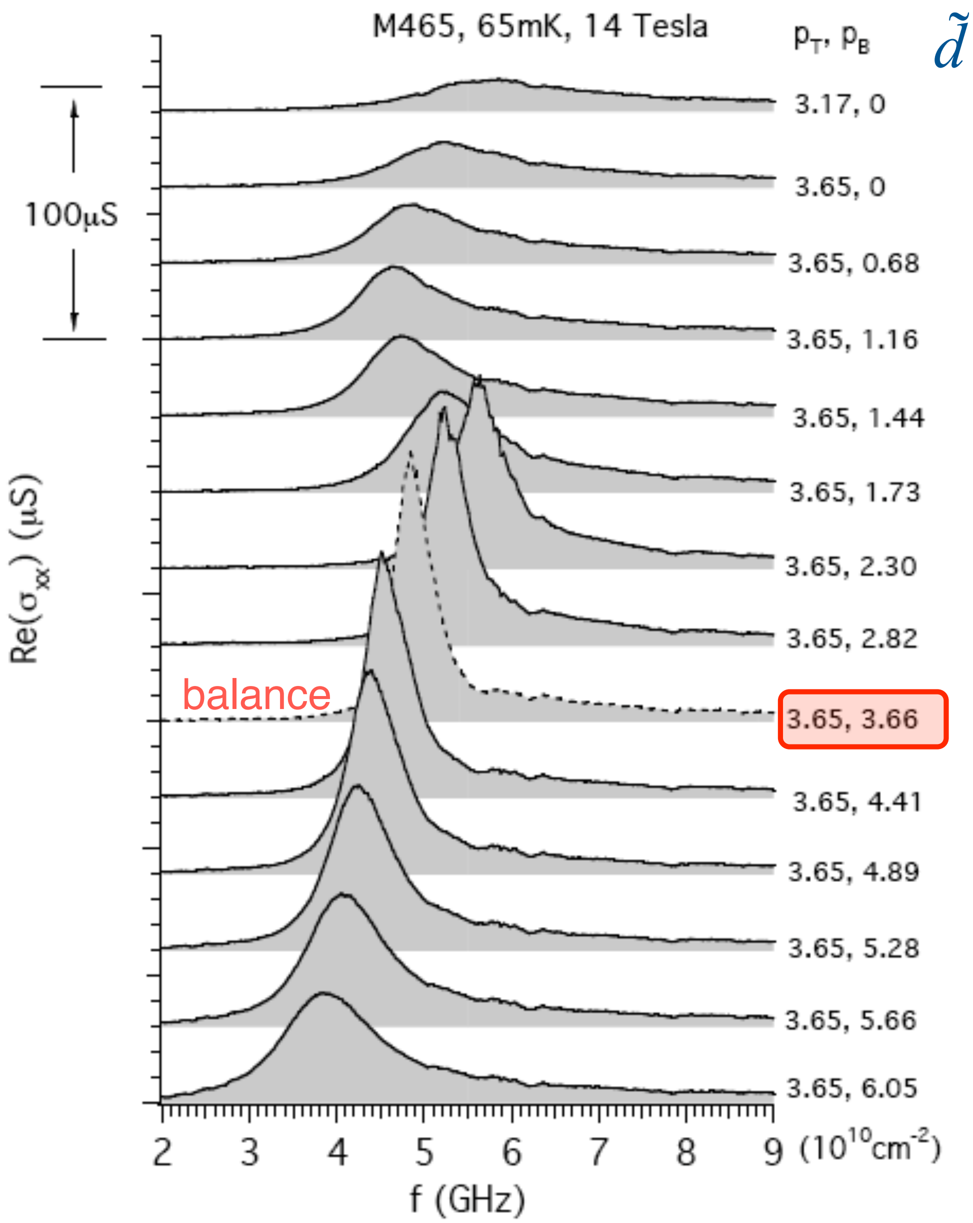
Dips in $f_{pp}$ , Q at ν= 1/2 , 2/3

Clearly in insulating phase

FQH liquid correlations affect BWC
1/2: interlayer

In weak pinning: not softening of BWC
Reduction of pinning?

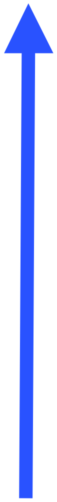
# Imbalanced layer densities: Spectra at many $p_{tot}$



$\tilde{d} = 1.56$

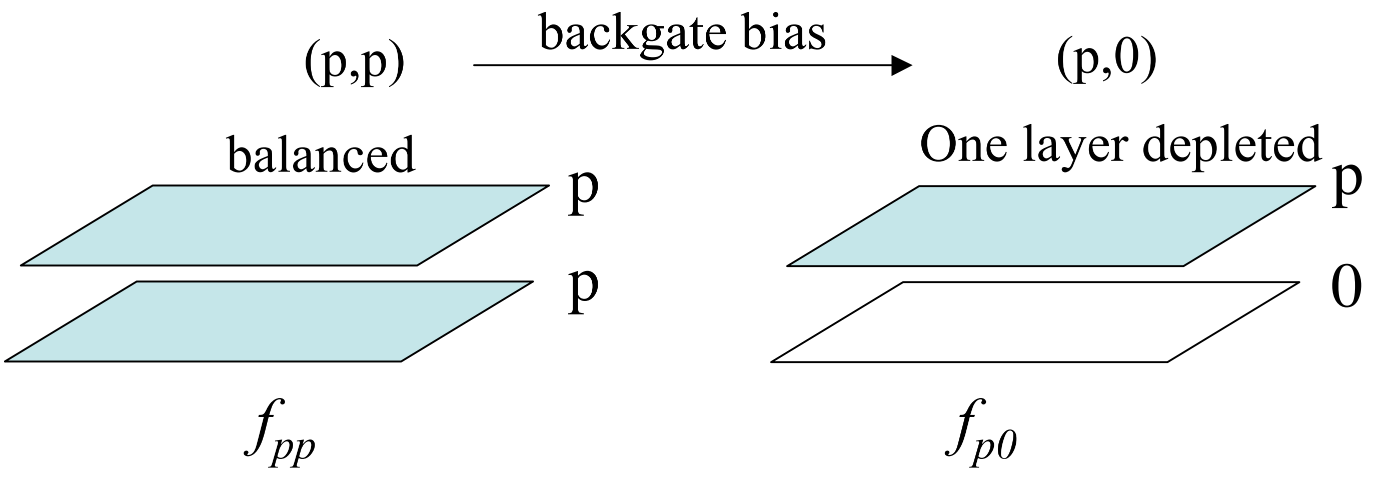
At imbalance,
significant density in each layer:

$\tilde{d} \leq 1.8$ One peak,
$\tilde{d} \geq 2.5$ Two peaks



$V_{backgate}$ increases

# Series of samples, varying $\tilde{d}$ : Single layer vs. Bi-layer



$$(p,p) \xrightarrow{\text{backgate bias}} (p,0)$$

balanced

p

p

$f_{pp}$

One layer depleted

p

0

$f_{p0}$

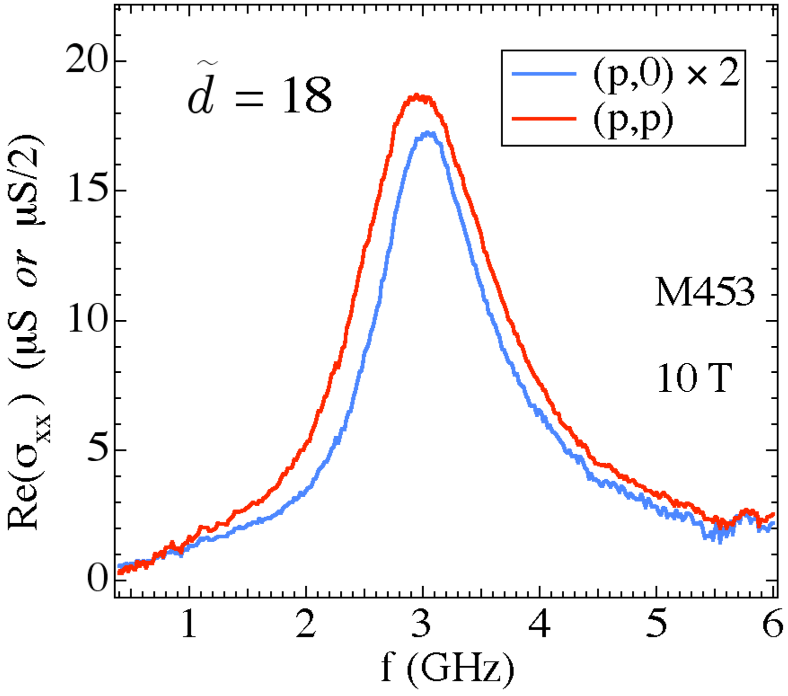
$$\boxed{\eta = f_{pp}/f_{p0}}$$

For independent layers ⇒

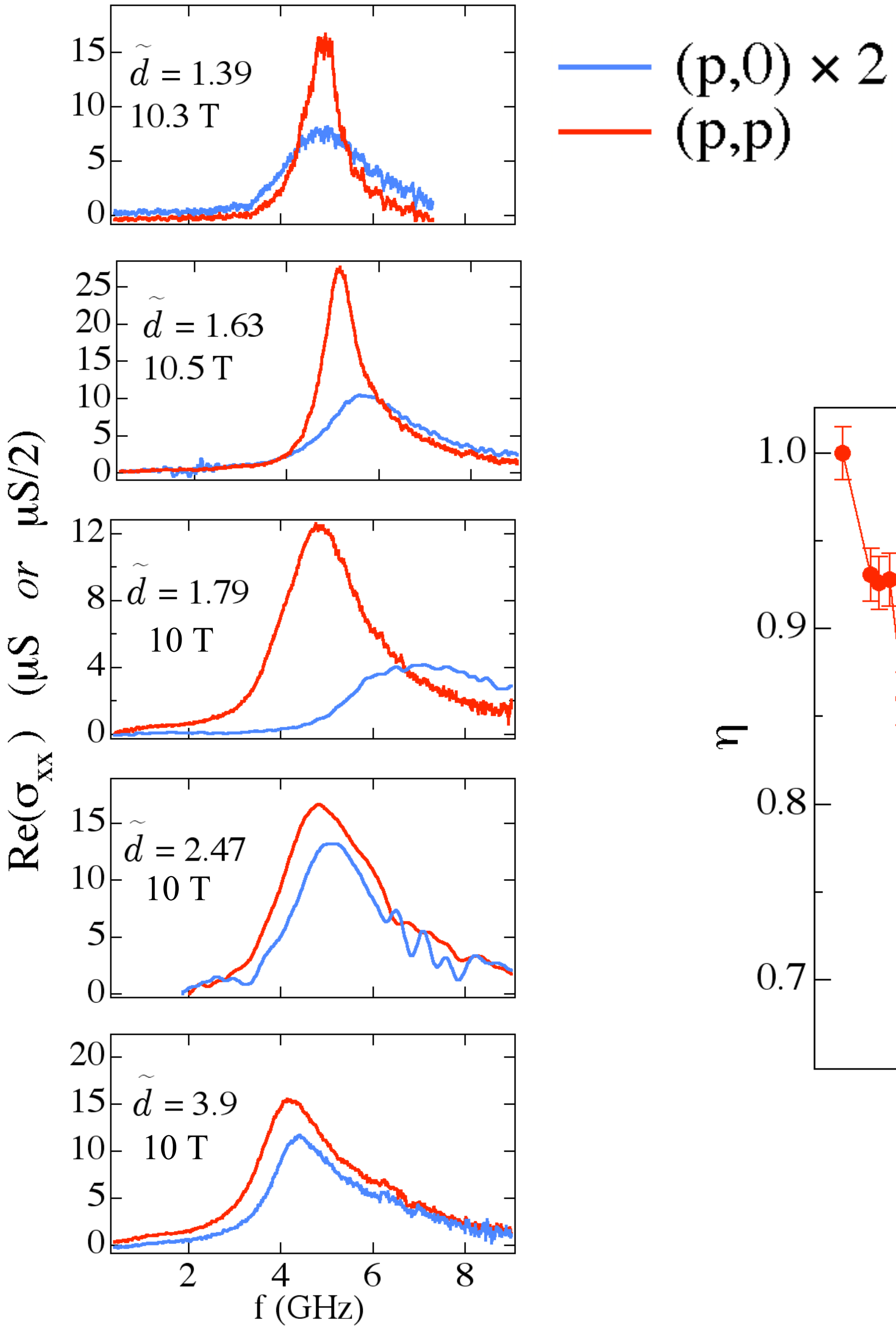
$$\sigma_{xx;pp} = 2 \times \sigma_{xx;p0}$$

$$\eta = 1$$

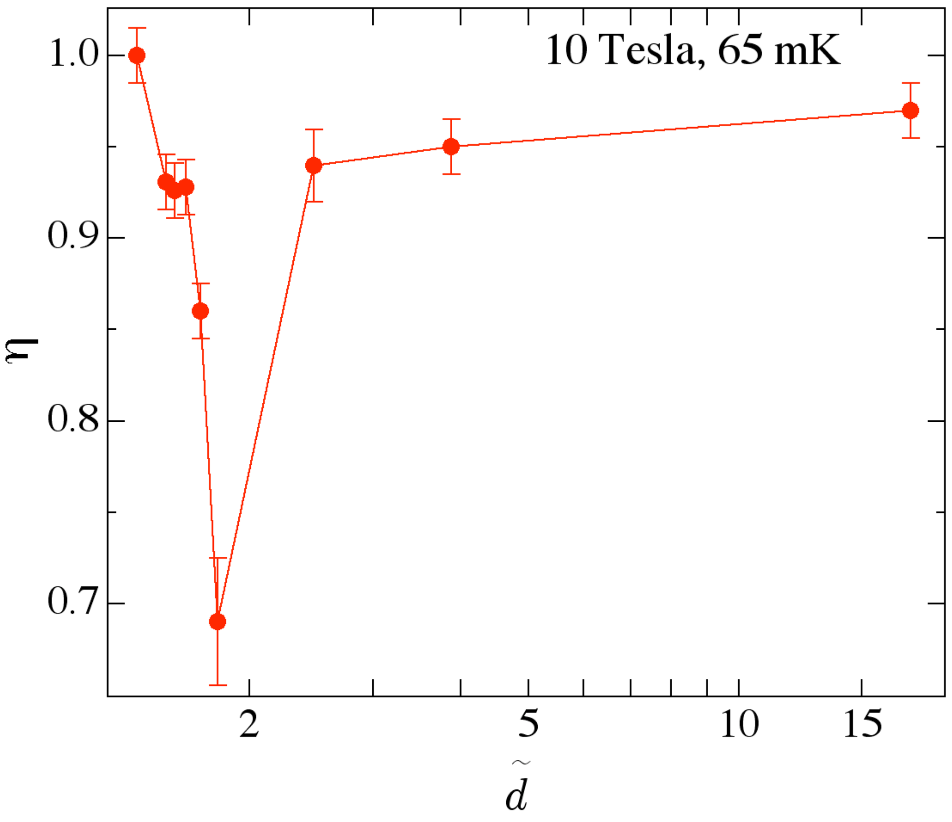
Conductivity of (p,0) state is doubled on graph



# Series of samples, varying $\tilde{d}$ : Single layer vs. Bi-layer

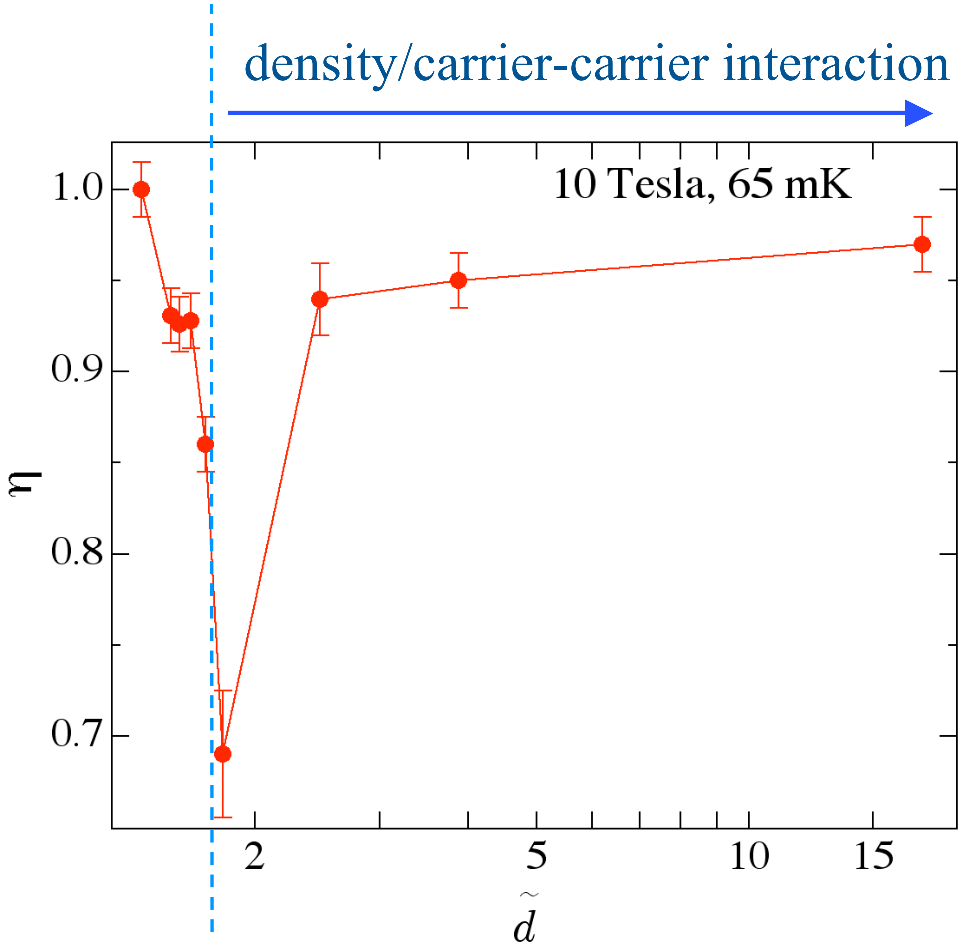


$$\eta = f_{pp}/f_{p0}$$



# Density/ Carrier-carrier interaction

$\boxed{f_{pk} \propto (\text{density})^{-\gamma}}$



Weak pinning, classical
State: (p,0) → (p,p)
$p_{total}$ : p → 2p
$f_{p0} > f_{pp}$

Small d̃ limit : like one layer

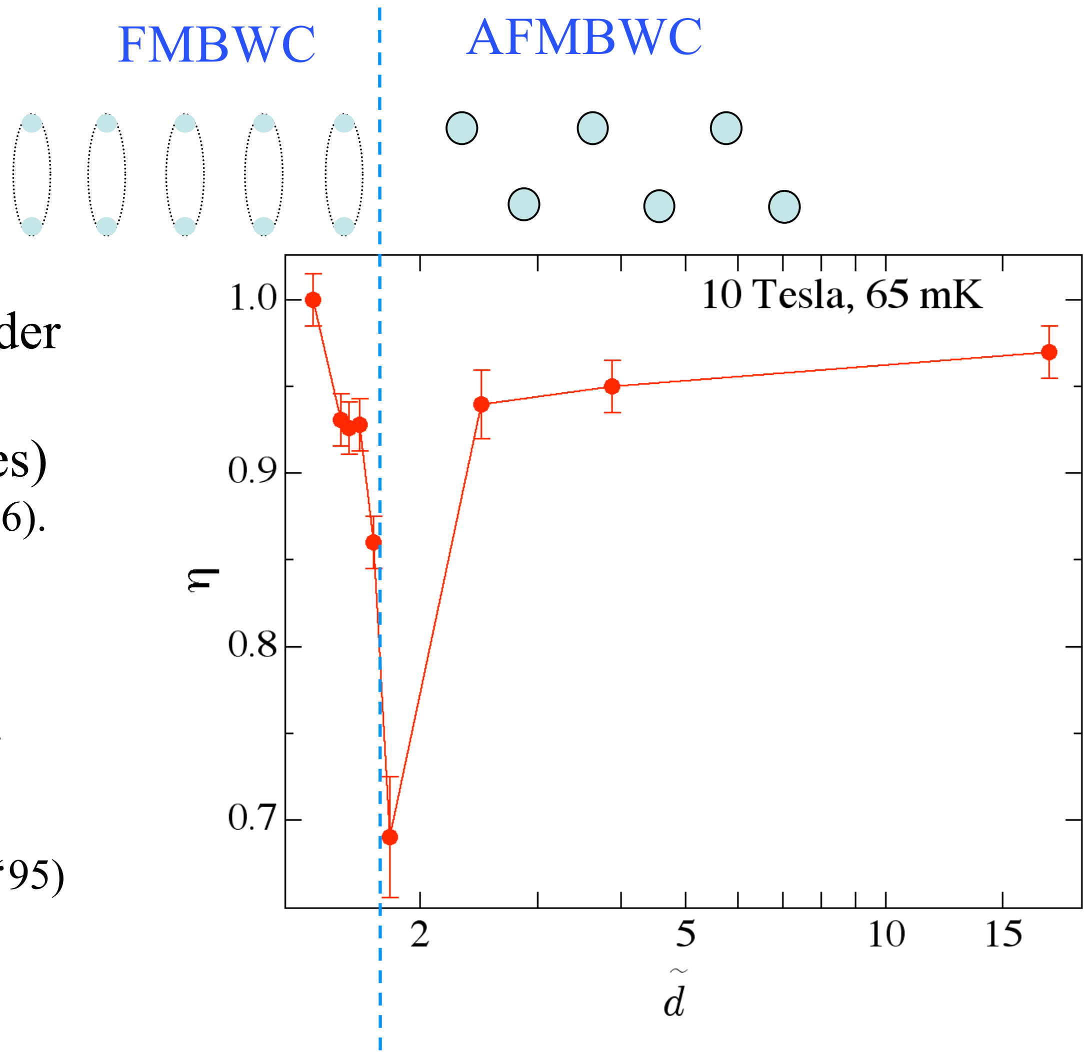
maximal effect: $f \propto p_{total}^{-\gamma}$

$\eta = f_{pp}/f_{p0} = \approx 0.71$ for $\gamma = 1/2$

Larger d̃ ⇒ softer (p,p) ⇒ larger η (closer to 1)

Question: how can η increase as d̃ decreases?

# Interpretation: AFM → FM BWC



- FMBWC: extra $f_{pp}$ from disorder spatially correlated btw. wells (e.g. local tunneling at impurities)
  Yong P. Chen, PRB **73**, 115314 (2006).
- Estimate transition at $\tilde{d}^* \sim 1.7$
- Exceeds theory predictions of $\tilde{d}^* \sim 0.4$
  (Narasimhan and Ho,Zheng and Fertig '95)
- Comparison : $\nu=1$ excitonic Bose-Einstein condensate exists for $\tilde{d}^* < 1.8$.

# “Applications” Summary

*Skyrmions in IQHWC around $\nu = 1$*

- Use rotation in field to vary $E_Z$, $\tilde{g}$
- $n^*$, $\tilde{g}$ dependence of $f_{pk}$ : consistent with $S \gtrsim 2$ in Côté et al theory, gives reduced $f_{pk}$
- No sign of square to triangular transition

*Bilayers*

- Surveyed samples, negligible tunneling, for many $\tilde{d}$
- Compare (p,p) and (p,0) states to isolate effects of interlayer interaction
  frequency ratio η shows striking minimum
- Interpretation:
  $\tilde{d} > 1.7$: carrier-carrier interaction effect
  $\tilde{d} < 1.7$: pinning enhanced in FMBWC