Non-Fermi liquid and Charge Fractionalization in low dimensions

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Low dimensional electron systems

<u>Collaborators:</u>
Bertrand I. Halperin
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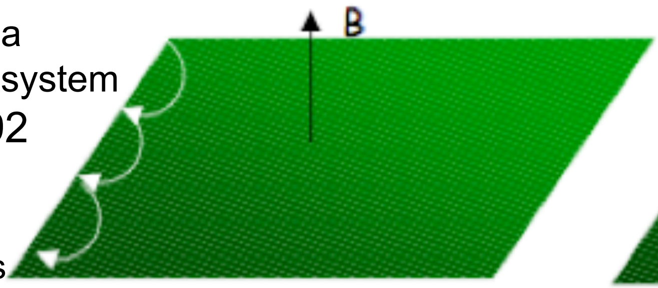
dimensional Reduction and new physics…

2D systems - Chiral edges of the QH and FQHE
Striped phase at high Landau levels

Edges as a
Chiral 1D system
Wen 1992



Laughlin
Quasiparticles
$\nu e = e/3$…

3D systems - Crystals of 1D molecules - Polyacetelene
Stripes in High Tc superconductors

Graphene

Pseudogap phase of high-Tc cuprates

Coupled wires systems

“modes confined to the same spatial channel”

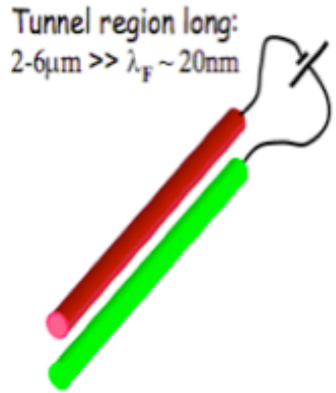
<u>Non-chiral systems & charge sector</u>

Main objectives of this talk

Understand better the breakdown of Fermi liquid in one dimension

K. Le Hur PRB 74, 165104 (2006)
Electron Decoherence and Applications to Interferometry:
Karyn Le Hur, Phys. Rev. Lett. 95, 076801 (2005) & PRB 65, 233314 (2002)

1st step towards Charge fractionalization in non-chiral Luttinger liquids using Momentum-Resolved tunneling
Novel transport measurement: asymmetry

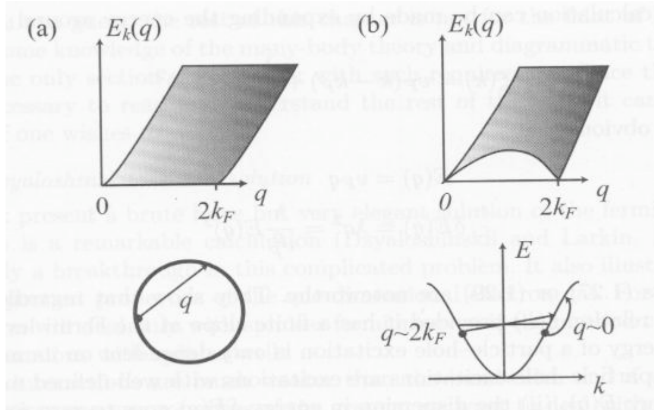


Hadar Steinberg, Gilad Barak, Amir Yacoby,
Loren N. Pfeiffer, Ken W. West,
Bertrand I. Halperin, Karyn Le Hur, Nature Physics 4, 116 (2008)

Theory: K. Le Hur, B. I. Halperin, A. Yacoby
Annals of Physics, 323, 3037-3058 (2008)

Luttinger Paradigm

Tomonaga 1950 following Bloch 1933, Haldane 1981



particle/hole pair "excitations"

$\rho^{e}_{+}(q) = \sum_k a^{\dagger}_{k+q} a_k$ and $\rho^{e}_{-}(q) = \sum_k b^{\dagger}_{k+q} b_k$

<u>Plasmon (boson) waves:</u>

$$\omega(q) = |q|\sqrt{\left(v_F + \frac{g_4(q)}{2\pi}\right)^2 - \left(\frac{g_2(q)}{2\pi}\right)^2}$$

g_4: forward scattering and g_2: backward scattering

$$g = \left(\frac{1 + y_4/2 - y_2/2}{1 + y_4/2 + y_2/2}\right) \qquad y_i = g_i/(\pi v_F)$$

$$g_2 = g_4 \longrightarrow \boxed{vg = v_F}$$

$$\boxed{Z(\omega) = \omega^{\frac{g+g^{-1}}{2} - 1}}$$

Non-Fermi liquid in 1D: a weak-interaction argument

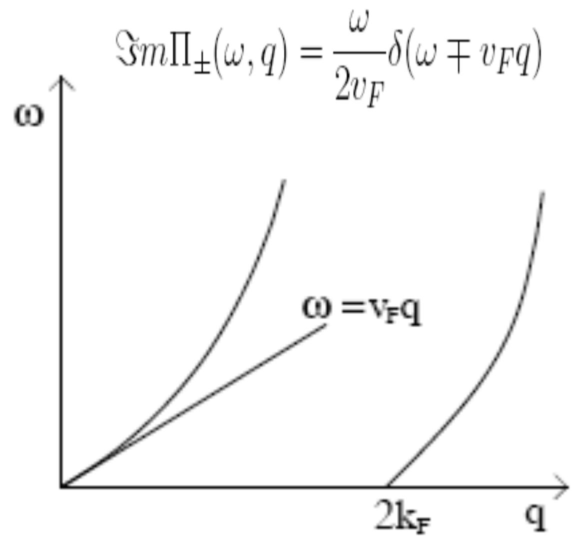
Inject a <u>spinless</u> electron at the right Fermi point $+k_F$

energy conservation = momentum conservation

$$\Im m\Sigma(k_F,\omega,T) = -\alpha^2 \max(\omega,T)$$
$$\Re e\Sigma(\omega) = -\alpha^2\omega\ln\omega$$

$$\boxed{Z(\omega)\propto\omega^{\alpha^2}}$$

$$\alpha = \frac{U}{E_F} = \frac{g_2}{v_F}$$

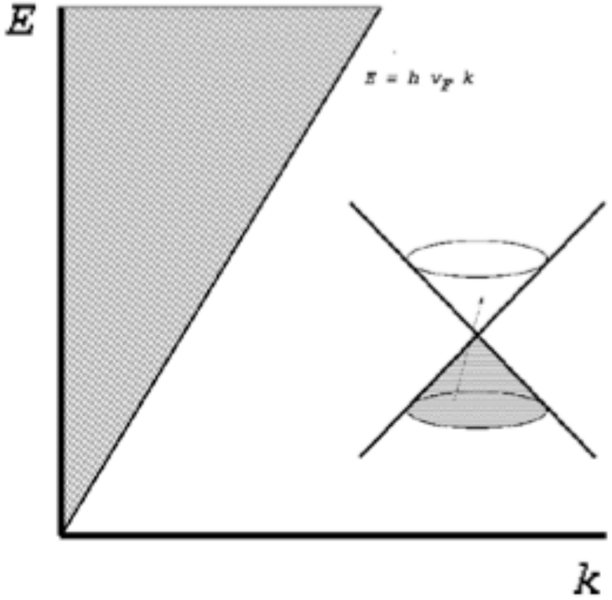


Quasiparticle weight vanishes at ω=0

For spinless fermions, the first 2 (forward) diagrams cancel each other…

+: right-moving particle
-: left-moving particle

Note: Non-Fermi liquid in graphene…



(in contrast to the electron gas)

Also renormalization of Fermi velocity

Decay of a quasiparticle:

$$\lim_{\omega \to \epsilon_p + 0^+} \Im m \Sigma(\omega, \vec{p}) = \frac{1}{48} \left(\frac{e^2}{\epsilon_0 v_F} \right)^2 v_F |\vec{p}|$$

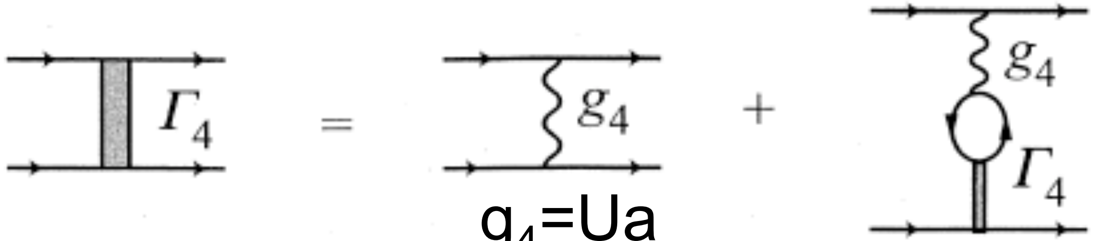
(perturbation theory in fine-structure cst)

$$\prod(k, E) \approx \frac{k^2}{\sqrt{v_F^2 k^2 - E^2}}$$

Imaginary for $E > v_F k$

Gonzalez, Guinea, Vozmediano 1994,96

Forward scattering and velocity renormalization



g_4=Ua

g_2=0

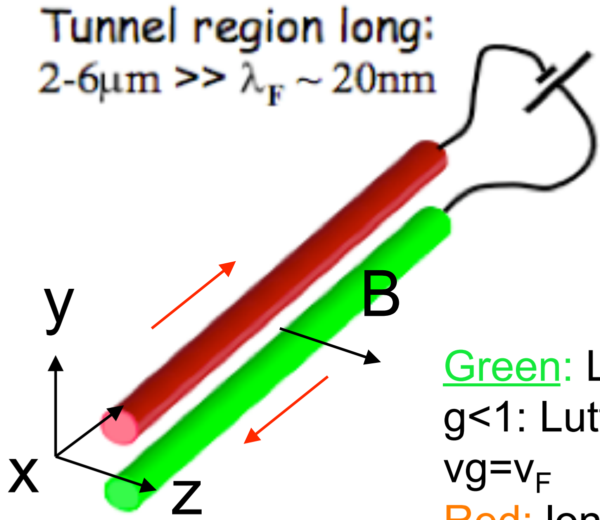
$$\Gamma_4 = \frac{g_4(\omega - v_F q)}{\omega - vq}$$

$$vq = \left(v_F + \frac{g_4}{2\pi}\right) q$$

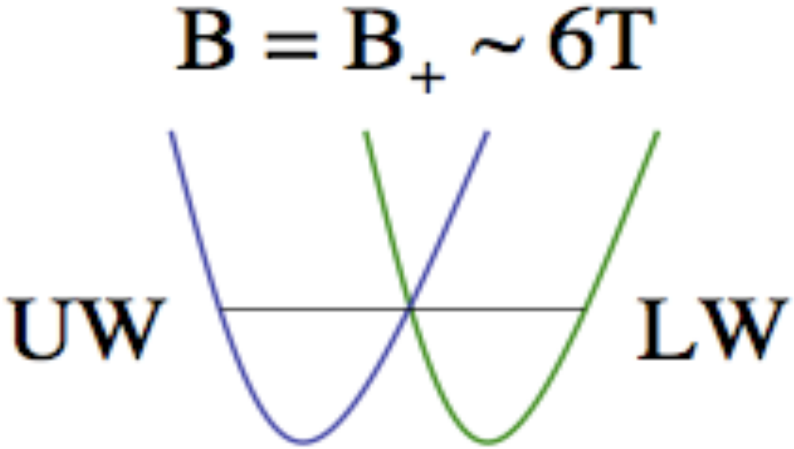
Understand better the non-Fermi liquid requires to inject
a bare electron at <u>one</u> Fermi point (uni-directional injection)

Unidirectional Injection:

Momentum-resolved Tunneling



Green: Luttinger liquid
g<1: Luttinger parameter
vg=v_F
Red: long probe (1D)



Momentum is conserved during the tunneling process

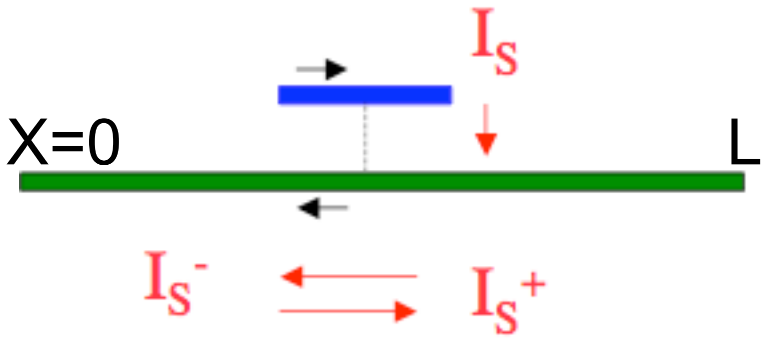
A transverse field B produces a momentum boost $q_B = 2\pi Bd/\phi_o$

e.g.: Landau gauge $A_y = xB$

Ideal situation when $k_{FL} + k_{FU} = q_B$

Is the current uni-directional?

K. Le Hur, B. I. Halperin, and A. Yacoby, Annals of Physics 323, 3037-3058 (2008)



$$H_t = \lim_{q \to 0} \int dx \sum_{s=\uparrow,\downarrow} t e^{-iqx} \Psi^{\dagger}_{-Ls}(x) \Psi_{+Us}(x) + h.c.$$

$$q = q_B - (k_{FL} + k_{FU}) \to 0$$

$$|\langle I(x=0) \rangle| = \frac{1+g}{2} I_S = I_S^-$$

$$\langle I(x=L) \rangle = \frac{1-g}{2} I_S = I_S^+$$

(those equalities are true beyond the weak-tunneling regime)

$$\frac{dI_S}{dV_{SD}} = |t|^2 \frac{2e^2}{h} \frac{1}{(\hbar v_F)^2} \frac{(T/\Lambda)^{\nu}}{\Gamma(\nu+1)} \frac{L_F^2}{1+q^2 L_F^2}$$

Understanding of the effect?

<u>Charge sector: chiral basis</u>

I. Safi & H. Schulz, 1995
M. Fisher & L. Glazman, 1996
T. Giamarchi's book,…

The Luttinger 1D Hamiltonian takes the general form

$$H = \int dx\, (vh/4g)[\rho_+^2 + \rho_-^2] - W\rho_- - Y\rho_+$$

W & Y are the associated chemical potentials

Here, $\rho_\pm$ represent the chiral densities
Notice that $\rho_+ = f(x-vt)$ and $\rho_- = f(x+vt)$

At equilibrium, we obtain $(Y,W) = (vh/2g)\rho_\pm$ (e=1)

Averaged chiral charges

Suppose we inject N electrons such that $J = N_+^e - N_-^e$
N_+ and N_- are the averaged charges of the chiral waves

Conservation laws:

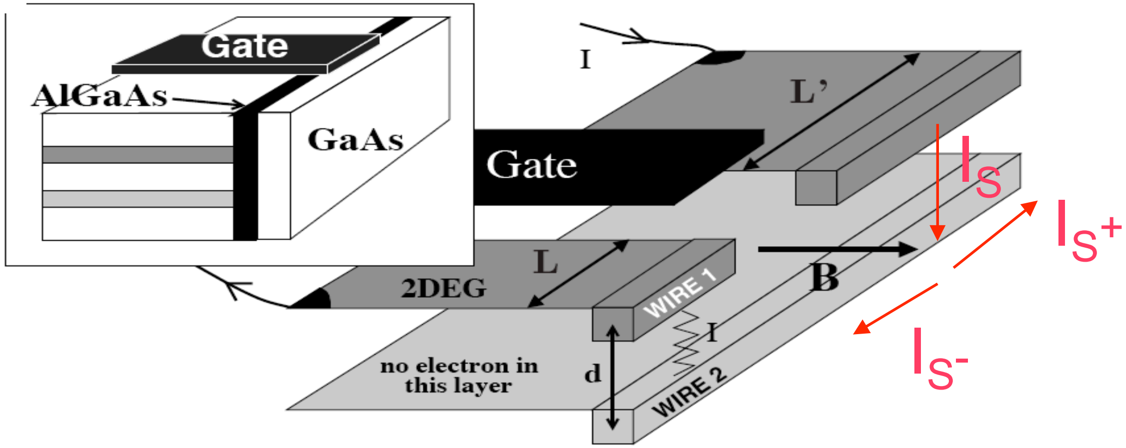
$$(N_+ + N_-) = N$$
$$v(N_+ - N_-) = vgJ$$

Current in a Luttinger liquid reads vgJ

Unidirectional electron: $N_- = (1+g)/2 = f$ & $N_+ = (1-g)/2 = (1-f)$
K.-V. Pham, Lederer, Gabay, 99; I. Safi & H. Schulz, 96; K. Le Hur, 2002;…

Weak backscattering from impurities results in $J = \pm 2$ and $N = 0$; thus $N_\pm = \pm g$
Attempt of shot-noise measurements in carbon nanotubes: Yamamoto et al. 2007, PRL (Stanford)

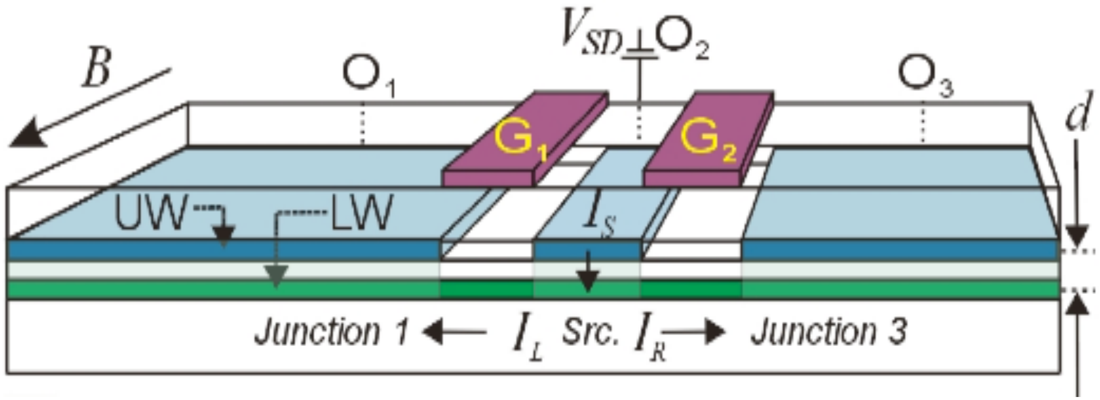
Asymmetry: novel quantity



$$\frac{I_S^- - I_S^+}{I_S} = (2f - 1) = g$$

Not so simple…

Currents detected at the left and right contacts are not I_S- and I_S+



For symmetric couplings with left and right leads, then:

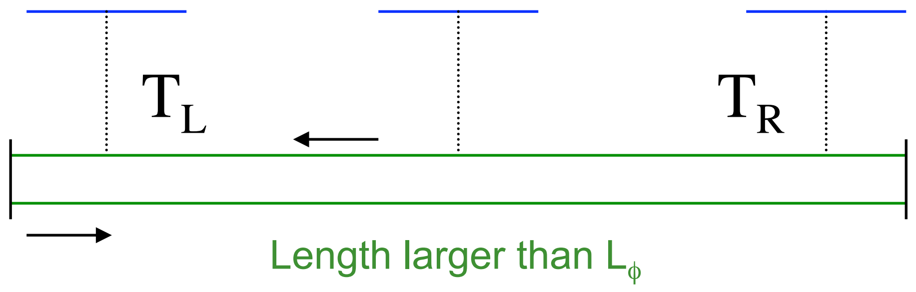
$$\frac{A_S(2e^2/h)}{G_2} = \frac{1}{g}\left(\frac{I_S^- - I_S^+}{I_S}\right) = 1$$

$A_S = I_L - I_R/(I_L + I_R)$

G_2: 2-terminal conductance

Glimpse on free electrons

$T_L = |t_L|^2$ and $T_R = |t_R|^2$



Free electrons:
Landauer picture

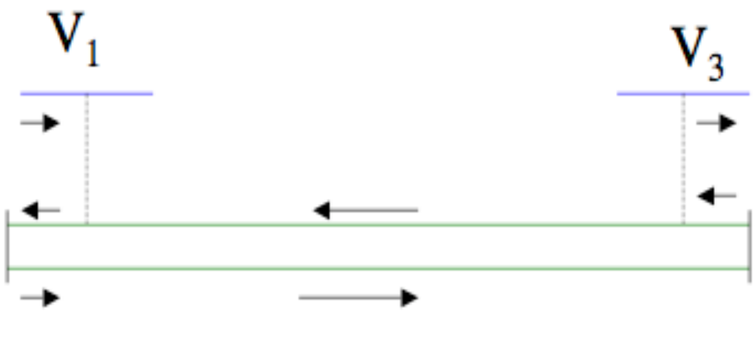
$R_L = 1 - T_L$, $R_R = 1 - T_R$

$$A_s = T_L - (1-T_L)T_R + (1-T_L)(1-T_R)T_L - (1-T_L)^2(1-T_R)T_R + \ldots$$

Sum over all possible paths in the lower wire

For $T_L = T_R$ $\quad A_S = (T_L)(2-T_L)^{-1}$

Two-terminal conductance



2 scatterers in cascade

$G_2 = (2e^2/h)T_{eff}$

A simple calculation leads to the celebrated result:

$$T_{eff} = \frac{T_L T_R}{1-(1-T_L)(1-T_R)}$$

Therefore for $T_L = T_R$, we observe that $A_S(2e^2)/(hG_2) = 1$

(For $T_L \gg T_R$, however $A_S \sim 1$ whereas $G_2 \propto T_R$)

Interacting case…

Including couplings with left and right wires (leads)

-General way of doing it: Using current conservation
-Not limited to weak or strong couplings
-Very general boundary condition for interacting « wires »
D. B. Chklovskii and B. I. Halperin, PRB 57, 3781 (1998)

Useful Formulas for a 1D wire:

The chiral currents are defined as: $I^{\pm} = e\, \rho_{\pm}\, v = (2eg/h)(Y,W)$

The current in a Luttinger liquid obeys:

$$I = I^{-} - I^{+} = (2eg/h)(W-Y)$$

$G_0 = 2e^2/h$

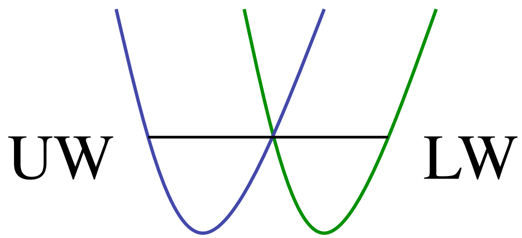
-current conservation: $I_S = I_L + I_R = I_S^- + I_S^+$

-currents on each part of the central junction in the wire

$I_L = I_L^- - I_L^+ = gG_0(W_1 - Y_1)$

$I_R = I_R^+ - I_R^- = gG_0(Y_3 - W_3)$

$B = B_+ \sim 6T$



-Fractionalization equations:

$gG_0(W_1 - W_3) = I_S^- = I_S f$

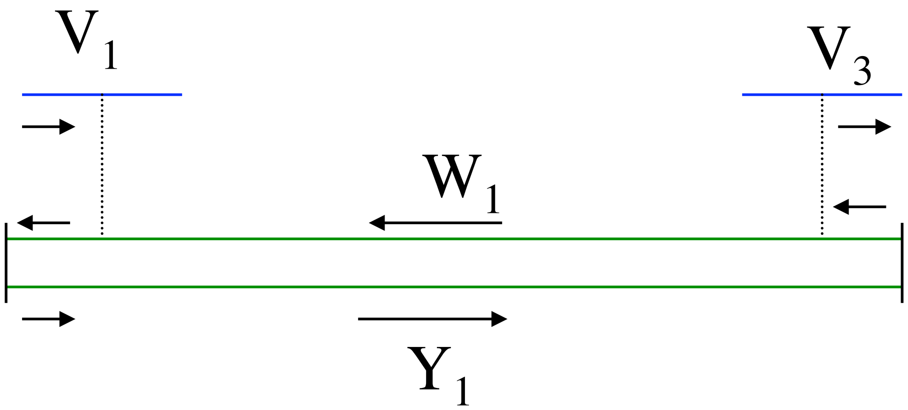
$gG_0(Y_3 - Y_1) = I_S^+ = I_S(1-f)$ $f = (1+g)/2$

-coupling with leads using current conservation

$y_1 = \beta V_1 + (1-\beta)W_1$

$W_3 = \beta V_3 + (1-\beta)Y_3$

Two-terminal conductance



$I_S = 0$

$$I_R = gG_0(Y_3 - W_3) = gG_0(Y_1 - W_1)$$

$$\beta(V_1 - V_3) = (2-\beta)\,(Y_1 - W_1)$$

$$G_2 = g\frac{2e^2}{h}\frac{\beta}{2-\beta}$$

g=1, free electrons: $T_i = 1-R_i = \beta_i$

- consistent with incoherent electron transport
- Maximum value of $G_2 = 2e^2/h$ and $\beta_{max} = 2/(1+g)$

β_{max} corresponds to $(2ge^2/h)(Y_1-W_1) = (2e^2/h)(V_1-V_3)$

Asymmetry and Universal Ratio

For symmetric couplings with left/right leads

$$A_S = \left(\frac{I_S^- - I_S^+}{I_S}\right)\frac{\beta}{2-\beta} = \left(\frac{I_S^- - I_S^+}{I_S}\right)\frac{G_2 h}{2e^2 g}$$

$$\frac{A_S 2e^2}{hG_2} = \left(\frac{I_S^- - I_S^+}{I_S}\right)\frac{1}{g} = 1$$

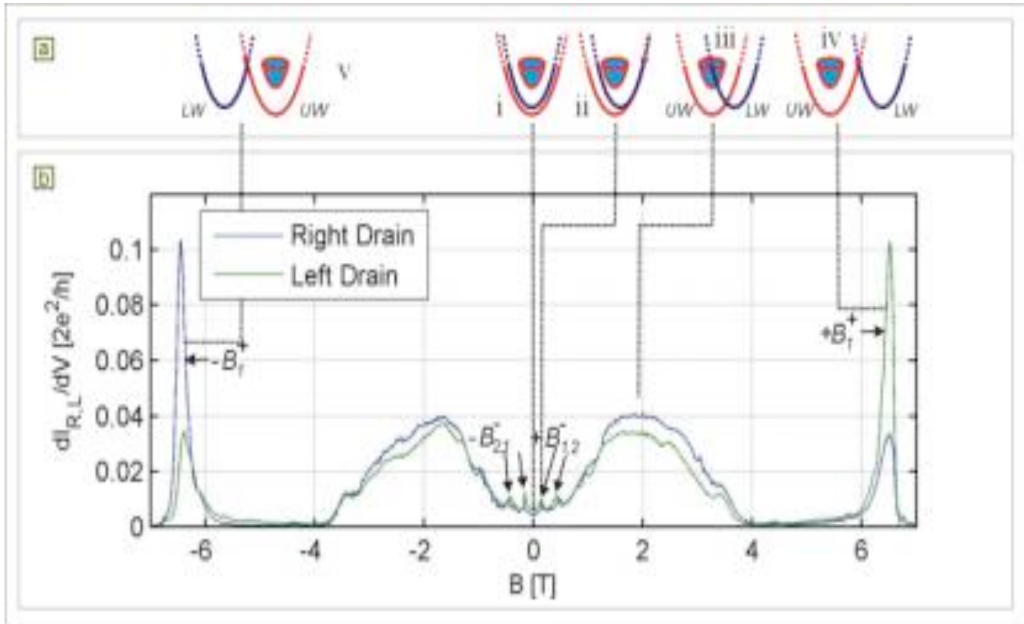
Test of counterpropagating currents

Microscopic justification of β both in weak & strong coupling limit

K. Le Hur, B. I. Halperin, and A. Yacoby, Annals of Physics 323, 3037-3058 (2008)

Momentum resolved tunneling

Hadar Steinberg, Gilad Barak, Amir Yacoby,
Loren N. Pfeiffer, Ken W. West,
Bert Halperin, Karyn Le Hur, Nature Physics 4, 116 Jan. 2008



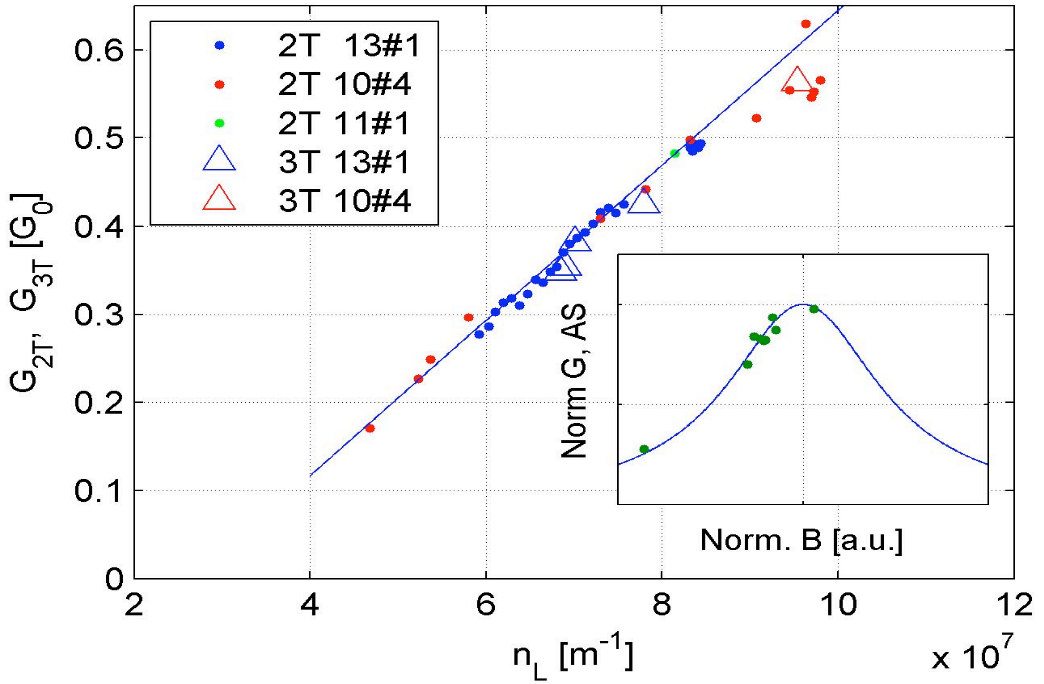
Prerequisite for unidirectional electron injection: $q = q_B - (k_{FU} + k_{FL}) \sim 0$

(Densities in the wires slightly different)

Experimental results:

$$\frac{A_S(2e^2/h)}{G_2} = \frac{1}{g}\left(\frac{I_S^- - I_S^+}{I_S}\right) = 1$$

$G_{3T} = A_s G_0$



Temperature is 0.25K

Luttinger exponent

Visible
counterpropagating
charge

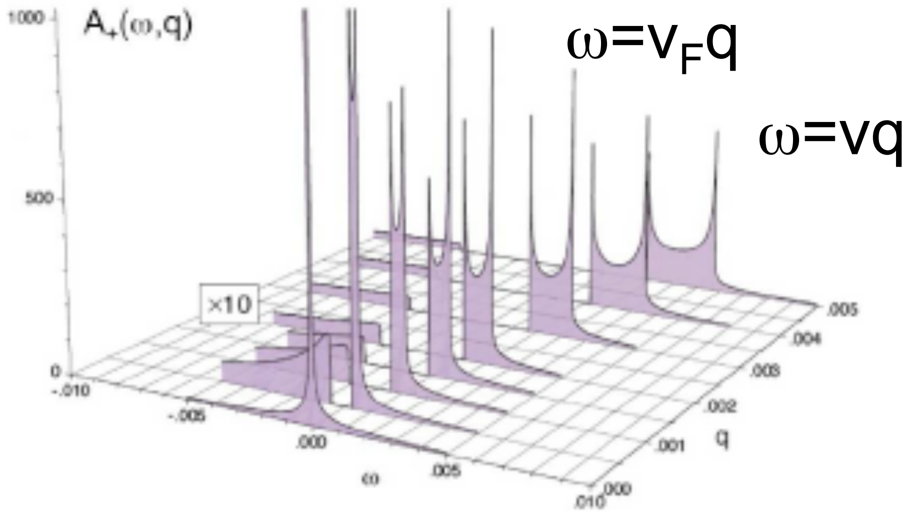


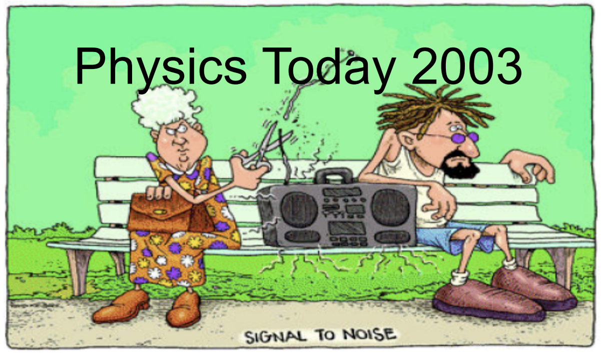
Fig. 5. Electron spectral function in the Luttinger theory (for an electron from the right Fermi branch) as a function of frequency ω for different wavevectors q measured from k_F ($\Lambda/\hbar = 1$ and $u = 1$). Temperature is zero, $g = 1/2$, and for simplicity, $L_F \to +\infty$ (a finite L_F produces the broadening of the different peak structures). The $\omega < 0$-part has been multiplied by 10 for clarity. Far from the Fermi "surface" (point), the electron spectral function reveals two peak features associated with the spin and right-moving charge mode (the counterpropagating charge mode also gives some spectral weight at negative ω). One can determine g from these two peaks.

$$0.4 < v_F/v = g < 0.5$$

See also D. Carpentier, C. Peca, L. Balents PRB 66, 153304 (2002)

noise …

“The noise is the signal” was a saying of Rolf Landauer, one of the founding fathers of mesoscopic physics. What he meant is that fluctuations in time of a measurement can be a source of information that is not present in the time-averaged value. A physicist may delight in noise, in a way reminiscent of figure 1.



Carlo Beenakker & Christian Schönenberger

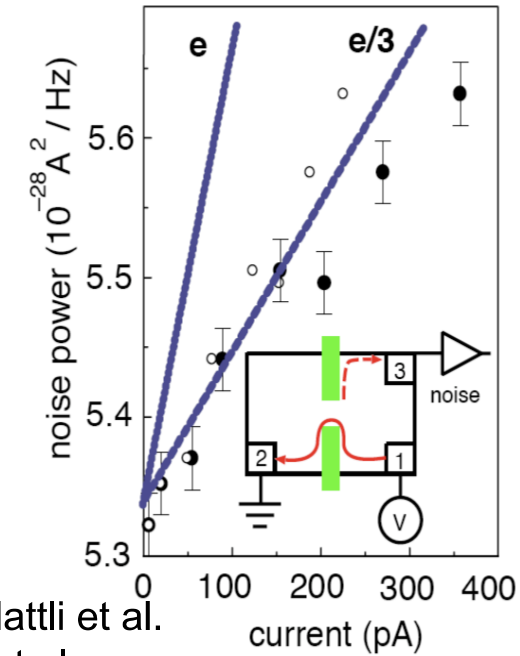
(Ignoring lead effects)

$S(\omega=0) = 2e^*\langle I\rangle$

e^*: Transferred charge in unit of e

In the nonchiral Luttinger liquid
$e^* = fe$ or $e^* = (1-f)e$

$$f = \frac{1+g}{2}$$



Saclay’s experiment
L. Saminadayar, C. Glattli et al.
See also M. Heiblum et al.

Charge: Beyond the quantum average

Pham, Lederer, Gabay, 1999 following M. Stone & M.P.A. Fisher, 1994

Eigenstates are built from the chiral vertex operators

$$V^{\pm}_{N_\pm}(x) = \exp -i\sqrt{\pi} N_\pm \Theta_\pm$$

$$[\hat{Q}, V^{\pm}_{N_\pm}(x)] = N_\pm V^{\pm}_{N_\pm}(x)$$

$$\longrightarrow [\rho(x), V^{\pm}_{N_\pm}(y)] = N_\pm \delta(x-y) V^{\pm}_{N_\pm}(x)$$

$$[\hat{J}, V^{\pm}_{N_\pm}(x)] = \frac{N_\pm}{g} V^{\pm}_{N_\pm}(x)$$

This operator identities suggest that the charge is sharp (not the result of a quantum average)

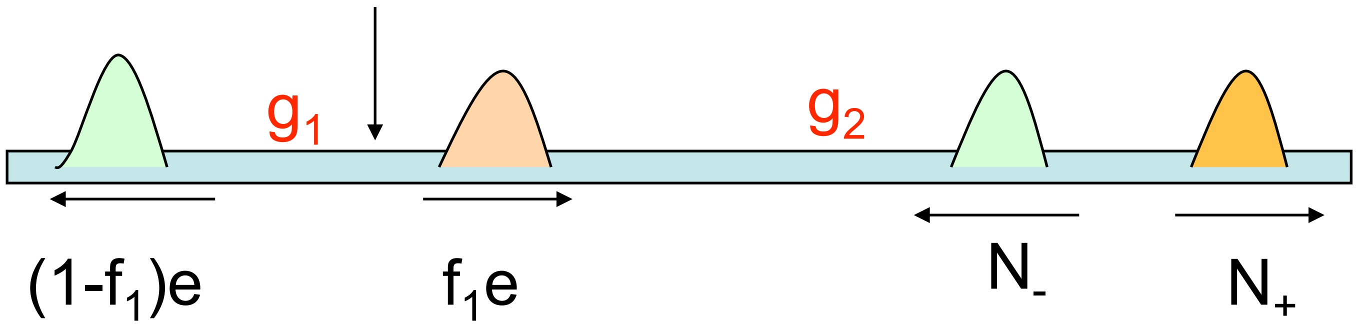
$$H_\pm = \sum_{\pm q>0} v|q| b^\dagger_q b_q + \frac{\pi v}{Lg}\left(\frac{\hat{Q} \pm g\hat{J}}{2}\right)^2$$

$\hat{Q}$ = total charge operator

$\hat{J}$ = total current operator

Excitations with arbitrary charge

K. Le Hur, B. I. Halperin, and A. Yacoby, Annals of Physics 323, 3037-3058 (2008)



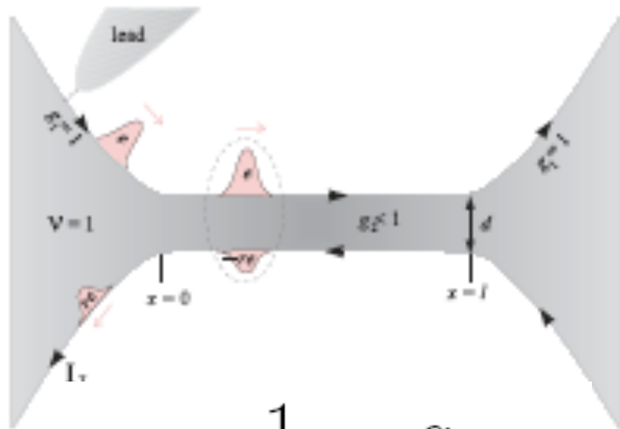
$f_1 = (1+g_1)/2$

$$N_- = f_1 r$$

$$N_+ = f_1 \frac{2g_2}{g_1+g_2} = f_1(1-r)$$

$$r = \frac{g_1 - g_2}{g_1 + g_2}$$

E. Berg et al, arXiv:0812.4321



$$r = \frac{1-g_2}{1+g_2}$$

Summary

Charge Fractionalization in 1D non-chiral systems

First step: current isnot unidirectional (unidirectional injection)
New Universal Ratio reflecting charge Fractionalization

Charge excitations in gapless systems can be arbitrary

Those charges exist beyond the quantum average:
Current noise? (strongly modified by leads at ω=0)