Various “novel” electronic phases of matter

Steve Kivelson, Stanford University

1) Characterizing the phase.
(Broken symmetries. How far “beyond” Landau …)

~~2) What is it “good for?”~~

3) Theoretical demonstration that the phase has “a right to exist.”

4) Convincing experimental sightings.

KITP Conference: New Directions in Low Dimensional Electron Systems   February 23, 2009

Electronic Liquid Crystalline Phases:

Novel Superconducting Phases from “competing” Superconducting Spin-Density-Wave and Charge-Density-Wave Order

Electronic Liquid Crystalline Phases:

Novel Superconducting Phases **Intertwining** Superconducting Spin-Density-Wave and Charge-Density-Wave Order

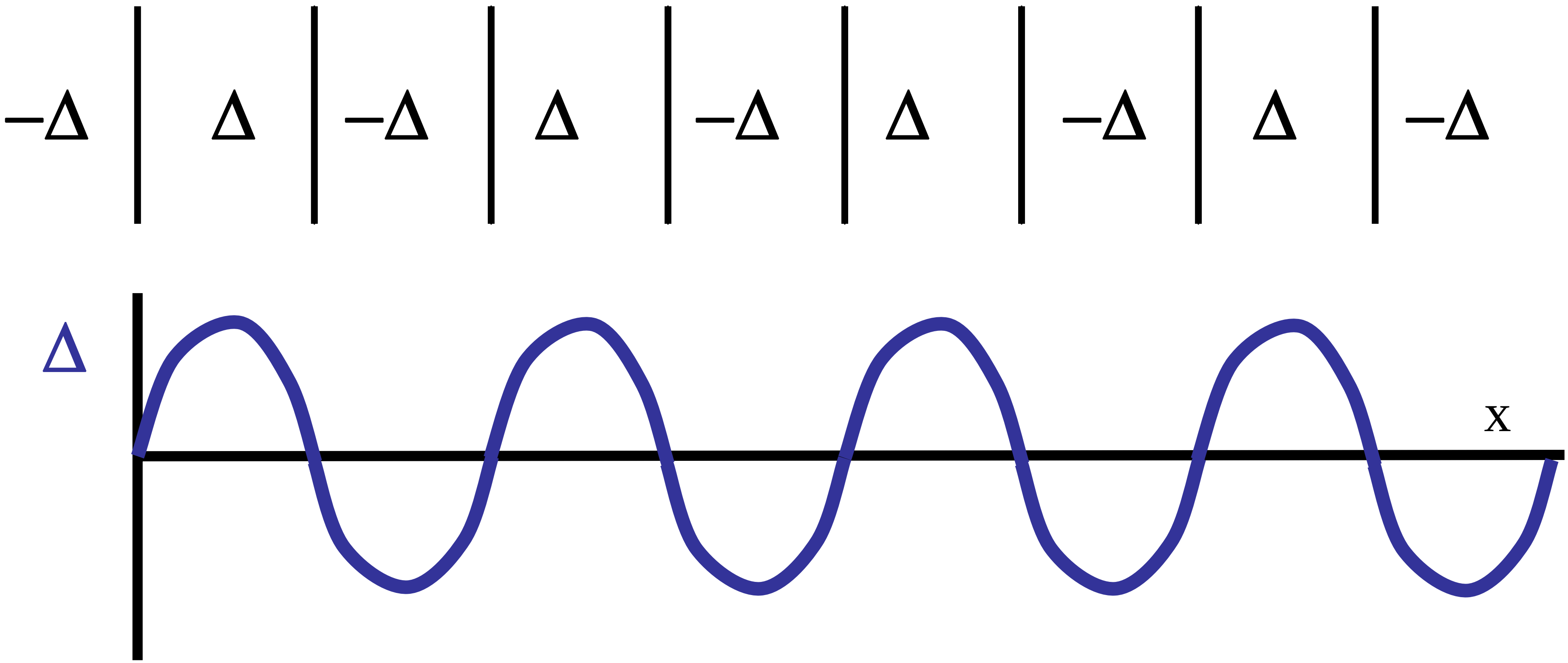
Pair density wave

or

Striped superconductor

Berg et al 2007, 2008; Baruch and Orgad 2008; Himeda et al 2002; Razkowski et al 2008; Capello et al 2008; Scalapino and White unpublished; Agterberg and Tsunetsugu 2008; Yang, Chen, Rice, Sigrist, Zhang unpublished; Berg, Fradkin, Kivelson, Tranquada unpublished.

Striped superconductor



$$< \Delta(x) > = \Delta \cos(Qx + \phi)$$

$$\overline{< \Delta(x) >} = 0 \qquad \overline{[< \Delta(x) >]^2} \neq 0$$

4

Striped superconductor

$$< \Delta(\mathbf{x}) > = \Delta \cos(\mathbf{Qx} + \theta)$$

Coupling to CDW order:

$$\Delta F = \lambda\, \rho(x)\, |\Delta(x)|^2 + \ldots$$

$$< \rho(\mathbf{x}) > = \rho_0 + \delta\rho \cos(\mathbf{2Qx} + \phi)$$

(Either the CDW order is the first harmonic of the striped SC order,
or the SC order is a subharmonic of the CDW order.)

Striped superconductor

$$< \Delta(x) > = \Delta \cos(Qx + \theta)$$

Gaps only the "nested" portions of the Fermi surface, leaving Fermi pockets with a finite density of states.

Like other density wave states.

Striped superconductor

Why might it want to exist?

Some reasons that it seems implausible:

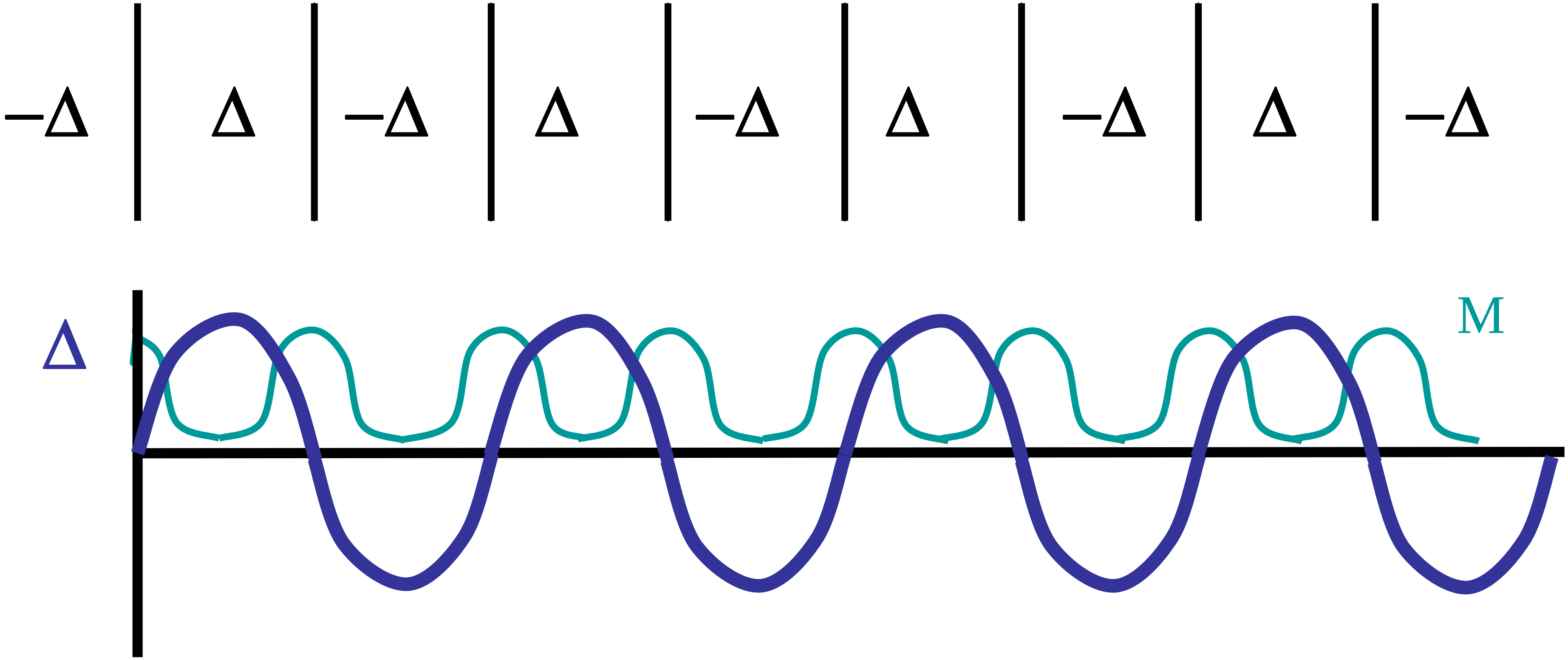
1) Cannot occur as the ground state of bosons.
2) Unlikely in BCS because perfect nesting at
   k & -**k** beats imperfect nesting at
   k & -**k** + **Q**
3) Hasn't been reported in the many numerical
   and experiental studies of superconductors.

Striped superconductor

Where does it come from?

I. FFLO (actually LO) state in presence of finite magnetization.

Striped superconductor (LO State)



$$< \Delta(x) > = \Delta \cos(Qx + \theta)$$

$$\phi - \theta = -\pi/2$$

$$< S(x) > = M_0 + \delta M \cos(2Qx + \phi)$$

Striped superconductor

Where does it come from?

I. FFLO (actually LO) state in presence of finite magnetization.

I am not sure of the status of this state in experiment.
It is a distinct state with explicit time reversal symmetry breaking and finite magnetization.

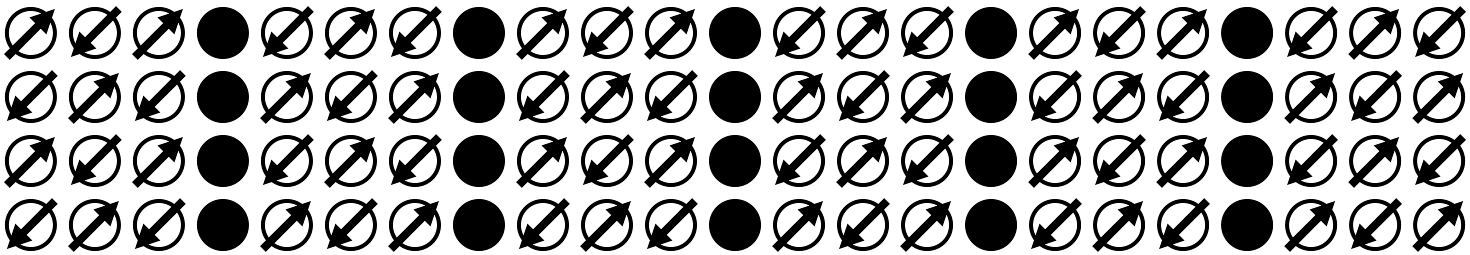
However, closely related phenomena, maybe in cold atoms, result from such a state.

Radzihovsky and Vishwanath - 2009

Striped superconductor

Where does it come from?

Related to stripe phases (unidirectional CDW and SDW) seen in some cuprates.



$$\langle \rho(\mathbf{x}) \rangle = \rho_0 + \delta\rho \, \cos(2Qx + \phi)$$

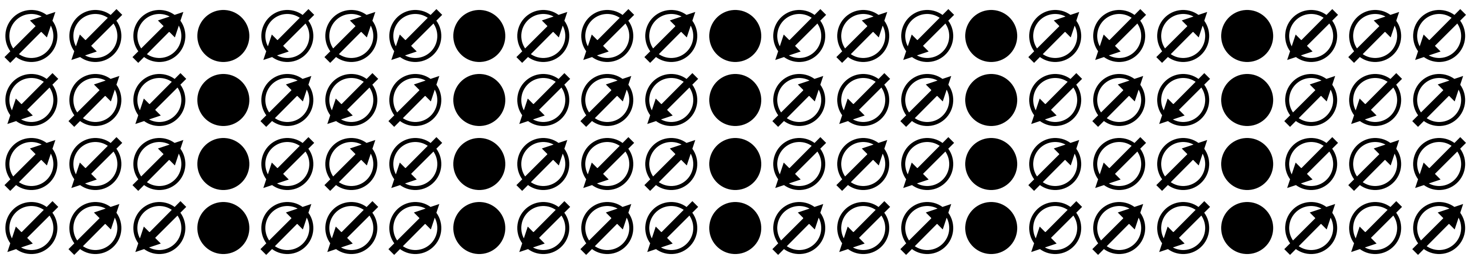
$$\langle \mathbf{S}(\mathbf{x}) \rangle = \mathbf{S} \cos(\pi x + \pi y) \, \cos(Qx + \phi_s)$$

“Topological doping”

Striped superconductor

Where does it come from?

Related to stripe phases (unidirectional CDW and SDW) seen in some cuprates.



$$< \rho(\mathbf{x}) > = \rho_0 + \delta\rho \, \mathbf{\cos(2Qx)}$$

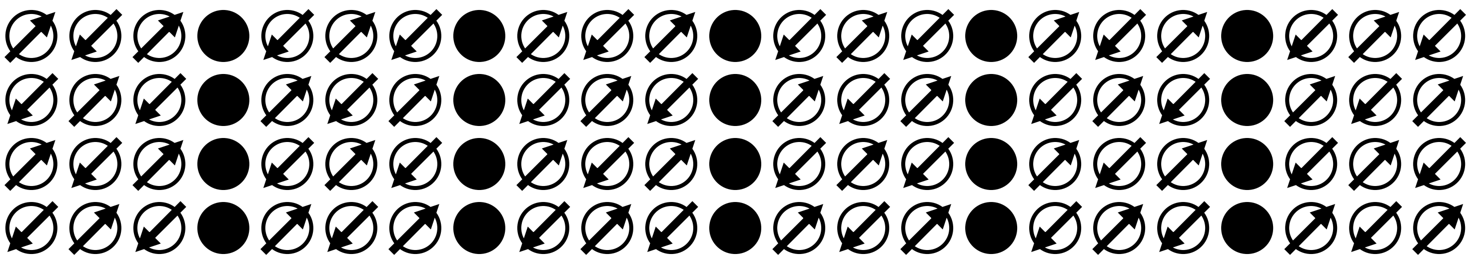
$$< \mathbf{S}(\mathbf{x}) > = \mathbf{S} \cos(\pi \mathbf{x} + \pi \mathbf{y}) \, \mathbf{\cos(Qx} + \pi/2\mathbf{)}$$

$$\boxed{< \Delta(\mathbf{x}) > = \Delta_0 + \Delta_1 \, \mathbf{\cos(2Qx)} + \ldots}$$

Striped superconductor

Where does it come from?

Related to stripe phases (unidirectional CDW and SDW) seen in some cuprates.



$$< \rho(\mathbf{x}) > = \rho_0 + \delta\rho \ \mathbf{cos(2Qx)}$$

$$< \mathbf{S(x)} > = \mathbf{S\ cos(\pi x + \pi y)}\ \mathbf{cos(Qx + \pi/2)}$$

$$\boxed{< \Delta(\mathbf{x}) > = \quad \Delta\ \mathbf{cos(Qx + \theta)} \quad + \ \ldots}$$

Striped superconductor

Does it have a right to exist?

$U' > 10t$ $n=1$

$U < 0$ $n < 1$

$U' > 10t$ $n=1$

$U < 0$ $n < 1$

$U' > 10t$ $n=1$

$U < 0$ $n < 1$

14

$U' > 10t$ $n=1$

Striped superconductor

Does it have a right to exist?

We have “solved” this model using interchain perturbation theory and using DMRG to treat up to 5 x long systems.

-U/t

QuickTime™ and a decompressor are needed to see this picture.

<n>

Striped superconductor

What is it good for?

1) It accounts for some dramatic experimental observations in $La_{1.875}Ba_{0.125}CuO_4$

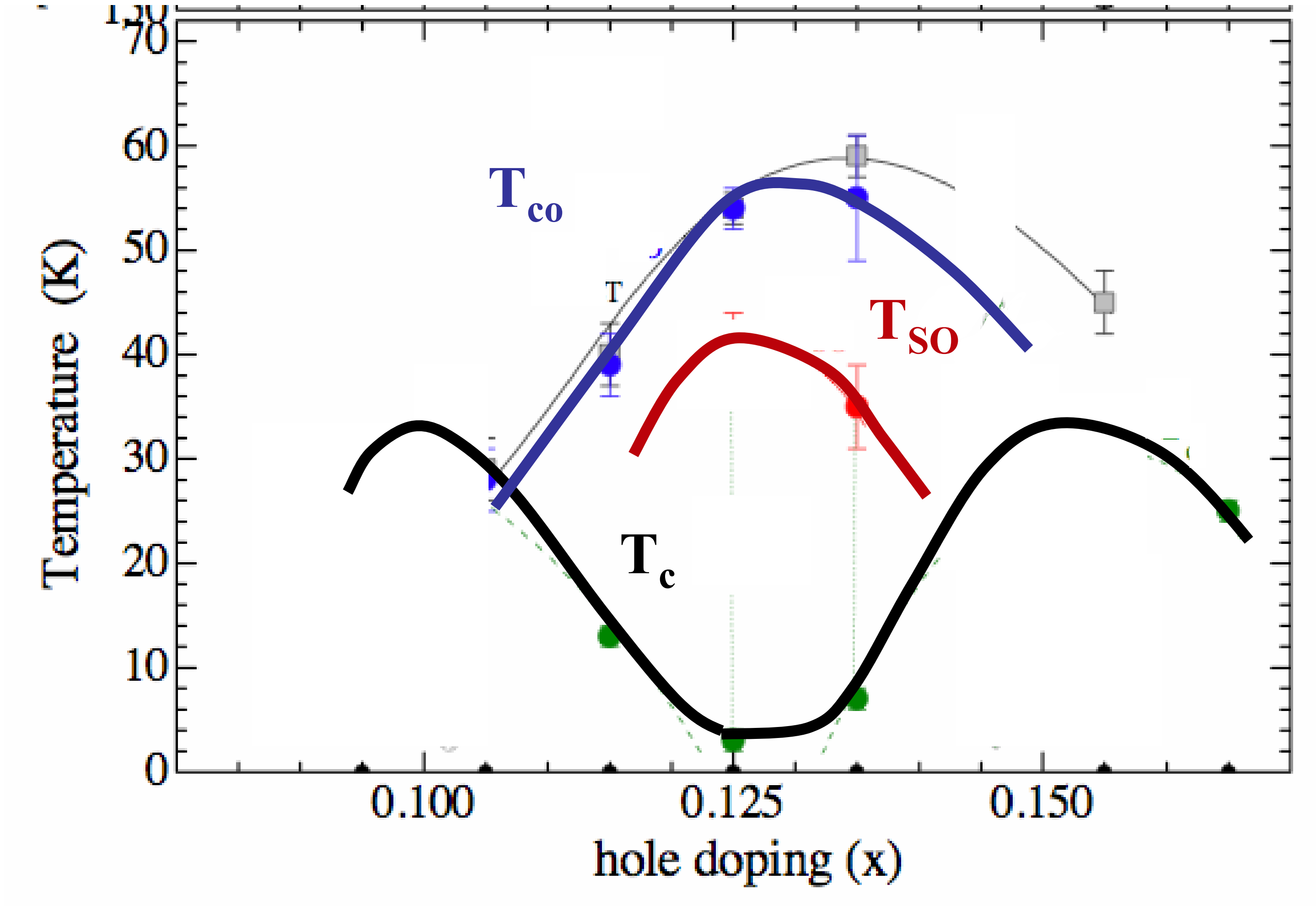
2) It is the "mother" of other interesting phases.

Striped superconductor

Convincing experimental sightings?

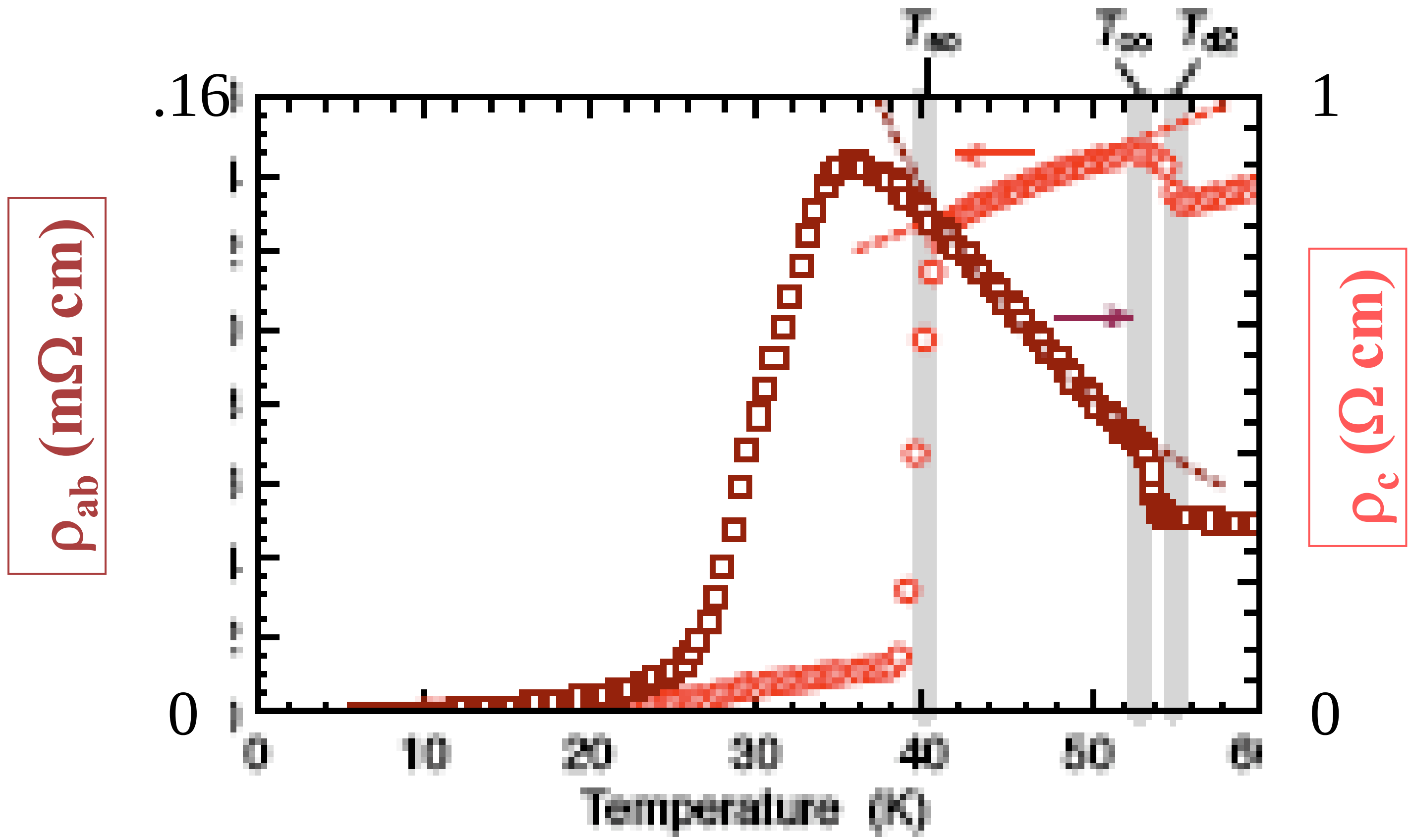
The striped superconductor:

“1/8 anomaly” in $La_{2-x}Ba_xCuO_4$



Tranquada, Hücker, and Gu 2008

In plane and interplane resisitivity of LBCO with x=1/8



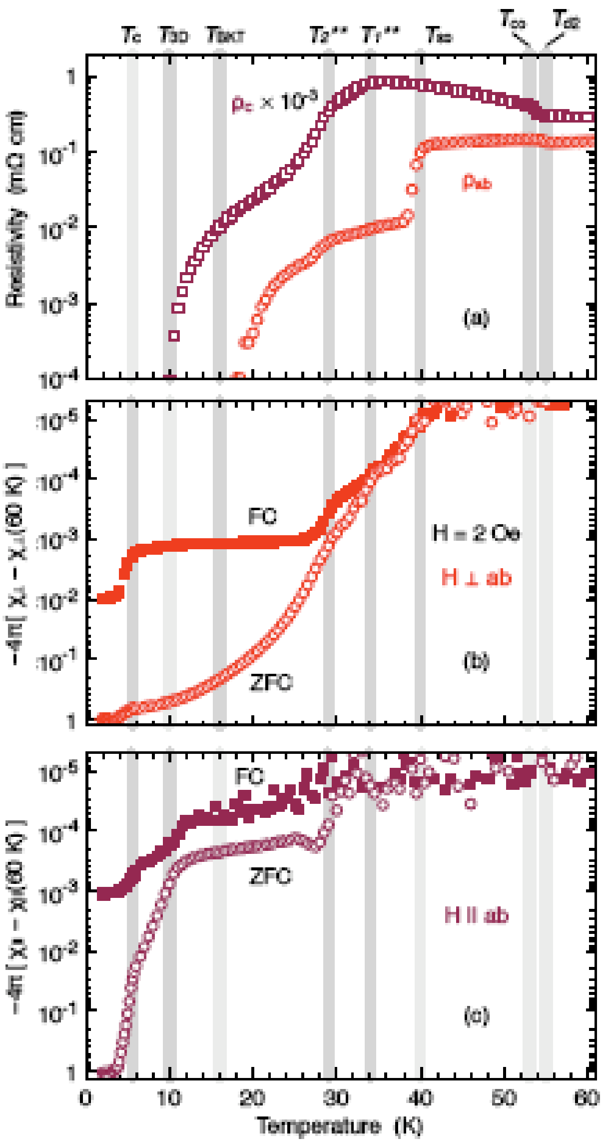
Tranquada *et al*, Phys. Rev. B 2008

Resistivity, thermal transport, specific heat, and diamagnetism in LBCO T

Tranquada *et al*, Phys. Rev. B 2008

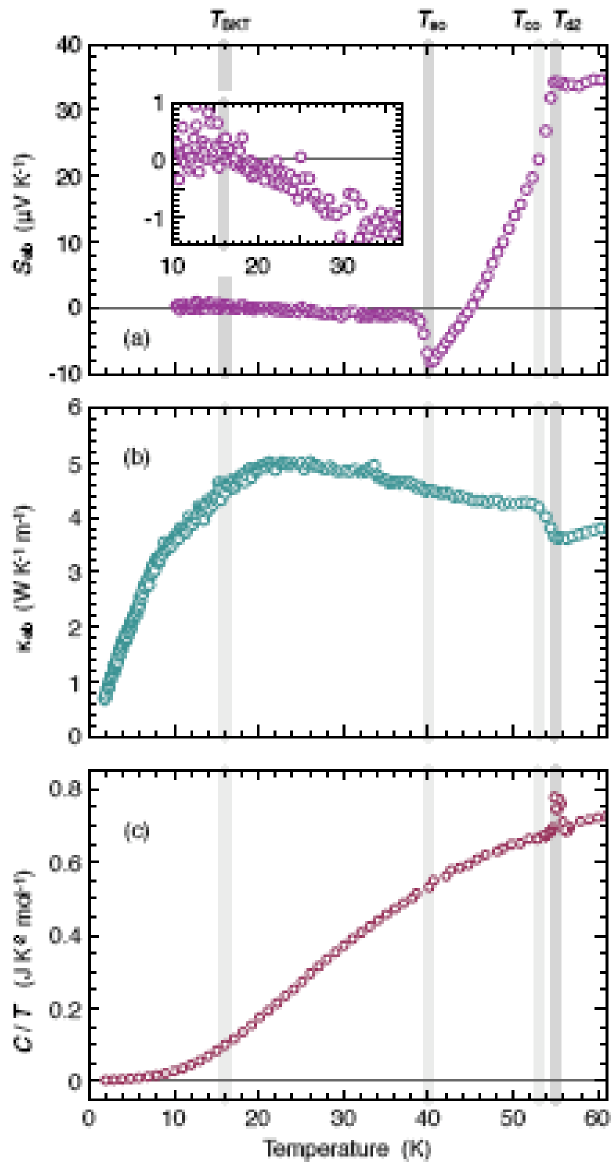
ρ_{ab}

ρ_c



$\chi_{ab}(T) - \chi_{ab}(60K)$

$\chi_{ca}(T) - \chi_{ca}(60K)$



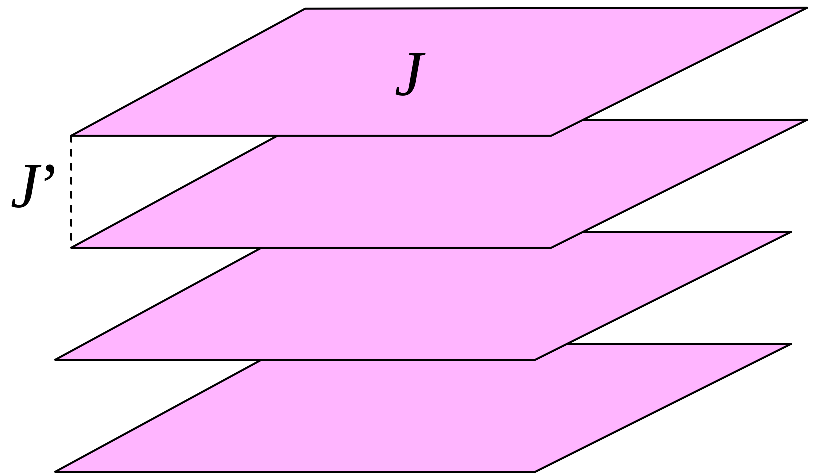
S_{ab}

κ_{ab}

C/T

2D S.C. in a 3D system

Anisotropic xy model:



$J >> J'$

Single layer coherence length, ξ, diverges exponentially at T_{KT}

Effective interlayer coupling

21

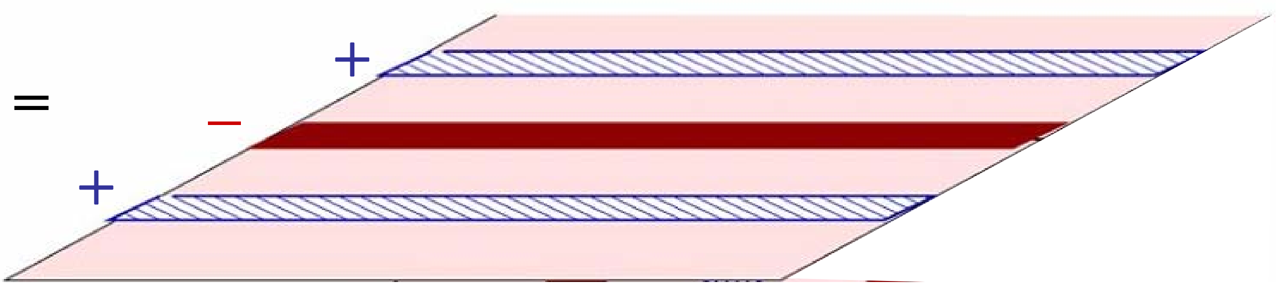
$J^{eff} \sim J' (\xi/a)^2$ Gets rapidly large neat T_{KT}

Vanishing inter-layer coupling?

- **Inter-layer coupling "frustration"?**
  - **(This is what makes the undoped AF parent compound, La_2CuO_4, so strongly 2D.)**
- **Superconducting order usually occurs at *q* = 0 (i.e., *uniform*) and so cannot be frustrated.**
- **However, if in LBCO the in-plane SC order occurs at finite *q* (*modulated SC order*)…**

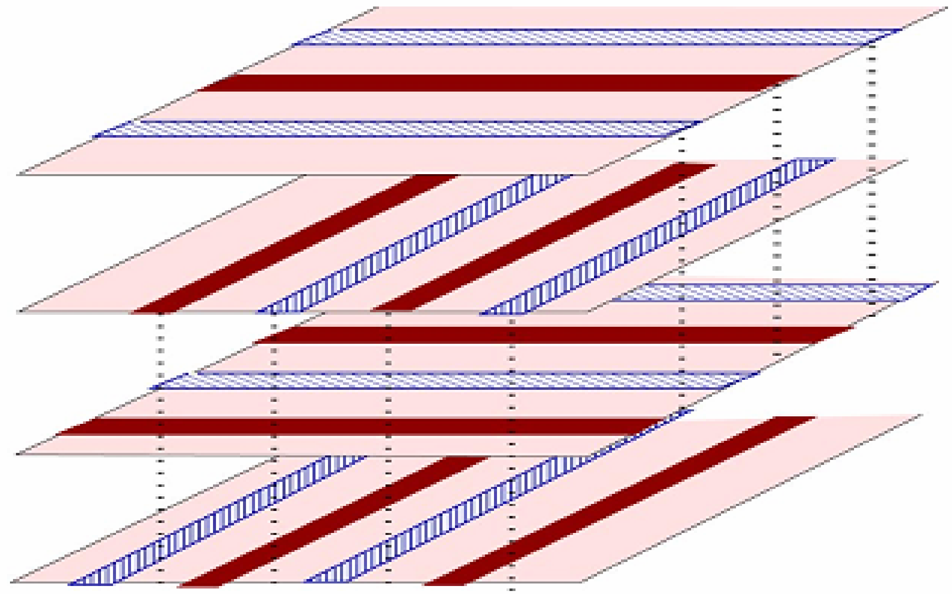
A "striped superconductor":

$\Delta(\mathrm{r}) =$



Dynamical layer decoupling

LBCO LTT structure:



Interlayer Josephson coupling:

$$J_1' = J_2' = J_3' = 0!$$

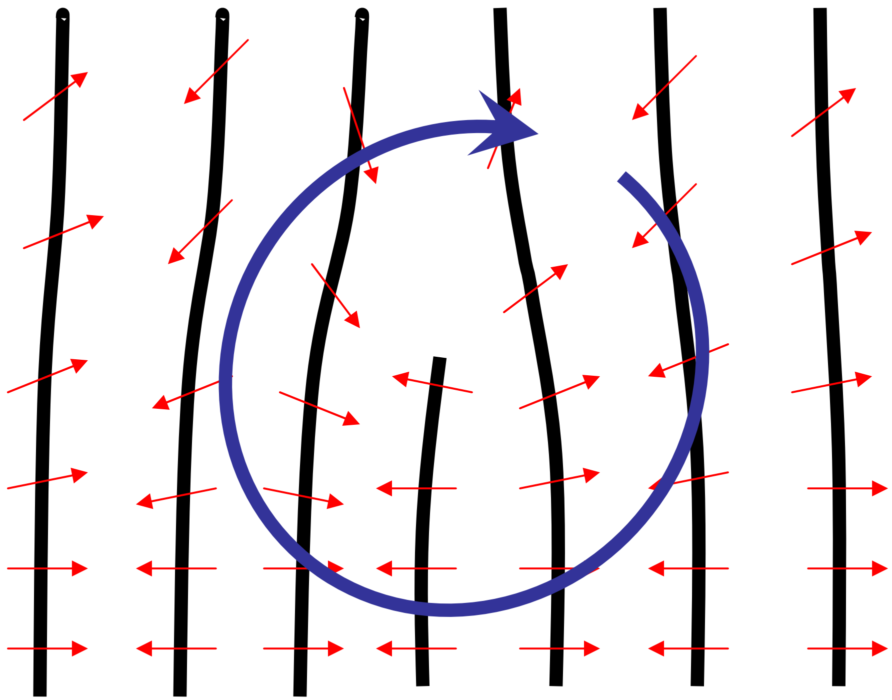
E. Berg, E. Fradkin, E-A. Kim, S. Kivelson, V. Oganesyan, J. Tranquada, S-C. Zhang, PRL (2007)

Coupling to disorder

Defect in the stripe order:

S.C. phase

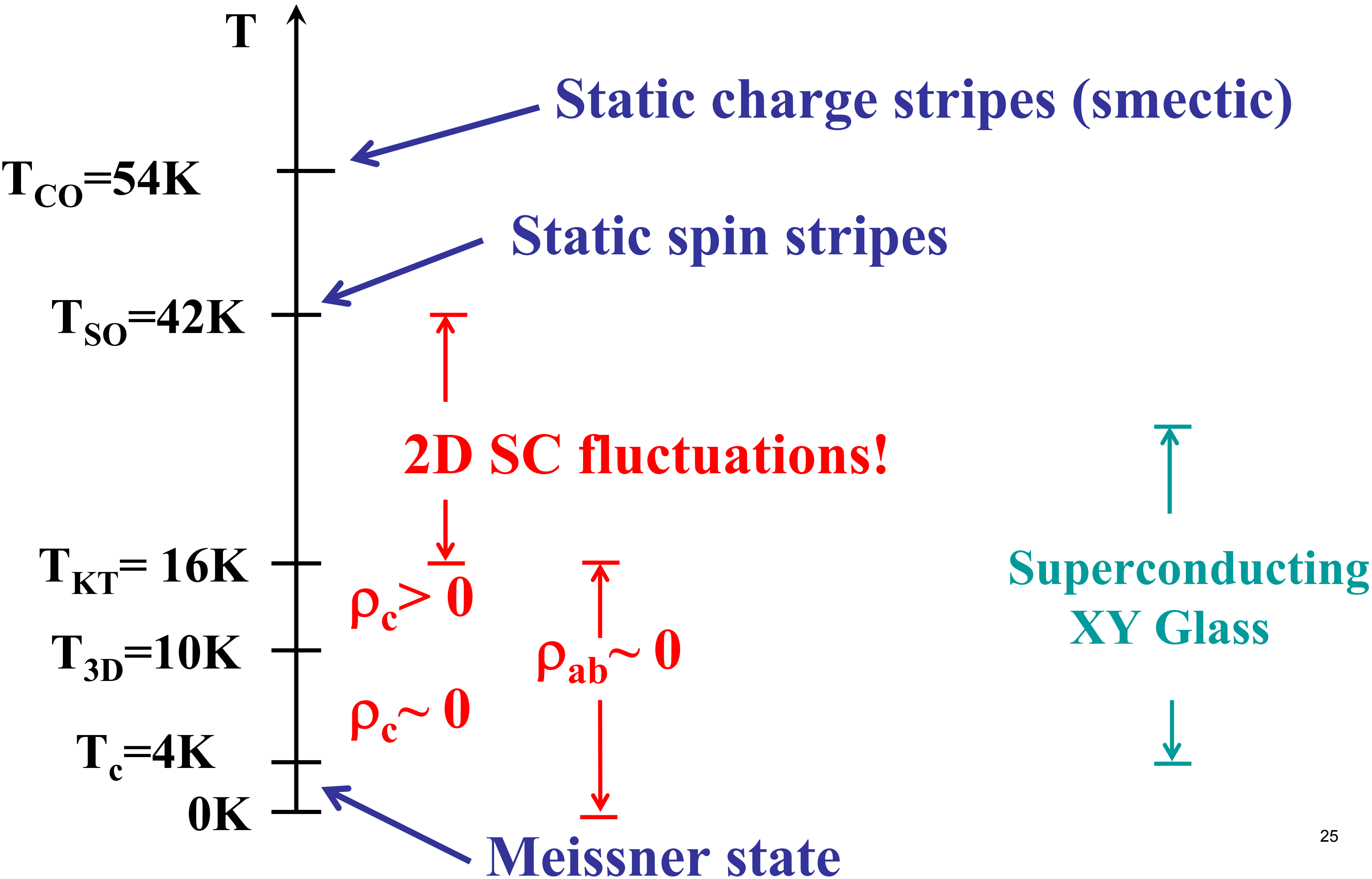
(pseudo-spin):



- **binding of ±π vortices to dislocations**
- **Prevents macroscopic phase coherence**

- **XY spin glass physics, time reversal symmetry breaking, …**

Cascade of thermal transitions/crossovers



Compelling indirect evidence of striped superconductor.

1) Dynamical interlayer decoupling:
   Frustrated interlayer Josephson coupling

2) Arrested superconducting coherence in plane:
   Frustration produced by quenched disorder

3) A variety of unusual glassy phenomena associated with the frustration of SC order.

Striped superconductor

New phases obtained by partial melting of striped SC

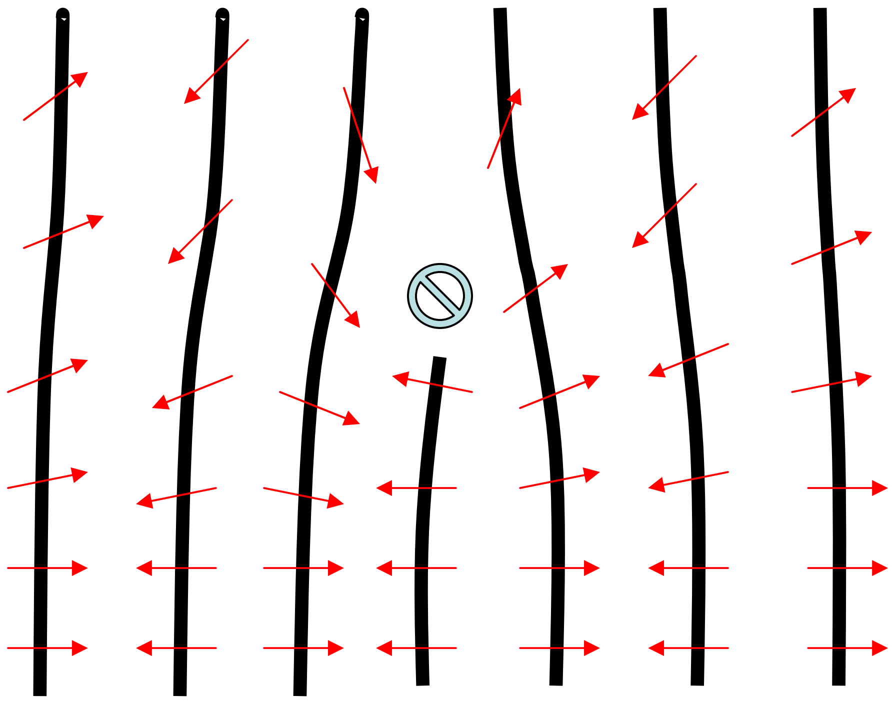
Topological defects in a striped SC:

Vortex - hc/2e - ordinary

But dislocations in the CDW order are a bit more interesting

Bound-state of a half vortex and a dislocation

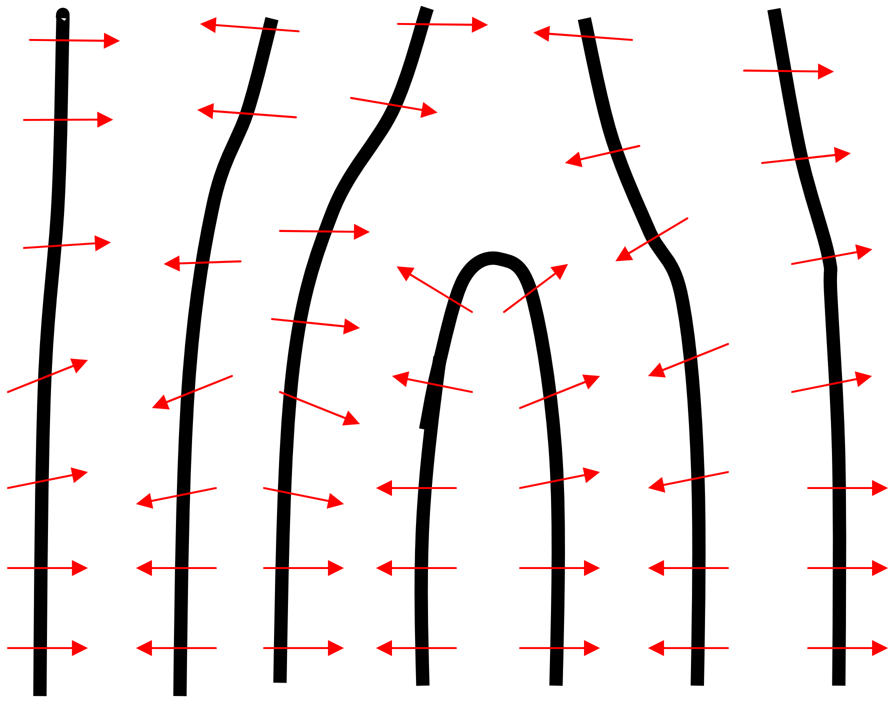
S.C. phase:



$\Phi_Q = hc/4e$

Double dislocation

S.C. phase:



(Bound state of two dislocations)

Striped superconductor

New phases obtained by partial melting of striped SC

Topological defects in a striped SC:

Double Dislocation: $\mathbf{E_{2xD} \sim 4\,\kappa\ \log [\, R/a \,]}$

Vortex : $\mathbf{E_V \sim \rho_s\ \log [\, R/a \,]}$

Dislocation + 1/2 vortex: $\mathbf{E_{D+V/2} \sim [(1/4)\,\rho_s + \kappa]\ \log [\, R/a \,]}$

(Also disclinations etc., assumed to be unimportant until higher temperatures)

Depending on ρ_s/κ, melting can proceed in various ways.

Striped superconductor

New phases obtained by partial melting of striped SC

Thermal transitions with increasing T from striped SC at T=0:

$$\boxed{\begin{array}{l} E_{2xD} \sim \mathbf{4\,\kappa}\ \log[R/a]; \qquad E_V \sim \boldsymbol{\rho_s}\ \log[R/a]; \\ \qquad E_{D+V/2} \sim \mathbf{[(1/4)\,\boldsymbol{\rho_s} + \boldsymbol{\kappa}]}\ \log[R/a] \end{array}}$$

If $\kappa >> \rho_s$ Striped SC: striped CDW: nematic: isotropic

If $\kappa \sim \rho_s$ Striped SC: nematic: isotropic

If $\rho_s >> \kappa$ Striped SC: charge 4e nematic SC:
nematic: isotropic

Charge 4e Superconductor

Charge 4e nematic superconductor obtained by partial melting of striped SC

$$< \Delta(\mathbf{x}) > = 0 \qquad [< [\Delta(\mathbf{x})]^2 >] \neq 0$$

Remaining vortices have $\Phi_0 = hc/4e$

Does not require formation at a microscopic level of Cooper quartets.

Cannot be described by a quadratic (BCS-like) mean-field effective Hamiltonian.

We have proposed experiments to search for this state in LBCO.

New phases obtained by partial melting of striped states

Nemematic Fermi Fluid

Spontaneous breaking of the point group symmetry,
e.g. from C_4 to C_2.

(Familiar in complex classical fluids, but
where are the "nematigens?")

(Where crystal field effects are significant,
hard to distinguish from "orthorhombicity.")

(In some cases, this can also be viewed as
"orbital ordering.")

Nematic Fermi Fluid

What's it good for?

Can account for some striking experimental results.

Can produce non-Fermi liquid behavior.

Nematic Fermi Fluid

Theoretical proof of its right to exist.

Thermal dislocation melting of stripe (smectic) phase produces classical nematic. (No controlled theory of quantum melting exists as far as I know.)

Strong coupling limit of Cu-O model - *Fradkin, Kivelson, Geballe 2003*

Pomeranchuk instability for bands near a van-Hove singularity - *Metzner, Kee, Kim, Raghu, SAK, Oganesyan, Fradkin, Kee and Wu …*

Frustrated magnets - *many people, but rigorous proof in Biskup, Chayes, and SAK 2006*

Nematic Fermi Fluid

Experimental sightings:

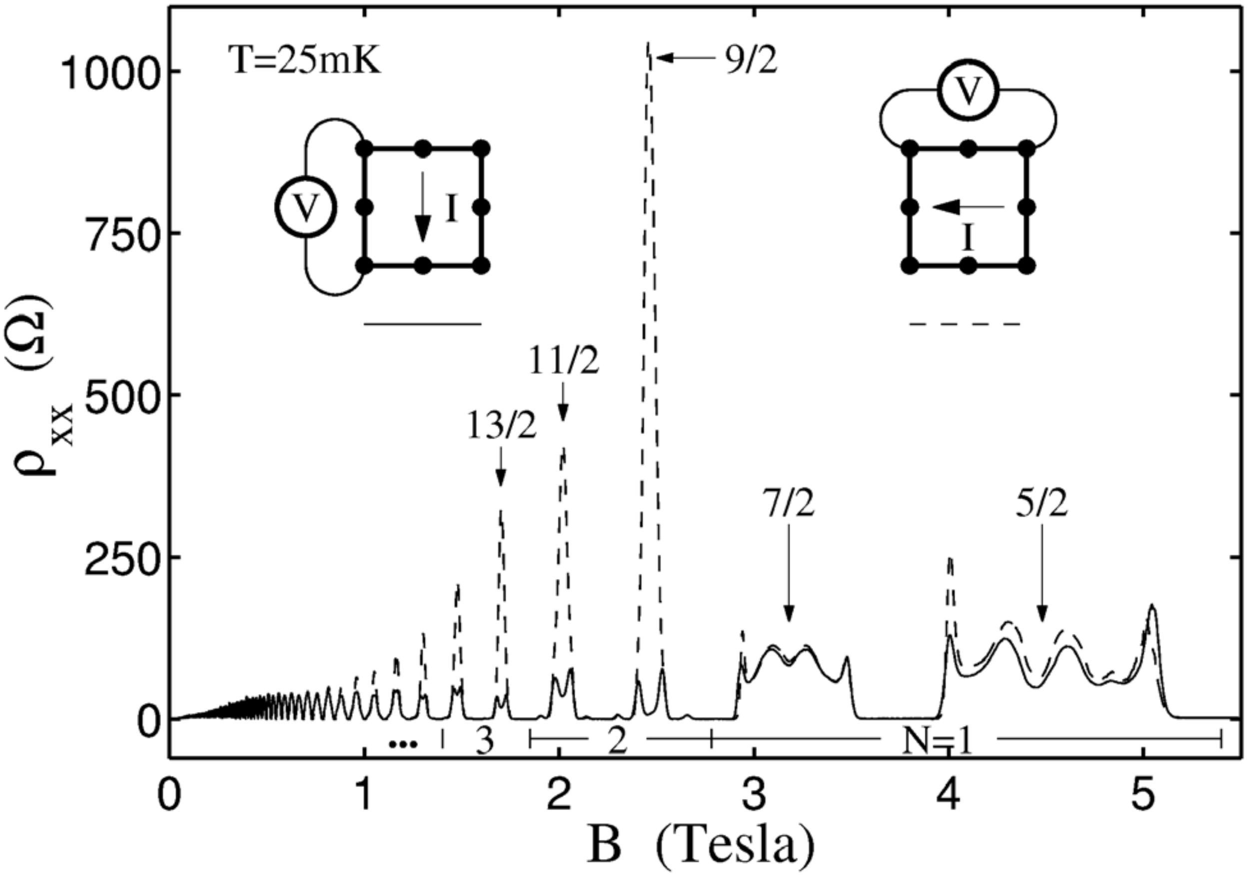
Quantum Hall systems at $\nu = n + 1/2$

Anisotropic State in Higher Landau Levels

Eisenstein and collaborators

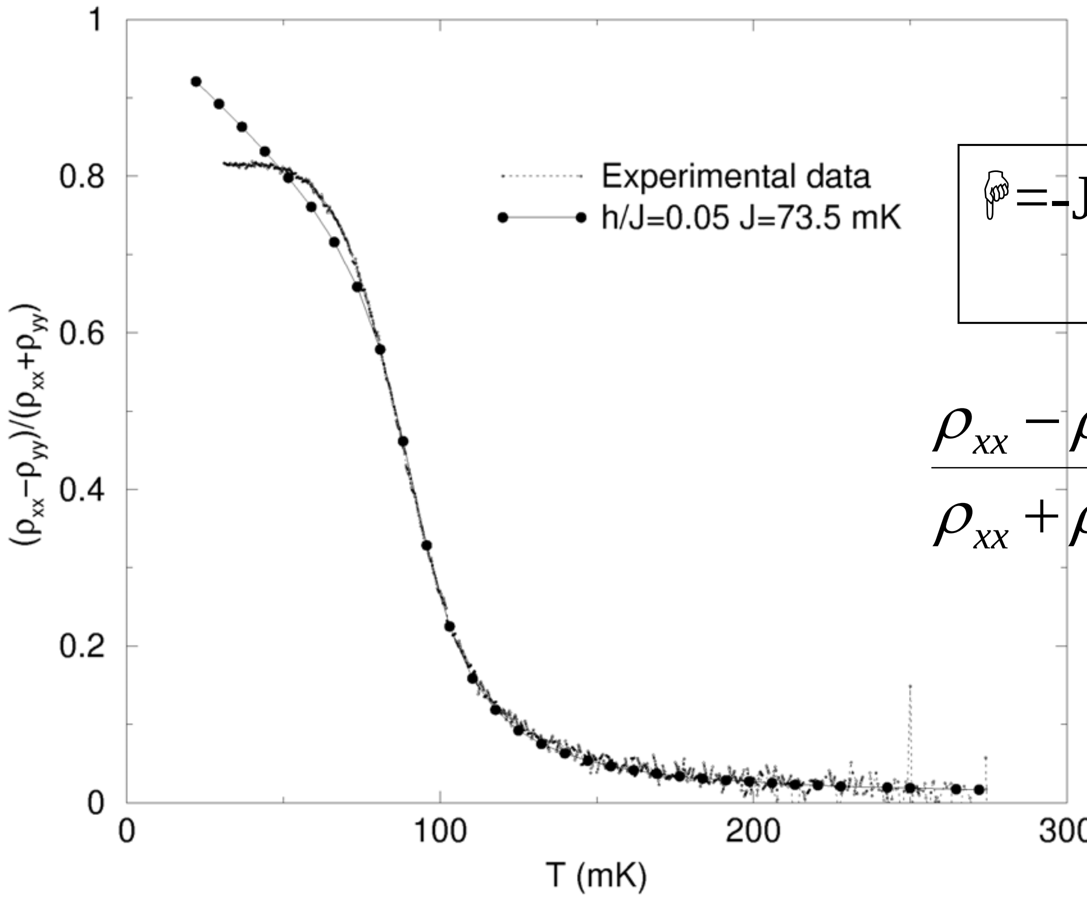
For a review, see Eisenstein et al 1999

$\nu = n_{el}\,\phi_o$



Ultra high mobility 2DEG in GaAs-GaAlAs heterostructures

Comparison with classical XY model of Nematic transition



☟$= -J\cos(2\theta_i - 2\theta_j)$
$-h\cos(2\theta_i)$

$$\frac{\rho_{xx} - \rho_{yy}}{\rho_{xx} + \rho_{yy}} \sim \langle \cos(2\theta) \rangle$$

Eisenstein and collaborators
For a review, see Eisenstein et al 1999

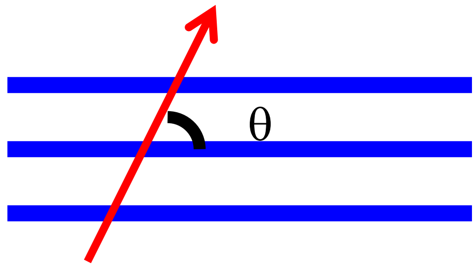
Nemataic Fermi Fluid

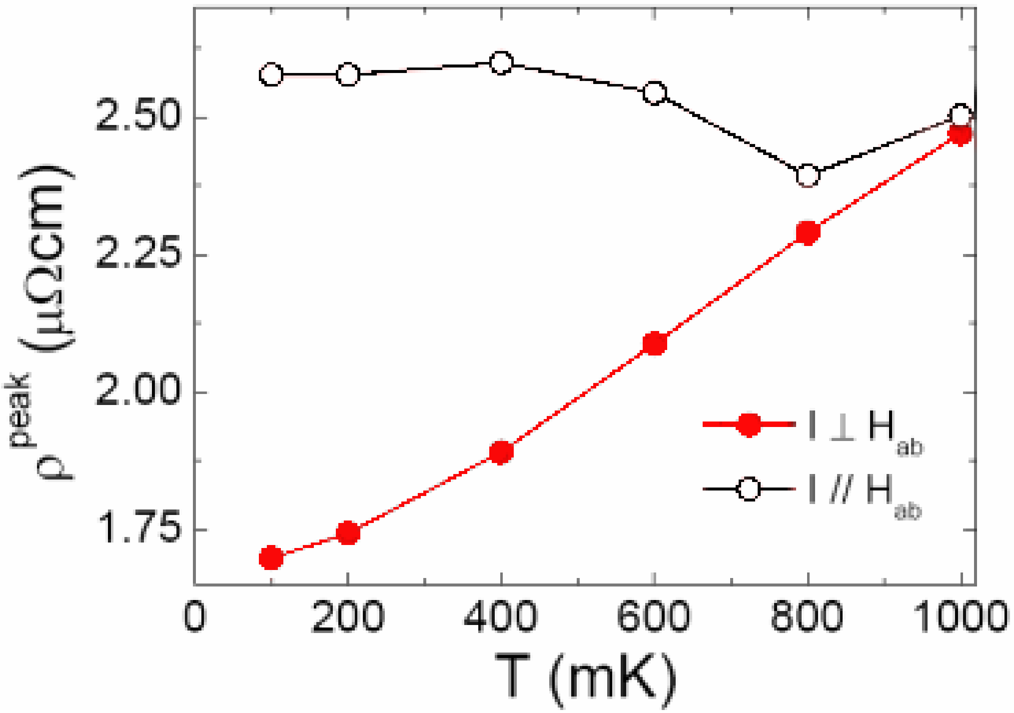
Experimental sightings:

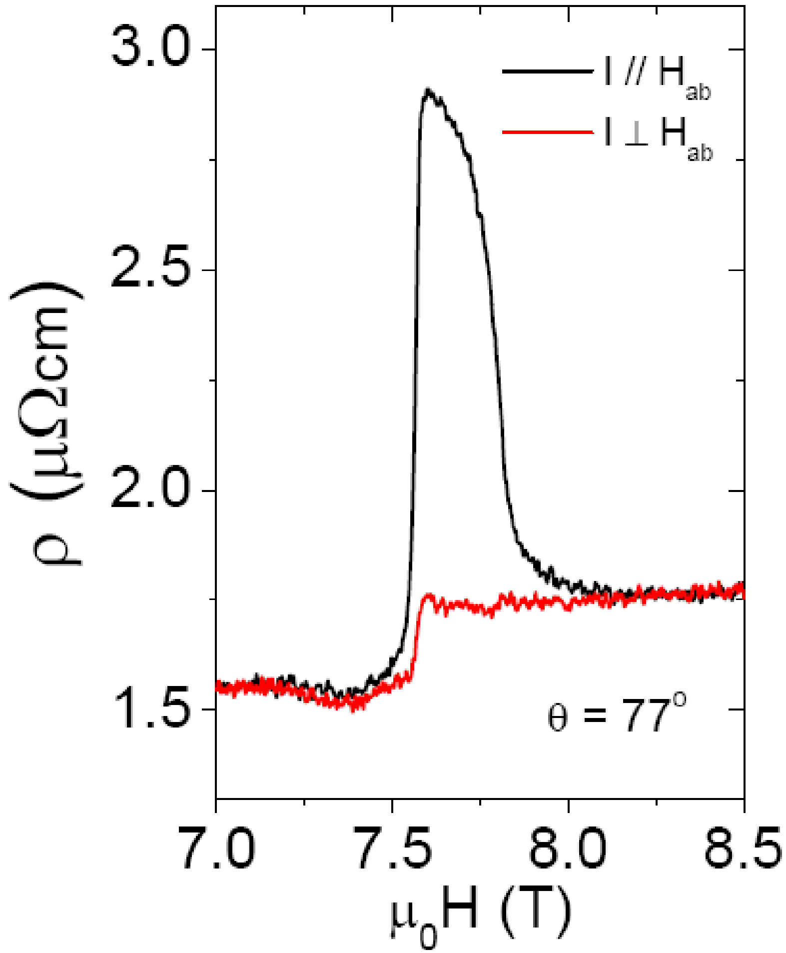
Quantum Hall systems at $\nu = n + 1/2$

Metamagnetic phase in $Sr_3Ru_2O_7$

Anisotropic resistivity: temperature dependence







No orthorhombicity detectable in X-ray scattering

Borzi *et al*, Science 2007

Under consideration (and debate)

Nematic phase in the cuprates-
especially $YBa_2CuO_{7-\delta}$

Nematic phase in the oxy-pnictides

A long list of novel electronic phases in 2D (or quasi 2D)

1) **Striped SC** - "seen" in LBCO

2) **Charge 4e nematic superconductor** - a mechanism found and a candidate material identified.

3) Nematic Fermi fluid - "seen" in $\nu = n+1/2$ QHE, $Sr_3Ru_2O_7$ probably seen in YBCO, maybe seen in Fe-pnictides

4) **Spin liquids** - Several new poofs of principle.

QuickTime™ and a
decompressor
are needed to see this picture.

QuickTime™ and a
decompressor
are needed to see this picture.

Continuous topological phase transition as a function J'/J from a non-Abelian to an Abelian phase

QuickTime™ and a
decompressor
are needed to see this picture.

QuickTime™ and a
decompressor
are needed to see this picture.

For topological computing:

Spontaneous time reversal symmetry breaking eliminates the need for large magnets.

A long list of novel electronic phases in 2D (or quasi 2D)

1) **Striped SC** - “seen” in LBCO
2) **Charge 4e nematic superconductor** - a mechanism found and a candidate material identified.
3) **Nematic Fermi fluid** - “seen” in $\nu = n+1/2$ QHE, $Sr_3Ru_2O_7$ probably seen in YBCO, maybe seen in Fe-pnictides
4) **Nodal spin liquid** insulator with 3 electrons per unit cell - found in solvable model.
5) **Chiral spin liquid** (with 2 electrons per unit cell) which spontaneously breakes time reversal symmetry, and can undergo a phase transition from an Abelian to a non Abelian state - found in solvable model.
6) **BZA - spin liquid** with 3 electrons per unit cell and spinon pseudo-Fermi surface found in solvable model with fine tuning to a critical line.