Two-loop QCD corrections to heavy-to-light quark transitions

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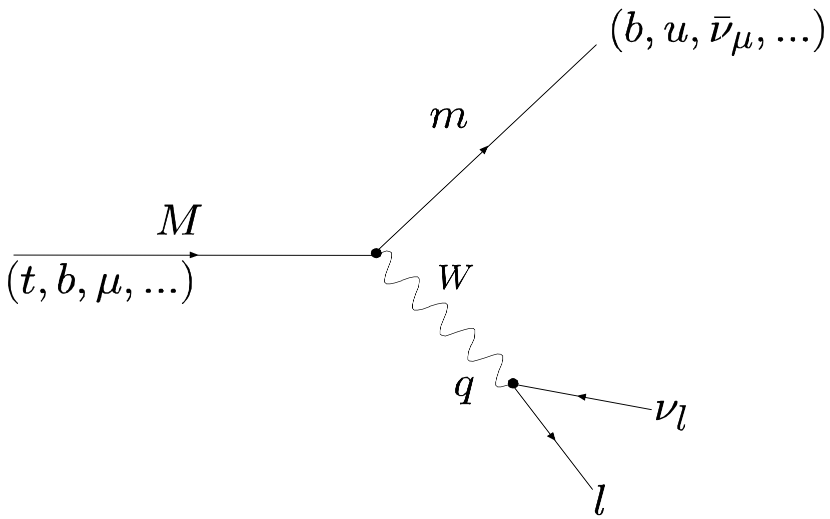
Outline

1. Introduction & motivation
2. Methods: optical theorem, asymptotic expansions, recurrence relations
3. Results
4. Conclusions

Introduction

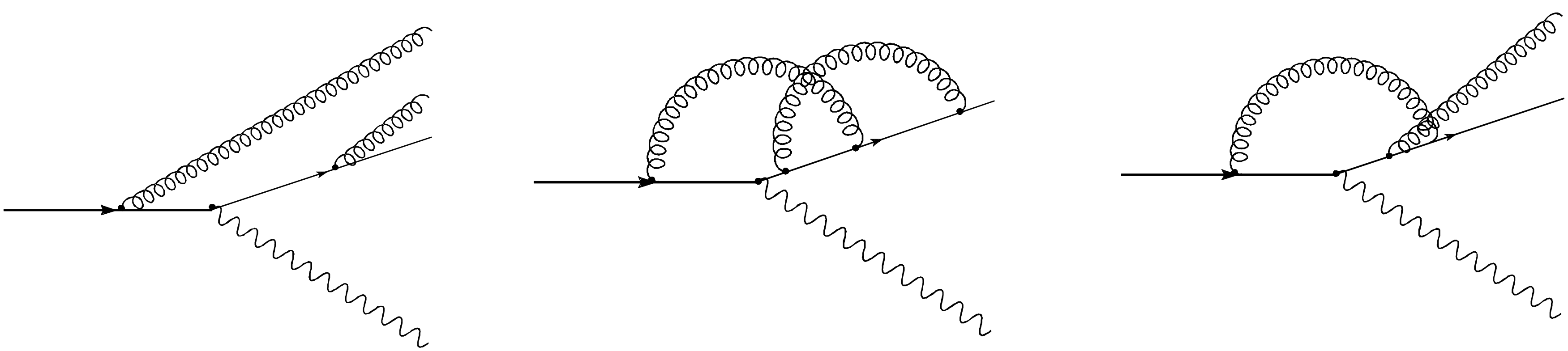
Generic situation:



$M \gg m$

q = invariant mass of leptons

2-loop corrections:



Motivation for studying heavy-to-light decays

- Many important applications:
  - $t \to bW$ decay rate
  - $b \to u$ and muon decay differential width
- Real challenge – charged massive particle in the initial state
- They become feasible thanks to a recent breakthrough in computing methods
- The same methods can be used for other processes: b non-leptonic decays, B_s mixing …

Applications of 2-loop corrections to heavy-to-light decays

- Top quark decay rate
  - ➢ Very short lifetime:

$$\frac{1}{\tau_t} = 175 \text{ MeV } \left(\frac{m_t}{m_W}\right)^3 \simeq 1.5 \text{ GeV } \gg \Lambda_{QCD}$$

much shorter than typical confinement scale:

top behaves almost like a free quark !

  - ➢ t→bW dominant decay channel: $|V_{tb}| \simeq 1$

Can shed light on new physics but high accuracy SM predictions needed !

- What is known so far from theory side
  - Tree level: $\Gamma_0 \simeq 1.5\ \mathrm{GeV}$
  - NLO QCD corrections: $\simeq -8.4\,\%\ \Gamma_0$

    (Jezabek, Kuhn 1989)
  - NLO electroweak: $< +2\,\%\ \Gamma_0$

    (Denner, Sack; Eilam *et al.* 1991)

Up to now theoretical uncertainty mainly due to NNLO QCD contributions.

Estimated at $\simeq -2\,\%\ \Gamma_0$ (Czarnecki, Melnikov 1999; Chetyrkin *et al.* 1999)

- $b \longrightarrow u l \bar{\nu}_l$ decay

  - Total decay rate known (van Ritbergen, Stuart 1999) – our result provides a crosscheck

  - differential decay rate:

$$\frac{d\Gamma(b \longrightarrow u l \bar{\nu}_l)}{dq^2}$$

known in expansion around $q^2 = m_b^2$

- expansion around $q^2 = 0$ missing to date

- 2-loop QED corrections to $\mu \longrightarrow \nu_\mu e \bar{\nu}_e$ decay

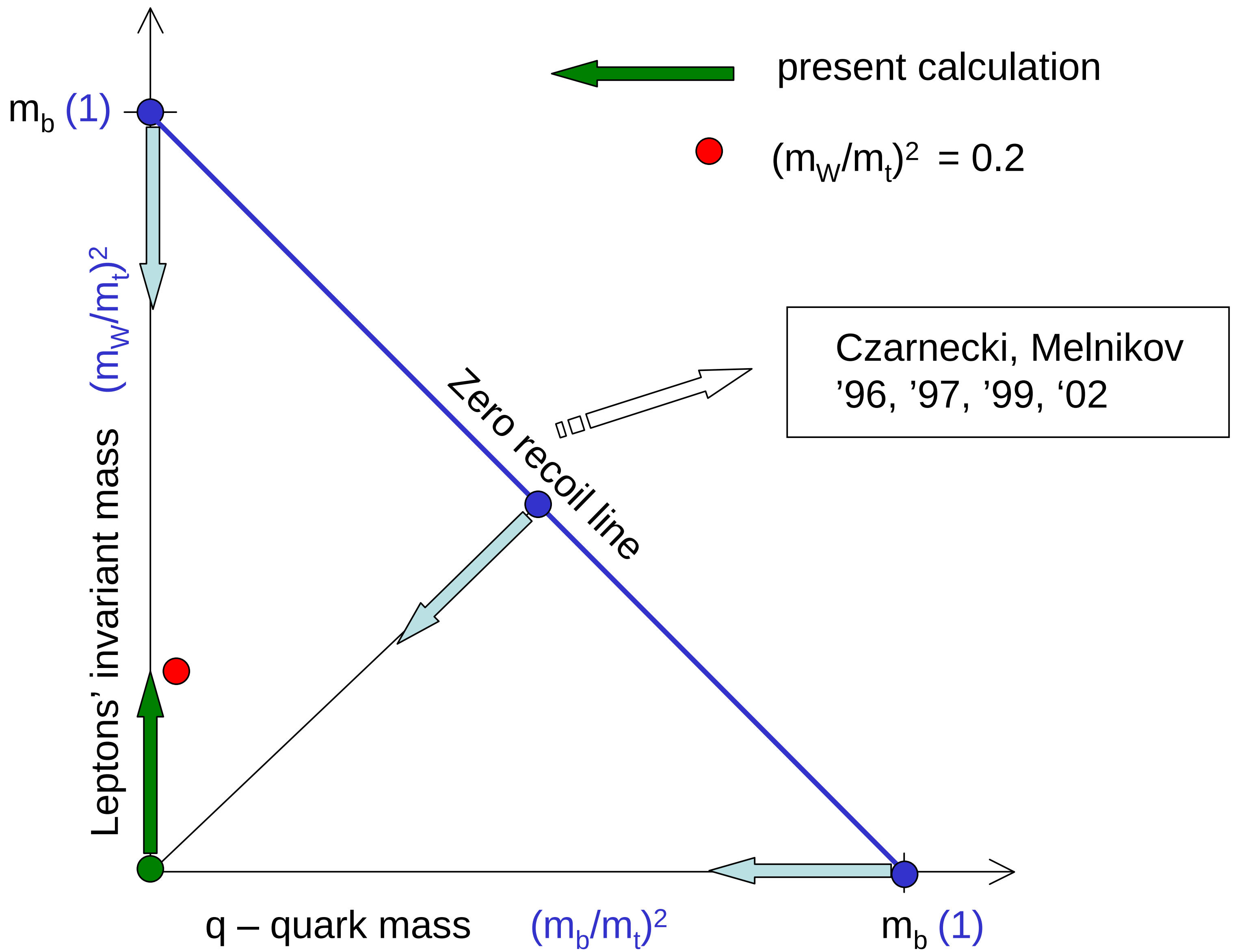
  - Charged particles on different fermion lines

charge retention order

  - Total decay rate known (van Ritbergen 1999)

  - We calculated differential decay rate in the full range of q^2

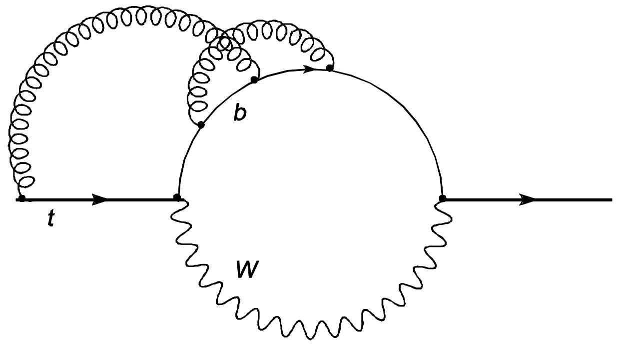
Summary of applications



Methods

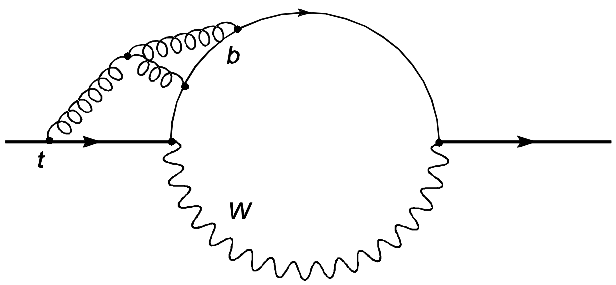
(a) Diagrams to be computed for $O(\alpha_s^2)$

Abelian



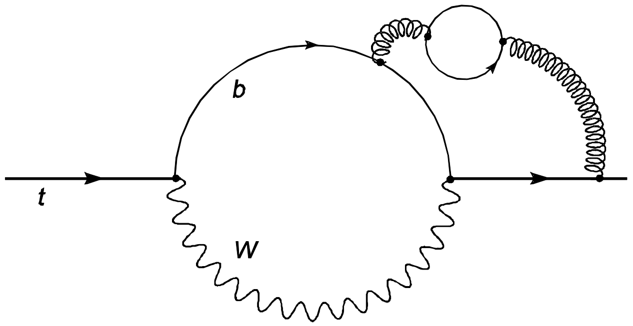
+ 18 terms

Non-abelian



+ 10 terms

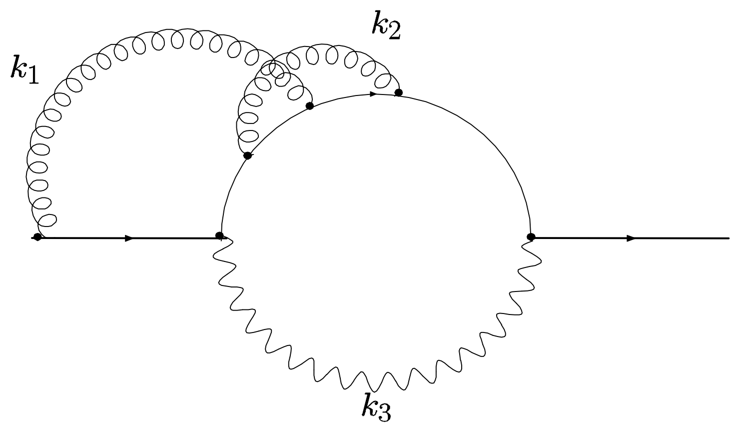
Vacuum polarization



+ 5 terms

(b) Asymptotic expansion

- two scales in the problem: m_t, m_W



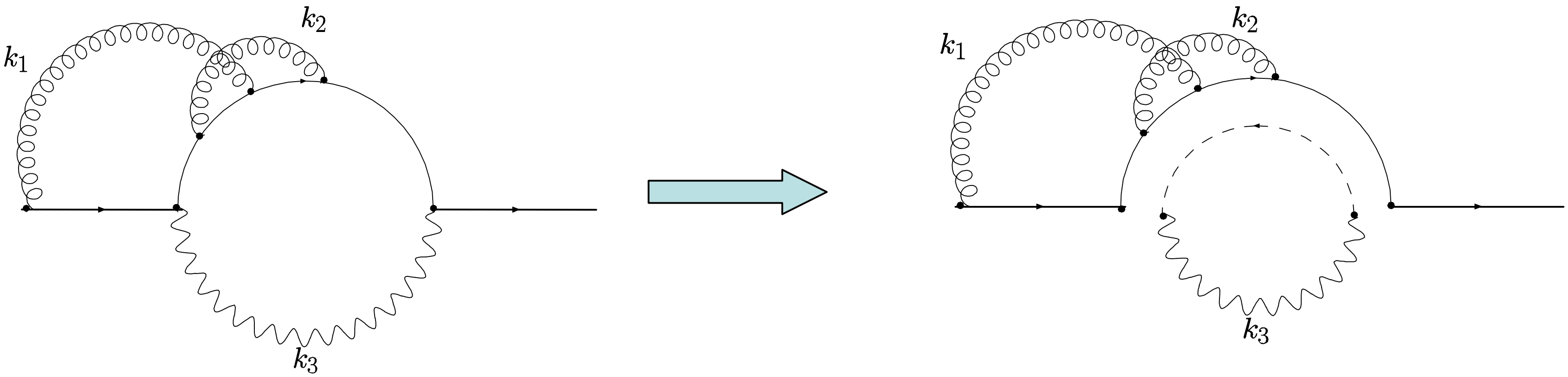
$$k_1, k_2 \sim m_t$$

- Hard region: $k_3 \sim m_t$

W propagator can be expanded as a series in powers of $(m_W/m_t)^2$:

$$\frac{k_\mu k_\nu - m_W^2 g_{\mu\nu}}{k^2 - m_W^2} = \frac{k_\mu k_\nu}{k^2} + \frac{m_W^2}{k^2}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) + \ldots$$

- Soft region: $k_3 \sim m_W$



- problem factorizes – much simpler than hard part
- does not arise in the leading order $(m_w/m_t)^2$

We end up with single-scale integrals

(c) Lorenz algebra, traces of γ matrices

- Performed automatically, each diagram reduced to a linear combination of scalar integrals

- 9 basic topologies in our problem

(d) Scalar integrals reduced to master integrals (MI) using recurrence relations

$$\int \frac{d^D k}{(2\pi)^D} \frac{\partial}{\partial k_\mu} \left[l_\mu f(p; k_1, \ldots, k_n) \right] = 0$$

How to solve the system of recurrence relations ?

- Traditional method “by inspection” – very time consuming
  - ➢ We programmed reduction procedures for all 9 basic topologies in FORM
- Fully automated and process independent approach – the Laporta algorithm (2001)
  - ➢ Generate integration-by-parts identities for all possible combinations of propagators
  - ➢ Solve large system of linear equations using Gauss elimination with a given ordering function
  - ➢ Modified version of the Laporta algorithm in dedicated computer algebra system PolarBear

- First time both approaches used simultaneously to obtain a new result

Traditional FORM implementation:

- ➢ much faster for simple topologies (a few minutes to calculate 6 terms of expansion)
- ➢ crashes for higher expansion terms
- ➢ prone to human mistakes

PolarBear:

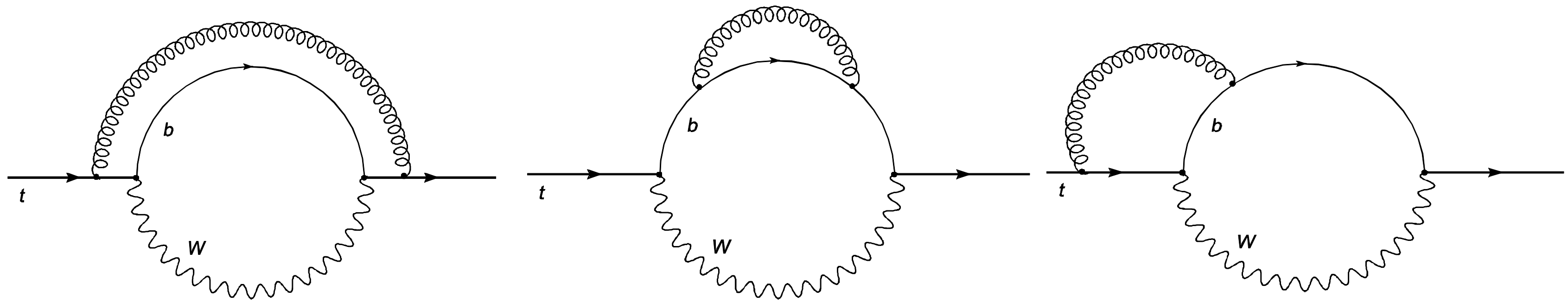
- ➢ each topology requires several hours computing time
- ➢ topologies can be computed in parallel
- ➢ very reliable

Plans for future: completely automatic tool

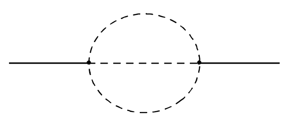
- Generate diagrams
- Identify distinct topologies and distribute them on the parallel computer cluster
- Reduction to MI independently on each node

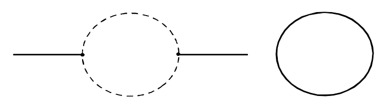
Some parts already exist (new graph generator, hyper efficient algorithm for Dirac traces in D-dim, symbolic solver…)

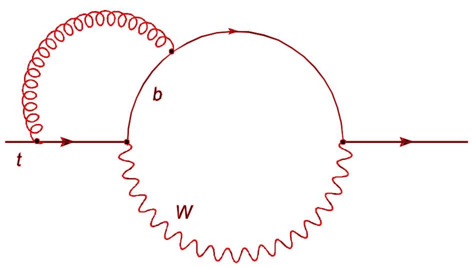
Example: NLO QCD Correction



- Can be reduced to 2 simple master integrals:

M_1 = 

M_2 = 

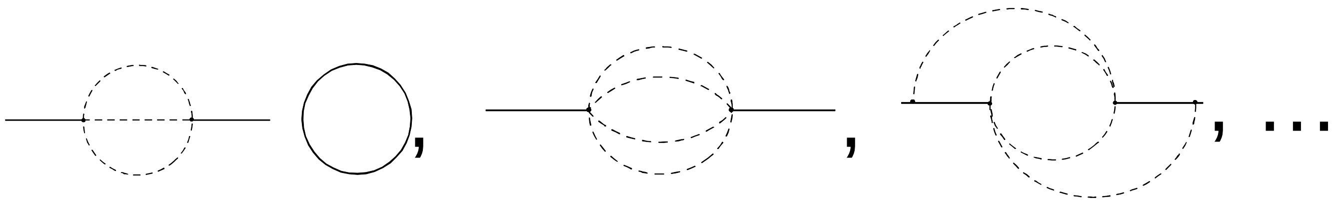


$$= \frac{1}{2\varepsilon^2}M_2 + \frac{1}{4\varepsilon}\left(M_1 - 5M_2\right) + \frac{1}{6}M_2 + \left(\frac{3}{4}M_1 - \frac{5}{12}M_2\right)\varepsilon + O(\varepsilon^2)$$

$$D = 4 - 2\varepsilon$$

Results

- Any integral in the hard region expressed in terms of 24 MI



- t $\rightarrow$ bW decay rate:

$$\Gamma(t \to bW) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 \right]$$

$$\Gamma_0 \equiv \frac{G_F m_t^3 \left|V_{tb}\right|^2}{8\sqrt{2}\pi}$$

X_0, X_1 known

- NNLO QCD contribution

$$X_2 = C_F \left(T_R N_L X_L + T_R N_H X_H + C_F X_A + C_A X_{NA}\right)$$

$$T_R = \frac{1}{2} \quad C_F = \frac{4}{3}, \quad C_A = 3, \quad N_L = 5, \quad N_H = 1$$

- Leading coefficients compared with numerical prediction

$$X_L = -\tfrac{4}{9} + \tfrac{23}{108}\pi^2 + \zeta_3 + \ldots \quad (\simeq 2.8594\ldots)$$ Num. 2.85(7)

$$X_H = \tfrac{12991}{1296} - \tfrac{53}{54}\pi^2 - \tfrac{1}{3}\zeta_3 + \ldots \quad (\simeq -0.06359\ldots)$$ Num. -0.06360(1)

$$X_F = 5 - \tfrac{119}{48}\pi^2 - \tfrac{11}{720}\pi^4 + \tfrac{19}{4}\pi^2 \log 2 - \tfrac{53}{8}\zeta_3 + \ldots \quad (\simeq 3.575\ldots)$$ Num. 3.5(2)

$$X_A = \tfrac{521}{576} + \tfrac{505}{864}\pi^2 + \tfrac{11}{1440}\pi^4 - \tfrac{19}{8}\pi^2 \log 2 + \tfrac{9}{16}\zeta_3 + \ldots \quad (\simeq -8.154\ldots)$$ Num. -8.15(7)

Expansion parameter: $\omega = (m_W/m_t)^2$

We constructed expansion up to ω^5

- Final result for top decay rate:

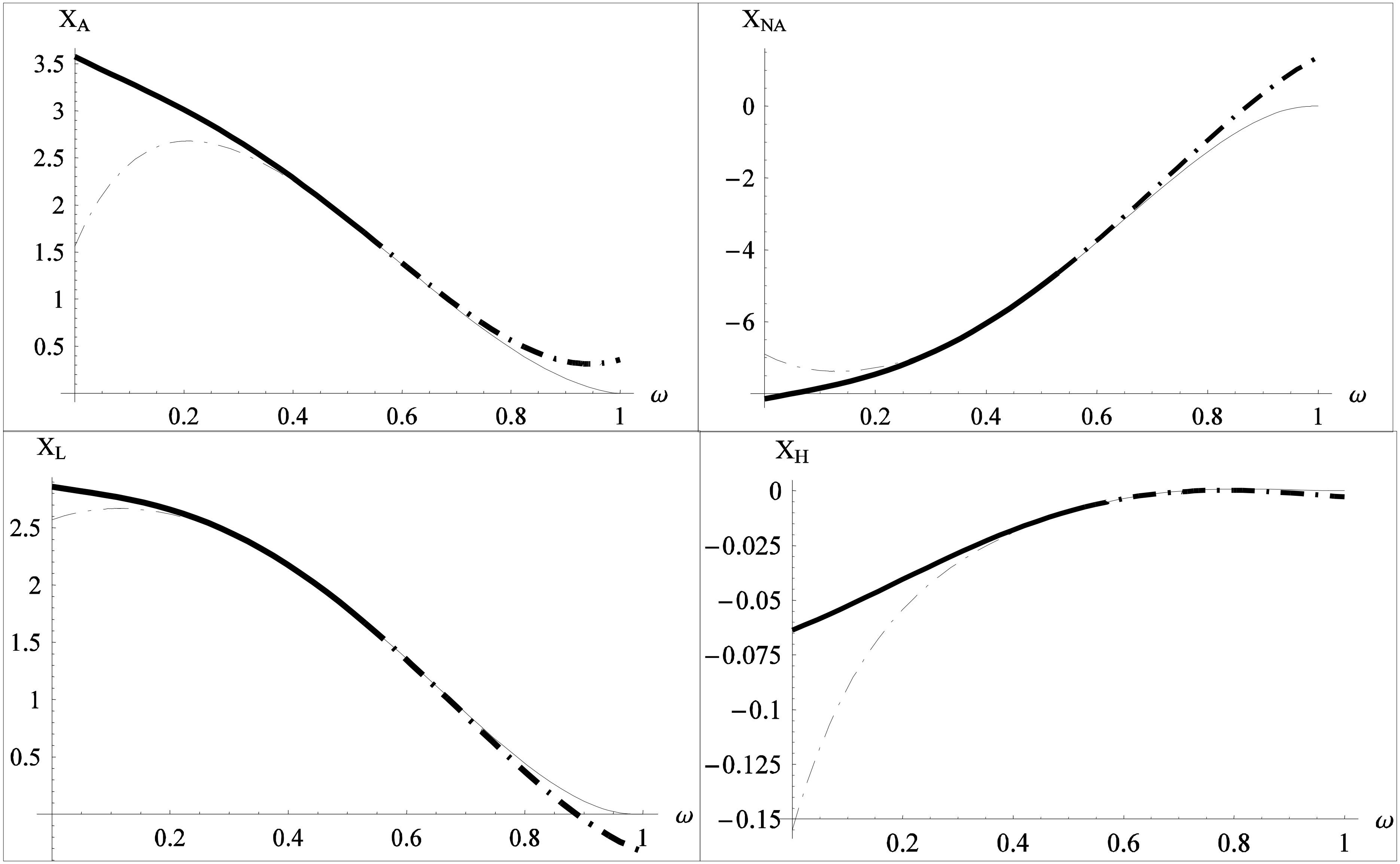
$$\omega \simeq 0.213 \longrightarrow X_2 = -15.5(1)$$

Error almost entirely due to inaccurate determination of m_t

- theoretical uncertainty 20 times smaller

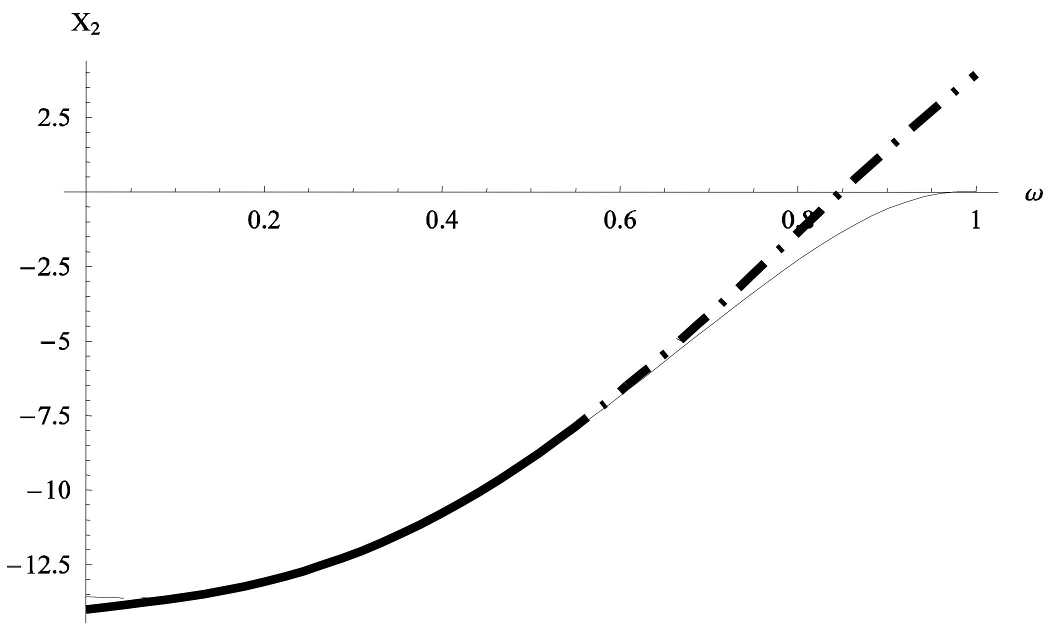
- NNLO QCD correction to t decay $\simeq -2.15\% \;\; \Gamma_0 X_0$

- Matching procedure with $\omega = 1$ for N_L=4



$\omega = 1$ - thin line $\omega = 0$ - thick line

- Differential decay rate for $b \longrightarrow ul\bar{\nu}_l$ (N_L=4)



Total rate for b and muon decay: $\Gamma_{sl} = \int_0^{M^2} dq^2 \frac{d\Gamma_{sl}}{dq^2}$

$b \longrightarrow ul\bar{\nu}_l \qquad \int_0^1 d\omega X_2(\omega) = -10.644$ (-10.648)

$\mu \longrightarrow \nu_\mu e\bar{\nu}_e \qquad \int_0^1 d\omega X_A(\omega) = 1.7797$ (1.7794)

Conclusions

- Modern computing methods made some previously unreachable calculations accessible
- We calculated new, analytical prediction for top decay rate
- Matching procedure enable us to obtain differential decay rate in the full range of leptons' invariant mass
- Crosscheck for b and muon total width
- Still much space for code optimizations and many physical applications ahead