Electroweak gauge boson rapidity distributions at NNLO

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- Physics motivations
- Description of the calculation
- Results: LHC, Tevatron, fixed target
- Conclusions

Physics motivations

- Why Drell-Yan at NNLO?
  - Extraction of parton distribution functions
    - At fixed target energies, where α_s is large
    - At high luminosities (LHC), where $\Delta\sigma_{stat}$ is small
  - LHC luminosity monitor Dittmar *et. al.*
  - Measurement of precision EW parameters: M_W, s_W^2

$\Rightarrow$ These require percent-level precision

Physics motivations

- Why differential distributions at NNLO?
  - For pdf extraction

$$\frac{d\sigma}{dY} = f_q\left(\sqrt{\frac{m_V^2}{s}}e^{Y}\right) f_{\bar{q}}\left(\sqrt{\frac{m_V^2}{s}}e^{-Y}\right) + \mathcal{O}(\alpha_S)$$

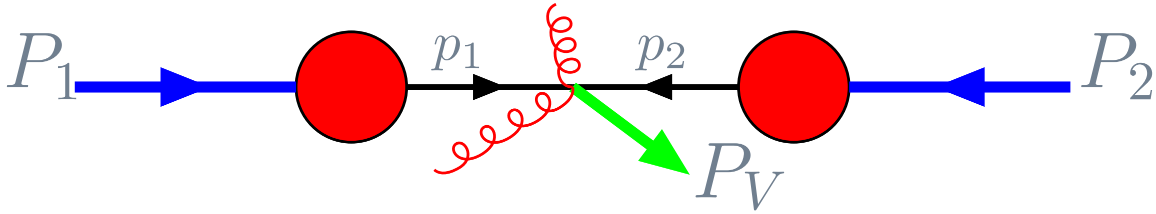
  - Partonic energy fractions fixed by M^2, Y

$$m_V^2 = x_1 x_2 s, Y = \ln(x_1/x_2)/2$$

  $\Rightarrow$ Need rapidity to reconstruct pdfs

$\Rightarrow$ Need distributions for most applications

Drell-Yan rapidity distribution



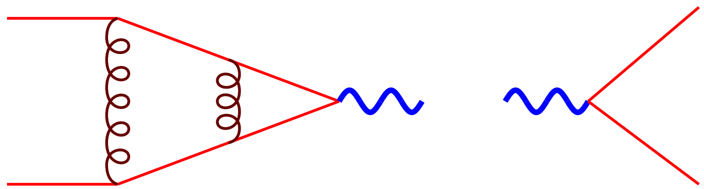
$$P_1 = \tfrac{1}{2}(1, \vec{0}, 1) \qquad p_1 = x_1 P_1$$
$$P_2 = \tfrac{1}{2}(1, \vec{0}, -1) \qquad p_2 = x_2 P_2$$
$$P_V = (E, \vec{p_T}, p_z)$$

Rapidity: $Y = \frac{1}{2} \log\left(\frac{E+p_z}{E-p_z}\right)$

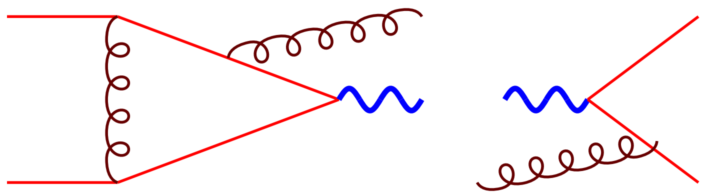
$$u = \frac{x_1}{x_2} e^{-2Y} = \frac{p_1 \cdot p_V}{p_2 \cdot p_V}$$

Anatomy of a NNLO calculation

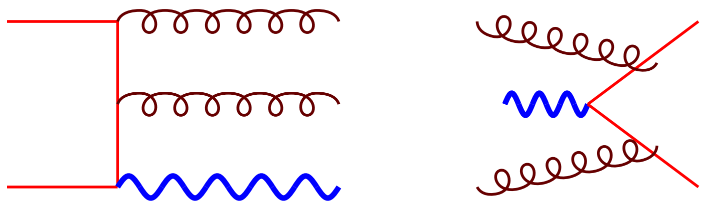
- Virtual-Virtual



- Real-Virtual



- Real-Real



Anatomy of a NNLO calculation

- Real-Virtual
  - One-loop $\times$ 2-particle PS $\Rightarrow$ simple
- Virtual-Virtual
  - Two-loop integrals $\Rightarrow$ not simple, but well studied
  - Loop integrals satisfy recurrence relations arising from Poincare invariance
  - Can reduce to a small set of independent master integrals
  - Can calculate using differential equations

Anatomy of a NNLO calculation

- Real-Real
  - Difficult and not well studied
  - Can we adapt multi-loop techniques to PS integrals?
  - Yes- use unitarity C. Anastasiou, K. Melnikov

$$\sigma_{\alpha\beta\to 1\ldots n} \propto \int \left[\prod_{i=1}^{n} d^d q_i \delta\left(q_i^2 - m_i^2\right)\right] \delta\left(p_{\alpha\beta} - q_{1\ldots n}\right) \times \left|\mathcal{M}_{\alpha\beta\to 1\ldots n}\right|^2$$

  - Cutkosky rules: $\delta\left(q_i^2 - m_i^2\right) \Rightarrow \frac{1}{q_i^2 - m_i^2 - i\epsilon} - \frac{1}{q_i^2 - m_i^2 + i\epsilon}$

$\Rightarrow$ Maps phase space integrals $\Rightarrow$ cut loop integrals

Differential distributions

- Can extend to differential quantities

$$\frac{d\sigma}{dY} \propto u \int \left[\prod_{i=1}^{n} d^d q_i \delta\left(q_i^2 - m_i^2\right)\right] \delta\left(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right)$$
$$\times \delta\left(p_{\alpha\beta} - q_{1\ldots n}\right) \left|\mathcal{M}_{\alpha\beta\to 1\ldots n}\right|^2$$

  - Replace the $\delta\left(q_i^2 - m_i^2\right)$ as before; also replace

$$\delta(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}) \Rightarrow \frac{p_2 \cdot P_h}{(p_1 - up_2)\cdot P_h - i\epsilon} - (+i\epsilon)$$

  - mass-shell condition $\rightarrow$ rapidity constraint

Extraction of singularities

- Matrix elements contains terms which behave as

$$|\mathcal{M}|^2 \propto \frac{1}{u-z}, \quad |\mathcal{M}|^2 \propto \frac{1}{1-uz} \quad \left(z = M^2/\hat{s}\right)$$

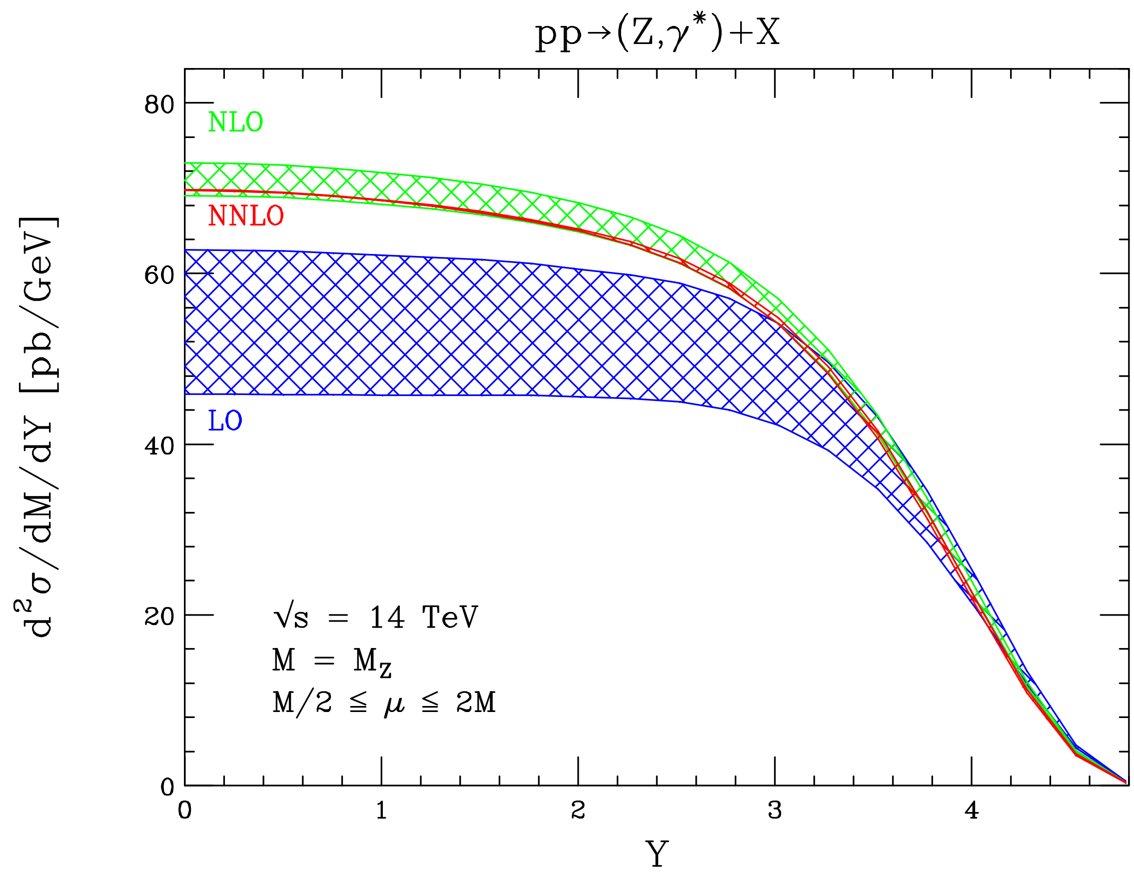
- Phase space contains the factor $\left[(u-z)\,(1-uz)\right]^{-2\epsilon}$
- Can separate singularities in u, z by setting

$$y = \frac{u-z}{(1-z)(1+u)}$$

- Phase space becomes $\left[y(1-y)(1-z)^2 f(y,z)\right]^{-2\epsilon}$

$\Rightarrow$ Can extend to more differential quantities

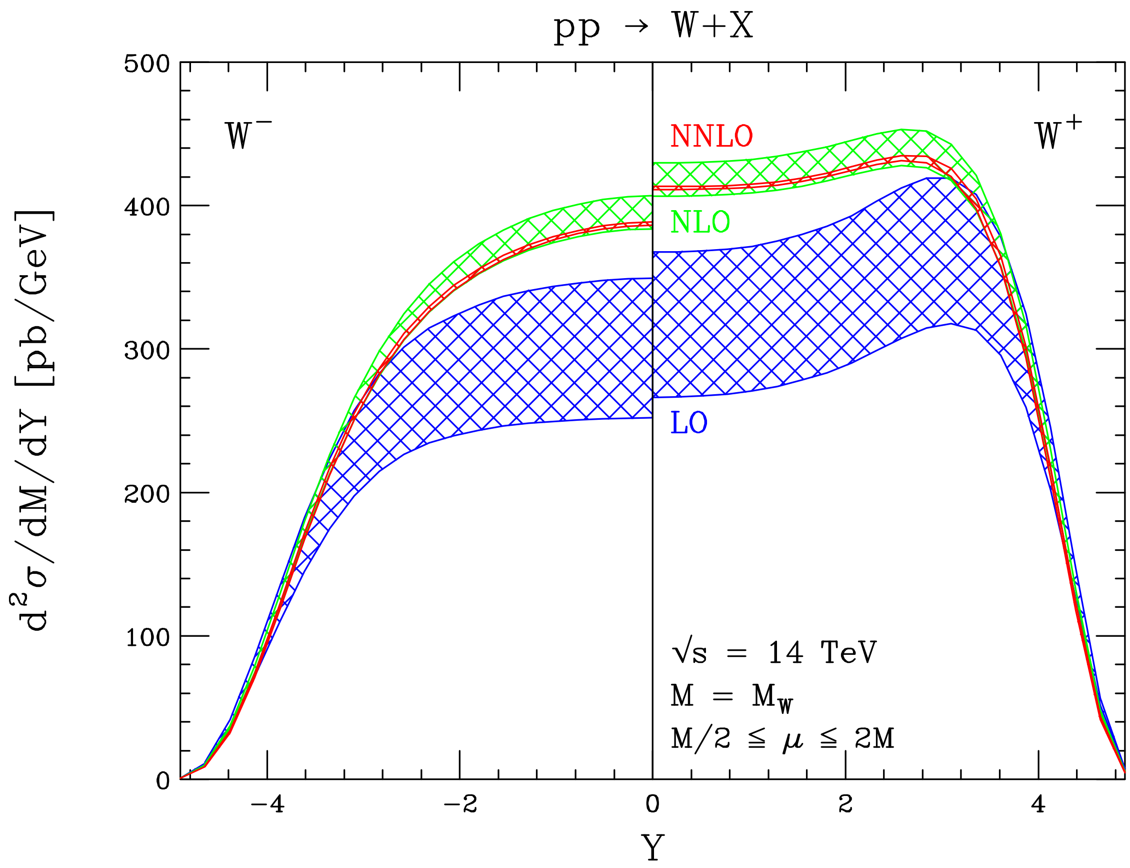
Z production at the LHC



- Result completely stable against μ variation at NNLO

⇒ $25 - 30\%$ at LO; 6% at NLO; $0.1 - 1\%$ at NNLO

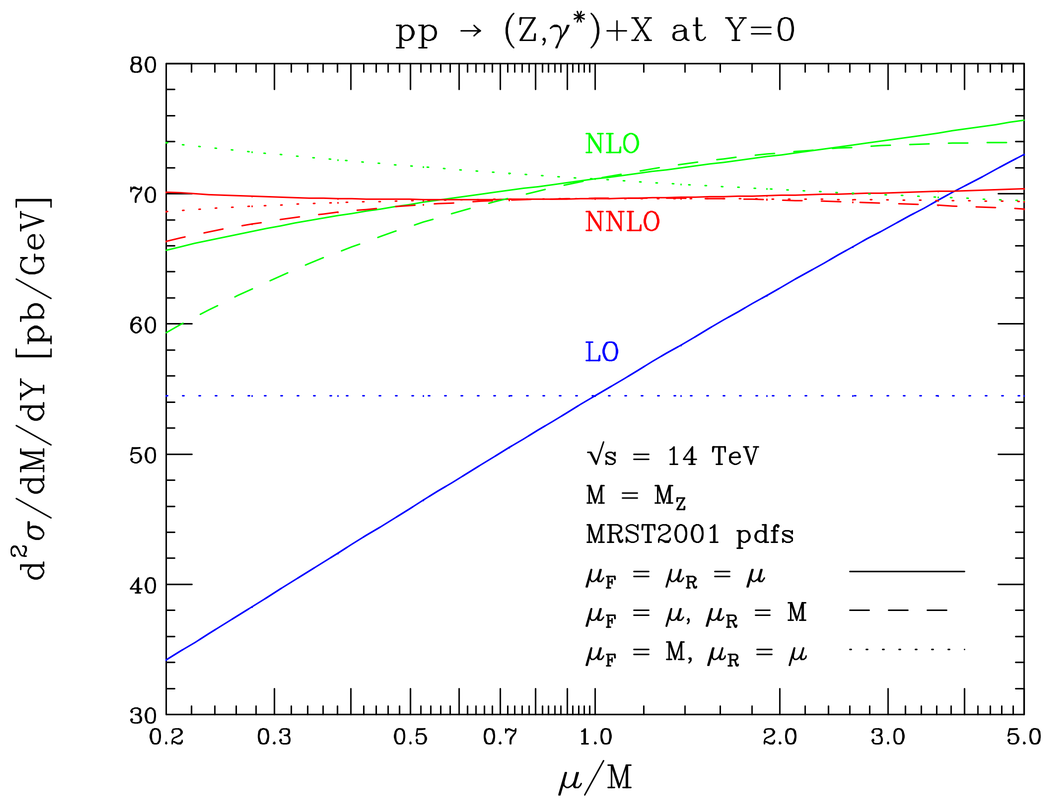
W production at the LHC



- Similar scale dependences as Z production

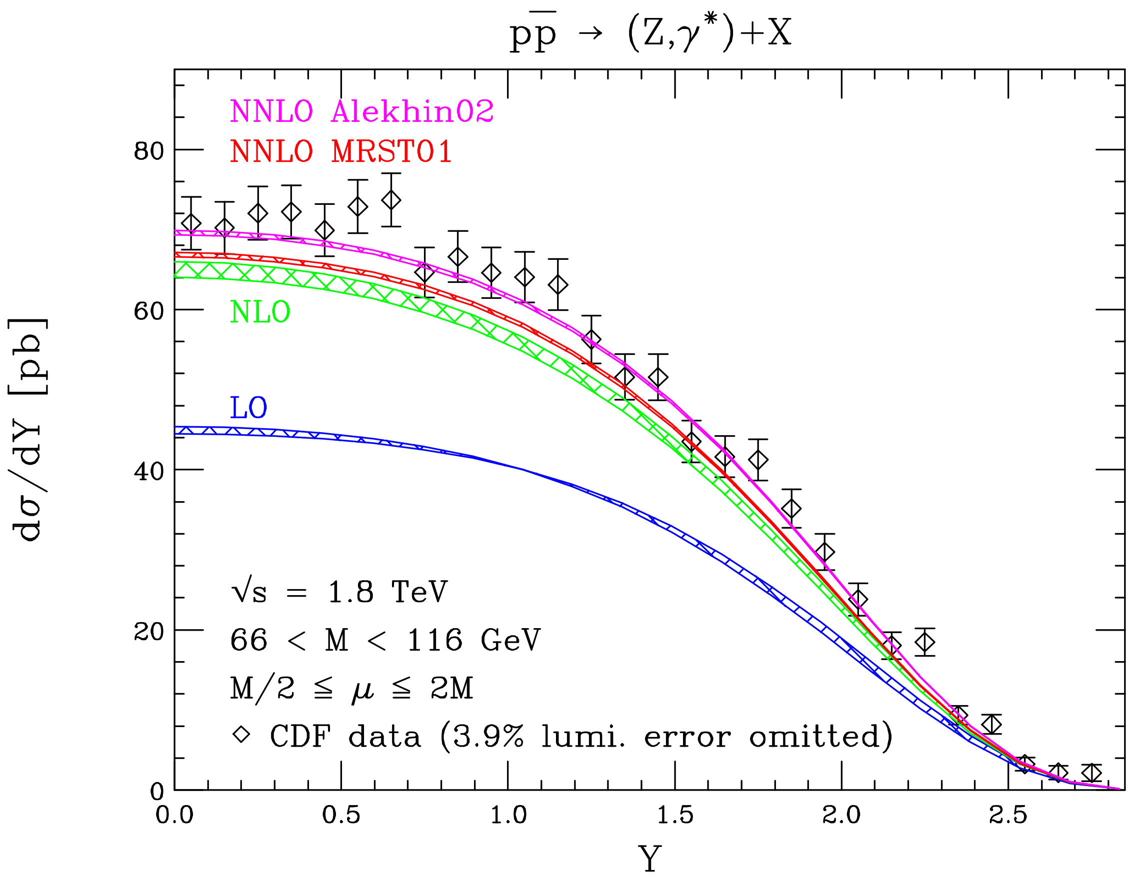
⇒ $25 - 30\%$ at LO; 6% at NLO; $0.1 - 1\%$ at NNLO

Scale variations at the LHC



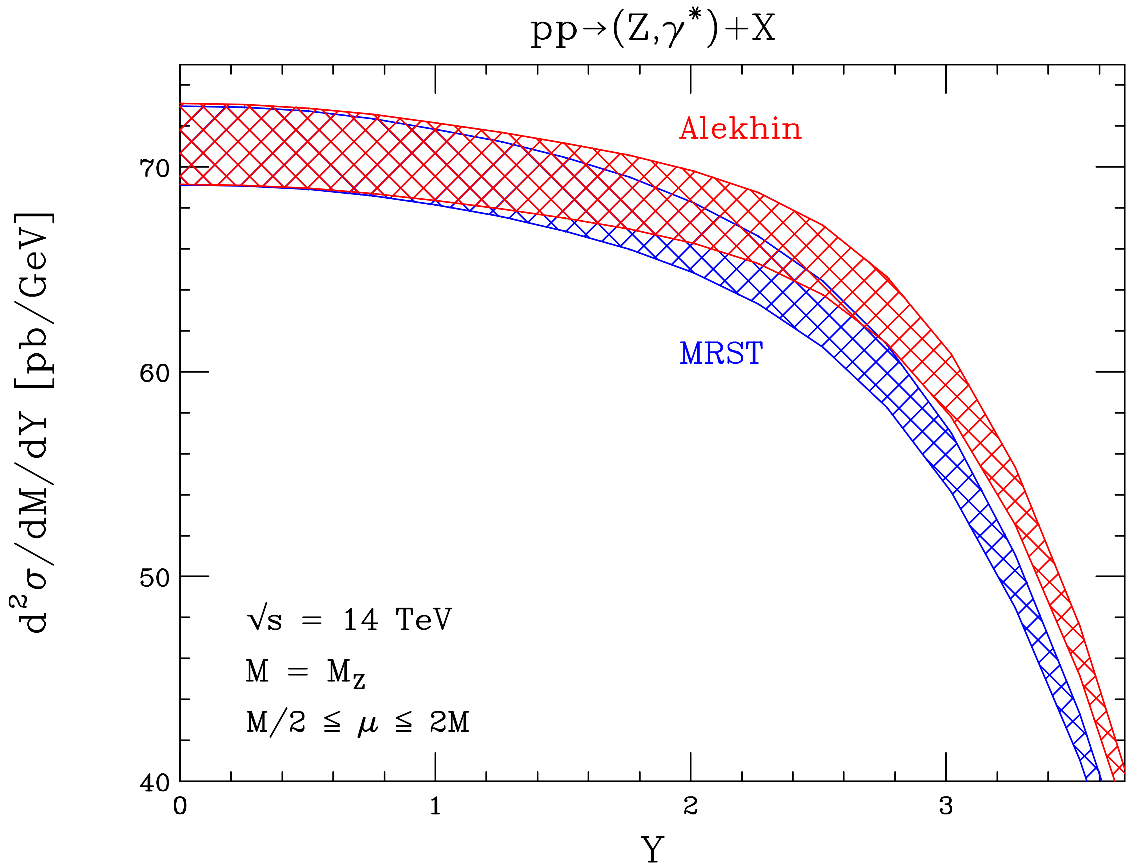
- Varying μ_R alone: $\leq 0.5\%$
- Varying both μ_R and μ_F: $\leq 1\%$
- Varying μ_F alone: $\leq 1\%$ for $M/2 \leq \mu_F \leq 2M$, $\leq 5\%$ for $M/5 \leq \mu_F \leq 5M$

Z production at the Tevatron



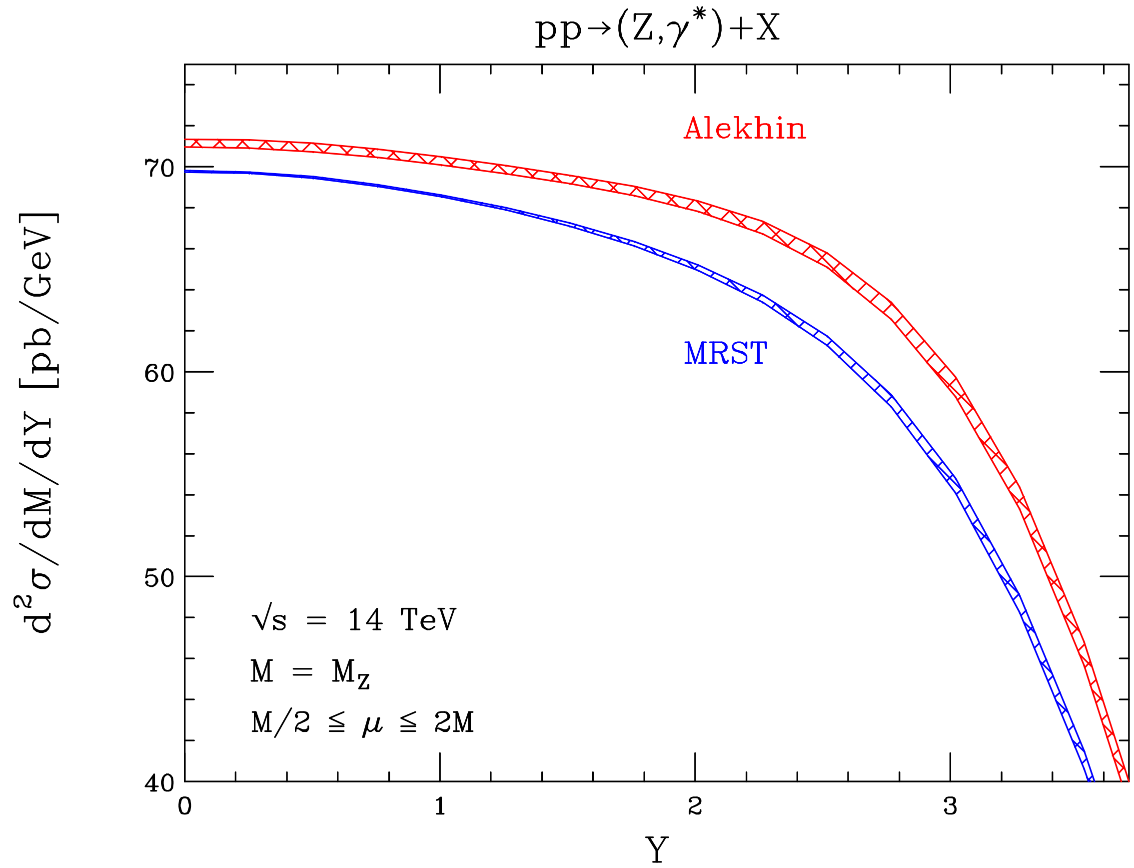
- Scale variations $3 - 6\%$ at NLO, $< 1\%$ at NNLO
- NNLO corrections increase cross section by $3 - 5\%$

PDF comparisons



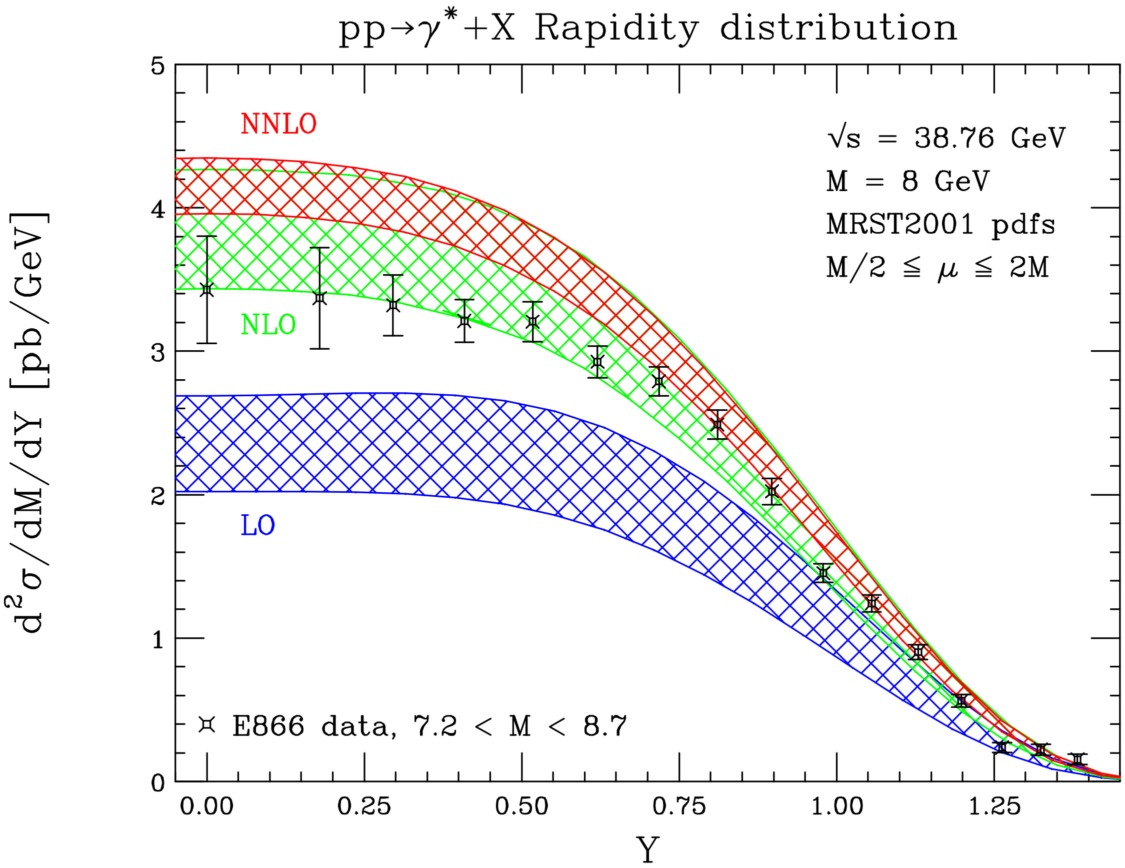
- Alekhin parameterization fits only to DIS data; MRST fits to DIS, DY, jets
- Scale variations render undistinguishable at NLO

PDF comparisons



- Alekhin parameterization fits only to DIS data; MRST fits to DIS, DY, jets
- Scale variations render undistinguishable at NLO
- Resolved at NNLO

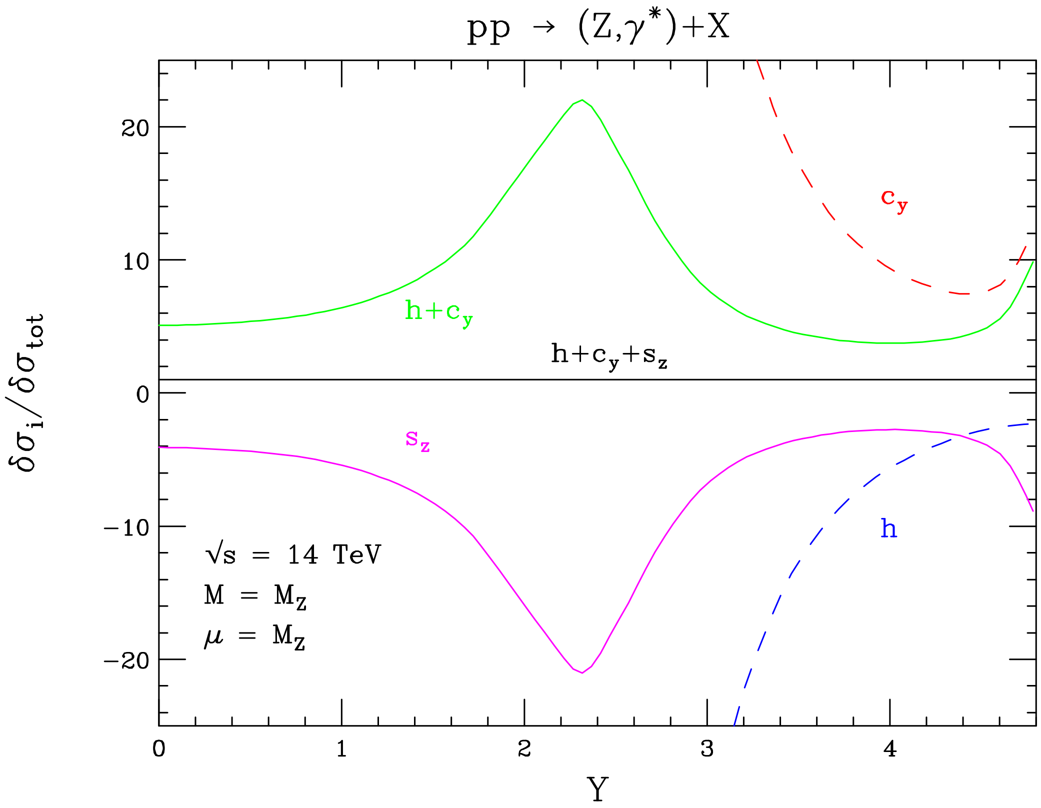
Fixed target DY (E866)



- Strong constraint on $\bar{q}$ and $x \to 1$ q_{val} distribution functions
- Reduced μ dependence at NNLO reveals discrepancy with data

$\Rightarrow$ Tune $\bar{q}$ pdfs

Soft and collinear approximations



- Split $\mathcal{O}(\alpha_s^2)$ corrections into hard, soft, and collinear pieces
- ⇒ denotes behavior of additional partons in final state
- No reasonable approximation to the full result; persists until very high energies

Conclusions

- Have presented a calculation of the Drell-Yan rapidity distribution at NNLO
- Have described a new method for computing real radiation contributions at higher orders
  - Maps real radiation $\Rightarrow$ cut loop integrals
  - Can apply machinery developed for reduction, calculation of virtual corrections
  - Useful for "semi-inclusive" quantities
- Residual scale variations are less than 1% at the LHC

$\Rightarrow$ Drell-Yan is now a high precision probe of QCD