Subtraction method for numerical calculation of one loop QCD matrix elements

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INTRODUCTION

- The idea is to build a program that can calculate the NLO QCD correction to any process automatically.
- We can test the perturbative QCD and measure the α_s strong coupling and the parton distribution function ($f_{a/H}(x, \mu_F^2)$) of the proton more accuracy.
- The QCD corrections to any process which involves hadron in the initial and/or final state is always significant.
- The LO predictions strongly depends on the renormalization and factorization scales. This strong dependence is always a big uncertainty. The NLO correction can stabilize the theoretical prediction.
- At the LHC we expect complicated or even strange final states with large number of final state objects. We can check the standard model or any model beyond the standard model e.g.: minimal SUSY, any extension of the SUSY,... To calculate the necessary matrix elements analytically is always a huge task even at LO level $\Longrightarrow$ Let us try to do it numerically!

NLO CROSS SECTION

$$\sigma(p_A, p_B) = \sum_{a,b} \int_0^1 d\eta_a d\eta_b f_a(\eta_b, \mu_F^2) f_b(\eta_b, \mu_F^2)$$
$$\times \left[\hat{\sigma}^{LO}_{a,b}(\eta_a p_A, \eta_b p_B) + \hat{\sigma}^{NLO}_{a,b}(\eta_a p_A, \eta_b p_B)\right]$$

where

$$\hat{\sigma}^{LO}_{a,b}(p_a, p_b) = \int_m d\hat{\sigma}^B_a(p_a, p_b) = \int d\Gamma^{(m)} |M_{a,b}|^2 F_J^{(m)}(p_a, p_b, p_1, .., p_m)$$

and the NLO correction

$$\hat{\sigma}^{NLO}_{a,b}(p_a, p_b) = \int_{m+1} d\hat{\sigma}^R_{a,b}(p_a, p_b) + \int_m d\hat{\sigma}^V_{a,b}(p_a, p_b) + \int_m d\hat{\sigma}^C_{a,b}(p_a, p_b)$$

This integrals (R,V,C) are separately divergent but their sum is finite in $d = 4$ dimension.

BORN CONTRIBUTION

The leading order contribution is an m-parton phase space integral:

$$\hat{\sigma}_{ab}^{LO}(p_a, p_b) = \int_m d\hat{\sigma}_{ab}^{B}(p_a, p_b) = \int d\Gamma^{(m)} |M_{ab}|^2 F_J^{(m)}(p_a, p_b; p_1, .., p_m)$$

- Trivially no UV singularities. (No integral over infinite phase space.)
- No IR singularities from the phase space integral ensured by the $F_J^{(m)}$ measurement function.
- At this level the main task (challenge) is calculating out the matrix element square.
  - the matrix element automatically generated up to $2 \to 6$ or even $2 \to 8$ (MADGRAPH, ALPGEN, HELAC, AMEGIC++,...)
  - plus automatic integration over the phase space (PHEGAS, MADEVENT, SHERPA,...)
  - they are matched to the parton shower – CKKW (proven only in the e^+e^- case)

NLO CORRECTION: REAL CONTRIBUTION

To eliminate the IR singularities from the real part the best way is the dipole method:

$$\sigma_{a,b}^{NLO} = \int_{m+1} [d\sigma_{a,b}^{R}|_{\epsilon=0} - d\sigma_{a,b}^{A}|_{\epsilon=0}] + \int_{m} [d\sigma_{a,b}^{V} + d\sigma_{a,b}^{C} + \int_{1} d\sigma_{a,b}^{A}]_{\epsilon=0}$$

massless case : S. Catani and M.H. Seymour
massive case : S. Catani S. Dittmaier, M.H. Seymour, Z. Trócsányi

The $d\sigma_{a,b}^{A}$ is a local counterterm for $d\sigma_{a,b}^{R}$ with same pointwise behaviour as $d\sigma_{a,b}^{R}$. Furthermore it is integrable in $d = 4 - 2\epsilon$ dimension over the single parton subspaces.

$$\sigma^{NLO} = \int_{m+1} [d\sigma^{R}|_{\epsilon=0} - \sum_{\text{dipoles}} d\sigma^{B} \otimes dV|_{\epsilon=0}] + \int_{m} [d\sigma^{V} + d\sigma^{B} \otimes \boldsymbol{I}^{R}(\epsilon)]_{\epsilon=0}$$
$$+ \int_0^1 dx \int_0^1 dy \int_m d\sigma^{B}(xp_a, yp_b) \otimes (\boldsymbol{P} + \boldsymbol{K})(x,y)$$

where the $\boldsymbol{I}(\epsilon)$ singular factor in the massless case is

$$\boldsymbol{I}^{R}(\epsilon) = -\frac{\alpha_s}{2\pi} \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \sum_{j \neq i} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{\boldsymbol{T}_i^2} \left(\frac{\mu^2}{s_{ij}}\right)^{\epsilon} \left[\boldsymbol{T}_i^2 \left(\frac{1}{\epsilon^2} - \frac{\pi^2}{3}\right) + \gamma_i \frac{1}{\epsilon} + \gamma_i + K_i + \mathcal{O}(\epsilon)\right]$$

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- several program: EKS,JETRAD, EVENT(2), EERAD, DISENT, DISASTER++,MEPJET,..
- in some case we can calculate up to $2 \rightarrow 3$ (NLOJET++, MCFM)
- some interesting process is computed but many of them still missing

NLO CORRECTION (VIRTUAL PART)

The virtual contribution to the NLO correction is

$$\sigma^{NLO} = \cdots + \int_m \left[d\sigma^V + d\sigma^B \otimes \boldsymbol{I}^R(\epsilon)\right]_{\epsilon=0} + \cdots$$

where

$$\int_m d\hat{\sigma}^V = \int d\Gamma^{(m)}\, 2\mathcal{R}e\langle M^{(0)}(\{p\}_m)|M^{(1)}(\{p\}_m)\rangle\, F_J^{(m)}(\{p\}_m)$$

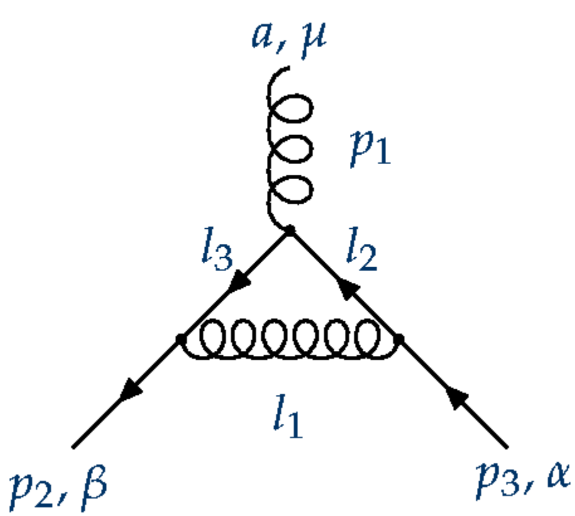
There are several method for calculating $|M^{(1)}(\{p\}_m)\rangle$:

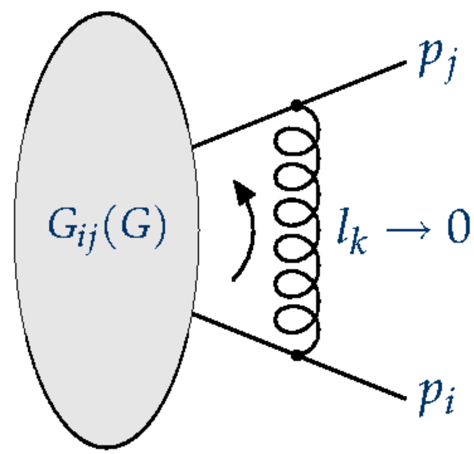
- Calculating analytically. Perform every algebra, integral analytically. — Bern, Dixon,Kosower, Kunszt, Signer, Trócsányi,...
- Combining the real and virtual contributions. — Krämer, Soper
- Numeric/analytic method — Binoth, Heinrich, Kauer
- More analytic/less numeric way. — Giele, Glover
- Subtraction method. — This work

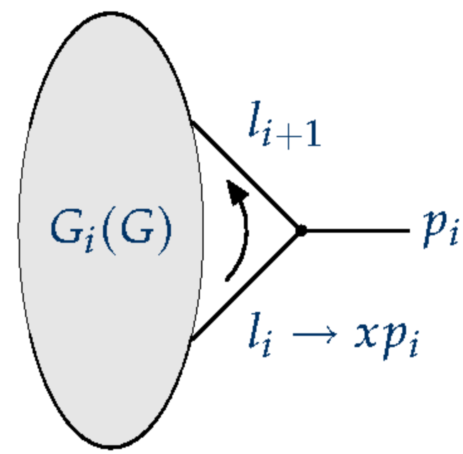
SINGULARITIES

Generally a one-loop matrix element has the following singularities:

- UV singularities — the loop momentum become infinitively large —
- soft singularities — internal loop gluon line becomes soft —
- collinear singularities — two internal loop lines become collinear —







SUBTRACTION OF THE SINGULARITIES

$$
\begin{aligned}
\sigma^{NLO} = \; & \cdots + \sum_{\{m\}} \sum_{G\text{ graphs}} \int d\Gamma^{(m)}(\{p\}_m) F_J^{(m)}(\{p\}_m)\, 2\mathcal{R}e \int \frac{d^4 l}{(2\pi)^4} \\
& \times \Big\{ \langle M^{(0)}(\{p\}_m) | \Big[|\widetilde{\mathcal{G}}_R(G;\{l\}_n,\{p\}_m)\rangle - |\widetilde{\mathcal{G}}^A(G;\{l\}_n,\{p\}_m)\rangle \Big] \Big\}_{\epsilon=0} \\
+ \; & \Big[\sum_{\{m\}} \int d\Gamma^{(m)}(\{p\}_m) F_J^{(m)}(\{p\}_m) \\
& \qquad \times \int \frac{d^d l}{(2\pi)^d} \sum_{G\text{ graphs}} 2\mathcal{R}e \langle M^{(0)}(\{p\}_m) | \widetilde{\mathcal{G}}^A(G;\{l\}_n,\{p\}_m)\rangle \\
& + \int_m d\sigma^B \otimes (\boldsymbol{I}^s(\epsilon) + \boldsymbol{I}(\epsilon)) \Big]_{\epsilon=0} + \cdots
\end{aligned}
$$

where $d\sigma^B \otimes \boldsymbol{I}^s(\epsilon)$ is the contribution of 1-loop graphs with self energy on the external lines

$$
\boldsymbol{I}^s(\epsilon) = \frac{\alpha_s}{2\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \frac{1}{\epsilon} \sum_{i=1}^{m} \gamma_i^s \,, \qquad \gamma_q^s = \gamma_{\bar q}^s = \frac{C_F}{2} \,, \qquad \gamma_g^s = -\frac{5}{6} C_A + \frac{4}{6} T_R n_f
$$

Numerical renormalization

The ultraviolet divergences come from the 1-loop vertex corrections. We have to define UV counterterms with the following properties:

- It has to be local counter term.
- It has to exactly match the singular behaviour of the original vertex correction.
- It has to be free from any IR (soft and/or collinear) singularities.
- It has to be integrable in $d = 4 - 2\epsilon$ dimension analytically and the results has to be cancelled by the corresponding renormalization constant without any finite remnant.

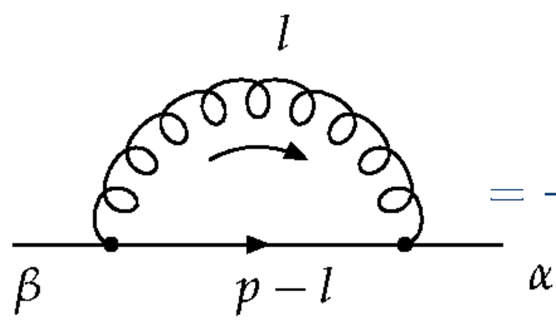
Any renormalized 1PI diagram at 1-loop level is the following:

$$\begin{aligned}
\Gamma_R(\{p\}_m) &= \Gamma^{(0)}(\{p\}_m) + \int \frac{d^d l}{(2\pi)^d} \left[\tilde{\Gamma}^{(1)}(\{l\}_n;\{p\}_m) - \tilde{\Gamma}_{UV}(\{l\}_n;\{p\}_m)\right] \\
&+ \left[(Z_\Gamma^{(1)} - 1)\Gamma^{(0)}(\{p\}_m) + \int \frac{d^d l}{(2\pi)^d} \tilde{\Gamma}_{UV}(\{l\}_n;\{p\}_m)\right] \\
&= \Gamma^{(0)}(\{p\}_m) + \int \frac{d^d l}{(2\pi)^d} \left[\tilde{\Gamma}^{(1)}(\{l\}_n;\{p\}_m) - \tilde{\Gamma}_{UV}(\{l\}_n;\{p\}_m)\right]
\end{aligned}$$

AN EXAMPLE: QUARK PROPAGATOR

The quark selfenergy is defined by

$$-\mathrm{i}\Sigma_{\alpha\beta}(p) =$$



$$= -g_s^2\mu^{2\epsilon}C_F\delta_{\alpha\beta}\int\frac{d^d l}{(2\pi)^d}\frac{\gamma^\nu(\not p-\not l)\gamma_\nu}{(l^2+\mathrm{i}0)((p-l)^2+\mathrm{i}0)}$$

and the corresponding UV counterterm is

$$\widetilde{\Sigma}^{UV}_{\alpha\beta}(l,p) = -g_s^2\mu^{2\epsilon}C_F\delta_{\alpha\beta}\frac{\gamma^\nu(\not p-\not l)\gamma_\nu}{\left(\left(l-\frac{p}{2}\right)^2-\mu^2e^{-1}+\mathrm{i}0\right)^2}$$

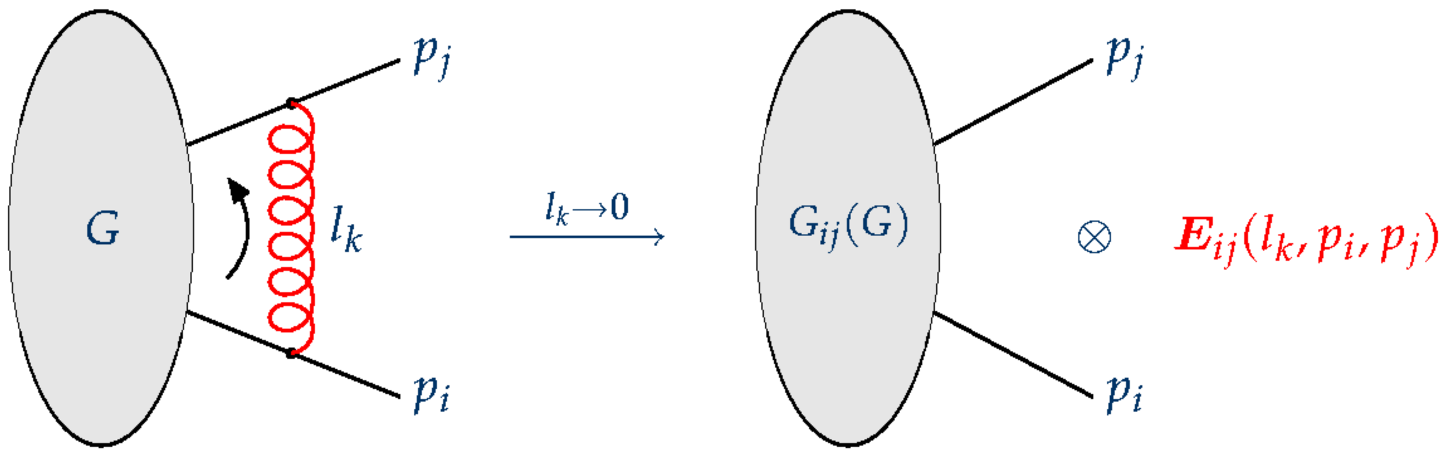
The renormalized quark propagator is

$$\mathrm{i}S_{\alpha\beta}(p) = \delta_{\alpha\beta}\frac{\mathrm{i}\not p}{p^2+\mathrm{i}0}+\int\frac{d^4 l}{(2\pi)^4}\frac{\mathrm{i}\not p}{p^2+\mathrm{i}0}\left[\widetilde{\Sigma}_{\alpha\beta}(l,p)-\widetilde{\Sigma}^{UV}_{\alpha\beta}(l,p)\right]_{\epsilon=0}\frac{\mathrm{i}\not p}{p^2+\mathrm{i}0}$$

$$\frac{1}{Z_S} = Z_\psi = 1-\frac{\alpha_s}{4\pi}C_F\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}\ , \qquad \int\frac{d^d l}{(2\pi)^d}\widetilde{\Sigma}^{UV}_{\alpha\beta}(l,p) = \mathrm{i}\frac{\alpha_s}{4\pi}C_F\delta_{\alpha\beta}\not p\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}$$

SOFT SINGULARITIES

When a gluon loop line become soft the loop integral over this region become singular. In this limit the integrand of the graph has the following form



or in more explicit form

$$\left|\tilde{\mathcal{G}}(G; l_1, \ldots, l_n; p_1, \ldots, p_m)\right\rangle \quad \sim \quad E_{ij}(l_k, p_i, p_j)\, \boldsymbol{T}_i \cdot \boldsymbol{T}_j\, \boldsymbol{H}_{ij} \left|\tilde{\mathcal{G}}(G_{ij}(G); p_1, \ldots, p_m)\right\rangle \quad ,$$

where the eikonal factor is

$$E_{ij}(l_k, p_i, p_j) = -\mathrm{i} g_s^2 \mu^{2\epsilon} \frac{4 p_i \cdot p_j}{(l_k^2 + \mathrm{i}0)((l_k + p_i)^2 + \mathrm{i}0)((l - p_j)^2 + \mathrm{i}0)}$$

SOFT SUBTRACTION

Using the soft factorization formulae the subtraction term can be defined straitforward by

$$\sum_{\{i,j\}} \big|\mathcal{S}_{ij}(G;l_k,\{p\}_m)\big\rangle = \sum_{\{i,j\}} E_{ij}(l_k,p_i,p_j)\boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\boldsymbol{H}_{ij}\big|\tilde{\mathcal{G}}(G_{ij}(G);\{p\}_m)\big\rangle$$

and the integral of the soft counterterm is

$$\int\frac{d^dl}{(2\pi)^d}\big|\mathcal{S}_{ij}(G;l_k,\{p\}_m)\big\rangle = \mathcal{V}_{ij}^{\rm soft}(\epsilon)\,\boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\boldsymbol{H}_{ij}\big|\tilde{\mathcal{G}}(G_{ij}(G);p_1,\dots,p_m)\big\rangle$$

where the $\mathcal{V}_{ij}^{\rm soft}(\epsilon)$ singular factor is the integral of the eikonal factor

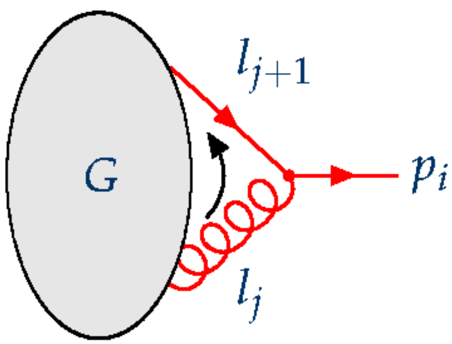
$$\mathcal{V}_{ij}^{\rm soft}(\epsilon) \equiv \int\frac{d^dl}{(2\pi)^d}\,E_{ij}(l) = \frac{\alpha_s}{4\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{-2p_i\cdot p_j}\right)^\epsilon\left(\frac{2}{\epsilon^2}+\mathcal{O}(\epsilon)\right)$$

The graph sum of the integrated counterterm is

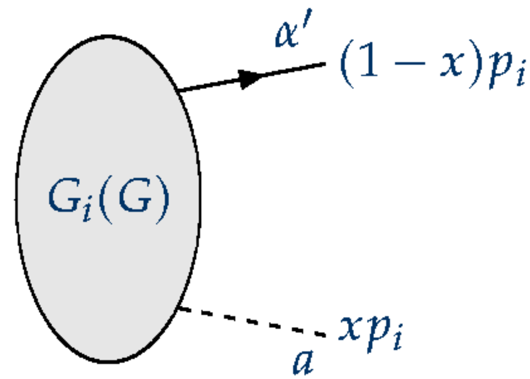
$$\sum_{\{i,j\}}\int\frac{d^dl}{(2\pi)^d}\sum_G\big|\mathcal{S}_{ij}(G;l_k,\{p\}_m)\big\rangle = \sum_{\{i,j\}}\mathcal{V}_{ij}^{\rm soft}(\epsilon)\,\boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\big|\mathcal{M}_{\rm tree}(p_1,\dots,p_m)\big\rangle$$

COLLINEAR SINGULARITIES

When two internal loop lines become collinear the integral become divergent. In this limit the integrand has the following factorization property:



$$\xrightarrow{l_j \to x p_i} \frac{-2\mathrm{i} g_s \mu^\epsilon \sqrt{1-x}\, T^a_{\alpha\alpha'}}{(l_j^2+\mathrm{i}0)((l_j-p_i)^2+\mathrm{i}0)}$$



Defining the collinear coefficient function

$$\left|f_i^{C,0}(G;x;p_1,\ldots,p_m)\right\rangle = \lim_{l_j \to x\, p_i} l_j^2\,(p_i-l_j)^2\,\left|\tilde{\mathcal{G}}(G;l_1,\ldots,l_n;p_1,\ldots,p_m)\right\rangle$$

The naive collinear formulae is

$$\left|\tilde{C}_i(G;l_j,\{p\}_m)\right\rangle = \frac{1}{(l_j^2+\mathrm{i}0)((p_i-l_j)^2+\mathrm{i}0}\int_0^1 dx\,\delta\left(x-\frac{l_j\cdot n_i}{p_i\cdot n_i}\right)\left|f_i^{C,0}(G;x;\{p\}_m)\right\rangle$$

$$n_i^\mu = -p_i^\mu + \frac{2\,p_i\cdot w}{w^2}w^\mu\ , \qquad w^\mu = \sum_{k\in\text{final state}} p_k^\mu$$

COLLINEAR SUBTRACTION

There are two problem with this factorization formula:

- It is soft singular. We already subtract all the soft poles including the soft-collinear double poles. We have to eliminate this soft poles

$$\begin{aligned} \big|f_i^C(G;x;\{p\}_m)\big\rangle &= \big|f_i^{C,0}(G;x;\{p\}_m)\big\rangle - \frac{1}{x}\lim_{y\to 0} y\big|f_i^{C,0}(G;y;\{p\}_m)\big\rangle \\ &- \frac{1}{1-x}\lim_{y\to 1}(1-y)\big|f_i^{C,0}(G;y;\{p\}_m)\big\rangle \end{aligned}$$

- It is UV divergent. This UV divergence is artificial. We can easily fix this by introducing a UV regulator

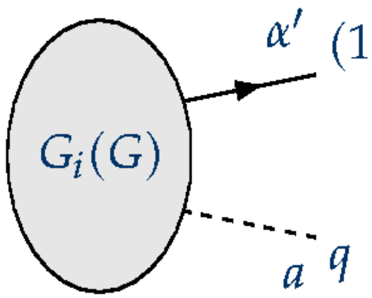
$$f_{UV}(l_j, l_j - p_i) = \frac{1}{2}\left(\frac{-\mu^2 e}{l_j^2 - \mu^2 e + \mathrm{i}0} + \frac{-\mu^2 e}{(l_j - p_i)^2 - \mu^2 e + \mathrm{i}0}\right)$$

The collinear subtraction is given by

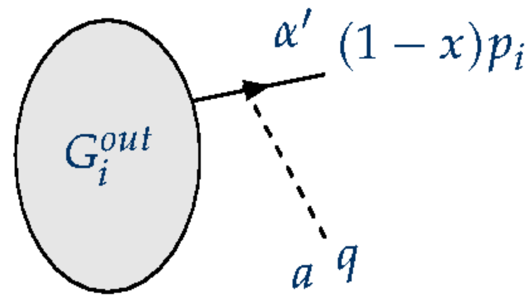
$$\big|\tilde{C}_i(G;l_j,\{p\}_m)\big\rangle = \frac{f_{UV}(l_j, l_j - p_i)}{(l_j^2 + \mathrm{i}0)((p_i - l_j)^2 + \mathrm{i}0}\int_0^1 dx\,\delta\left(x - \frac{l_j\cdot n_i}{p_i\cdot n_i}\right)\big|f_i^C(G;x;\{p\}_m)\big\rangle$$

GRAPH SUM OF THE COLLINEAR SUBTRACTION

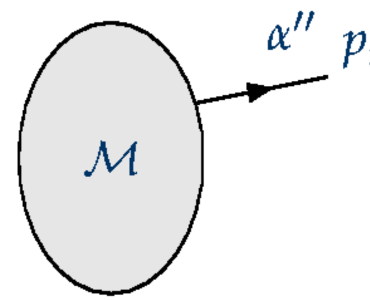
$\sum_G \lim_{q \to x p_i}$



$= \; - \lim_{q \to x p_i} \sum_{G_i^{out}}$



$= \; -g_s \mu^\epsilon \sqrt{1-x}\, T^a_{\alpha'\alpha''}$



In the general case

$$\sum_G \big| f_i^{C,0}(G; x, \{p\}_m) \big\rangle = i g_s^2 \mu^{2\epsilon} \frac{f_i^C(x)}{x} \gamma_i^C \, \big| \mathcal{M}^{\rm tree}(\{p\}_m) \big\rangle$$

where

$$f_q^C(x) = 1 - x \;, \qquad f_g^C(x) = 2 - x \;, \qquad \gamma_q^C = 2C_F \;, \qquad \gamma_g^C = C_A$$

INTEGRAL OF THE COLLINEAR SUBTRACTION

The integral of the collinear subtraction term for the i leg is

$$\int \frac{d^d l}{(2\pi)^d} \sum_G \big|\tilde{C}_i(G; l_j, \{p\}_m)\big\rangle = -\mathrm{i} g_s^2 \mu^{2\epsilon} \int \frac{d^d l_j}{(2\pi)^d} \frac{f_{UV}(l_j, l_j - p_i)}{(l_j^2 + \mathrm{i}0)((p_i - l_j)^2 + \mathrm{i}0)} \gamma_i^C \big|\mathcal{M}^{\text{tree}}(\{p\}_m)\big\rangle$$

and performing the integral and sum over the all possible collinear configuration we have

$$\sum_{i=1}^m \int \frac{d^d l}{(2\pi)^d} \sum_G \big|\tilde{C}_i(G; l_j, \{p\}_m)\big\rangle = \frac{\alpha_s}{4\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(-\frac{1}{\epsilon} + \mathcal{O}(\epsilon) \right) \sum_{i=1}^m \gamma_i^C \big|\mathcal{M}^{\text{tree}}(\{p\}_m)\big\rangle$$

Final formulae

$$\begin{aligned}\sigma^{NLO} \quad = \quad & \cdots + \sum_{\{m\}} \sum_{G \text{ graphs}} \int d\Gamma^{(m)}(\{p\}_m) F_J^{(m)}(\{p\}_m)\, 2\mathcal{R}e \int \frac{d^4 l}{(2\pi)^4} \\ & \times \Big[\langle M^{(0)}(\{p\}_m) | \widetilde{\mathcal{G}}^R(G;\{l\}_n,\{p\}_m) \rangle \\ & \qquad - \sum_{\substack{i,j \\ \text{pairs}}} \langle M^{(0)}(\{p\}_m) | \widetilde{\mathcal{S}}_{ij}(G;\{l\}_n,\{p\}_m) \rangle \\ & \qquad - \sum_i \langle M^{(0)}(\{p\}_m) | \widetilde{\mathcal{C}}_i(G;\{l\}_n,\{p\}_m) \rangle \Big]_{\epsilon=0} \\ + \quad & \int_m d\sigma^B \otimes \left[\boldsymbol{I}^V(\epsilon) + \boldsymbol{I}^R(\epsilon) \right]_{\epsilon=0} + \cdots \end{aligned}$$

where

$$\boldsymbol{I}^V(\epsilon) = \frac{\alpha_s}{4\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{1}{\epsilon^2} \sum_{\substack{i,j=1 \\ i\neq j}}^{m} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \left(\frac{\mu^2}{-2p_i \cdot p_j} \right)^\epsilon - \frac{1}{\epsilon} \sum_{i=1}^{m} \gamma_i \right)$$

$$\gamma_q = \gamma_{\bar{q}} = \frac{3}{2} C_F \;, \qquad \gamma_g = \frac{11}{6} C_A - \frac{4}{6} T_R n_f$$

AUTOMATIC NLO PROGRAM (VIRTUAL PART)

We have a subtraction method for calculating 1-loop matrix elements numerically. This method works in the massive case too.

What we have to do :

- Implementing the subtraction method for 1-loop amplitudes including massive fermions.
- Finding the optimal way to perform the loop integral numerically.
- Optimizing the evaluation of the graph sum.
- Building a 1-loop level matrix element generator.
- .....