

# Full one-loop electroweak radiative corrections for LC

*LAPTH-Minamitateya Collaboration*

LoopFest III

2004.4.1-3, KITP UCSB

presented by K.Kato(Kogakuin U.)

# Introduction

LHC → LC (NLC, Tesla, GLC,  
.....)

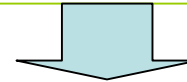
Higgs, SUSY, ...

Discovery is not the goal

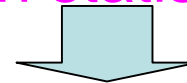
→ Detailed SM study

→ Find *something*  
beyond SM

Good accelerator  
& detectors



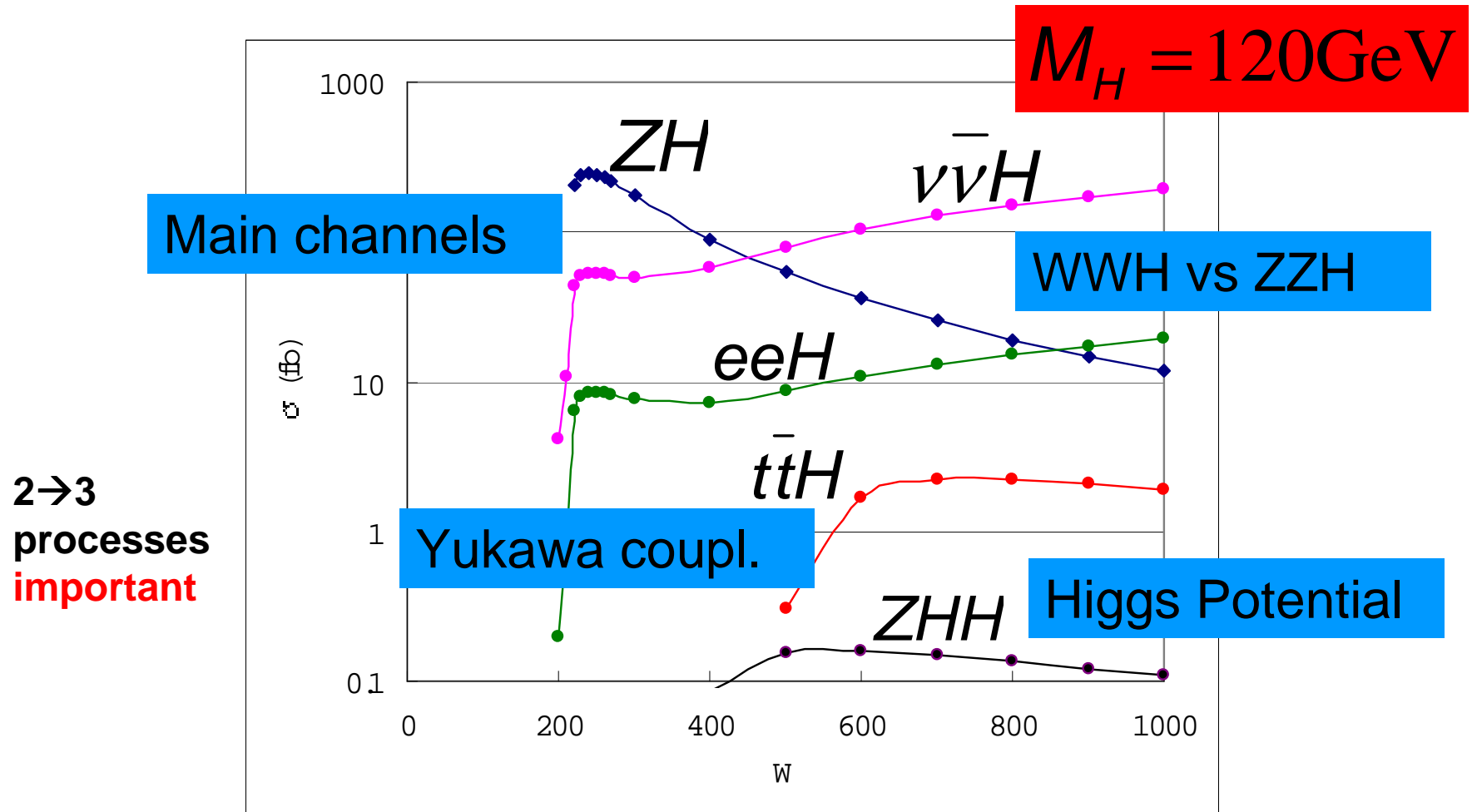
High statistics



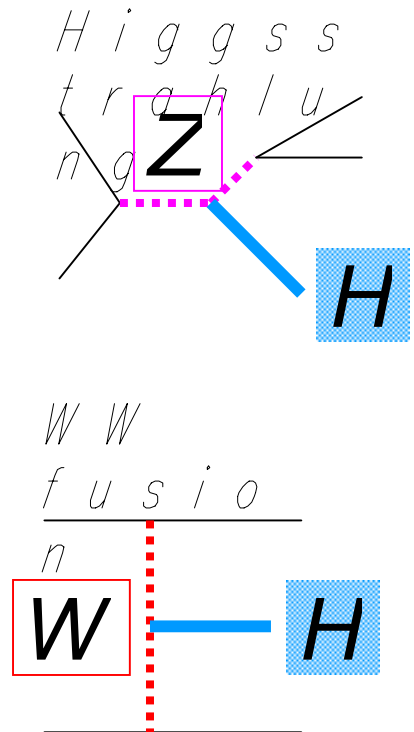
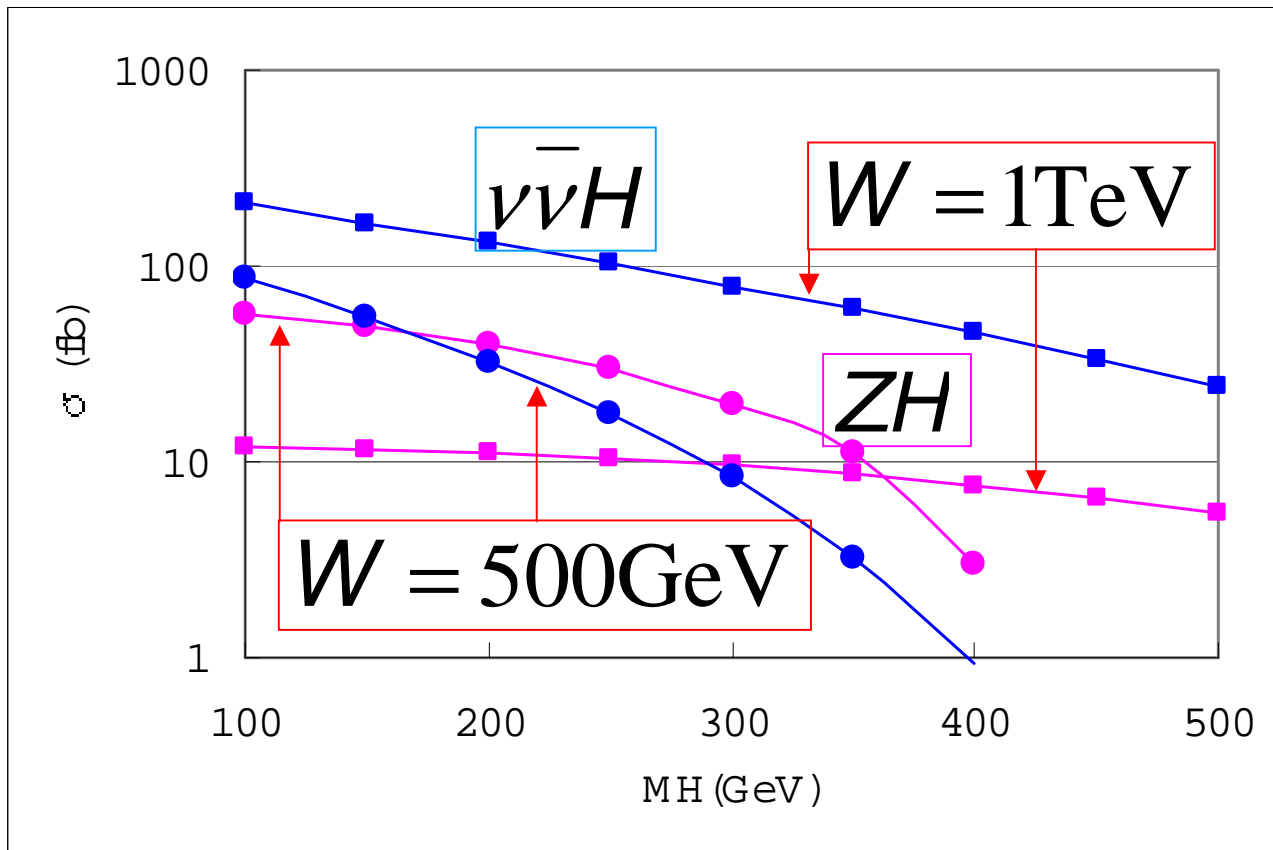
Requires accurate  
theoretical values  
→ R.C.

2 → 3, 4 channels  
important

# Higgs@LC: tree cross sections



# tree cross sections



**Interference**  
**→ Full calculation**

# number of diagrams (GRACE:NLG model)

A.Denner, J.Kublbeck,  
R.Mertig, M.Bohm,  
Z.Phys.C56(1992) 261;  
B.A.Kniehl,  
Z.Phys.C55(1992) 605.

$e^+ e^- \rightarrow$	tree	1-loop
$ZH$	4(1)	341(119)

10 years  
required to  
develop 2→3  
tools

with(without) e-scalar couplings

# Now...Full 1-loop RC available

$$e^+ e^- \rightarrow \nu \bar{\nu} H$$

GRACE, PLB559(2003)252  
Denner et al., NPB660(2003)289

$$e^+ e^- \rightarrow t \bar{t} H$$

GRACE, PLB571(2003)163  
You et al., PLB571(2003)85  
Denner et al., PLB575(2003)290

$$e^+ e^- \rightarrow Z H H$$

GRACE, PLB576(2003)152  
Zhang et al., PLB578(2004)349

## New results

$$e^+ e^- \rightarrow e^+ e^- H$$

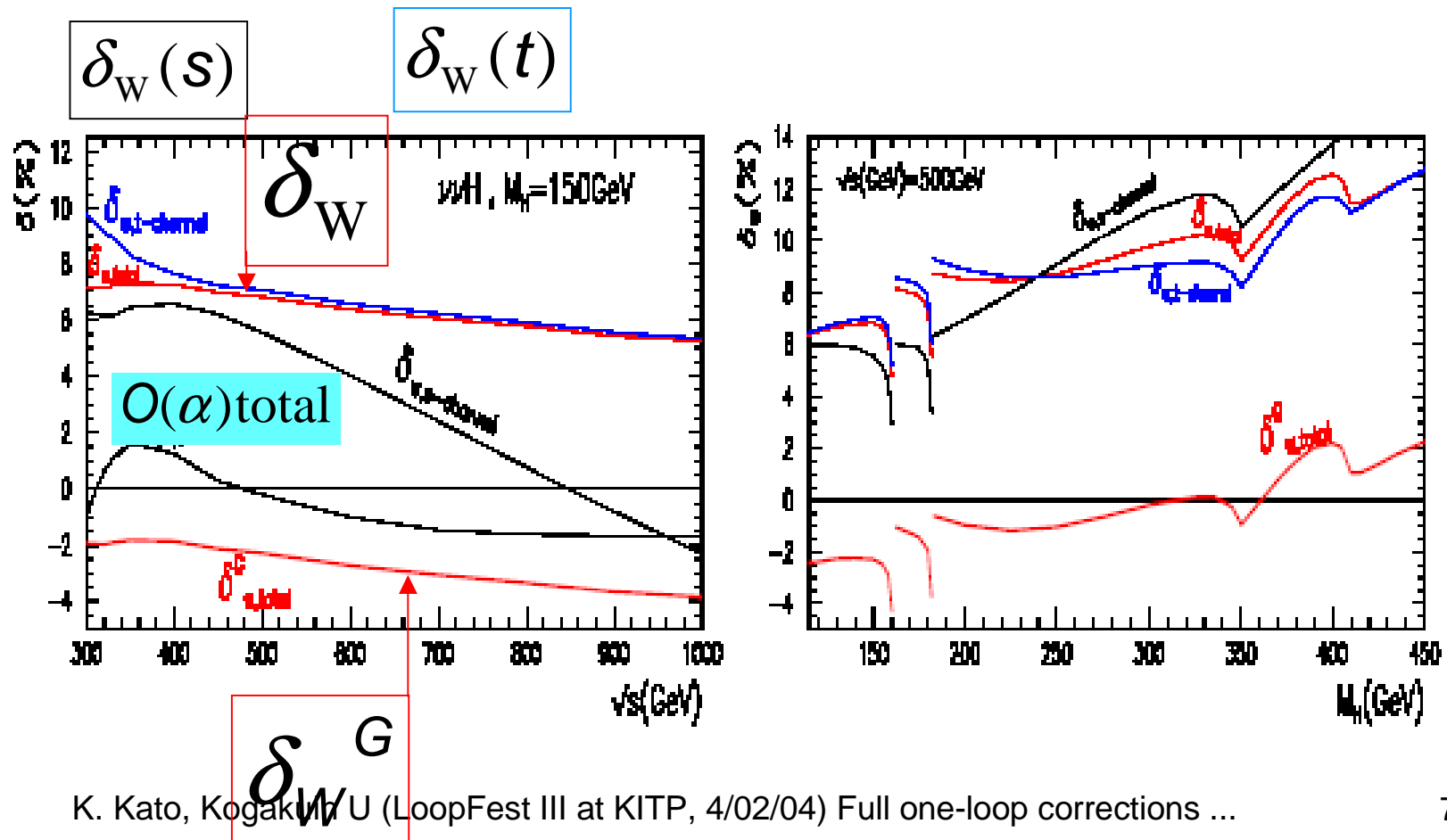
$$e^+ e^- \rightarrow \nu \bar{\nu} \gamma$$

$$\nu = \nu_\mu, \nu_e$$

$$e^+ e^- \rightarrow \nu \bar{\nu} H$$

hep-ph/0212261  
Phys.Lett.B 559(2003) 252.

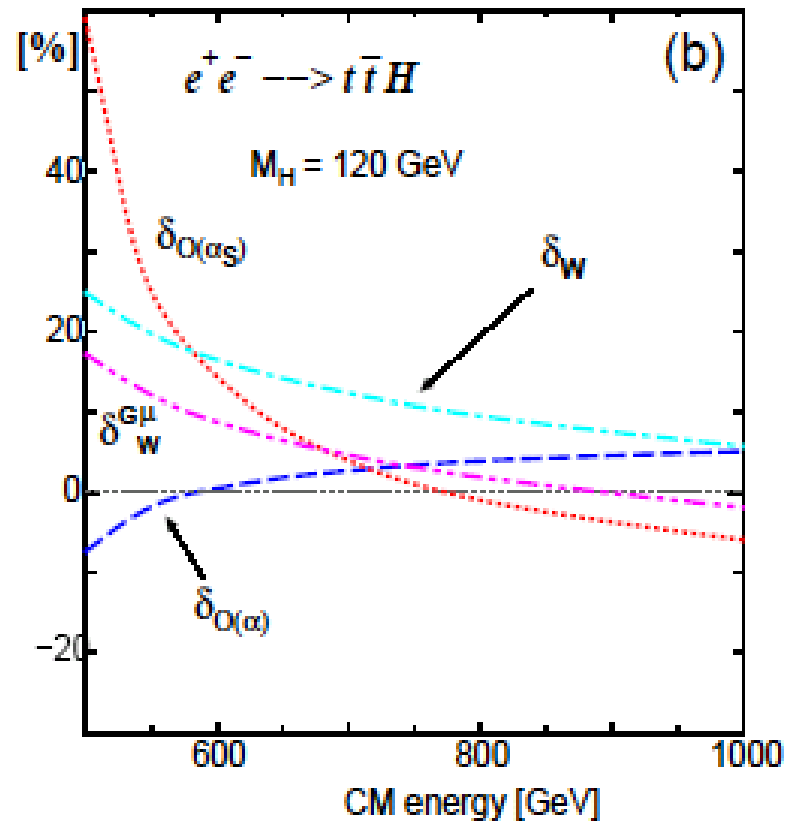
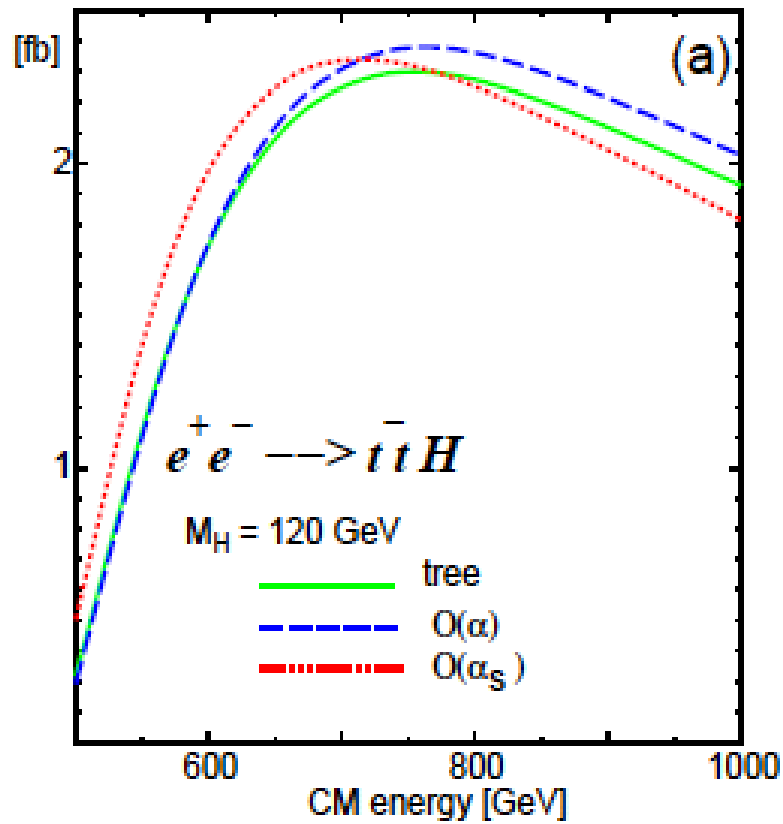
s,t channels show different behavior  
genuine weak correction in G-scheme is -2.-4%



$$e^+ e^- \rightarrow t\bar{t}H$$

hep-ph/0307029

Phys.Lett.B 571(2003) 163.

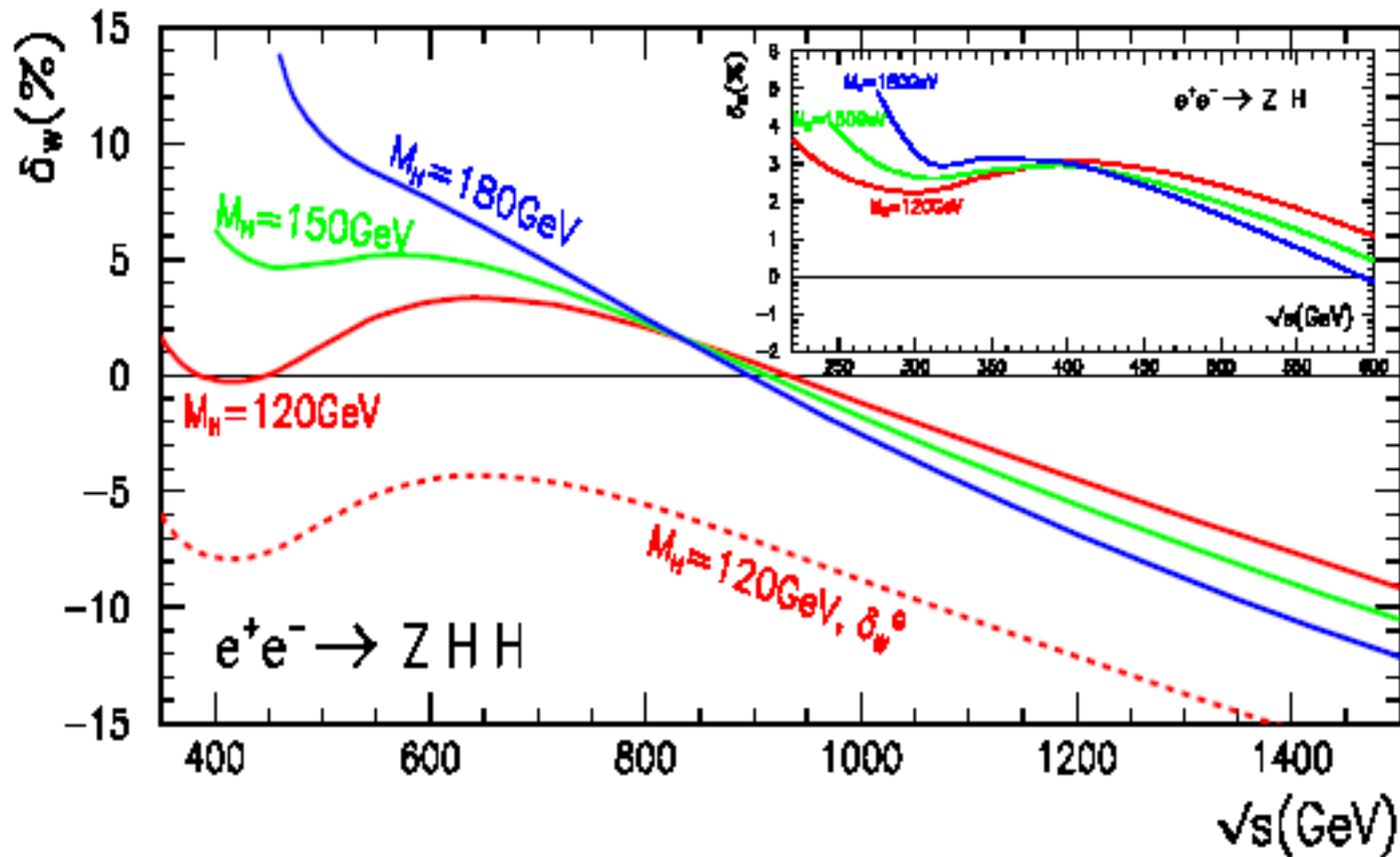
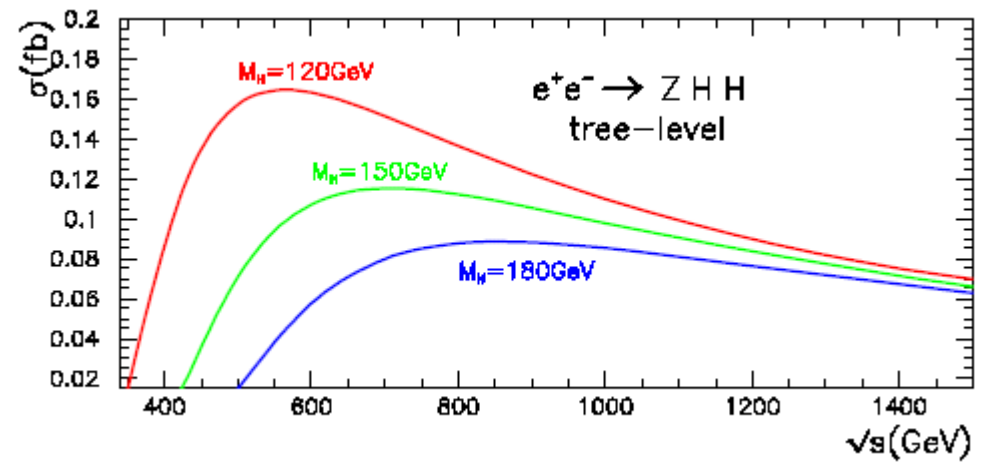




$$e^+ e^- \rightarrow Z H H$$

hep-ph/030910

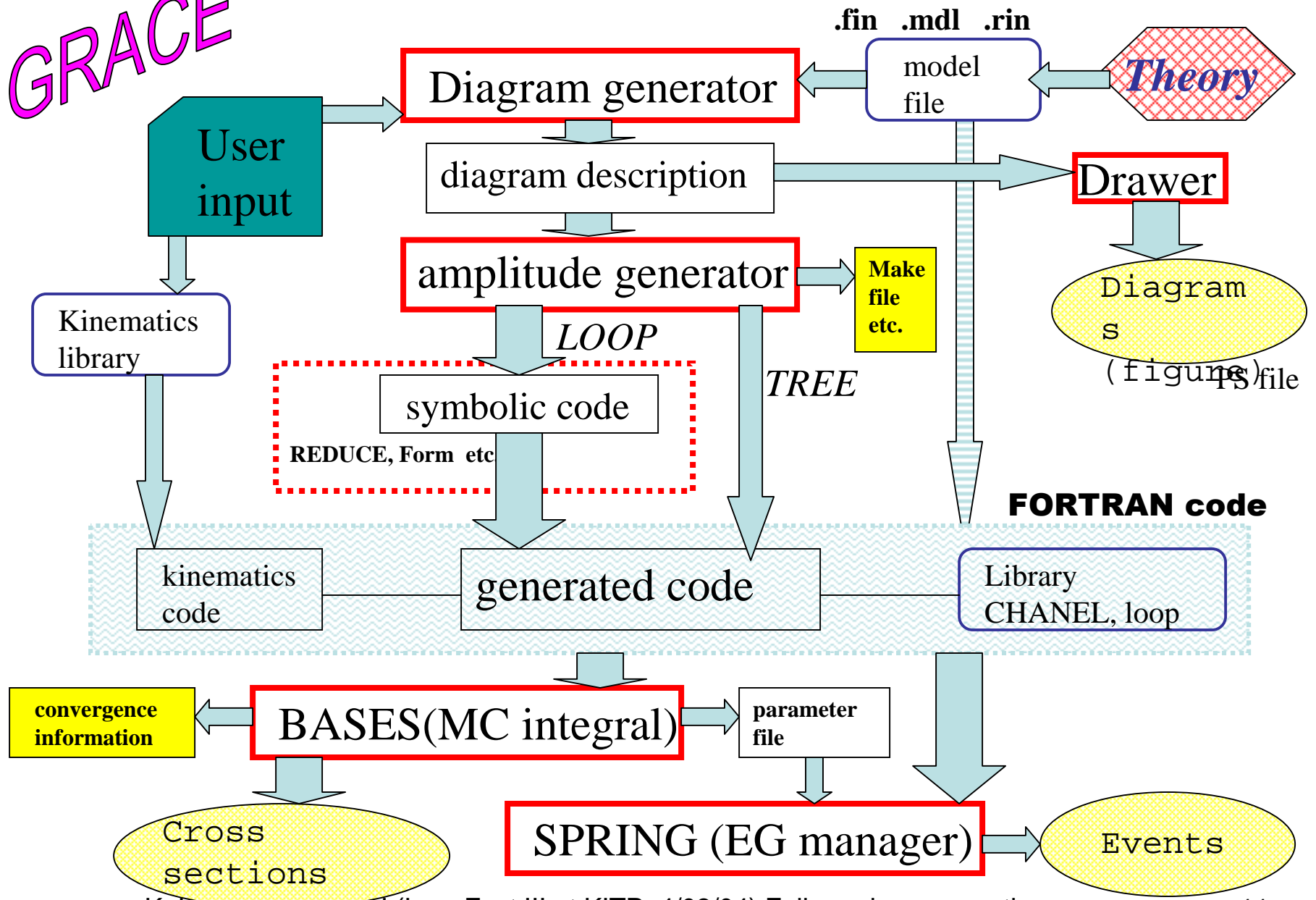
Phys.Lett.B 576(2003)152.



# system components

- Diagram generation for input process
- Amplitude/Matrix element generation
- Kinematics and Integration (efficiency)
- Event generation (efficiency & weight)
- Peripheral tools: rule generator, diagram selection, QED radiation, PDF, loop integral library, multi-process, color flow and interface for hadronization, etc.

# GRACE



# 5-point functions

$N$  rank M

$$I_5 = \sum \mathbf{G}_{\mu\nu\dots\sigma} \int d\ell \frac{\ell^\mu \ell^\nu \dots \ell^\sigma}{D_0 D_1 D_2 D_3 D_4} \quad \begin{aligned} D_0 &= \ell^2 + X_0 \\ D_j &= \ell^2 + 2\ell \cdot r_j + X_j \end{aligned}$$

$$A_{ij} = r_i \cdot r_j \quad g^{\mu\nu} = r_i^\mu A_{ij}^{-1} r_j^\nu \quad \ell^2 = D_0 - X_0$$

$$\ell^\mu = r_i^\mu A_{ij}^{-1} (r_j^\nu \ell) = \frac{1}{2} r_i^\mu A_{ij}^{-1} [D_j - D_0 + X_0 - X_j]$$

$$\rightarrow N = \sum_{\alpha=0}^4 E_\alpha(\ell) D_\alpha + F \quad 1 = \sum_{\alpha=0}^4 [a_\alpha + b_{\alpha j}(\ell r_j)] D_\alpha$$

scalar 5-pnt

BOX

BOX rank M-1

# $\delta(\text{QED}), \delta(\text{EW})$

$$\sigma = \sigma_0 (1 + \delta_{\text{QED}} + \delta_W)$$

$\delta_W$  non-QED virtual corrections

$$\delta_{\text{QED}} = \delta_{\text{QED}}^{\text{V}} + \delta_{\text{QED}}^{\text{soft}} + \delta_{\text{QED}}^{\text{hard}}$$

phase space subtraction  $f_{LL}$  = radiator

$$\delta_{\text{QED}} = \int (d\sigma_0 \delta_{\text{QED}}^{\text{V}} + d\tilde{\sigma}_0 \otimes f_{LL}) + \int_{\text{hard}} (d\sigma_{1\gamma} - d\tilde{\sigma}_0 \otimes f_{LL})$$

# How to check?

- Diagnostic step is highly important  
(example) Denner et al. : evaluate  $\sigma$  in 2 ( or more) independent ways and observe numerical agreement
- GRACE  
UV independence  
IR stability  
Gauge invariance → Non-linear gauge

# Non-linear gauge fixing terms

$$L_{\text{GF}} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z} (F^Z)^2 - \frac{1}{2\xi} (F^A)^2$$

$$F^\pm = \left( \partial^\mu \mp i e \tilde{\alpha} A^\mu \mp i \frac{e c_W}{s_W} \tilde{\beta} Z^\mu \right) W_\mu^\pm \quad F^A = \partial^\mu A_\mu$$

$$+ \xi_W \left( M_W \chi^\pm + \frac{e}{2s_W} \tilde{\delta} H \chi^\pm \pm i \frac{e}{2s_W} \tilde{\kappa} \chi_3 \chi^\pm \right)$$

$$F^Z = \partial^\mu Z_\mu + \xi_Z \left( M_W \chi_3 + \frac{e}{2s_W c_W} \tilde{\epsilon} H \chi_3 \right)$$

# Samples of NLG Feynman rules

W – W – A

$$e[g^{\mu\nu}(p_1 - p_2)^\rho$$

$$+ (1 + \tilde{\alpha} / \xi_W)(p_3^\nu g^{\mu\rho} - p_3^\mu g^{\nu\rho})$$

$$+ (1 + \tilde{\alpha} / \xi_W)(p_2^\mu g^{\nu\rho} - p_1^\nu g^{\mu\rho})]$$

W –  $\chi$  – A

$$\mp ieM_W(1 - \tilde{\alpha})g^{\mu\nu}$$

modified

$\bar{c}^{\mp} - c^A - A - W^{\pm}$

$$- e^2 \tilde{\alpha} g^{\mu\nu}$$

$\bar{c}^{\mp} - c^A - \chi^{\pm} - H$

$$\mp ie^2 \frac{1}{2s_W} \tilde{\delta} \xi_W$$

ghost-ghost-vector-vector / ghost-ghost-scalar-scalar



# Non-linear gauge

- Numerator structure is the same as Feynman gauge  
→ Loop integral library
- Vertices modified
- general values → #diagrams

$$g^{\mu\nu} \quad (\text{for } \xi = 1)$$

“old” usage

→ reduce #diagram

$$\tilde{\alpha} = 1 \Rightarrow \text{no } \Lambda W \chi$$

$$\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa}$$

Check gauge invariance  
→ Independence on gauge parameters

# Gauge invariance check

- **full set** (all diagrams) **quadruple precision**  
at a few phase-space points  
compute values of ME for a several sets of  
gauge parameters

*if check is OK*

- **production set** (kill e-scalar couplings)  
**double precision**  
integrate in phase-space

# $ee \rightarrow eeH$

- diagrams

full set (for NLG check) tree 42, 1-loop 4470 (inc. C.T.)

production set (for integration) tree 2, 1-loop 510 (inc. C.T.)

- $M_Z = 91.1876 \text{ GeV}$   $\Gamma_Z = 2.4952 \text{ GeV}$  (appear at resonant poles only)

$$m_t = 174 \text{ GeV}, G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$$

$$E_{\text{CM}} = 200 \cdot 1000 \text{ GeV}, k_{\text{cut}} = E * 0.05$$

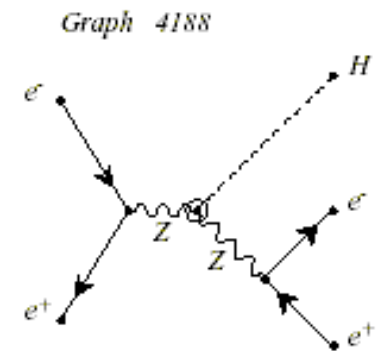
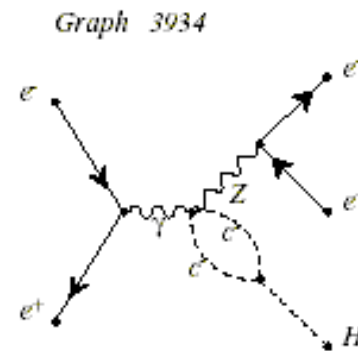
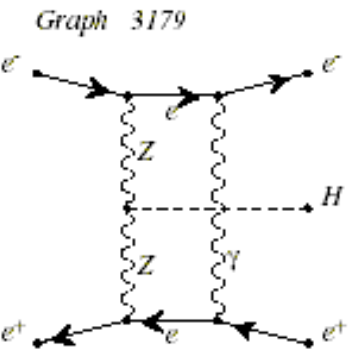
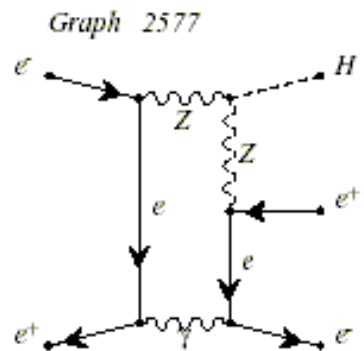
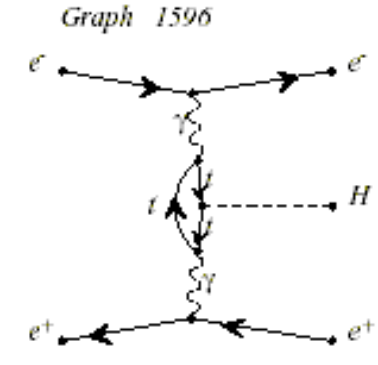
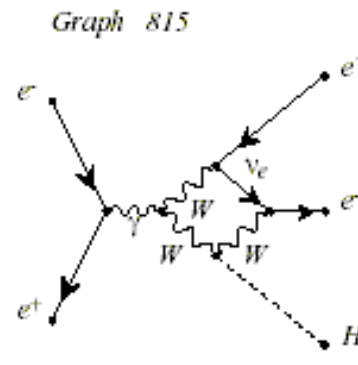
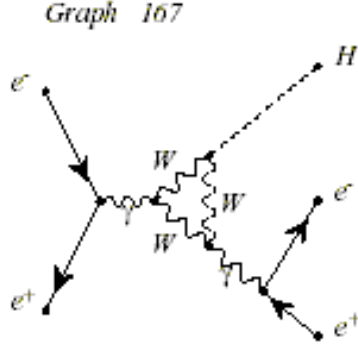
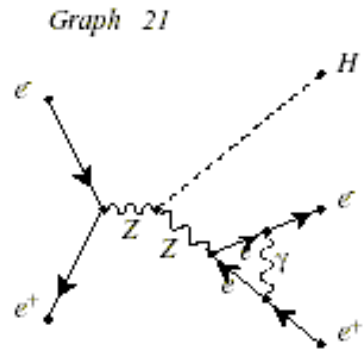
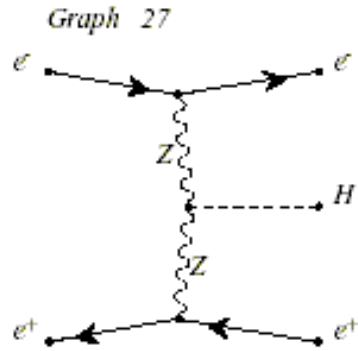
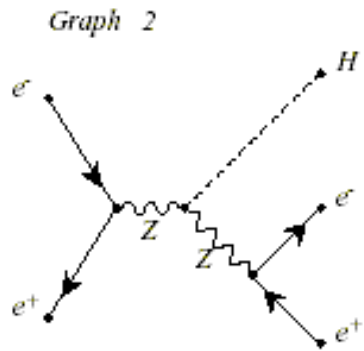
$$M_H = 120 \text{ GeV} \rightarrow M_W = 80.3766 \text{ GeV}, \Delta r = 2.55\%$$

$$M_H = 150 \text{ GeV} \rightarrow M_W = 80.3611 \text{ GeV}, \Delta r = 2.63\%$$

$$M_H = 180 \text{ GeV} \rightarrow M_W = 80.3477 \text{ GeV}, \Delta r = 2.70\%$$

(by Z.Hioki's formulas,  $\Delta r$  in 1-loop)

$$e^+ e^- \rightarrow e^+ e^- H$$



# check by NLG (ee→eeH)

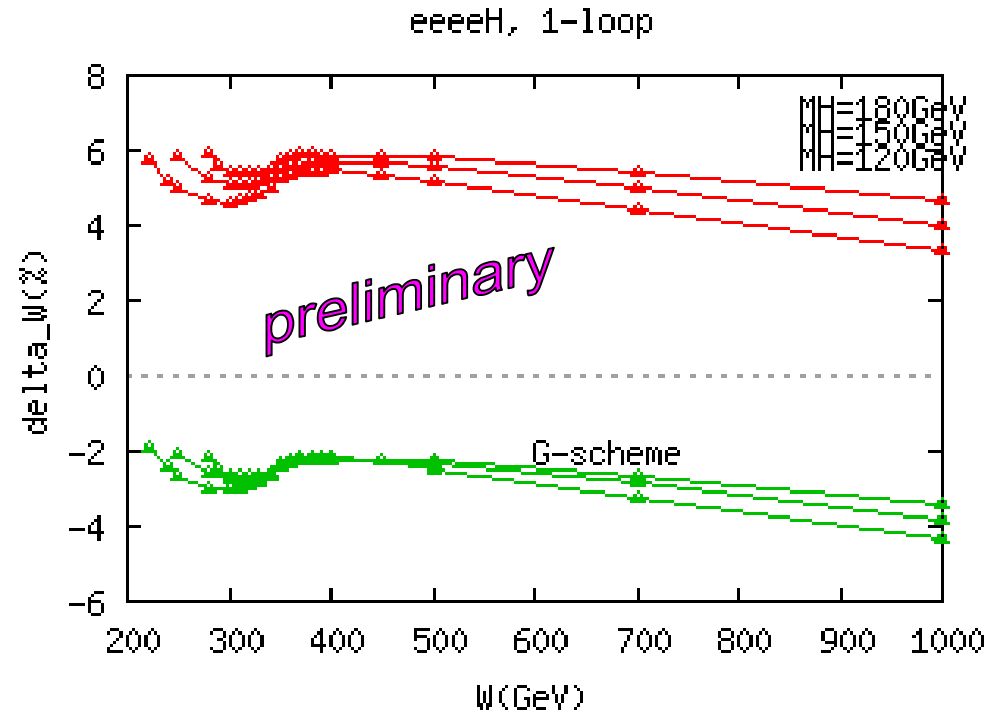
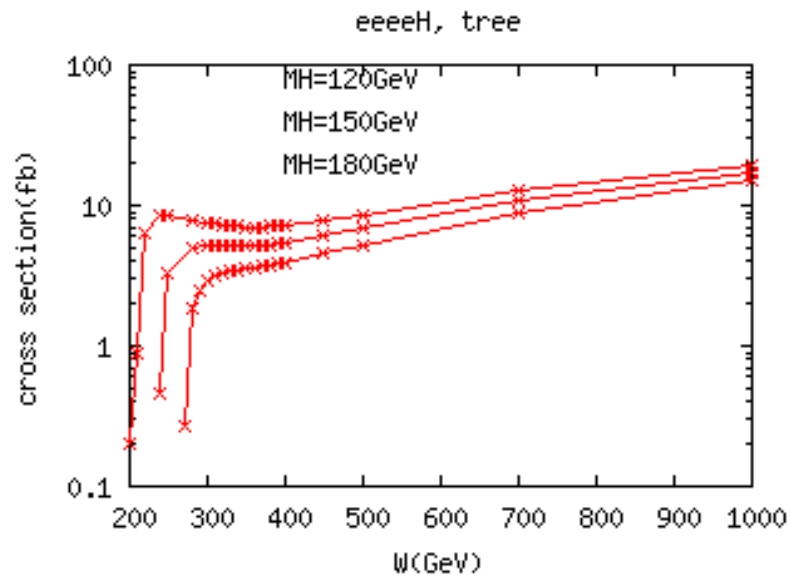
compute for several  $\xi$ , at a point in PS

$$f = \sum_{k=0}^4 a_k \xi^k \quad (\xi = \tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa})$$

a^0	a^4	a^3	a^2	a^1
166 @			.2626107E-05	-.5842526E-05
.3216419E-05				
167 @			-.1247760E-04	.1306251E-04
.2558662E-04				
168				.1083887E-05
.1083887E-05				
169 @			.3625232E-06	-.7250464E-06
.3625232E-06				
170 @			.2626107E-05	-.5842526E-05
.3216419E-05				
171 @			-.1247760E-04	.1306251E-04
.2558662E-04				
172				.1083887E-05
.1083887E-05				
173 @			.3625232E-06	-.7250464E-06
.3625232E-06				



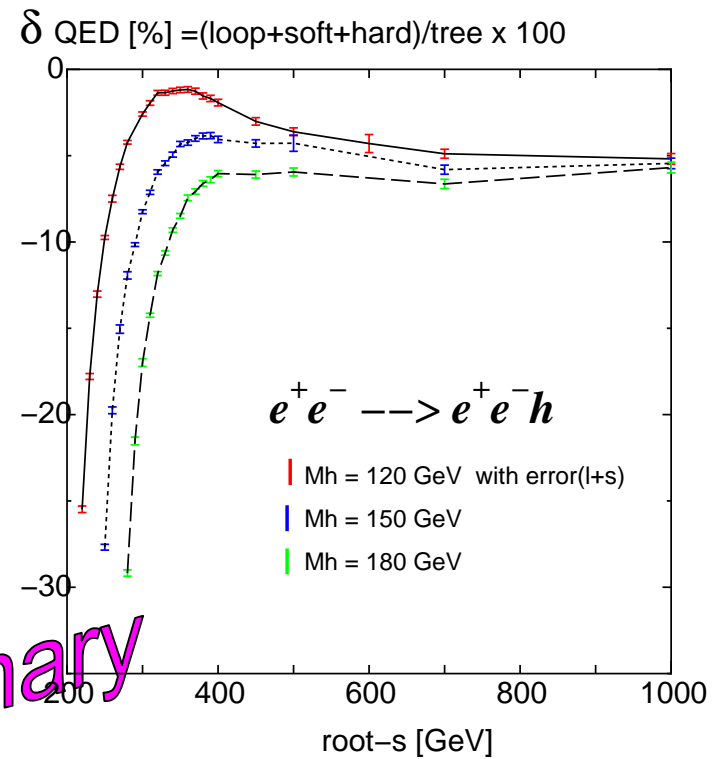
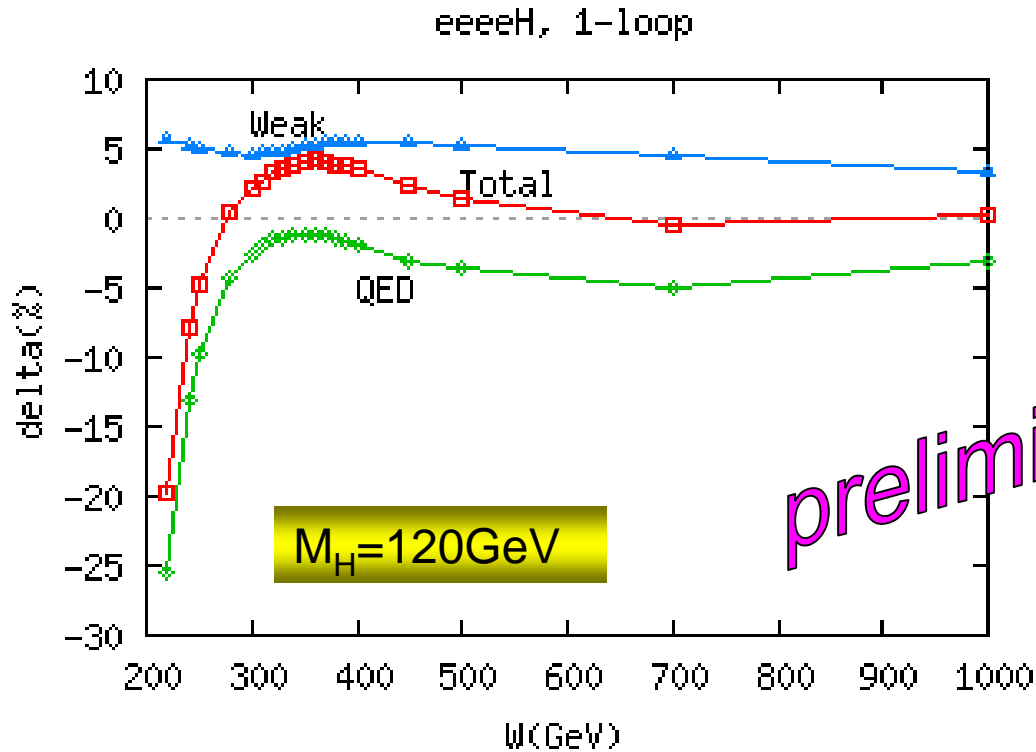
# $ee \rightarrow eeH$



**G-scheme absorbs MH-dep,  
correction is -2 to -4%**

# $ee \rightarrow eeH$

results look similar to  $ee \rightarrow \nu\nu H$  case ...



preliminary

another progress

$$e^+ e^- \rightarrow \nu \bar{\nu} \gamma$$

- diagrams  
full set (for NLG check)  
tree 10, 1-loop 1099 (inc. C.T.)  
production set (for integration)  
tree 5, 1-loop 331 (inc. C.T.)
- $M_W = 80.3766 \text{ GeV}$  ,  $M_Z = 91.1876 \text{ GeV}$   
 $\Gamma_Z = 2.4956 \text{ GeV}$   
 $M_H = 120 \text{ GeV}$  ,  $m_t = 174 \text{ GeV}$   
 $E_{\text{CM}} = 200 \text{ - } 3000 \text{ GeV}$   
OPAL cut:  $(\gamma) p_T > 0.05 E_B$  ,  $15^\circ < \theta < 165^\circ$

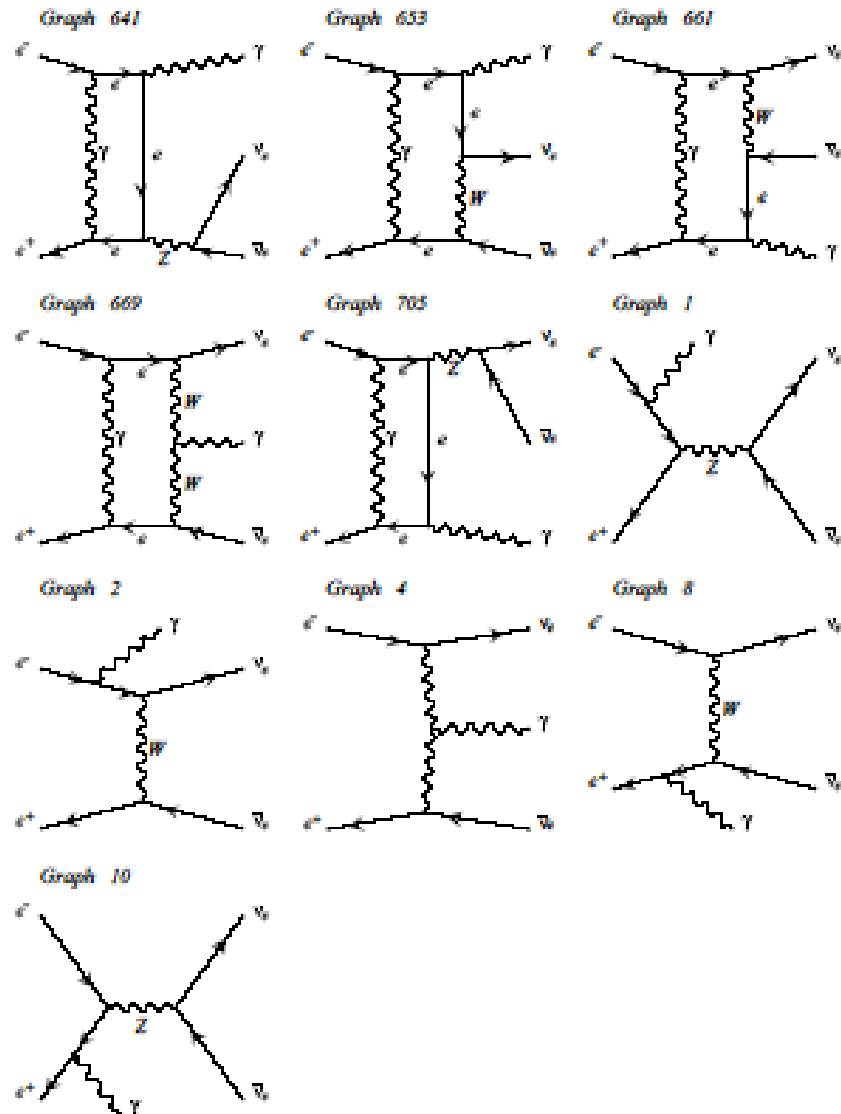
for SUSY



$$e^+ e^- \rightarrow \nu \bar{\nu} \gamma$$

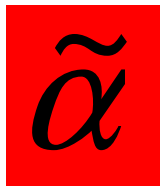
IR diagrams

tree diagrams



# check by NLG<sub>(ee→nunuA)</sub>

	a^4	a^3	a^2	a^1	a^0
1V				-.5252278E-03	.7952048E-03
2V				.1872623E-14	-.1872623E-14
6V				.1872623E-14	-.1872623E-14
□					
18V			.4986017E-11	.7865753E-10	-.8364354E-10
19V			.3779384E-10	.1104061E-09	-.1482000E-09
20V			.1208277E-10	-.2416554E-10	.1208277E-10
22V				.6561564E-10	-.6561564E-10
23V				-.4995059E-42	.4995059E-42
24V			.4797596E-09	-.9595192E-09	.4797596E-09
26V				-.6561564E-10	.6561564E-10
27V				.4995059E-42	-.4995059E-42
28V			-.4797596E-09	.9595192E-09	-.4797596E-09
29V	-.4773915E-37	.2482467E-36	.5633036E-36	-.7638111E-36	-.1405948E-07
30V	-.7346840E-39	.4897893E-39	.3673420E-38	-.3428525E-38	.5827357E-08
31V	-.1040802E-37	.9896805E-37	.1900383E-36	-.2785983E-36	-.1405949E-07
□					
854P				-.1180226E-14	.1180226E-14
855B			.7634114E-04	-.3060816E-03	.8768512E-03
sum1	.14148E-30	-.40640E-30	-.86324E-25	.17416E-23	.51997
max	.12998E-30	.32070E-04	.96820E-04	.59183E-01	.15505
cnt	15	25	70	381	

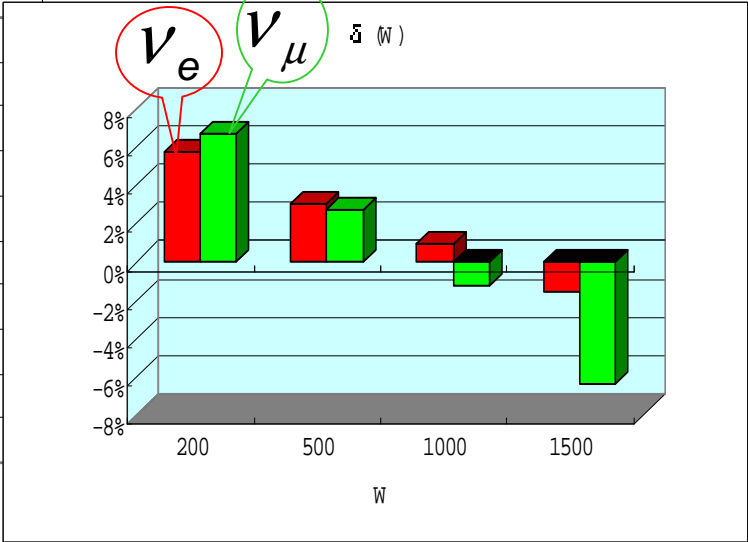
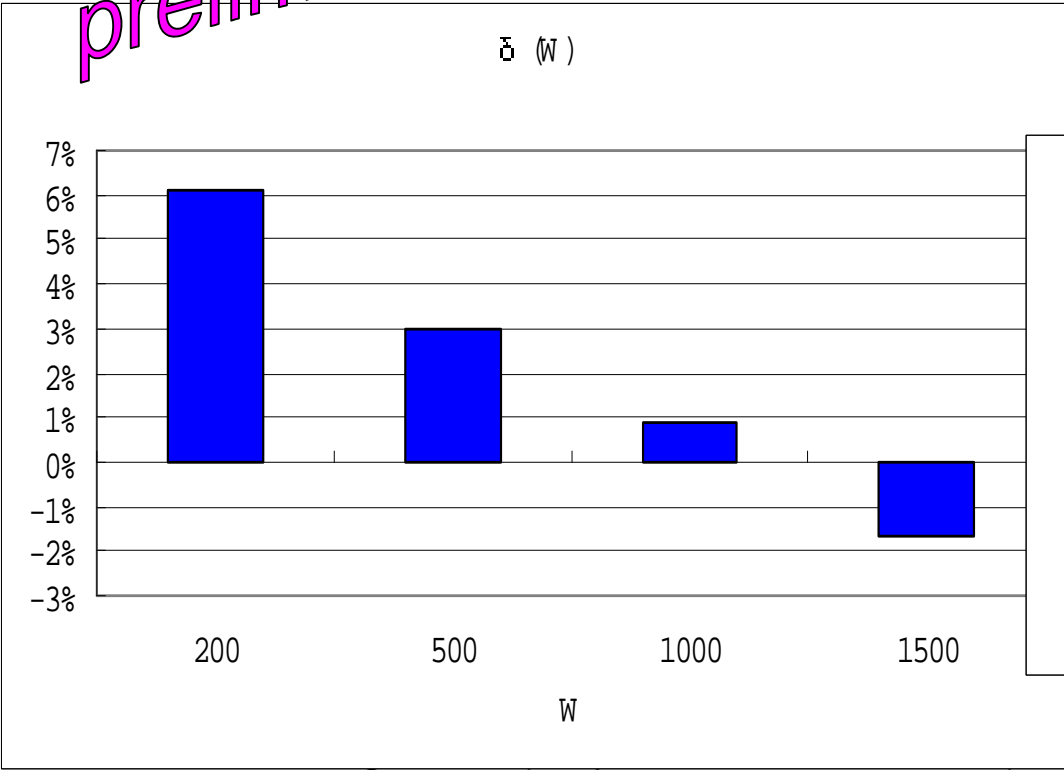
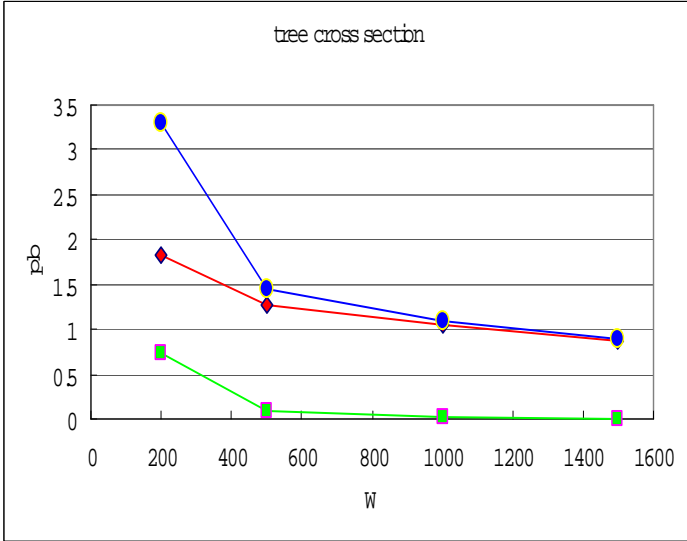


$$e^+ e^- \rightarrow \nu \bar{\nu} \gamma$$

photon distribution ... in progress

preliminary

$M_H = 120 \text{ GeV}$



all one-loop corrections  $\Delta r = 2.55\%$

# conclusion

- Technology is established to handle full-EW RC for  $2 \rightarrow 3$  processes.
- Gauge parameter independence in NLG is powerful to confirm the results.
- $e^+e^- \rightarrow e^+e^-H$  calculated, size and behavior is similar to  $e^+e^- \rightarrow \nu\bar{\nu}H$
- study of  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  in progress