LoopFest III Santa Barbara, April 2004

The LoopVerein: (not only) European activities

Stefan Dittmaier

MPI Munich

Contents

1 Introduction

Experiments at LEP/SLC/Tevatron

- confirmation of Standard Model as quantum field theory
  (quantum corrections significant)
- top mass m_t indirectly constrained by quantum corrections
  $\leftrightarrow$ in agreement with m_t measurement of Tevatron
- Higgs mass M_H indirectly constrained by quantum corrections
  $\hookrightarrow$ impact on Higgs searches

Great success of precision physics

$\hookrightarrow$ even greater potential at future $\mathrm{e}^+\mathrm{e}^-$ linear collider

- GigaZ/MegaW: precision increases by factor ~ 10 w.r.t. LEP/SLC
  EXP: $\Delta\sin^2\theta_\mathrm{eff}^\mathrm{lept} \sim 0.00001, \qquad \Delta M_\mathrm{W} \sim 7\,\mathrm{MeV}$
  TH: go from a few 10^2 to a few 10^4 (more complicated) diagrams
- high-energy runs ($\sqrt{s} = 350, 500, 800, ...\,\mathrm{GeV}$):
  various cross sections at %-level, SUSY particle production (?)

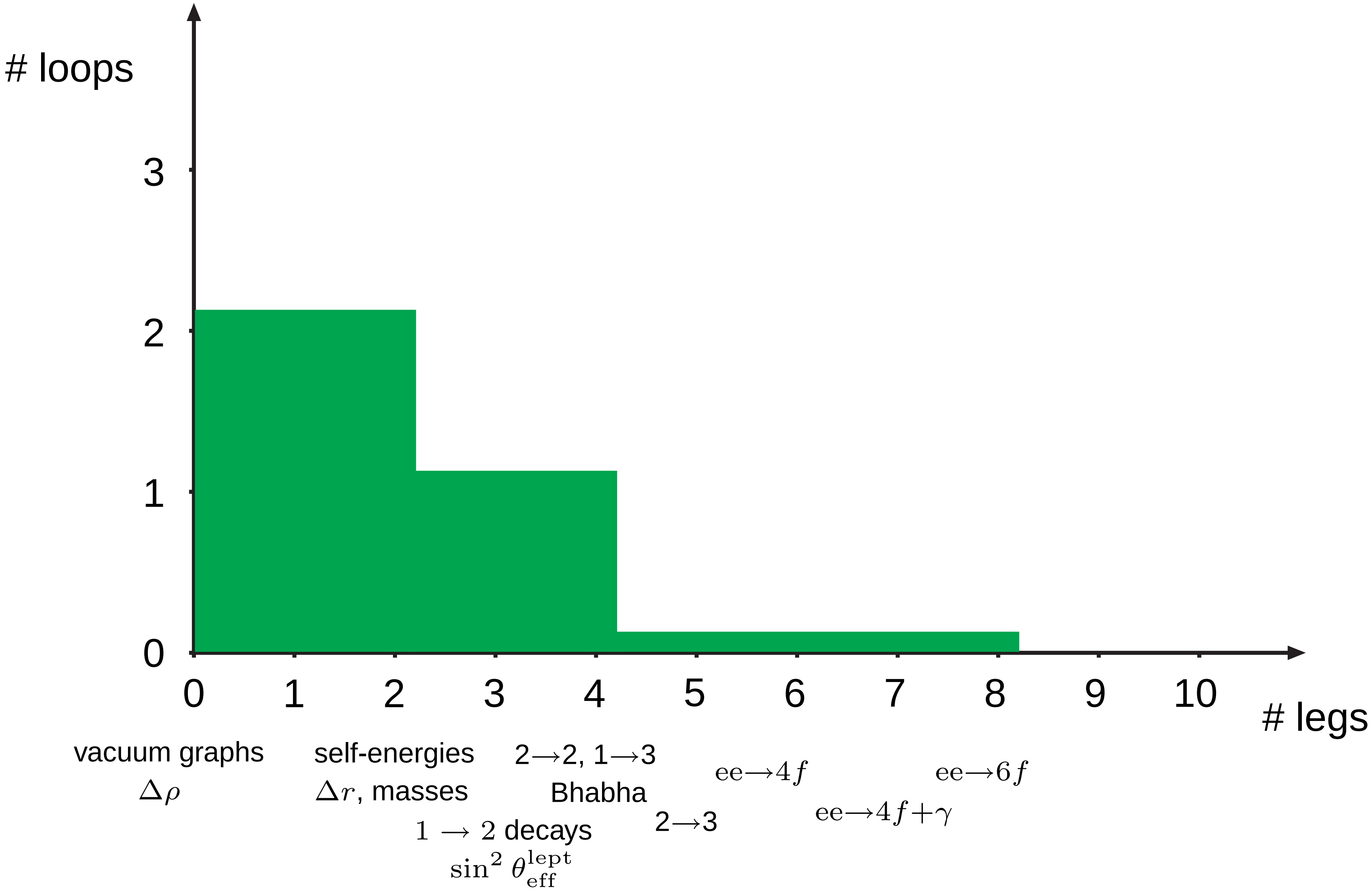
LC physics program demands precision calculations

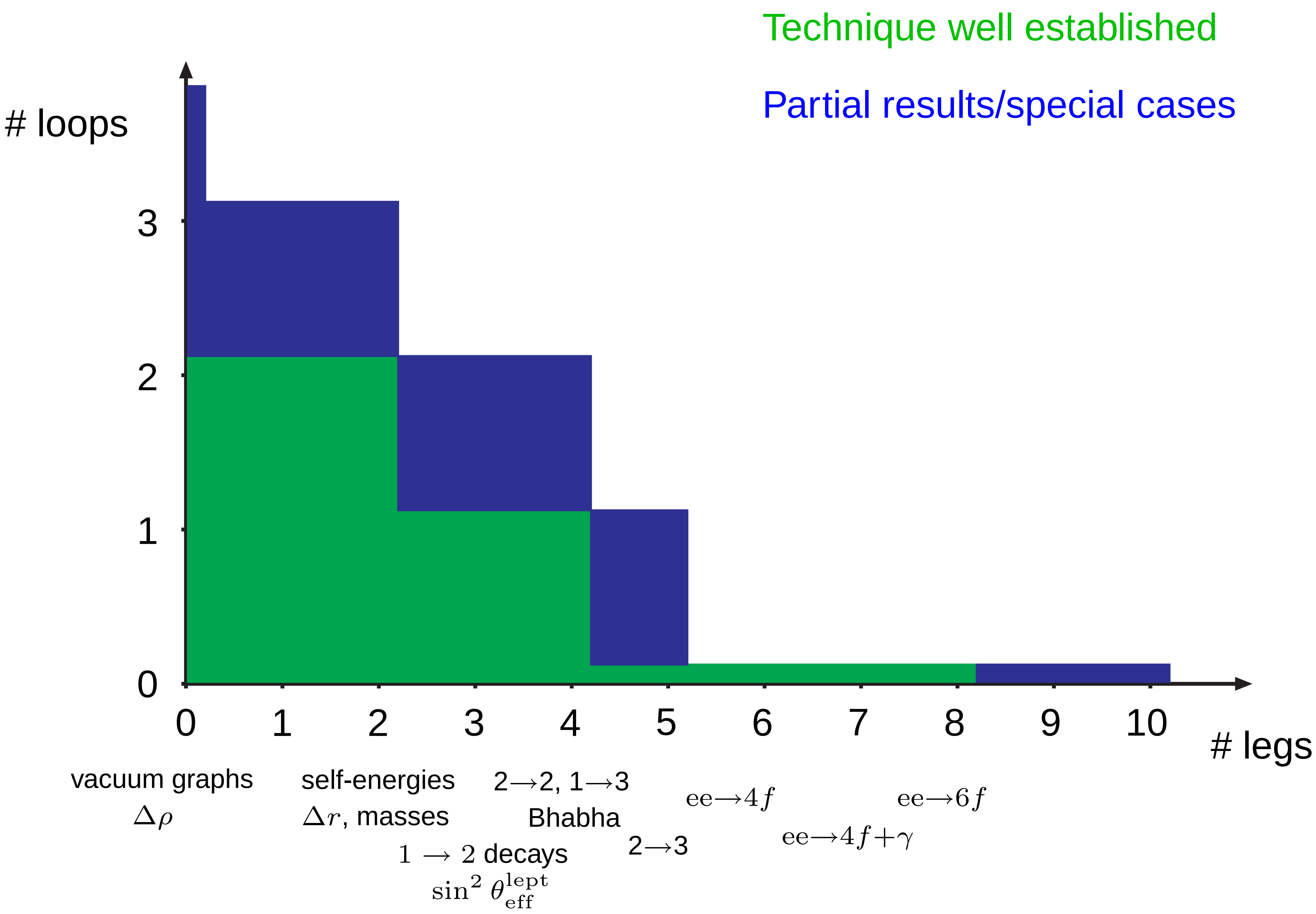
- progress on higher-order corrections
  - ◇ 2-loop accuracy for $2 \to 2$ and $1 \to 3$ reactions
  - ◇ full 1-loop calculations for $2 \to 3, 4, ...$ processes
- progress on simulation tools
  - ◇ Monte Carlo generators for multi-particle final states

This talk: summary of developments (more topical than comprehensive) presented in the LoopVerein (conveners: S. Dittmaier, W. Hollik)

- restriction to physics at future $\mathrm{e}^+\mathrm{e}^-$ linear colliders
- brief review of time after spring 2001 (TESLA TDR)
- emphasis on progress after LoopFest II, May 2003

Technique well established




Technique well established
Partial results/special cases
loops
3
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legs
vacuum graphs
$\Delta\rho$
self-energies
Δr, masses
$1 \to 2$ decays
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$
2→2, 1→3
Bhabha
2→3
$\mathrm{ee}{\to}4f$
$\mathrm{ee}{\to}4f{+}\gamma$
$\mathrm{ee}{\to}6f$

2 Multi-loop calculations — progress on $2 \to 2$ and $1 \to 3$ reactions

2.1 Precision calculations for μ decay

- 2-loop contribution with closed fermion loops — Freitas, Hollik, Walter, Weiglein '00

  $\Delta M_{\mathrm{W}} = \mathcal{O}(50\,\mathrm{MeV})$

  – diagram generation by *FeynArts* — Böhm, Denner, Eck, Küblbeck '90–'92; Hahn '98–'02

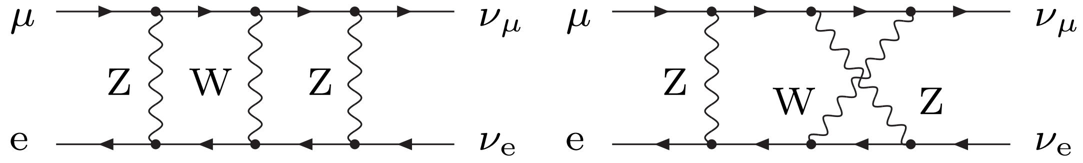
  – algebraic reduction by *TwoCalc* — Weiglein

  – semianalytical evaluation of 2-loop master integrals — Bauberger, Berends, Böhm, Buza '95

  recalculation by Awramik, Czakon '03

- M_{H} dependence of 2-loop diagrams — Freitas, Hollik, Walter, Weiglein '02

- Complete 2-loop calculation of boson loops — Awramik, Czakon '02; Onishchenko, Veretin '02



  $\Delta M_{\mathrm{W}} = \mathcal{O}(1\,\mathrm{MeV})$

  – diagram generation by *Diagen* (AC) and *DIANA* (OV) — Fleischer, Tentyukov '99–'02

  – algebraic reduction by *FORM*

  – semianalytical evaluation (AC) and asymptotic expansion (OV)

Recent progress: 3-loop contributions to $\Delta\rho$

Faisst, Kühn, Seidensticker, Veretin '03

– corrections of $\mathcal{O}(G_\mu^2 m_t^4 \alpha_s)$ and $\mathcal{O}(G_\mu^3 m_t^6)$

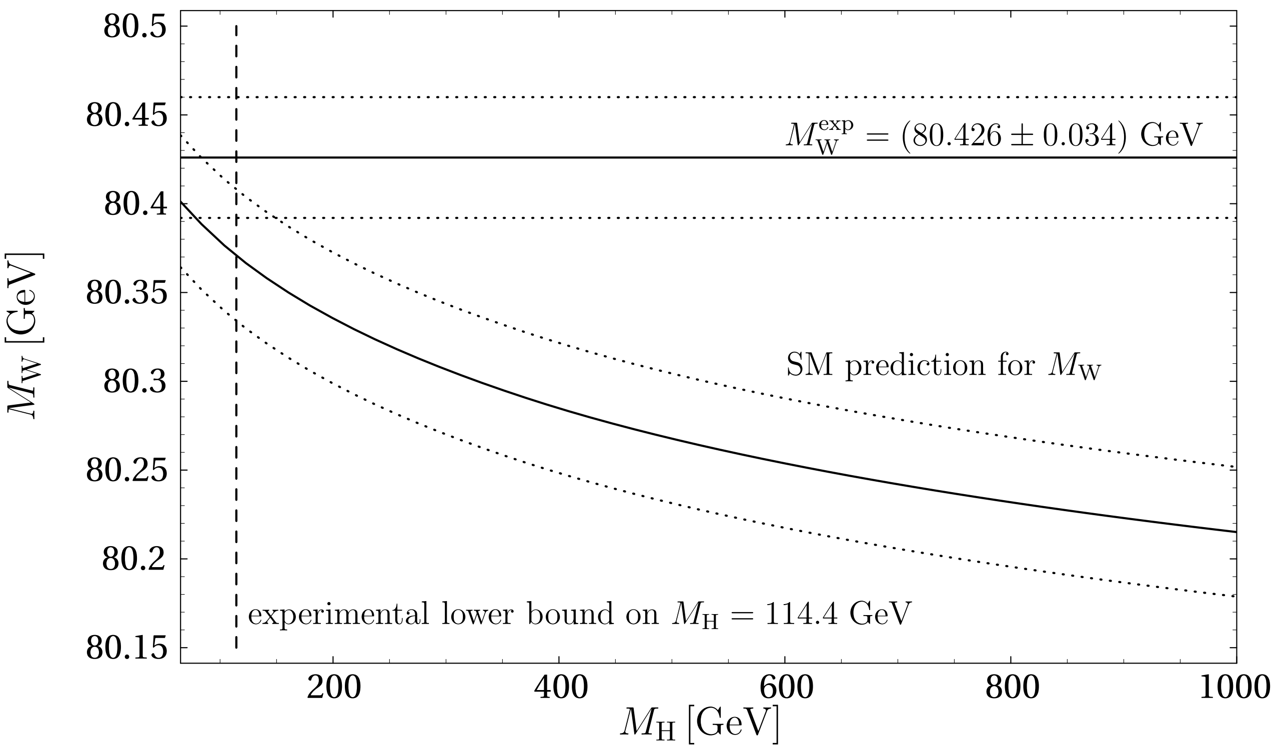
$\hookrightarrow \Delta M_W = \mathcal{O}(5\,\mathrm{MeV})$ $\hookrightarrow \Delta M_W = \mathcal{O}(0.5\,\mathrm{MeV})$

– diagrams generated by *QGRAF* and asymptotically expanded in *FORM*

Nogueira '93

$\hookrightarrow$ tadpole diagrams computed with *MATAD* Steinhauser '00

Prediction of M_W from muon decay:



Awramik, Czakon, Freitas, Weiglein '03

Theoretical uncertainty:

status 2000: $\Delta M_W \sim 6\,\mathrm{MeV}$

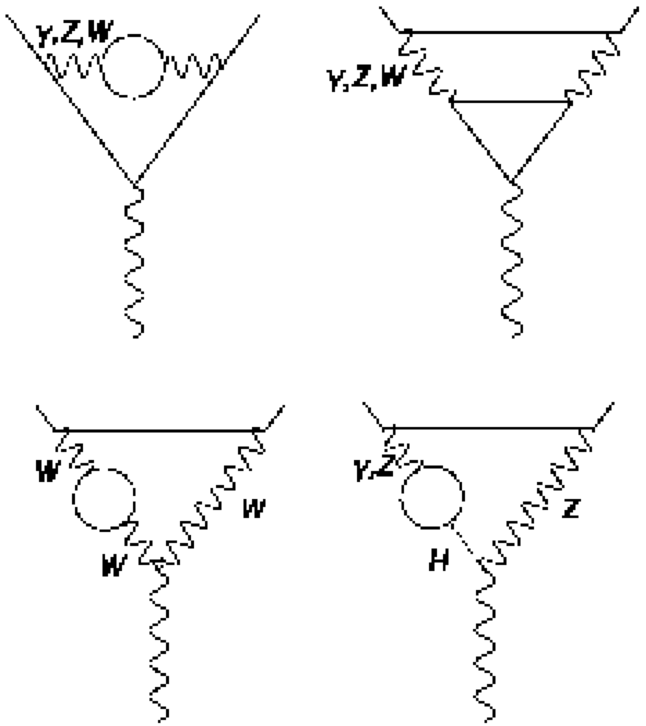
status 2003: $\Delta M_W \sim 2\,\mathrm{MeV}$

2.2 Precision calculation of effective weak mixing angle

Corrections to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = \frac{1}{4}\left(1 - \mathrm{Re}\left\{\frac{g_V}{g_A}\right\}\right)$

$\hookrightarrow$ very important contribution to high-precision physics at GigaZ

Features of 2-loop fermionic diagrams: Awramik, Czakon '03



- diagrams with closed fermion loops form gauge-invariant subset
- main complication: 2-loop vertex diagrams
- Higgs diagrams vanish by CP symmetry

Status of calculations: 2-loop fermionic corrections in progress

- calculation completed by Awramik, Czakon '03
  $\hookrightarrow$ numerical results to be expected soon ($\rightarrow$ talk of M. Awramik)
- independent calculation by Freitas, Weiglein in progress
  based on *FeynArts/TwoCalc* and (semi-)numerical integration

Some details of the calculation by Awramik, Czakon:

- diagrams with closed top-quark loops
  $\hookrightarrow$ asymptotic large-mass expansion for $M_{\mathrm{Z}}^2/m_{\mathrm{t}}^2 \to 0$
- diagrams with closed loops of light (massless) fermions
  $\hookrightarrow$ integration-by-parts techniques for reduction to master integrals
  (Laporta algorithm, symmetry identities)
  development of program IBPSOLVER
  $\hookrightarrow$ master integrals from differential equations
- subtlety: γ_5 problem, complicated by collinear divergences

2.3 Recent results from the 2-loop frontier

Genuine 2-loop corrections to $2 \to 2$, $1 \to 3$ processes

- Algebraic reduction to master integrals — Anastasiou, Gehrmann, Glover, Laporta, Oleari, Remiddi, Smirnov, Tausk, Veretin '00

  by integration by parts, Lorentz invariance identities

  $\hookrightarrow$ calculation of master integrals by Mellin–Barnes technique, (Anastasiou, Smirnov, Tausk, Tejeda-Yeomans '99–'02)
  differential equations, numerical techniques
  (Gehrmann, Remiddi '00, '01; Binoth, Heinrich '00)

- Direct reduction of full 2-loop amplitudes — Moch, Uwer, Weinzierl '02

  $\hookrightarrow$ higher transcendental functions $\to$ nested harmonic sums

- Explicit algebraic results:
  - ◇ 2-loop amplitudes for massless $2 \to 2$ processes (Bhabha, QCD parton scattering, etc.) — Anastasiou, Bern, v.d.Bij, De Freitas, Dixon, Ghinculov, Glover, Oleari, Schmidt, Tejeda-Yeomans, Wong '01, '02
  - ◇ 2-loop QCD amplitudes for $\mathrm{e}^+\mathrm{e}^- \to 3$ jets — Garland, Gehrmann, Glover, Koukoutsakis, Moch, Remiddi, Uwer, Weinzierl '02

Further activities related to the LoopVerein:

- $\mathcal{O}(\alpha)$ correction to $e^+e^- \to f\bar{f} + \gamma$ — Jadach, Melles, Ward, Was, Yost '99–'01; Czyz, Kühn, Rodrigo '02
- Numerical approach to 2-loop diagrams
  $\hookrightarrow$ self-energies, some vertex corrections — Actis, Ferroglia, Passarino, Passera, Uccirati '01–'04
- Progress on universal QED corrections to polarized $e^{\pm}$ scattering in higher orders — Blümlein, Kawamura '02
- Part of 2-loop renormalization of massive Bhabha process:
  (1-loop graphs) $\times$ (1-loop counterterms) — Fleischer, Riemann, Tarasov, Werthenbach '02
- 2-parton bremsstrahlung in NNLO
  - ◇ leading colour terms for dipole subtraction approach — Weinzierl '03
  - ◇ complete IR structure of $e^+e^- \to 2$jets — Binoth and Heinrich '04; Anastasiou, Melnikov, Petriello '04; Gehrmann-De Ridder, Gehrmann, Glover '04

2.4 Multi-loop calculations in the MSSM

Precision observables in the MSSM

- known: – precision observables in 1-loop order since '94–'96
  – $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to $\Delta\rho$
  Djouadi, Gambino, Heinemeyer, Hollik, Jünger, Weiglein '97, '98
- more recently: Yukawa corrections to $\Delta\rho$ of $\mathcal{O}(y_\mathrm{t}^2)$, $\mathcal{O}(y_\mathrm{t}y_\mathrm{b})$, $\mathcal{O}(y_\mathrm{b}^2)$
  Heinemeyer, Weiglein '02

  obtained with *FeynArts / TwoCalc*

  $\hookrightarrow$ induced corrections: $\Delta M_\mathrm{W} \sim 2\,\mathrm{MeV}, \quad \Delta\sin^2\theta_\mathrm{eff}^\mathrm{lept} \sim 0.00001$

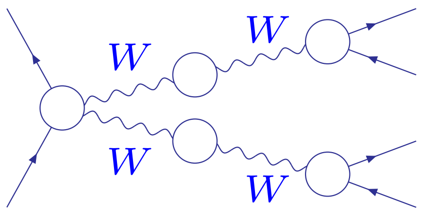
Precision calculations for MSSM Higgs masses and widths

- Higgs masses: progress in *FeynHiggs* Frank, Hahn, Heinemeyer, Hollik, Weiglein '98–'03
  – subleading 2-loop corrections
  – Δm_b resummation
  – new 1-loop renormalization (mainly $\tan\beta$)
  – corrections of $\mathcal{O}(y_\mathrm{t}^2)$, $\mathcal{O}(y_\mathrm{t}y_\mathrm{b})$, $\mathcal{O}(y_\mathrm{b}^2)$ taken from Dedes, Degrassi, Slavich '03
- Higgs decays: progress in *HDECAY* and *FeynHiggsDecay*
  Djouadi, Kalinowski, Spira '97–'03 Heinemeyer, Hollik, Weiglein '00–'03

  $\hookrightarrow$ Higgs and SUSY working groups of the ECFA workshop

3 Radiative corrections to $2 \to 3, 4, \ldots$ processes

3.1 Status of $e^+e^- \to 4f$



W-pair production $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f(+\gamma)$

State-of-the-art generators
RacoonWW (Denner, S.D.,Roth,Wackeroth) and
KoralW $\oplus$ *YFSWW* (Jadach,Płaczek,Skrzypek,Ward) include

- full lowest-order matrix elements for $\mathrm{e^+e^-} \to 4f(+\gamma)$
- non-universal electroweak corrections in “double-pole approximation”
- improvements by leading higher-order corrections

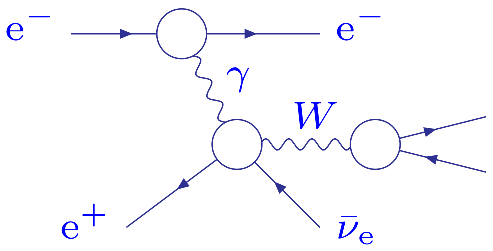
Estimates of theoretical uncertainties (TU) for

- total cross section (Denner et al., Jadach et al.)

$$\Delta\sigma/\sigma \lesssim \begin{cases} 2\% & \text{for} \quad \sqrt{s} < 170\,\mathrm{GeV} \quad \text{“improved Born approximation” (IBA)} \\ 0.7\% & \text{for} \quad 170\,\mathrm{GeV} < \sqrt{s} < 180\,\mathrm{GeV} \\ 0.5\% & \text{for} \quad 180\,\mathrm{GeV} < \sqrt{s} < 500\,\mathrm{GeV} \end{cases}$$

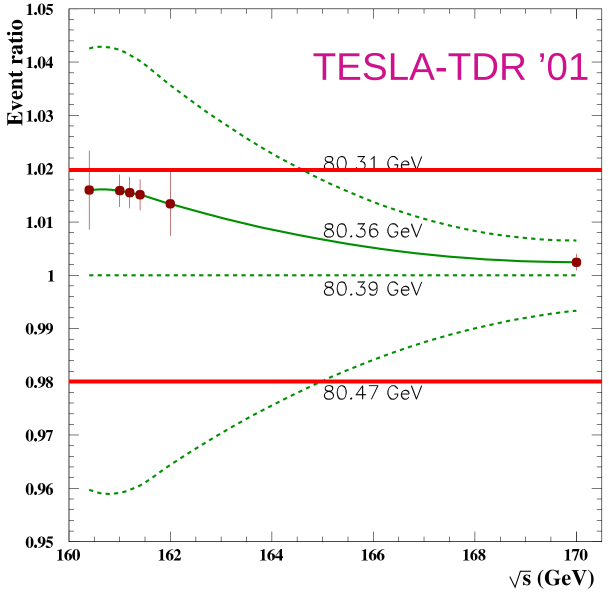
- direct M_W reconstruction (Jadach et al. ’01) $\quad \Delta M_\mathrm{W} \lesssim 5\,\mathrm{MeV}$
- bounds on anomalous TGC λ (Brunelière et al. ’02) $\quad \Delta\lambda \lesssim 0.005$

Single-W production $e^+e^- \to e\nu_e W \to e\nu_e + 2f(+\gamma)$



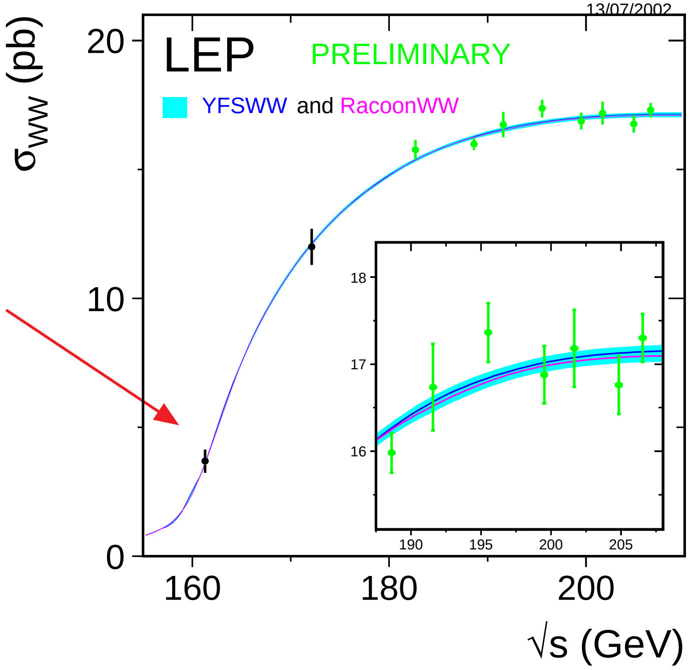
- State-of-the-art generators:
  grc4f (Kurihara et al.), *KoralW* (Jadach,Płaczek,Skrzypek,Ward,Was),
  NEXTCALIBUR (Berends et al.), *SWAP* (Montagna et al.),
  WPHACT (Accomando et al.), *WTO* (Passarino)
- Predictions include
  - ◇ full lowest-order matrix elements (with finite m_e)
  - ◇ ISR–FSR interference (“electric charge screening”) in leading-log approximation
  - ◇ improvement of running $\alpha(Q^2)$ in t-channels
  - ◇ no non-universal electroweak corrections
- Theoretical uncertainty: $\Delta\sigma/\sigma \sim 5\%$

WW-threshold scan at MegaW:

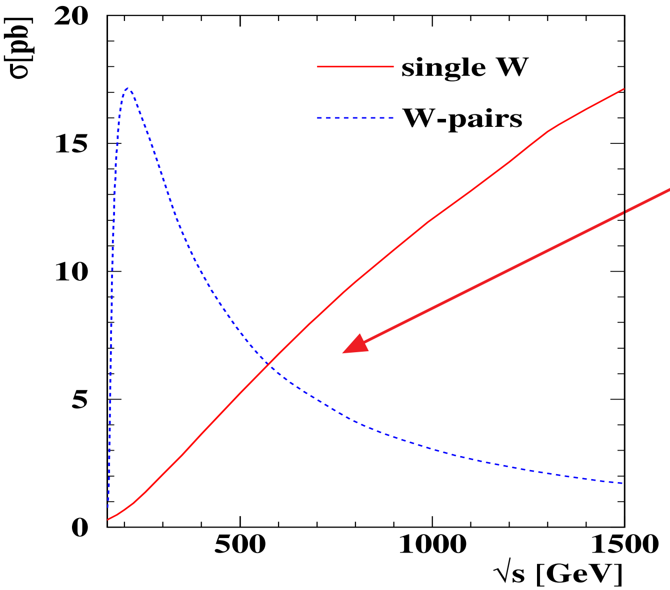


$\pm 2\%$ TU of IBA near threshold

WW cross section at LEP2:



σ_{WW} and σ_{W} at LC energies:



Measurements at %-level

Requirements for LC physics:

– Full $\mathcal{O}(\alpha)$ correction for $\mathrm{e}^+\mathrm{e}^- \to 4f$

– Leading corrections beyond $\mathcal{O}(\alpha)$

3.2 Electroweak corrections to $2 \to 3$ processes

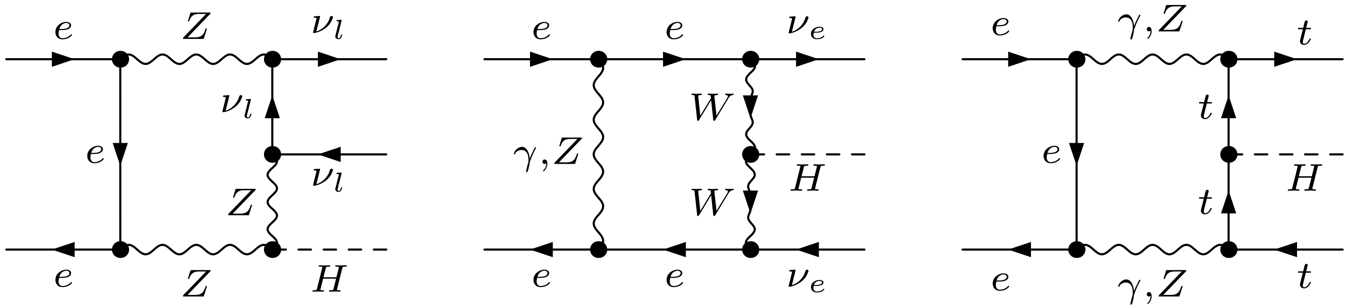
Results for $\mathcal{O}(\alpha)$ corrections to Higgs-production processes

- Three competing groups:
  - ◇ Bélanger, Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Shimizu
    $\hookrightarrow$ GRACE / 1-LOOP package
  - ◇ Denner, S.D., Roth, Weber
  - ◇ You, Ma, Chen, Zhang, Yan-Bin, Hou
- Full $\mathcal{O}(\alpha)$ corrections calculated for
  - ◇ $\mathrm{e^+e^-} \to \nu\bar{\nu}\mathrm{H}$ Bélanger et al. '02; Denner et al. '03
  - ◇ $\mathrm{e^+e^-} \to \mathrm{t\bar{t}H}$ You et al. '03; Bélanger et al. '03; Denner et al. '03
  - ◇ $\mathrm{e^+e^-} \to \mathrm{ZHH}$ You et al. '03; Bélanger et al. '03
  - ◇ $\gamma\gamma \to \mathrm{t\bar{t}H}$ You et al. '03
- Total cross sections and distributions investigated
- Features of the corrections:
  - ◇ electroweak corrections typically of $\mathcal{O}(10\%)$
  - ◇ simple approximation for corrections seems unfeasible

$\Rightarrow$ Full calculations needed for LC physics

Features of the calculations

- # of 1-loop diagrams: $\mathcal{O}(350,\ 700,\ 1600)$ for $\mathrm{e}^+\mathrm{e}^- \to \nu\bar{\nu}\mathrm{H},\ \mathrm{t\bar{t}H},\ \mathrm{ZHH}$
- gauge-invariance checks (calculation in different gauges)
  - ◇ ‘t Hooft–Feynman gauge (default)
  - ◇ non-linear gauge (Bélanger et al.)
  - ◇ background-field gauge (Denner et al.)
- combination of virtual and real corrections in different ways
  - ◇ phase-space slicing (default)
  - ◇ subtraction methods (Bélanger et al., Denner et al.)
- main complication: pentagon diagrams

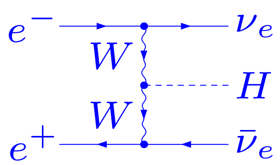


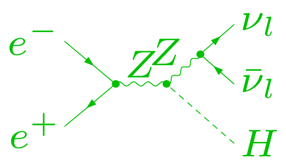
etc.

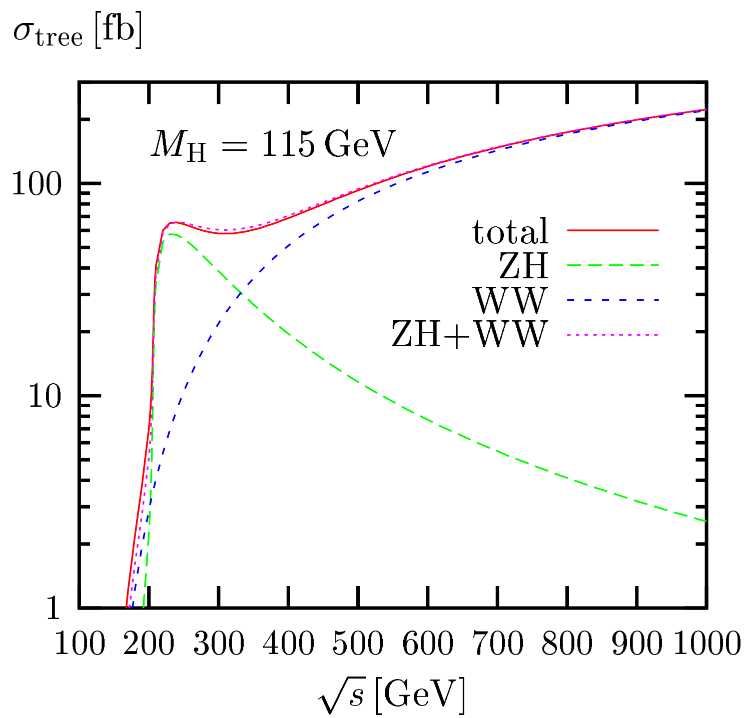
technically: algebraic reduction of 5-point integrals to 4-point integrals

Total cross section for $e^+e^- \to \nu\bar{\nu}H$

Lowest order:

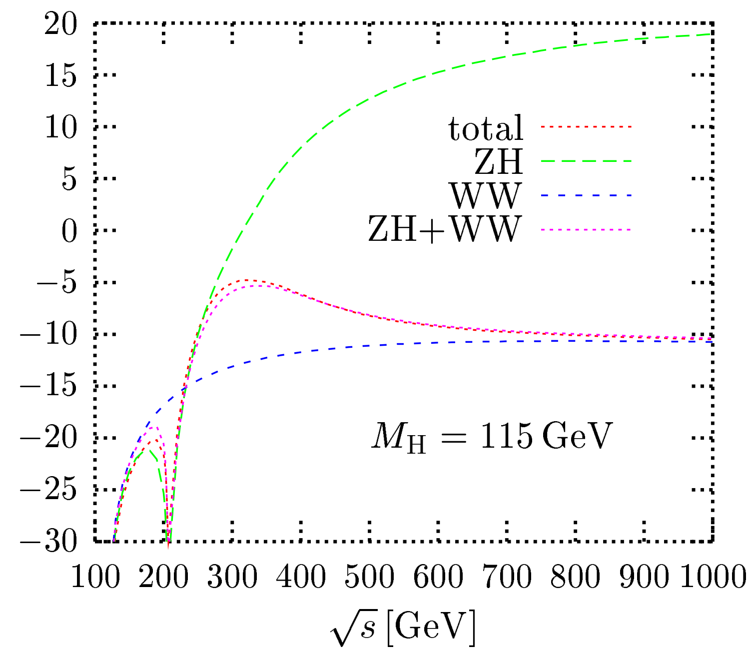






Relative corrections (G_μ scheme):

Denner, S.D., Roth, Weber '03



Comparison of results: ($\sqrt{s} = 500\,\text{GeV}$, $\alpha(0)$ scheme)

M_H [GeV]	σ_{tree} [fb]	σ [fb]	$\sigma/\sigma_{\text{tree}} - 1$[%]	
150	61.074(7)	60.99(7)	−0.2	Bélanger et al.
	61.076(5)	60.80(2)	−0.44(3)	Denner et al.
300	10.758(1)	10.30(1)	−4.2	Bélanger et al.
	10.7552(7)	10.282(4)	−4.40(3)	Denner et al.

Agreement within $\mathcal{O}(0.2\%)$

Total cross section for $\mathrm{e^+e^-} \to \mathrm{t\bar{t}H}$

Comparison of results: ($M_\mathrm{H} = 120\,\mathrm{GeV}$, $\alpha(0)$ scheme)

$\sqrt{s}$ [GeV]	σ_tree [fb]	σ [fb]	$\sigma/\sigma_\mathrm{tree} - 1[\%]$	
600	1.7293(3)	1.738(2)	0.5	Bélanger et al.
	1.7292(2)	1.7368(6)	0.44(3)	Denner et al.
800	2.2724(5)	2.362(4)	3.9	Bélanger et al.
	2.2723(3)	2.3599(6)	3.86(2)	Denner et al.
1000	1.9273(5)	2.027(4)	5.2	Bélanger et al.
	1.9271(3)	2.0252(5)	5.09(2)	Denner et al.

Agreement within $\approx 0.1\%$ (=statistical error of Bélanger et al.)

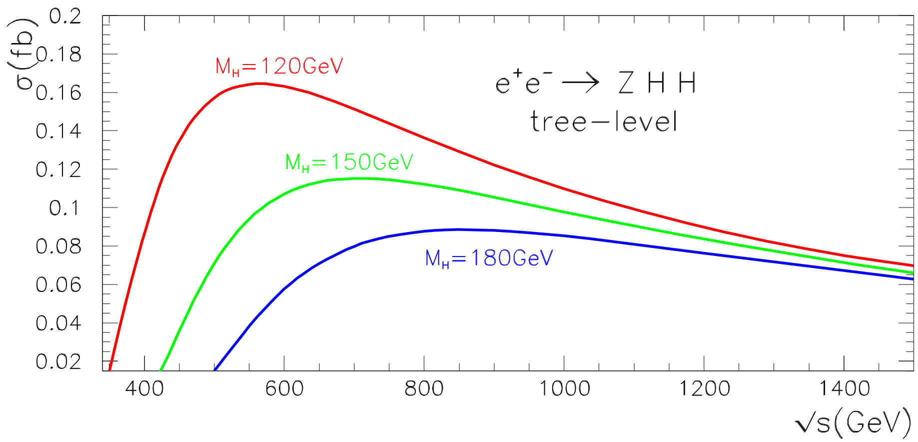
More detailed comparisons reveal

- agreement between Bélanger et al. and Denner et al. within integration errors
- agreement with You et al. up to differences at low and high energies

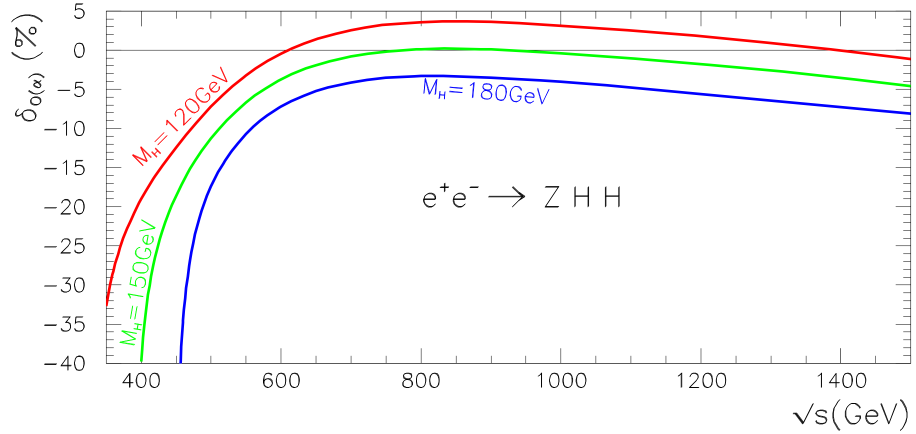
$\Rightarrow$ $\mathcal{O}(\alpha)$ corrections are under control

Total cross section for $e^+e^- \to ZHH$

Lowest order:



Relative corrections: Bélanger et al. ’03



Comparison of results: $(M_{\mathrm{H}} = 115\,\mathrm{GeV}$, $\alpha(0)$ scheme)

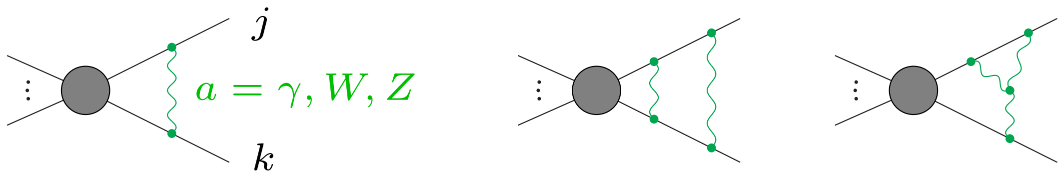
$\sqrt{s}$ [GeV]	σ_{tree} [fb]	σ [fb]	$\sigma/\sigma_{\mathrm{tree}} - 1[\%]$	
500	0.17491(2)	0.16282(2)	$-6.91(1)$	Bélanger et al.
	0.17493(2)	0.1629(2)	$-6.9(1)$	You et al.
800	0.14155(1)	0.14705(2)	$+3.89(1)$	Bélanger et al.
	0.14156(3)	0.1471(3)	$+3.9(2)$	You et al.
2000	0.05021(1)	0.04773(2)	$-4.95(4)$	Bélanger et al.
	0.05021(1)	0.0473(2)	$-5.8(4)$	You et al.

Agreement within $\mathcal{O}(0.1\%)$ for small CM energies,
but differences of $\mathcal{O}(1\%)$ in TeV range

4 Other theoretical developments

4.1 Electroweak radiative corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



etc.

+ sub-leading logarithms from mass singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta_{\mathrm{LL}}^{1-\mathrm{loop}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta_{\mathrm{NLL}}^{1-\mathrm{loop}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta_{\mathrm{LL}}^{2-\mathrm{loop}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta_{\mathrm{NLL}}^{2-\mathrm{loop}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still significant at 2-loop level

NOTE: differences to QED / QCD where Sudakov log's cancel

- massive gauge bosons W, Z can be reconstructed
  $\hookrightarrow$ no need to add "real W, Z radiation"
- non-Abelian charges of W, Z are "open" $\rightarrow$ Bloch–Nordsieck theorem violated

$\Rightarrow$ Aim: universal prescription for improving (e.g. 1-loop) calculations

Progress at the 2-loop level

Explicit calculations for

- LL level Melles '00; Hori, Kawamura, Kodaira '00; Beenakker, Werthenbach '00, '01
- LL + angular-dependent NLL level Denner, Melles, Pozzorini '03
- form factor for gauge-boson–fermion vertex to LL+NLL level Pozzorini '04

Structure of result at ang.-dep. NLL level:

$$\mathcal{M} \sim \mathcal{M}_0 \otimes \left(\delta_{\mathrm{ew}} + \delta_{\mathrm{em}} + \frac{1}{2}\delta_{\mathrm{ew}}^2 + \delta_{\mathrm{ew}} \otimes \delta_{\mathrm{em}} + \frac{1}{2}\delta_{\mathrm{em}}^2\right)$$

with

$$\delta_{\mathrm{ew}} = \frac{\alpha}{4\pi}\left\{-\frac{1}{2}\sum_j C_j^{\mathrm{ew}} \ln^2\frac{M_{\mathrm{W}}^2}{s} + \sum_{k\neq j}\sum_{a=\gamma,\mathrm{Z},\mathrm{W}^\pm} I_j^a I_k^{\bar{a}} \ln\frac{s}{|2p_j p_k|}\ln\frac{M_{\mathrm{W}}^2}{s}\right\}$$

$$\delta_{\mathrm{em}} = \frac{\alpha}{4\pi}\left\{-\frac{1}{2}\sum_j Q_j^2\left[2\ln\frac{m_j^2}{s}\ln\frac{m_\gamma^2}{M_{\mathrm{W}}^2} - \ln^2\frac{m_j^2}{M_{\mathrm{W}}^2}\right] + \sum_{k\neq j} Q_j Q_k \ln\frac{s}{|2p_j p_k|}\ln\frac{m_\gamma^2}{M_{\mathrm{W}}^2}\right\}$$

Agreement with proposed resummations

based on a symmetric SU(2)$\times$U(1) theory at high energies
matched with QED at electroweak scale

$$\mathcal{M} \sim \mathcal{M}_0 \otimes \exp\left(\delta_{\mathrm{ew}}\right) \otimes \exp\left(\delta_{\mathrm{em}}\right)$$

Fadin et al. '99; Melles '00, '01;
Kühn, Moch, Penin, Smirnov '01

Progress in the MSSM

Explicit 1-loop results with proposed exponentiations Beccaria et al. '01, '02

4.2 Precision calculations for SUSY-particle production

Calculations widely performed with packages

FeynArts (with MSSM extension) (Hahn, Schappacher '01), *FormCalc/LoopTools* (Hahn '98–'04)

or *GRACE/SUSY* (Fujimoto, Ishikawa, Jimbo, Kaneko, Kon, Kuroda '00–'04)

- Complete $\mathcal{O}(\alpha)$ corrections within MSSM to pair production of
  - selectrons/smuons (Freitas, v.Manteuffel, Zerwas '02, '03)
  - staus/squarks (Arhrib, Hollik '03; Kovarik, C.Weber, Eberl, Majerotto '04)
  - charginos (Blank, Hollik '00; Baer, Diaz, Rivera, Ross '02; Fujimoto et al. '04)
  - neutralinos (Öller, Eberl, Majerotto '04; Fritzsche, Hollik '04)
- Chargino / neutralino mass spectrum (Fritzsche, Hollik '02; Öller et al. '03)
  input masses of $\chi_1^+, \chi_2^+, \chi_1^0$ $(\leftrightarrow\ M_1, M_2, \mu)$
  $\hookrightarrow$ 1-loop-corrected (by some GeV) predictions for masses of $\chi_2^0, \chi_3^0, \chi_4^0$
- Sfermion mass spectrum (Hollik, Rzehak '03)
  one sfermion mass of each doublet predictable
  $\hookrightarrow$ 1-loop corrections by some GeV
- QCD and SUSY-QCD corrections of $\mathcal{O}(\alpha_s)$ to $e^+e^- \to q\bar{q}g,\ \tilde{q}\bar{\tilde{q}}g,\ \tilde{q}\bar{q}\tilde{g},\ q\bar{\tilde{q}}\tilde{g}$
  Brandenburg, Maniatis, Weber '02
  $\hookrightarrow$ test of SUSY relation between gauge and Yukawa couplings

4.3 Automatization of radiative corrections and knowledge storing

Perturbative calculations are widely algorithmic $\rightarrow$ ideal for automatization

Feynman graph and amplitude generation

Diana (Tentyukov, Fleischer '00), *FeynArts, GRACE/1-loop, GRACE/SUSY, QGraf* (Nogueira '91)

Algebraic reduction of amplitudes and tools for numerical evaluations

FeynCalc, FormCalc/LoopTools, GRACE/1-loop, GRACE/SUSY

(+ many specialized or "in-house" packages using *Form* or *Mathematica*)

Recent progress in widely automatized one-loop calculations of

- SM $2 \rightarrow 3$ processes by *GRACE/1-loop*
- MSSM $2 \rightarrow 2$ processes by *FormCalc/LoopTools* and *GRACE/SUSY*

A completely different approach: "knowledge storing"

SANC = automatic calculation of $\mathcal{O}(\alpha)$ corrections based on database of precomputed diagrams

Andonov, Bardin, Bondarenko, Christova, Kalinovskaya, Nanava '02–'04

$\hookrightarrow$ database contains process types $2f \rightarrow 2f$, $\mathrm{e}^+\mathrm{e}^- \rightarrow V\gamma$, $\mathrm{e}\gamma \rightarrow \mathrm{e}V$, $q\bar{q} \rightarrow V\gamma$ and will be further extended

5 Conclusions

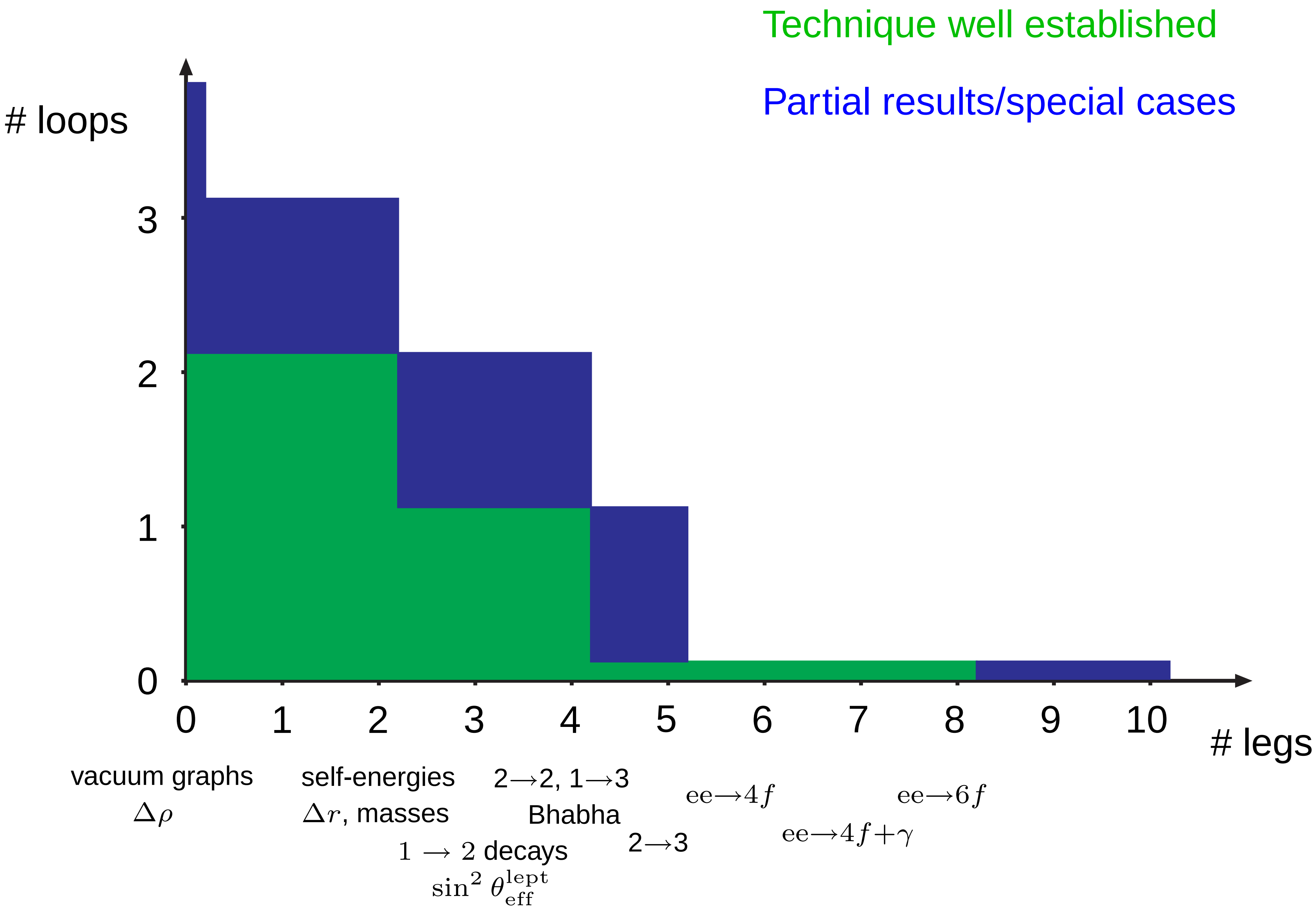
LC precision physics is great challenge

Good progress since TESLA TDR (3/2001) in various directions:

- 2-loop techniques for $2 \to 2$ and $1 \to 3$ reactions
- μ decay in full 2-loop accuracy and improved beyond
- 2-loop calculation of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ in progress
- 1-loop calculations for $2 \to 3$ processes:

  $$\mathrm{e^+e^-} \to \nu\bar{\nu}\mathrm{H},\ \mathrm{t\bar{t}H},\ \mathrm{ZHH}, \qquad \gamma\gamma \to \mathrm{t\bar{t}H}$$

- leading electroweak 2-loop corrections at high energies
- precision calculations of mass spectra and production cross sections for SUSY particles
- etc.


Technique well established
Partial results/special cases
loops
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10
legs
vacuum graphs
$\Delta\rho$
self-energies
Δr, masses
$1 \to 2$ decays
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$
2→2, 1→3
Bhabha
2→3
$\mathrm{ee}\to 4f$
$\mathrm{ee}\to 4f+\gamma$
$\mathrm{ee}\to 6f$

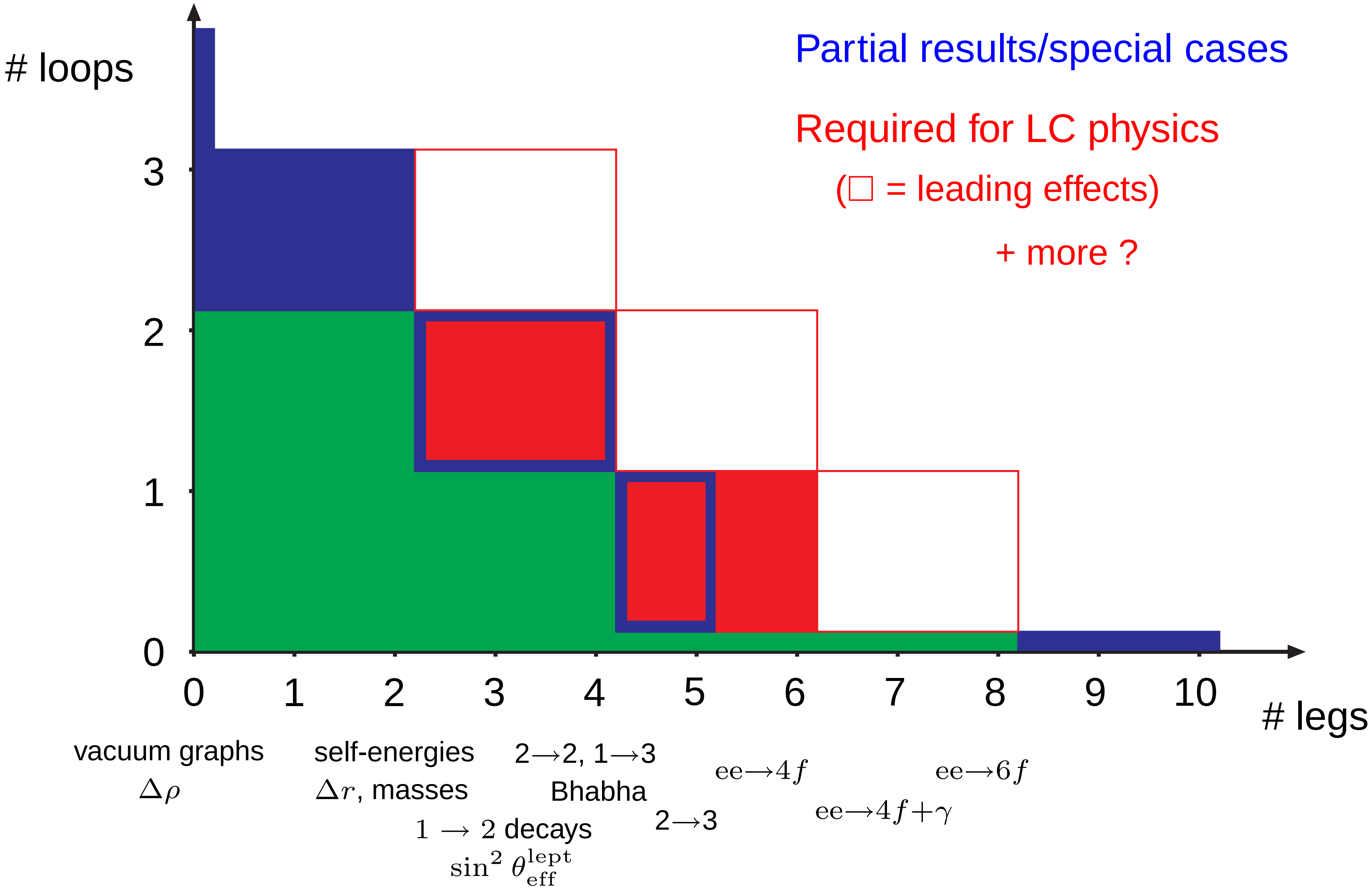
Technique well established

Partial results/special cases

Required for LC physics

(□ = leading effects)

+ more ?



Many issues for the future:

- complete 2-loop calculations for $1 \to 2, 3$ and $2 \to 2$ reactions
  - Z line shape and Z-pole pseudo-observables ($\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, etc.)
  - Bhabha scattering, $\mathrm{e}^+\mathrm{e}^- \to 3\mathrm{jets}$, etc.
- complete 1-loop calculations for $\mathrm{e}^+\mathrm{e}^- \to 4f$
- implementation of radiative corrections in Monte Carlo generators
- general issues
  - resummation of leading higher-order corrections
  - treatment of unstable particles
  - parallelization / automatization / standardization

$\Rightarrow$ Many long-termed projects — continuous support needed !