Unstable particle production with effective field theory

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Ref.: MB, A.P. Chapovsky (Aachen), A. Signer (Durham), G. Zanderighi (Fermilab), hep-ph/0312331 (brief) and hep-ph/0401002 (details)

Statement of the problem

- **Perturbation theory breaks down near resonance, because propagators become singular:**

$$\frac{g^2}{s-M^2} \sim 1 \quad \text{when} \quad s-M^2 \sim M\Gamma \sim (gM)^2$$

- **Two different scales: formation/decay time $1/M$, lifetime $1/\Gamma \gg 1/M$**

- **"Dyson" resummation of self-energy insertions**

$$\frac{1}{s-M^2} \rightarrow \frac{1}{s-M^2-\Pi(s)}$$

removes the singularity, since $\mathrm{Im}\,\Pi \sim -M\Gamma$.

- **Issues:**
  - **Rules for a systematic approximation (in g^2 and Γ/M) of the scattering amplitude/cross section**
  - **Gauge invariance**

State of the art

- **Next-to-leading order in g^2 and Γ/M:**
  - tree including Γ/M corrections
  - one-loop virtual in (double) pole approximation, leading order in Γ/M
  - real corrections done "exactly" or with approximations accurate to leading order in Γ/M

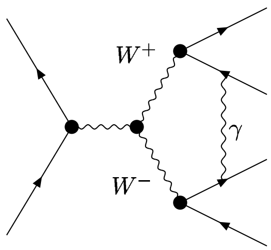
  Except for pair production near threshold.

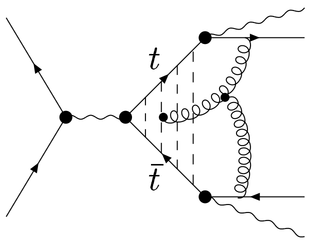
- A variety of often pragmatic approaches to deal with gauge invariance:
  - fermion-loop scheme (Argyres et al.,1995)
  - pinch technique (Papavassiliou et al., 1994)
  - complex mass scheme (Denner et al., 1999)
  - pole scheme (Stuart, 1991; Aeppli et al. 1994)
  - ...

- Not clear how to extend these beyond NLO calculations.

Motivation for further development

- **Precision calculations of W, Z and top production.**
- **Particularly of W^+W^- and $t\bar{t}$ production near threshold.**





- **Maybe the Higgs boson is heavy ...**
- **A problem of general interest: quantum field theory with unstable fundamental fields is understood in principle (Veltman, 1963), but not in weak coupling expansions.**

Setup (I)

- **Consider line-shape 1+2 $\rightarrow$ resonance $\rightarrow$ X**

$$\delta \equiv \frac{s - M^2}{M^2}$$

- **Off resonance, $\delta \sim 1$, conventional perturbation theory applies**

$$\sigma \sim g^4 f_1(\delta) + g^6 f_2(\delta) + \ldots$$

- **Near resonance, $\delta \ll 1$, expand in δ and reorganize**

$$\sigma \sim \sum_n \left(\frac{g^2}{\delta}\right)^n \times \{1\ (\mathrm{LO}); g^2, \delta\ (\mathrm{NLO}), \ldots\} = h_1(g^2/\delta) + g^2 h_2(g^2/\delta) + \ldots$$

- **The two approximations must be matched in an intermediate region.**
- **Construct the expansion resonant cross sections by integrating out the hard momentum scales ($\rightarrow$ effective field theory)**

Setup (II)

- **Inclusive line-shape ($\leftarrow$ use optical theorem)**

$$\bar{\nu}(q) + e^-(p) \to X$$

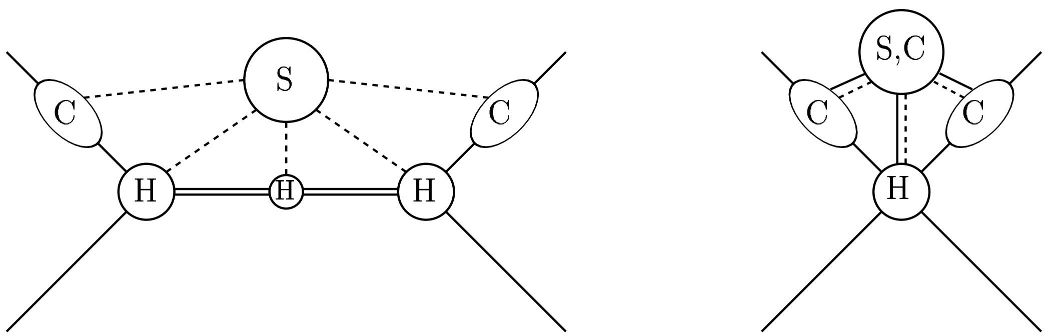
in a toy model

$$\begin{aligned}\mathcal{L} &= (D_\mu\phi)^\dagger D^\mu\phi - \hat{M}^2\phi^\dagger\phi + \bar{\psi}i\not{D}\psi + \bar{\chi}i\not{\partial}\chi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \\ &\quad + y\phi\bar{\psi}\chi + y^*\phi^\dagger\bar{\chi}\psi - \frac{\lambda}{4}(\phi^\dagger\phi)^2 + \mathcal{L}_{\rm ct},\end{aligned}$$

- **"Economy version" of $u\bar{d} \to W^- \to e^-\bar{\nu}$:**
  scalar resonance, Yukawa coupling to fermions, photons
  (The real process with traditional methods: (Wackeroth, Hollik, 1996; Dittmaier, Krämer, 2001; ...)**)**

Effective Theory (I)

- **Step 1: Integrate out hard fluctuations $k \sim M$**
  The EFT contains
  - **soft fields $k \sim \Gamma$: massless, scalar near resonance field ϕ_v ($p = Mv + k$, as in HQET)**
  - **hard-collinear fields (massless only) $k_+ \sim M, k_\perp \sim \sqrt{M\Gamma}, k_- \sim \Gamma$ and vice versa**
  - **Effective interactions**



- **Step 2: Integrate out hard-collinear fluctuations**
  which leaves
  - **soft fields as above**
  - **soft-collinear fields ψ_{n_-} ($p = Mn_-/2 + k$) and χ_{n_+} ($p = Mn_+/2 + k$)**

  i.e. only soft fluctuations around classical scattering trajectory.

Effective Theory (II)

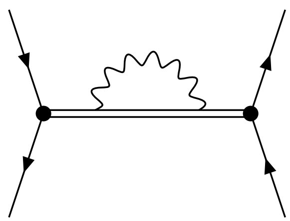
$$\mathcal{L}_{\rm eff} = 2\hat{M}\phi_v^\dagger\left(iv\cdot D_s - \frac{\Delta}{2}\right)\phi_v + 2\hat{M}\phi_v^\dagger\left(\frac{(iD_{s\top})^2}{2\hat{M}} + \frac{\Delta^2}{8\hat{M}}\right)\phi_v$$
$$- \frac{1}{4}F_{s\mu\nu}F_s^{\mu\nu} + \bar{\psi}_s i\not{D}_s\psi_s + \bar{\chi}_s i\not{\partial}\chi_s + \bar{\psi}_{n_-} in_- D_s\frac{\not{n}_+}{2}\psi_{n_-}$$
$$+ C\,[y\,\phi_v\bar{\psi}_{n_-}\chi_{n_+} + \text{h.c.}] + \frac{yy^* D}{4\hat{M}^2}(\bar{\psi}_{n_-}\chi_{n_+})(\bar{\chi}_{n_+}\psi_{n_-}) + \ldots$$

- **At NLO need**
  - **Δ to order g^4 (two-loop on-shell self-energy)**
    In the pole scheme $\Delta = -i\Gamma$ exactly with Γ the on-shell width
  - **$C = 1 + \ldots$ to one-loop**
  - **D at tree-level, $D = 1$**

- **The unstable particle propagator is** $$\frac{i}{2\hat{M}(v\cdot k - \Delta^{(1)}/2)}$$

- **After deriving $\mathcal{L}_{\rm eff}$ to the required accuracy by matching calculations, calculate the scattering amplitude in the effective theory – both is done in conventional PT**

Sample diagram



Separate hard and soft contributions to the 1-loop self-energy $\Pi(s) = \Pi_h(s) + \Pi_s(s)$, then expand

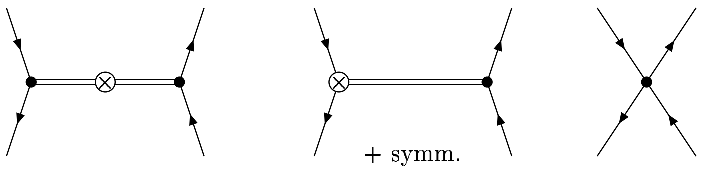
$$\Pi_h(s) = \hat{M}^2 \sum_l \delta^l \, \Pi^{(1,l)}$$

- **The different terms are distributed as follows:**
  - $\Pi^{(1,0)}$ (gauge-invariant) $\longrightarrow \Delta^{(1)}$ (LO)
  - $\Pi^{(1,1)}$ (gauge-dependent) $\longrightarrow C^{(1)}$ (NLO)
  - $\Pi^{(1,2)}$ (gauge-dependent) $\longrightarrow D^{(1)}$ (NNLO)
  - Π_s is reproduced by the effective theory self-energy

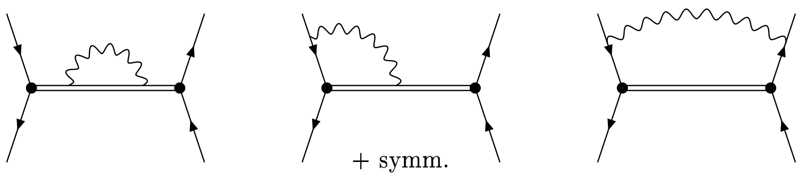
  And so on in higher order in δ and α

- **The matching procedure guarantees that the coefficients of the effective Lagrangian are automatically gauge-invariant (because so is the Lagrangian), and that no double-counting occurs.**

NLO line shape (I)



$$i\,\mathcal{T}_h^{(1)} = i\,\mathcal{T}^{(0)} \times \left[2\,C^{(1)} - \frac{[\Delta^{(1)}]^2}{8\mathcal{D}\hat{M}} + \frac{\Delta^{(2)}}{2\mathcal{D}} - \frac{\mathcal{D}}{2\hat{M}} \right]$$

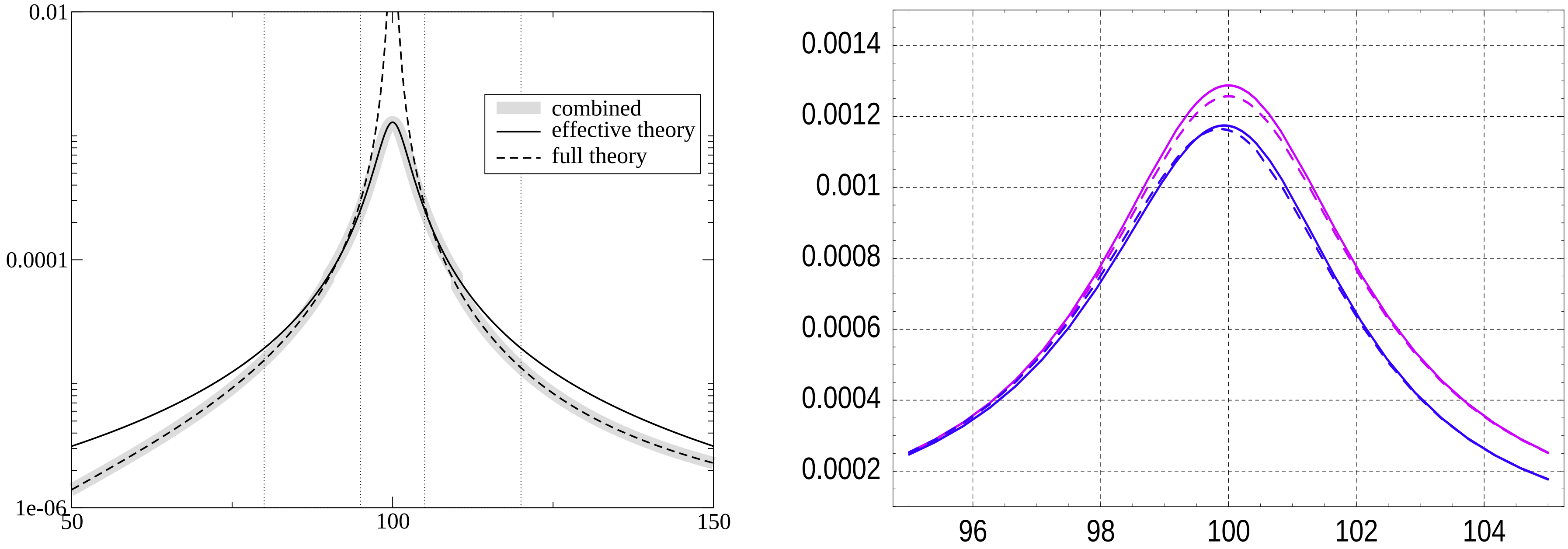


$$i\,\mathcal{T}_s^{(1)} = i\,\mathcal{T}^{(0)} \times a_g \left[4\ln^2\left(\frac{-2\mathcal{D}}{\mu}\right) - 4\ln\left(\frac{-2\mathcal{D}}{\mu}\right) + \frac{5\pi^2}{6} \right]$$

$$\mathcal{D} \equiv \sqrt{s} - \hat{M} - \frac{\Delta^{(1)}}{2}.$$

- **Leading-order line-shape $\mathcal{T}^{(0)}$ has exact Breit-Wigner form**
- **$1/\epsilon$ poles cancel when adding hard and soft contributions up to initial state collinear divergence (standard)**
- **Simple (single-scale) calculations**

NLO line shape (II)



Cross section in GeV^{-2} as function of $\sqrt{s}/\mathrm{GeV}$, $M = 100\,\mathrm{GeV}$.
Left: matching off-resonant and resonant cross section
Right: LO vs NLO (pole and $\overline{\mathrm{MS}}$ scheme)
Shown is the "partonic" cross section with initial state singularity minimally subtracted.

Electroweak theory (massive vector bosons)

- **Propagator in R_ξ gauge ($p = Mv + k$, k soft)**

$$\frac{i}{p^2 - M^2}\left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi M^2}\right) \rightarrow \frac{i}{2Mv\cdot k}\left(-g_{\mu\nu} + v_\mu v_\nu\right)$$

gauge-independent!
Describes three polarization states. Unphysical Higgs and longitudinal degree of freedom are integrated out.

- **Effective field and kinetic term**

$$W_v^\mu \equiv \left(-g_{\mu\nu} + v_\mu v_\nu\right)\, W^\mu, \qquad \mathcal{L}_{\rm eff} = 2\hat{M} W_v^{\mu\dagger}\left(iv\cdot D_s - \frac{\Delta}{2}\right) W_{v\mu} + \ldots$$

- **Just as for scalar.**
Non-renormalizability of massive vector boson theory is ok, because the EFT has a cut-off of order M anyway – implemented in dimensional regularization.
The EFT contains only massless particles and the resonance field, i.e. only photons and elecromagnetic gauge invariance.

Extensions of the formalism

- **Non-inclusive line-shapes $\longrightarrow$ phase space integrals/cut diagrams also expanded.**
- **Large logarithms of M/Γ can be summed with renormalization group equations.**
- **Extension to pair production conceptually straightforward, including pair production near threshold.**
  [work in progress]
- **High-energy limit $E \gg M$ in pair production, cf.** (Chapovsky et al., 2001)

Conclusion/Advantages of the EFT approach

- Breaks the calculation into several well-defined pieces (matching calculations, matrix element calculations) $\longrightarrow$ efficient and transparent calculation.

- It provides a systematic power-counting scheme in the small parameters (δ, couplings), which allows for an identification of the terms relevant for achieving a prescribed accuracy before actual calculations must be done.

- It provides a set of (Feynman) rules to compute the minimal set of terms necessary for a given accuracy. Since one does not calculate "too much", the calculation to a given order is presumably technically simpler than in any other approach.

- Gauge invariance is automatic at every order.

- Can be extended to any accuracy in the expansion in δ and in couplings at the expense of performing more complicated, but well-defined calculations. NNLO line shape calculations are feasible in practice.