Two Loop Fermionic Corrections to the Effective Weak Mixing Angle

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Plan

- Motivation
- Contributions to $\sin^2\theta_{eff}$
- Light Fermion Contributions
- Top Quark Contributions
- Triangle Fermion Loops and γ_5
- Conclusions & Outlook

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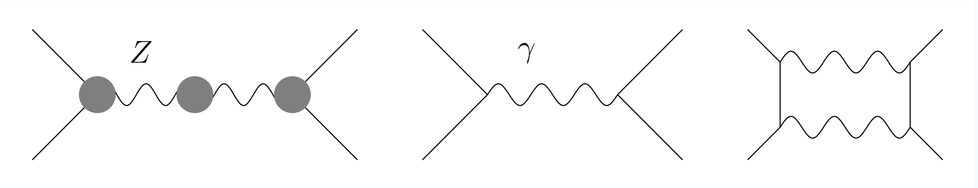
In collaboration with
M.Czakon (Zeuthen)
A.Freitas (Fermilab)
G.Weiglein (Durham)

Introduction & Motivation

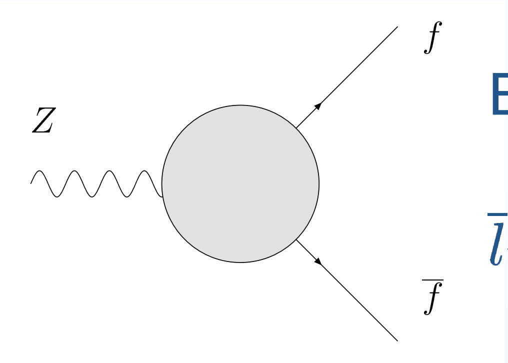
- four fermion processes at LEP



$$\mathcal{A}(e^+e^- \to Z \to f\overline{f}) = \frac{4ie^2 I_e^{(3)} I_f^{(3)}}{s - M_Z^2 + iM_Z\Gamma_Z}\rho_{ef}$$
$$\times [\gamma_\mu(1+\gamma_5) \otimes \gamma^\mu(1+\gamma_5)$$
$$-4|Q_e|s_W^2\kappa_e\gamma_\mu \otimes \gamma^\mu(1+\gamma_5) - 4|Q_f|s_W^2\kappa_f\gamma_\mu(1+\gamma_5) \otimes \gamma^\mu$$
$$+16|Q_eQ_f|s_W^4\kappa_{ef}\gamma_\mu \otimes \gamma^\mu]$$

- $\sin^2\theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(s = M_Z^2)]s_W^2$

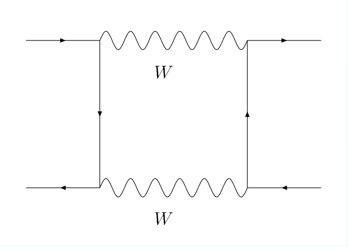
Introduction & Motivation



Effective Z boson vertex:

$$\bar{l}\gamma^{\mu}(g_V - g_A\gamma_5)lZ_{\mu}$$

- Adequate description through form factors at M_Z
- Necessary subtraction of
  - QED radiation
  - Small double boson exchange effects



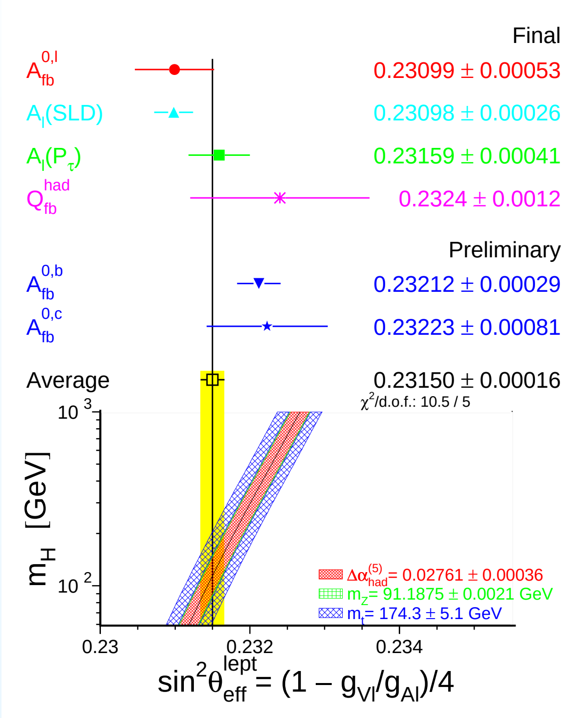
- Most interesting pseudoobservable:

$$\sin^2\theta^{\text{lept}}_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\left(\frac{g_V}{g_A}\right)\right) = \kappa\left(1 - \frac{M_W^2}{M_Z^2}\right)$$

Introduction & Motivation

- Very sensitive to top mass
- If new D0 value for m_t used, then $M_H < \sim 280$ GeV

- Current experimental precision 1.6×10^{-4}
- GigaZ foreseen 1×10^{-5}

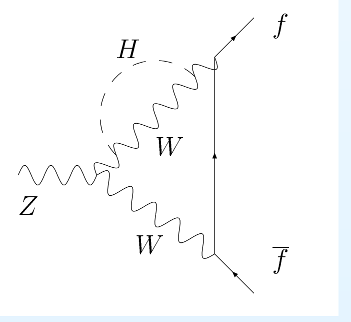


Introduction & Motivation

- Most sensitive to the Higgs boson mass:
  - at $m_t \sim 175$ GeV, gives $M_H < \sim 210$ GeV
  - from M_W measurement, $M_H < \sim 800$ GeV

- Dependence on M_H mass enters already at $\mathcal{O}(\alpha^0)$ through M_W
- At $\mathcal{O}(\alpha)$, through renormalization of $\sin^2\theta_W$

- At $\mathcal{O}(\alpha^2)$, through virtual exchanges

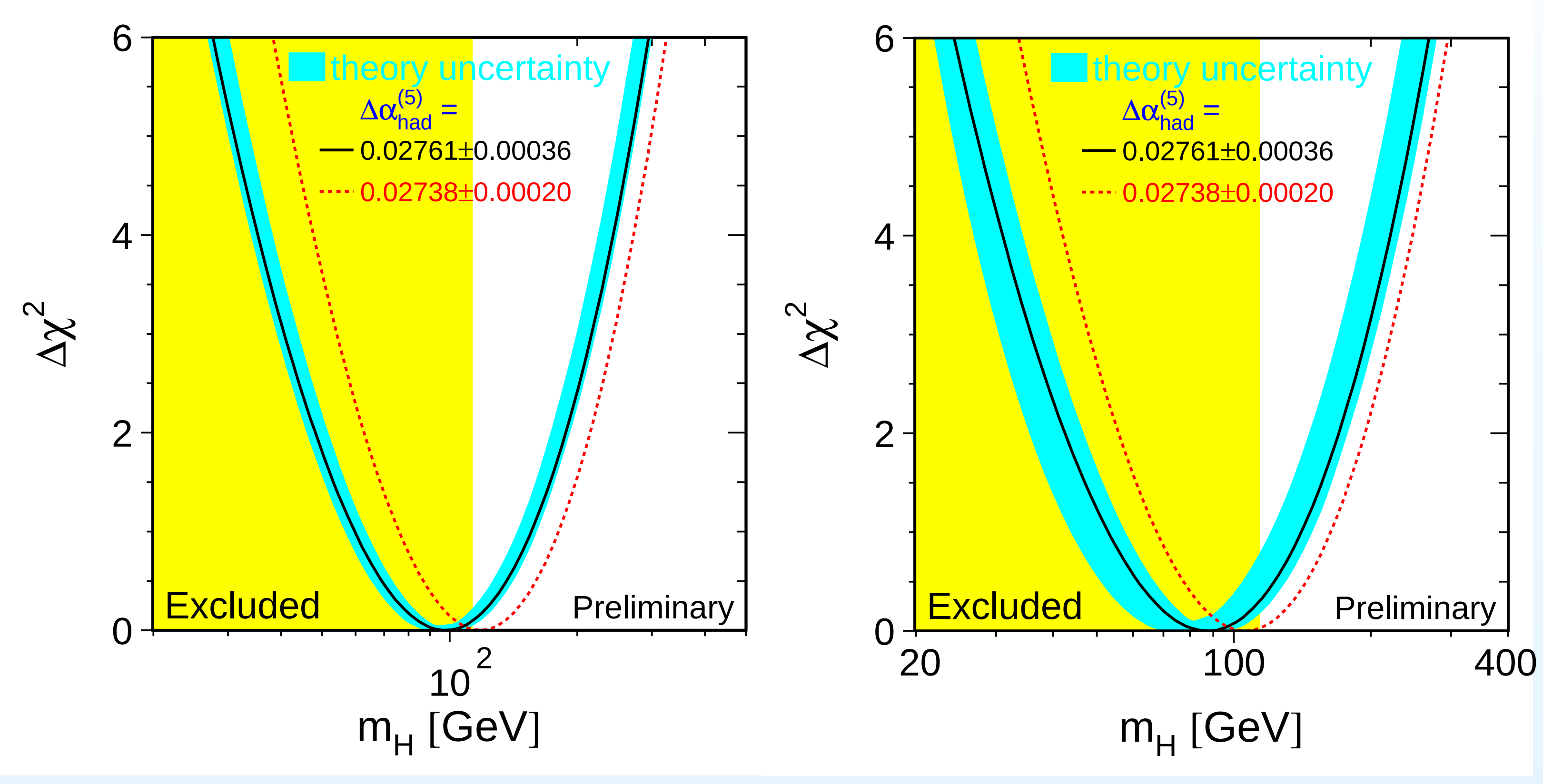


- Predominantly $\log M_H$

Motivation

- Direct motivation:

Winter '01 Summer '01



Motivation

error estimate from difference between

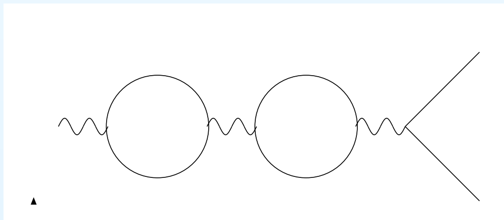
- $\sim m_t^2$ pred. of M_W
- full fermionic $\kappa(1 - M_W^2/M_Z^2)$
- Best electroweak prediction ($\sim m_t^2$)
  Degrassi, Gambino, Sirlin '97
- Gambino at Lepton Photon '03:
  "...a complete calculation of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ is nowhere in sight." (!)
- Recent calculations of M_W at $\mathcal{O}(\alpha^{\in})$ shift the prediction by 4MeV.

Contributions to $sin^2\theta_{eff}^{(2)}$

- At two loop order

$$\frac{1}{4}\frac{g_A^{(1)}}{g_A^{(0)\,2}}\left(g_V^{(1)}-\frac{g_V^{(0)}}{g_A^{(0)}}g_A^{(1)}\right)-\frac{1}{4}\frac{1}{g_A^{(0)}}\left(g_V^{(2)}-\frac{g_V^{(0)}}{g_A^{(0)}}g_A^{(2)}\right)$$

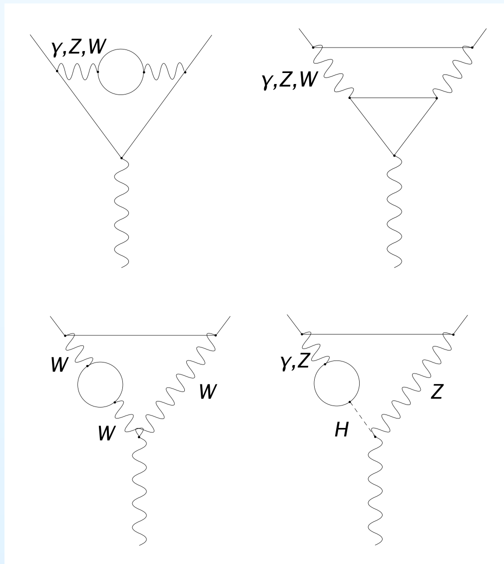
- Product of one loop contributions trivial, but necessary to cancel infrared divergence
- Generic two loop contribution contains products of imaginary parts one loop diagrams



Contributions to $sin\theta_{eff}$

Diagram classes:

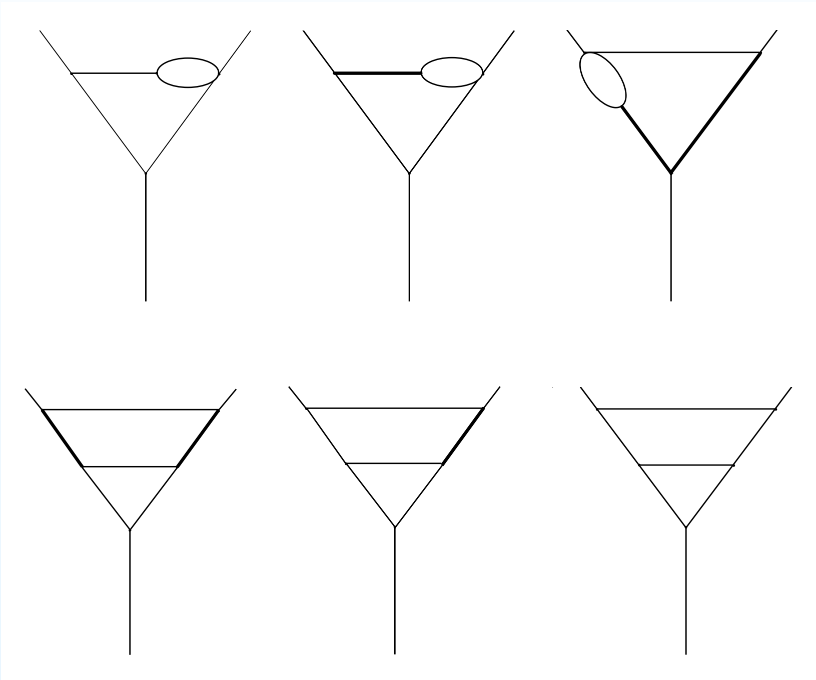
- Two loop on-shell propagators necessary for $\delta\sin^2\theta_W^{(2)}$ and $\delta Z_{\gamma Z}^{(2)}$
  - Calculated and extensively tested for muon decay
- Sole complication $\rightarrow$ two loop vertices



- Divided into two groups
  - containing top quark
  - containing only light fermions
- Diagrams with Higgs boson vanish by CP conservation

Light Fermion Contributions

- Possible prototypes:

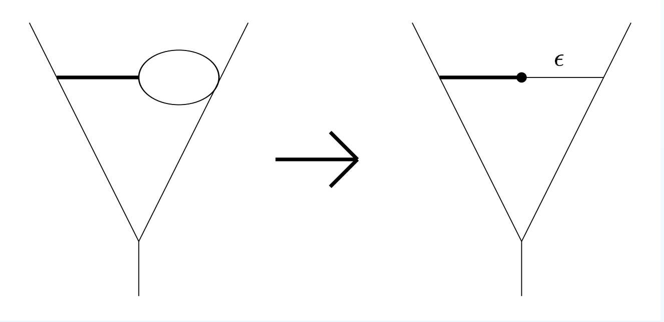


treated with:

- IBP ids to reduce to master integrals
- Differential equations to get analytical results

Light Fermion Contributions

Easier case: propagator type subloop



- Tensor reduction made trivial with use of harmonic tensors
- By dimensional analysis after partial fractioning: massless subloop becomes a propagator $1/p^{2\epsilon}$

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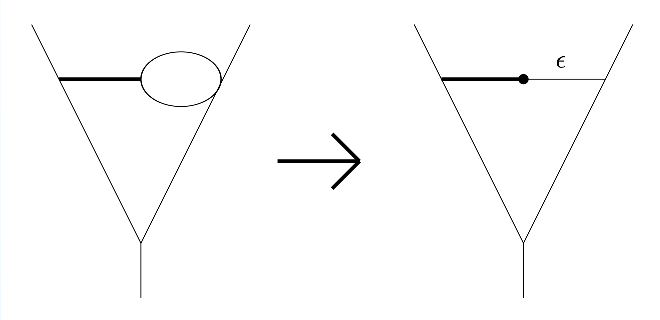

- Tensor reduction made trivial with use of harmonic tensors
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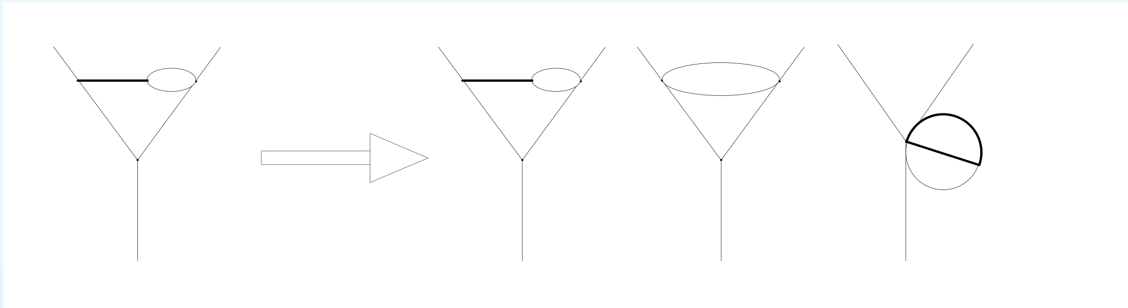
Consequence: IBP ids almost as easy as at one loop

Master integrals: 

Light Fermion Contributions

- Prototype satisfies differential equation

$$M^2 \frac{d}{dM^2} \ \mathsf{LF1}(M,m) =$$

$$\frac{1}{2}\frac{M^2}{M^2+m^2}((4-D)(4+5\frac{m^2}{M^2})\ \mathsf{LF1}(M,m)$$

$$+(10-3D)\ \mathsf{LF0}(M) - (2-D)\ \mathsf{T134}(0,0,m))$$

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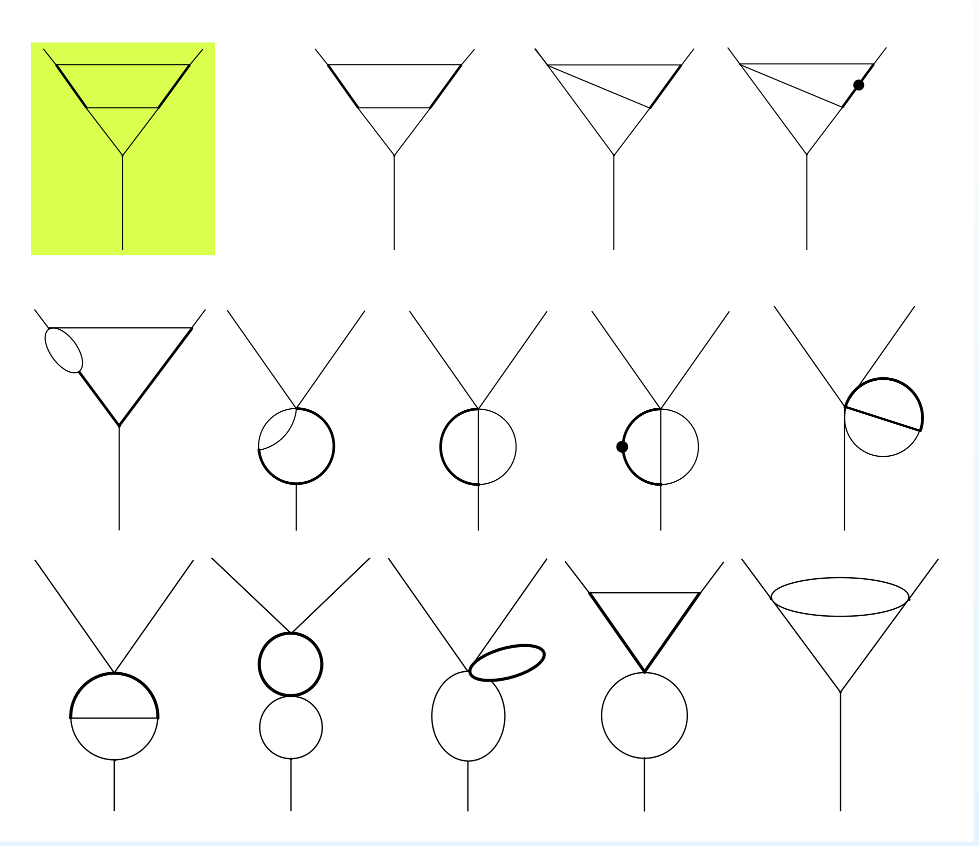
- After integration ($x = M^2/m^2$)

$$-\mathsf{Li}_2(-x)(-2+2\log(m^2)+3\log(-x)+\log(1+x))$$
$$+4\mathsf{Li}_3(-x) - \mathsf{S}_{1,2}(-x) + \frac{1}{2}\log(1+x)(+2\zeta_2$$
$$-\log(-x)(-4+4\log(m^2)+2\log(-x)+\log(1+x)))$$

Light Fermion Contributions

Harder case:



- 6 denominators
- 7 scalar products
- 8 IBP ids
- 1 Lorentz invariance id
- Even tensor reduction nontrivial

- Consequence: hardly feasible “by brain”

Light Fermion Contributions

- Strategy:
  - Develop multipurpose tool to solve IBP ids
  - Implement Laporta algorithm
  - Extend by symmetry identification and massless detachable subgraph elimination

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- Strategy:
  - Develop multipurpose tool to solve IBP ids
  - Implement Laporta algorithm
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- Result:
  - *IdSolver* (M.Czakon): C++ library based on DiaGen
  - $\sim 10^5$ integrals reduced within one hour
  - Used for a few other multiloop projects underway

Light Fermion Contributions

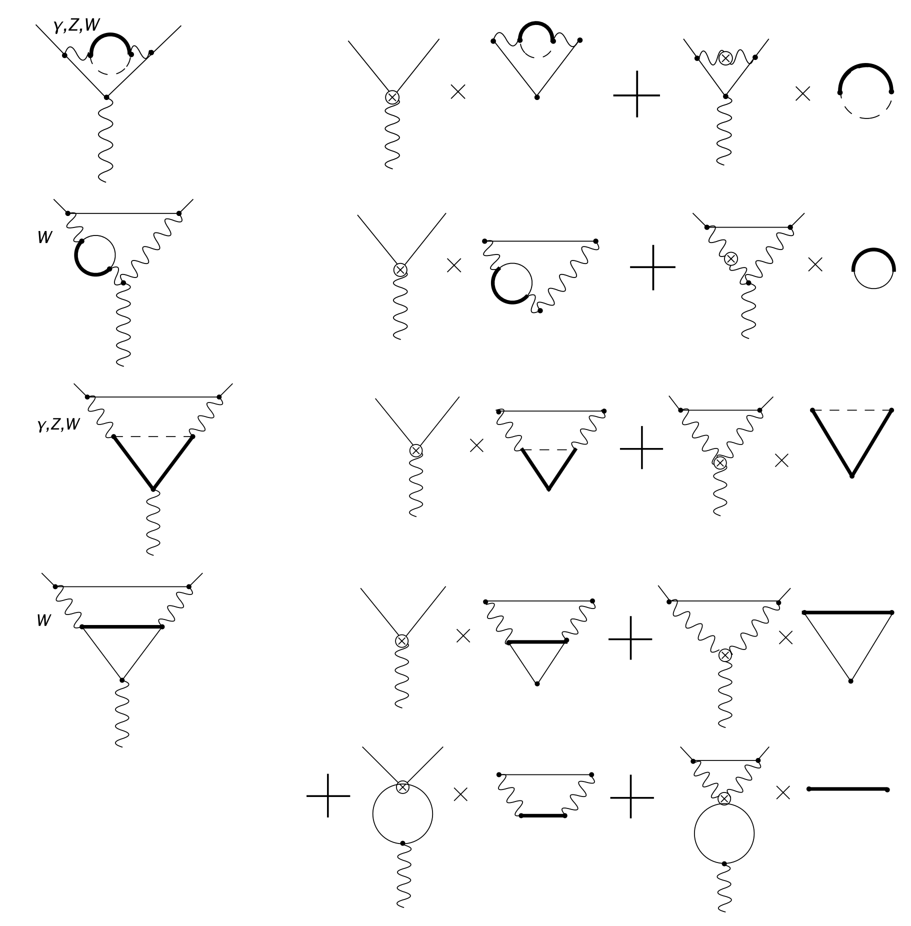
- At the end:

  Short analytic result expressed through at most polylogarithms
- internal tests:
  - Some master integrals tested by means of Padé resummed Mellin-Barnes representations
  - Others analytically by comparison with low momentum expansion
  - Complete diagrams tested by means of low momentum expansion

Light Fermion Contributions

- independent calculation (A.Freitas)
  - tensor reduction: IBPs ids, Lorentz invariance ids
  - topologies with self-energy sub-loop
    by dispersion relation reduced to 1 dimensional integrals
  - diagrams with triangle subloop:
    introduce Feynman parameters and integrate numerically (up to 3dimensional integrations for 2loop vertex)

Top Quark Contributions

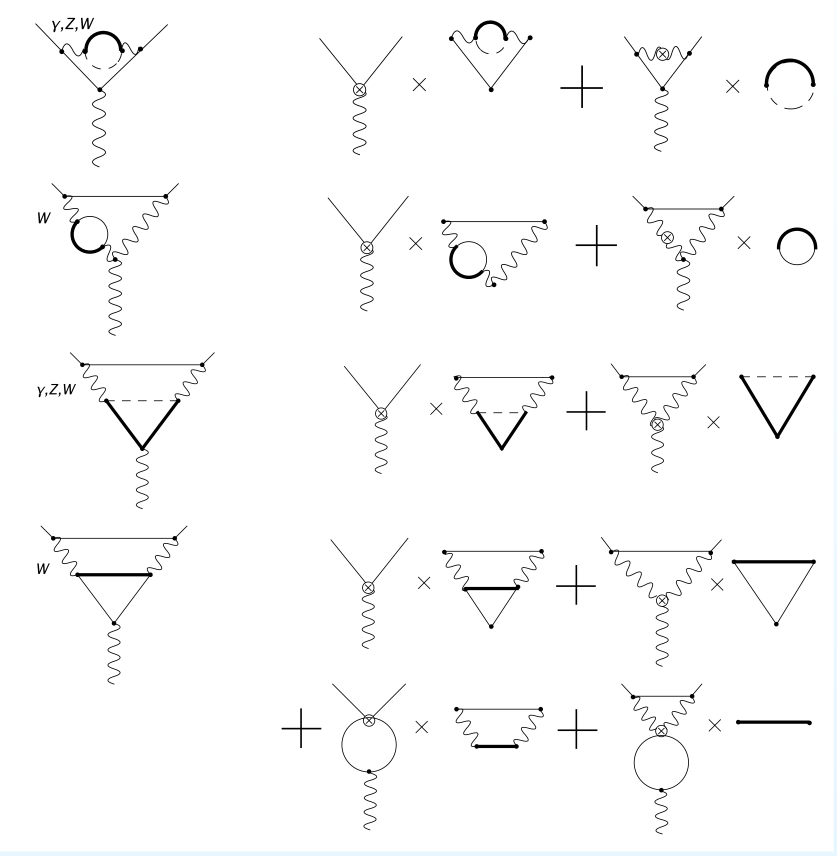


- Winning strategy: exploit large scale differences

$$M_Z^2/m_t^2 \approx 1/4$$

- Always simplifies diagrams to calculate: here at most two loop single scale tadpoles

Top Quark Contributions



Three subdiagrams:

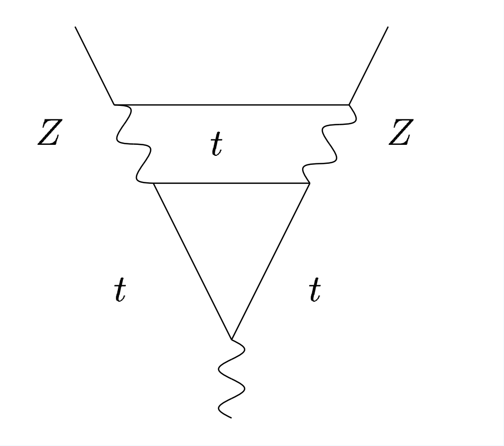
- Full diagram starts at m_t^0
- One loop subdiagram starts at m_t^2
- Single line subdiagram starts at m_t^{-2}

- Previously considered: m_t^2 contribution only
- Now, can go to arbitrary order, in practice: m_t^{-20}

Top Quark Contributions

- Example of excellent convergence ($x = M_Z^2/m_t^2$):

$$\frac{x}{3}\zeta_2+\frac{x^2}{4}\left(\frac{1}{3}\zeta_2-\frac{5}{9}+\frac{1}{3}\log x\right)+\frac{x^3}{5}\left(\frac{1}{9}\zeta_2-\frac{79}{240}+\frac{1}{4}\log x\right)+\ldots$$

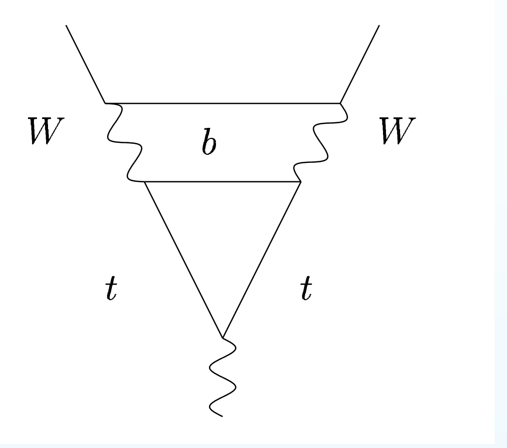


$$0.1483 \;-\; 0.0081 \;-\; 0.0019 \;+\; 0.0003 \;+\ldots$$

- Behaviour common to neutral current diagrams

Top Quark Contributions

- Example of asymptotic convergence:



- 8th order top quark mass expansion 0.34892
  relative error estimate $\pm 3.2 \times 10^{-4}$
- 8th order Taylor expansion 0.34877
  relative error estimate $\pm 4.9 \times 10^{-6}$
- Actual relative error on top quark mass expansion
  $\pm 4.4 \times 10^{-4}$
- Behaviour common to charged current diagrams

Triangle Fermion Loops and γ_5

- Old problem: lack of invariant regularization for chiral theories
- Experience from muon decay:
  - successful use of NDIM

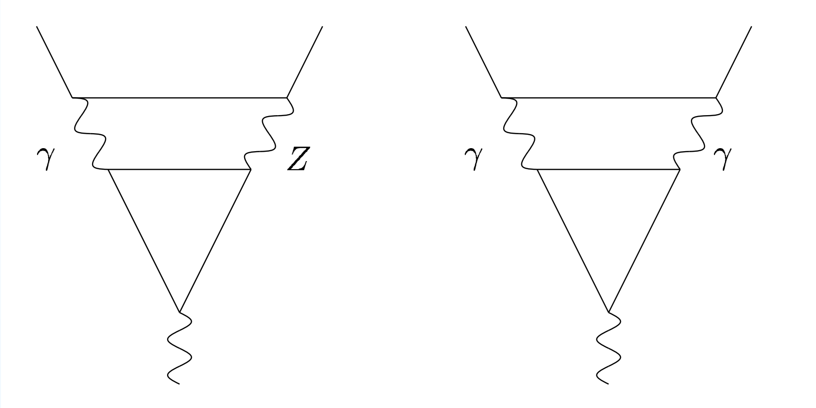
$$\{\gamma^\mu, \gamma_5\} = 0$$
$$\text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \gamma_5] = 4\, i\, \epsilon^{\alpha\beta\gamma\delta}$$

  - no poles in front of

$$\epsilon^{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta}$$

  - contribution comes solely from top quark diagrams

Triangle Fermion Loops and γ_5



- Situation complicated substantially by collinear divergences

- Collinear poles hit epsilon tensors
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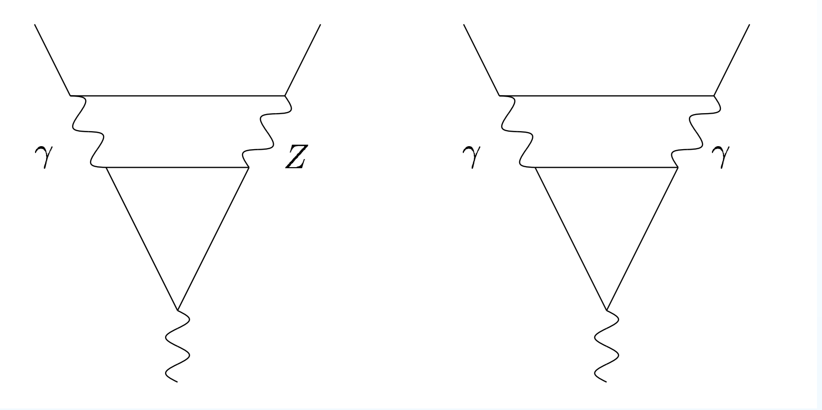

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Simplest solution:

- use photon mass regulator
- easily tractable with previous methods

Triangle Fermion Loops and γ_5

- Anomaly generates fermion mass independent infrared divergence
- Cancels within quark and lepton family
- Complete contribution vanishes for equal masses
- Different from zero for third family

Preliminary results

M_H	$\sin^{2(0)}_{eff}$	$\sin^{2(1)}_{eff}$	$\sin^{2(2)}_{eff}$	LF	Top	$Tr5$
100	0.222103	0.00746046	5.43035e-06	5.08323e-05	-0.000354522	?
200	0.222103	0.00746046	-2.64695e-05	3.40883e-05	-0.000355	?
600	0.222103	0.00746046	-8.53425e-05	-4.78e-05	-0.000346663	?
1000	0.222103	0.00746046	-7.91762e-05	-8.59563e-05	-0.00030234	?

M_H	$\sin^{\alpha\alpha_s}$	$\sin^{\alpha\alpha_s^2}$	$\Delta\rho^{\alpha^2\alpha_s}$	$\Delta\rho^{\alpha^3}$
100	-0.000804491	-0.000160793	2.63788e-05	3.26065e-06
200	-0.000804491	-0.000160793	4.3811e-05	1.79209e-06
600	-0.000804491	-0.000160793	8.52587e-05	1.72972e-06
1000	-0.000804491	-0.000160793	0.000104831	2.08883e-05

Conclusions

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- A fitting formula for use in experimental fits and application to ZFITTER is now under way
- Pure bosonic corrections will be approached with the same tools