POWHEG at NNLO using MiNLO

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Work done in collaboration with

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Based on 1206.3572, 1212.4504, and 1308.xxxx

LHC – The first part of the journey Santa Barbara, 8th-12nd July 2013

NLO

- Remarkable progress in NLO calculations over the past ten years. Goes under the name of the NLO revolution
- In fact (at least in my mind), the revolution started in '04 at a KITP workshop "Collider physics". Core of the revolution: understanding of how to compute NLO corrections without Feynman diagrams
- NLO wish-lists [ttbb,tttt, WWbb, bbbb, WWjj,W/Z+3,4,j, W+5j, 4j] are closed chapters
- Two main directions now
 - more legs: e.g. Blackhat focuses on pure n jets or W/Z + n jets pushing the frontier of n
 - more processes: towards a full automation of NLO calculations with codes like Helac, GoSam or MadLoop
- This progress went hand in hand with the development of merging of NLO and parton showers via MC@NLO (Frixione & Webber '02) or POWHEG (Nason '04)

NLO+PS

- Today, next-to-leading order parton showers (NLO+PS) have been realized as practical tools (POWHEG, MC@NLO, Sherpa) and are being today routinely used for LHC analyses
- First only processes with no associated jets in the final state, e.g. Drell-Yan, diboson, tt, VBF Higgs, ...
- Now associated jet production also included, e.g. for Higgs production in POWHEG there is
 - inclusive Higgs production (H)
 - Higgs plus one jet (HJ)
 - Higgs plus two jets (HJJ)

[same for W and Z]

NNLO

- we know Higgs and Drell-Yan since many years now (fully differential, with decays)
- for QCD, 2013 is the year of NNLO: full or partial results for associated Higgs production, top-pairs, H+jet, dijets [...]
- $^{\bullet}$ these calculations pave the way to all $2 \rightarrow 2$ processes relevant for LHC physics
- however, at the moment no method for NNLO+PS exists
- first ideas towards NNLO+PS for inclusive Higgs production presented in Hamilton et al. 1212.4504
- here: first practical implementation of those ideas & preliminary results
- method based on MiNLO procedure for NLO and is intimately connected to the merging problem. So, start discussing those.

MiNLO

Multiscale improved NLO

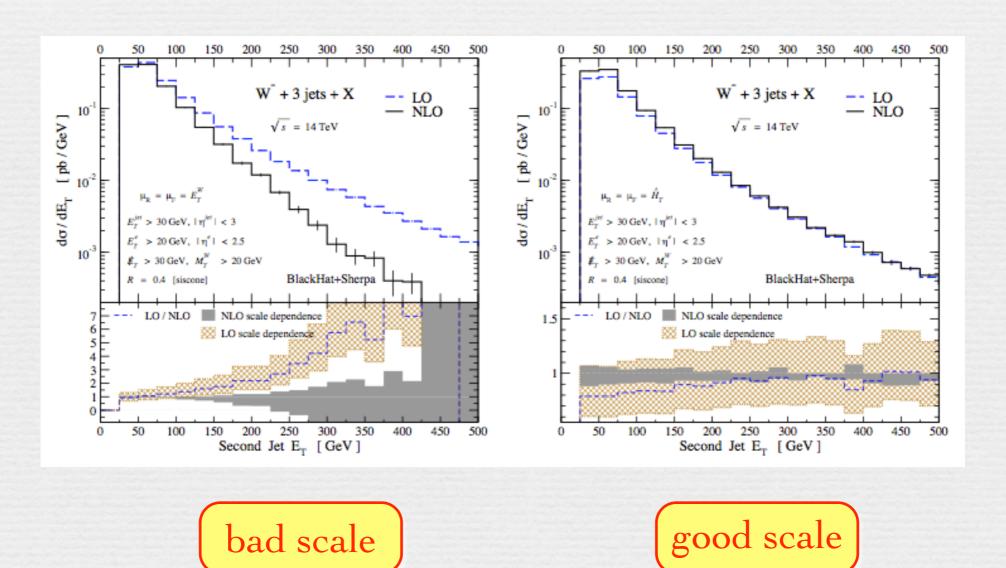
The observation triggering the first idea behind MiNLO was in a paper with K. Melnikov [0910.3671]

• the impact of NLO calculations is often discussed using the same scale choice at LO and NLO, however more advanced LO calculations exist that rely on the CKKW procedure for scale setting (see later) and inclusion of Sudakov effects

Even at NLO the scale choice is an issue and different choices can lead to a different picture/contrasting conclusions, so it seemed natural to look for an extension of the CKKW method to NLO

Scale choice at NLO

Often a "good scale" is determined *a posteriori*, either by requiring NLO corrections to be small, or by looking where the sensitivity to the scale is minimized



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Reason: bad scale is large logs large NLO, large scale dependence

But we also know that large NLO bad scale choice, since NLO corrections can have a "genuine" physical origin (new channels opening up, Sudakov logarithms, color factors, large gluon flux ...)

Furthermore, double logarithmic corrections can never be absorbed by a choice of scale (single log). So a "stability criterion" can be misleading.

Scale choice at LO

LO calculations in matrix elements generators that follow the CKKW procedure are quite sophisticated in the scale choice:

they use optimized/local scales at each vertex and Sudakov form factors at internal/external lines

Catani, Krauss, Kuehn, Webber '01 extension to pp collisions Krauss '02

Reminder:

a Sudakov form factor encodes the probability of evolving from one scale to the next without branching above a resolution scale Q_0

Recap of CKKW

The CKKW prescription in brief:

- $\label{eq:section}$ use the k_t algorithm to reconstruct the most likely branching history
- $\stackrel{>}{\triangleright}$ evaluate each α_s at the local transverse momentum of the splitting
- for each internal line between nodes at scale Q_i and Q_j include a Sudakov form factor $\Delta_{ij}=D(Q_0,Q_i)/D(Q_0,Q_j)$ that encodes the probability of evolving from scale Q_i to scale Q_j without emitting. For external lines include the Sudakov factor $\Delta_{i}=D(Q_0,Q_i)$
- match to a parton shower to include radiation below Q0

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- match to a parton shower to include radiation below Q0

Scale choice intertwined with inclusion of Sudakov form factors

MiNLO

Born as an extension to NLO of the CKKW procedure, such that the procedure to fix the scales is unbiased and decided *a priori*

In particular, the focus is on processes involving many scales (e.g. X+multi-jet production) and on soft/collinear branchings, i.e. on the region where it is more likely that associated jets are produced

Two observations

1. A generic NLO cross-section has the form

$$\alpha_{\rm S}^{n}(\mu_{R}) B + \alpha_{\rm S}^{n+1}(\mu_{R}) \left(V(Q) + nb_{0} \log \frac{\mu_{R}^{2}}{Q^{2}} B(Q) \right) + \alpha_{\rm S}^{n+1}(\mu_{R}) R$$

Adopting CKKW scales at LO, this becomes naturally

$$\alpha_s(\mu_1) \dots \alpha_s(\mu_n) B + \alpha_s^{n+1}(\mu_R') \left(V(Q) + b_0 \log \frac{\mu_1^2 \dots \mu_n^2}{Q^{2n}} B \right) + \alpha_s^{n+1}(\mu_R'') R$$

and the scale choices μ_R and μ_R are irrelevant for the scale cancelation

2. Sudakov corrections included at LO via the CKKW procedure lead to NLO corrections that need to be subtracted to preserve NLO accuracy

The original MiNLO

- 1. Find the CKKW n clustering scales $Q_1 < ... < Q_n$. Fix the hard scale of the process Q to the system invariant mass after clustering. Set Q_0 to Q_1 (inclusive on radiation below Q_1)
- 2. Evaluate the n coupling constants at the scales Q_i (times a factor to probe scale variation)
- 3. Set μ_R in the virtual to the geometric average of these scales and μ_F to the softest scale Q_1
- 4. Include Sudakov form factors for Born and virtual terms, and for the real term after the first branching
- 5. Subtract the NLO bit present in the CKKW Sudakov of the Born
- 6. The (n+1)th power of α_s in the real and virtual is evaluated at the arithmetic average of the n α_s in the Born term (since corrections can be thought of as additive at each vertex, but other choices possible)

MiNLO in one equation

Example: take e.g. HJ

In POWHEG it is customary to discuss the \overline{B} function, which is related to the differential NLO cross-section, for a given Born kinematics, integrated over radiation variables. For "normal" **HJ** it is given by

$$\bar{B} = \alpha_s^3(\mu_R) \left[B + V(\mu_R) + \int d\Phi_r R \right]$$

With MiNLO this function becomes

$$\Delta(Q_0,Q)$$
 $\Delta(Q_0,Q)$
 $\Delta(Q_0,Q)$
 $\Delta(Q_0,Q_0)=1$
 $\Delta(Q_0,Q_0)=1$

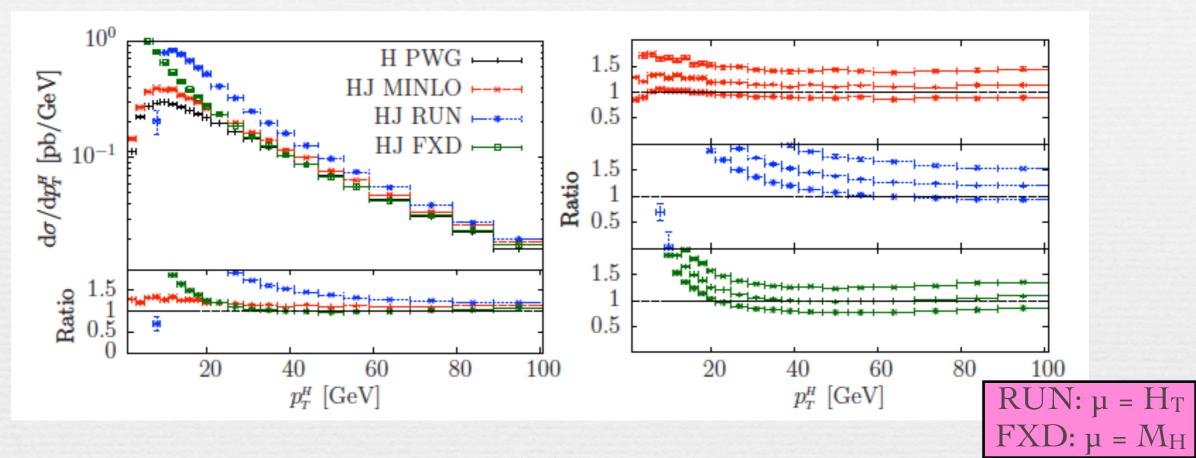
$$\bar{B} = \alpha_s^2 \left(M_H^2 \right) \alpha_s \left(q_T^2 \right) \Delta_g^2 \left(M_H, q_T \right) \left[B \left(1 - 2\Delta_g^{(1)} \left(M_H, q_T \right) \right) + V(\mu') + \int d\Phi_{\text{rad}} R \right]$$

Properties of MiNLO

MiNLO satisfies the following requirements

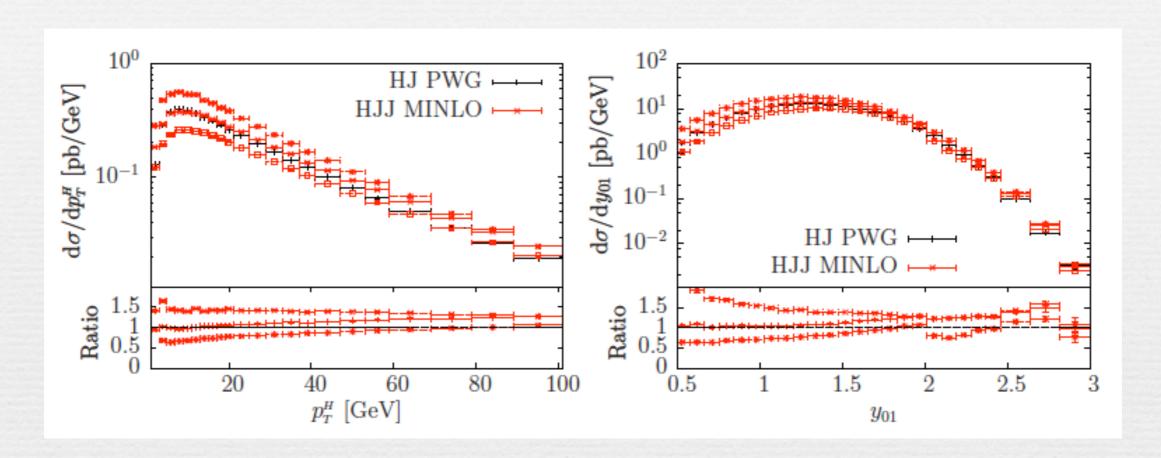
- the result is accurate at NLO, i.e. the scale dependence is NNLO
- the accuracy in the Sudakov region depends on the observable and the form of the Sudakov used
- the smooth behaviour of the CKKW scheme in the singular regions is preserved
 - X+n-jet cross-sections are finite even without jet cuts (do not need generation cuts or Born suppression factors)
 - X+n-jet cross-sections reproduce the inclusive cross-section accurate to LO (and better, see later)
- the procedure is simple to implement in any NLO calculation, i.e. the improvement requires only a very modest amount of work

First MiNLO results



- MiNLO mimics POWHEG all the way down to very small pt,H where standard H+j NLO calculations diverge
- MiNLO uncertainty band compatible with POWHEG all the way down to low transverse momenta
- MiNLO more compatible with fixed rather than running scales (surprising? No, running scale misses Sudakov)

H+2jets



- without cuts impossible to compare to standard NLO
- again, MiNLO uncertainty band compatible with POWHEG all the way down to low transverse momenta

Observation: NLO+PS calculations upgraded with MiNLO describes also inclusive distributions very well. How well really ...?

MiNLO & merging

- What is the accuracy of the MiNLO+PS calculation when looking at inclusive quantities?
 - **X** in the original MiNLO formulation terms neglected are $O(\alpha_s^{3/2})$, so almost NLO, but not quite ...

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- Can one modify the MiNLO procedure to guarantee NLO accuracy for also inclusive quantities?
 - ✓ yes, our explicit study of the case of H/V+jet shows that this is possible. This requires some changes that were part of the freedom in the formulation of MiNLO

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- Can one modify the MiNLO procedure to guarantee NLO accuracy for also inclusive quantities?
 - ✓ yes, our explicit study of the case of H/V+jet shows that this is possible. This requires some changes that were part of the freedom in the formulation of MiNLO
- Can one also solve the general case? The facts that
 - the simplest MiNLO already works well (see also later ...)
 - the HJ/VJ case could be solved in a relatively simple way make us confident that this is possible

Here I'll only sketch the idea (two versions of full proof in 1212.4504) Consider for simplicity the explicit case of H and H+j

The HJ-MiNLO formula reads

$$\bar{B} = \alpha_s^2 \left(M_H^2 \right) \alpha_s \left(q_T^2 \right) \Delta_g^2 \left(M_H, q_T \right) \left[B \left(1 - 2\Delta_g^{(1)} \left(M_H, q_T \right) \right) + V + \int d\Phi_{\text{rad}} R \right]$$

with

$$\Delta_g\left(Q, q_T\right) = \exp\left\{-\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \left[A\left(\alpha_s\left(q^2\right)\right) \log \frac{Q^2}{q^2} + B\left(\alpha_s\left(q^2\right)\right)\right]\right\}$$

$$\Delta_g(Q, q_T) = 1 + \Delta_g^{(1)}(Q, q_T) + \mathcal{O}(\alpha_s^2) \qquad \Delta_g^{(1)}(Q, q_T) = \alpha_s \left[-\frac{1}{2} A_1 \log^2 \frac{q_T^2}{Q^2} + B_1 \log \frac{q_T^2}{Q^2} \right]$$

The idea is to compare this with the NNLL resummation (including finite parts to achieve NLO accuracy for Higgs production, i.e. NLO⁽⁰⁾) and just see what is missing in the MiNLO formula

NNLL_Σ Higgs q_T resummation at fixed rapidity can be written as

$$\frac{d\sigma}{dydq_T^2} = \sigma_0 \frac{d}{dq_T^2} \left\{ \left[C_{ga} \otimes f_{a/A} \right] (x_A, q_T) \times \left[C_{gb} \otimes f_{b/B} \right] (x_B, q_T) \times \exp \mathcal{S} \left(Q, q_T \right) \mathcal{F} \right\} + R_f$$

Integrating in qT one gets

$$\frac{d\sigma}{dy} = \sigma_0 \left[C_{ga} \otimes f_{a/A} \right] (x_A, Q) \times \left[C_{gb} \otimes f_{b/B} \right] (x_B, Q) + \int dq_T^2 R_f + \dots$$

i.e. the formula is $NLO^{(0)}$ accurate if $O(\alpha_s)$ corrections to the coefficient functions are included and R_f is $LO^{(1)}$ accurate

Now, need to show that if the derivative is taken explicitly, and some higher orders are neglected, NLO⁽⁰⁾ accuracy is maintained.

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s, \, \alpha_s^2, \, \alpha_s^3, \, \alpha_s^4, \, \alpha_s L, \, \alpha_s^2 L, \, \alpha_s^3 L, \, \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2, \ \alpha_s^3, \ \alpha_s^4, \ \alpha_s L, \ \alpha_s^2 L, \ \alpha_s^3 L, \ \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$
 B_1

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2 \alpha_s^3, \ \alpha_s^4, \ \alpha_s L, \ \alpha_s^2 L, \ \alpha_s^3 L, \ \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$

$$B_1 B_2$$

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2 \alpha_s^3, \ \alpha_s^4, \alpha_s L \alpha_s^2 L, \ \alpha_s^3 L, \ \alpha_s^4 L \exp \mathcal{S} (Q, q_T)$$

$$B_1 \ B_2 \ \cdots \ A_1$$

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2 \alpha_s^3, \ \alpha_s^4, \alpha_s L \alpha_s^2 L, \ \alpha_s^3 L, \ \alpha_s^4 L \exp \mathcal{S} (Q, q_T)$$

$$B_1 \ B_2 \ \cdots \ A_1 \cdots$$

$$\sigma_0 \frac{1}{q_T^2} \left(\alpha_s \right) \left(\alpha_s^2 \right) \alpha_s^3, \ \alpha_s^4, \left(\alpha_s L \right) \alpha_s^2 L, \left(\alpha_s^3 L \right) \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$

$$B_1 \ B_2 \ \cdots \ A_1 \cdots \ C_1 \otimes C_1 \otimes A_1$$

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2, \alpha_s^3, \alpha_s^4, \alpha_s L \alpha_s^2 L, \alpha_s^3 L, \alpha_s^4 L \exp \mathcal{S} (Q, q_T)$$

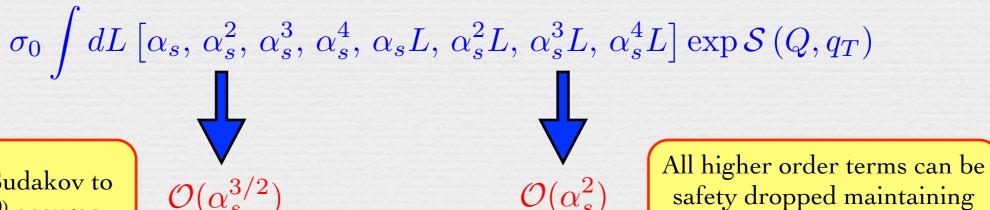
$$B_1 B_2 \cdots A_1 \cdots C_1 \otimes C_1 \otimes A_1 \cdots$$

Taking the derivative one gets

$$\sigma_0 \frac{1}{q_T^2} \left(\alpha_s \right) \left(\alpha_s^2 \right) \alpha_s^3, \ \alpha_s^4, \left(\alpha_s L \right) \alpha_s^2 L, \left(\alpha_s^3 L \right) \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$

$$B_1 B_2 \cdots A_1 \cdots C_1 \otimes C_1 \otimes A_1 \cdots$$

After integration with the Sudakov weight, the counting is set by $L \sim dL \sim 1/\sqrt{\alpha_s}$. So these terms contribute, e.g.

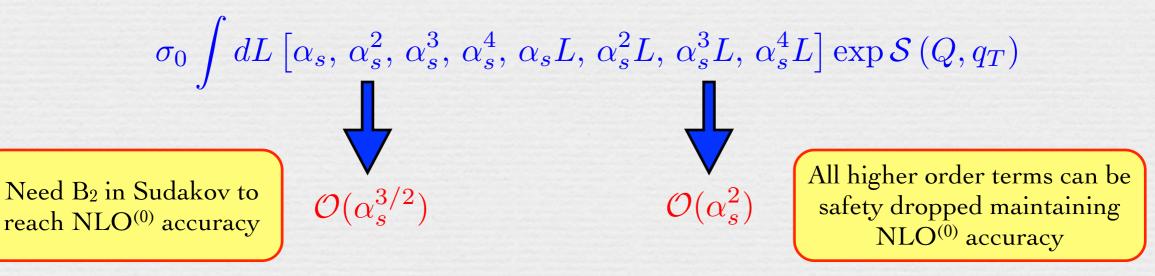


NLO⁽⁰⁾ accuracy

Need B₂ in Sudakov to reach NLO⁽⁰⁾ accuracy

Taking the derivative one gets

After integration with the Sudakov weight, the counting is set by $L \sim dL \sim 1/\sqrt{\alpha_s}$. So these terms contribute, e.g.



Similarly, the scale in V and R gives the largest difference in the $\alpha_s^2 L$ term, where they give an $\alpha_s^3 L^2$ variation. This contributes $O(\alpha_s^{3/2})$. So an effect of the same size to B_2 .

Q. e. d.

Conclusion:

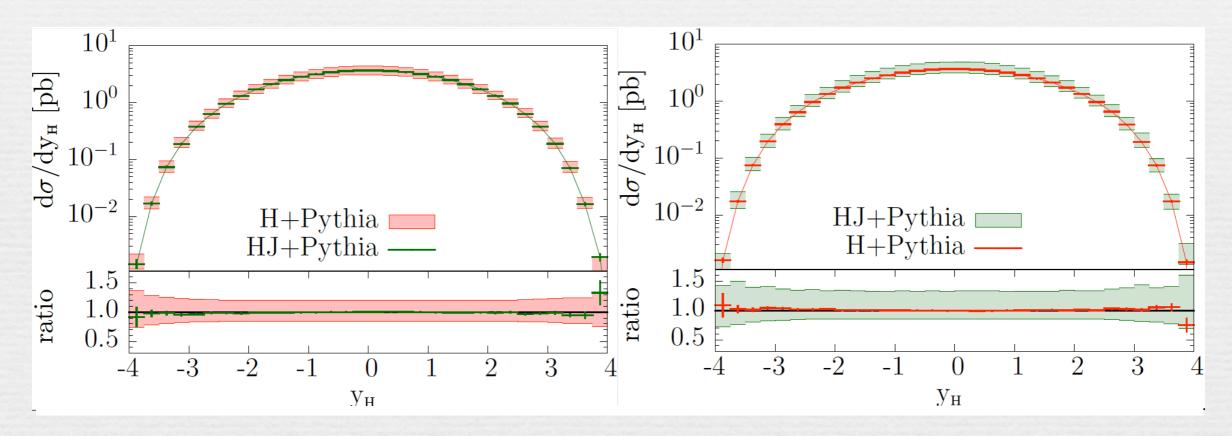
- The original MiNLO prescription is less than NLO accurate in the description of inclusive quantities, in that it neglects $O(\alpha_s^{3/2})$ terms
- achieve NLO accuracy from HJ also for inclusive Higgs observables by
 - ✓ including the B₂ term in the Sudakov form factors
 - ✓ taking the scale in the coupling constant in the real, virtual and subtraction terms equal to the Higgs transverse momentum

Provided this is done, the **HJ** describes both H and H+j at NLO, i.e. merging of H and HJ is effectively achieved without doing any merging!

NB: thus unlike other approaches, no merging scale is introduced

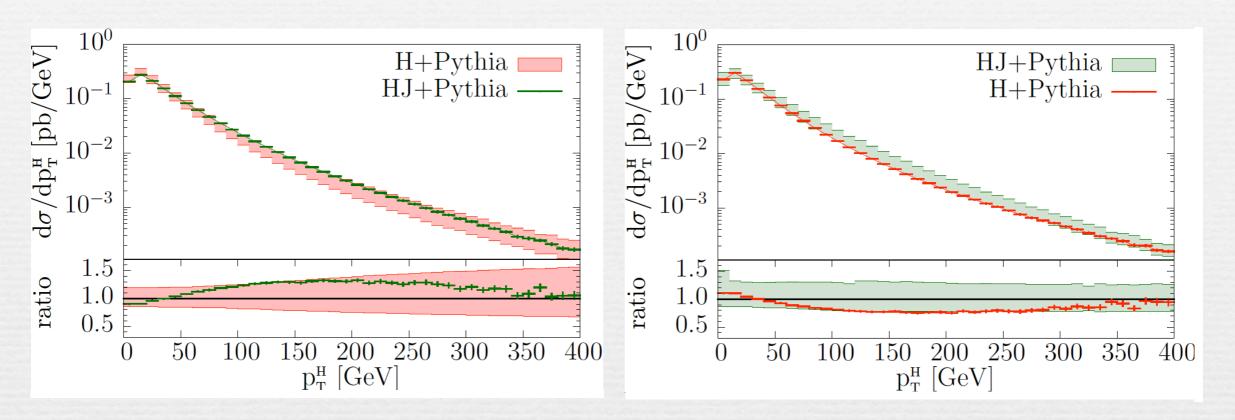
Phenomenology

Higgs (M_H=125 GeV) rapidity of the LHC (8 TeV). Use MSTW8NLO, bands are "7-scale" variation, hfact = 100 GeV in H



Excellent agreement in both in central value and in size of uncertainty bands (less so in W/Z)

Higgs pt



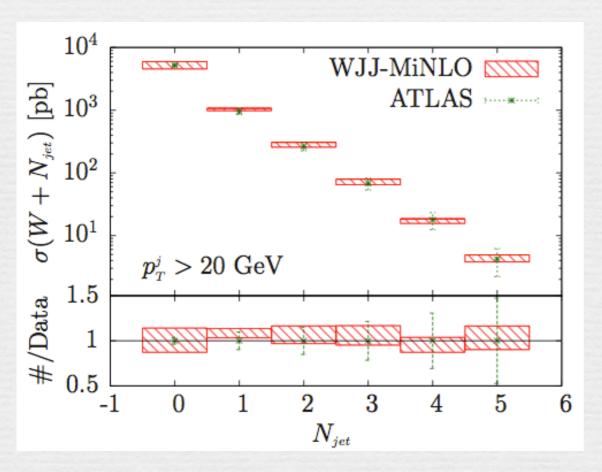
- overall good agreement over the whole region
- p_{t,H} described only at LO accuracy at high p_{t,H} in the H generator,
 (evident from the uncertainty band getting larger)

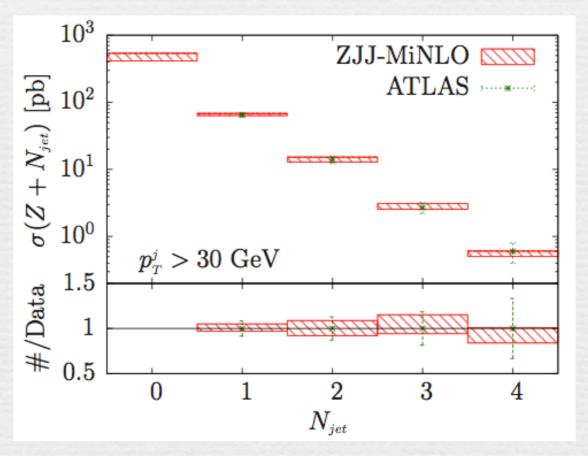
We looked at many more distributions, see 1212.4504 for more.

MiNLO-VJJ vs data

We recently implemented Wjj/Zjj in POWHEG, and compared the WJJ/ZJJ-MiNLO generators against ATLAS data from 0 to 5 jets.

Campbell, Ellis, Nason, Zanderighi 1303.5447 Wjj also in Frederix et al. 1110.5502; Zjj in Re 1204.5433

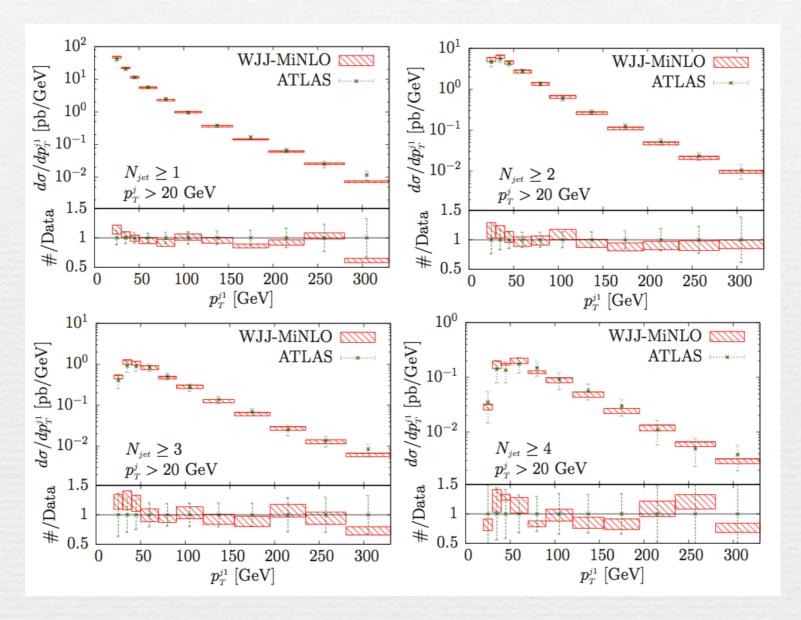




Results out of the box. Nothing has been tuned here.

Agreement really good.

MiNLO-VJJ vs data



We looked at all ATLAS distributions in 1201.1276 (Wjj) and 1111.2690 (Zjj) and always found a similar good agreement. These results are very encouraging in terms of extending the merging to more complex processes.

POWHEG@NNLO

Back to the main question:

How can one use MiNLO to upgrade POWHEG to NNLO?

POWHEG@NNLO

Consider the case of Higgs production



$$\left(\frac{d\sigma}{dy}\right)_{\text{HJ-MiNLO}}$$
 inclusive Higgs rapidity from **HJ-MINLO** generator

POWHEG@NNLO

Consider the case of Higgs production

$$\left(\frac{d\sigma}{dy}\right)_{\text{NNLO}}$$
 inclusive Higgs rapidity computed at NNLO

$$\left(\frac{d\sigma}{dy}\right)_{\text{HJ-MiNLO}}$$
 inclusive Higgs rapidity from **HJ-MINLO** generator

Since HJ-MINLO is NLO accurate, it follows that

$$\frac{\left(\frac{d\sigma}{dy}\right)_{\text{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\text{H I-MiNLO}}} = \frac{c_2\alpha_s^2 + c_3\alpha_s^3 + c_4\alpha_s^4}{c_2\alpha_s^2 + c_3\alpha_s^3 + d_4\alpha_s^4} \approx 1 + \frac{c_4 - d_4}{c_2}\alpha_s^2 + \mathcal{O}(\alpha_s^3)$$

Thus, reweighing HJ-MINLO results with this factor one obtains NNLO+PS accuracy, exactly in the same way as MC@NLO or POWHEG are NLO+PS accurate

Variants

It is also possible to split

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}q_\mathrm{T}\mathrm{d}y} = \frac{\mathrm{d}^2\sigma}{\mathrm{d}q_\mathrm{T}\mathrm{d}y}h(q;p^*) + \frac{\mathrm{d}^2\sigma}{\mathrm{d}q_\mathrm{T}\mathrm{d}y}\left[1 - h(q;p^*)\right]$$
 with h a function interpolating between 1 and 0, e.g.
$$h(q;p^*) = \frac{p^{*2}}{p^{*2} + q^2}$$

$$h(q; p^*) = \frac{p^{*2}}{p^{*2} + q^2}$$

And one can reweight only part of the cross-section:

$$\frac{\mathrm{d}^2 \sigma}{\mathrm{d}q_{\mathrm{T}} \mathrm{d}y} = \frac{\mathrm{d}^2 \sigma}{\mathrm{d}q_{\mathrm{T}} \mathrm{d}y} h(q, p^*) W(y) + \frac{\mathrm{d}^2 \sigma}{\mathrm{d}q_{\mathrm{T}} \mathrm{d}y} \left[1 - h(q, p^*) \right]$$

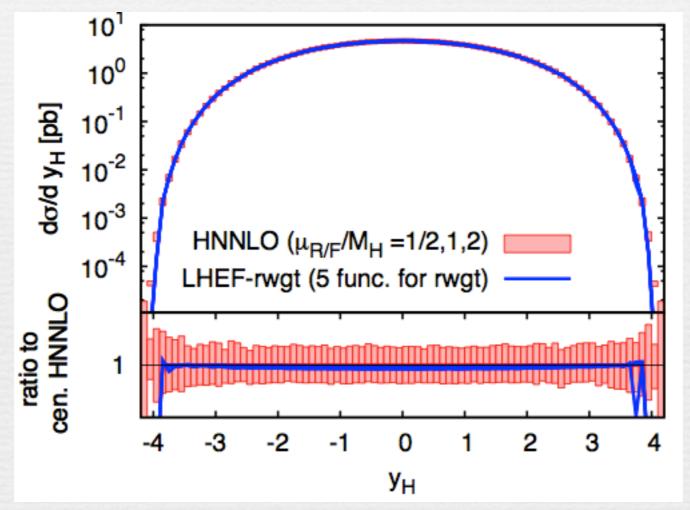
$$W(y) = \frac{\int dq_{\rm T} h(q; p^*) \left(\frac{d^2 \sigma}{dq_{\rm T} dy}\right)_{\rm NNLO}}{\int dq_{\rm T} h(q; p^*) \left(\frac{d^2 \sigma}{dq_{\rm T} dy}\right)_{\rm HJ}}$$

In the following: LHC 8 TeV, M_H=125.5 GeV, MSTW8nn PDFs.

All plots preliminary!

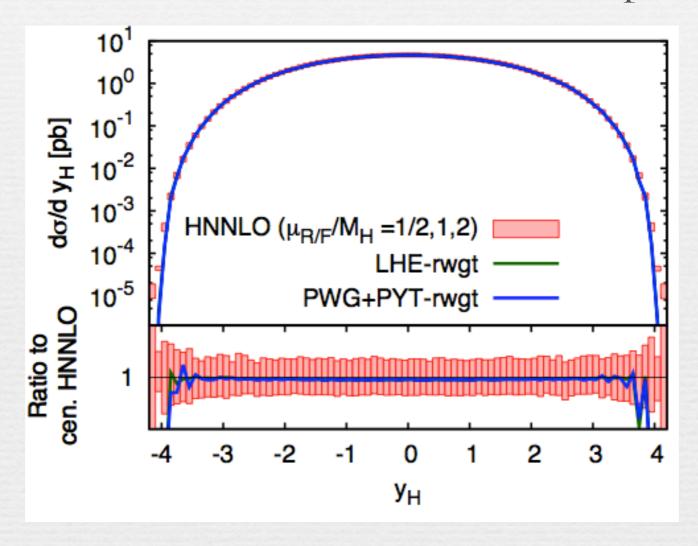
Reweighting using different profile functions — compared to HNNLO

scale-uncertainty



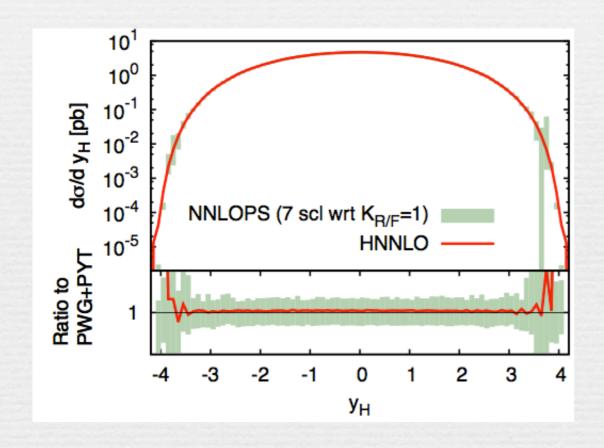
The uncertainty from varying $h(q;p_t^*)$ in the profile function is formally $O(\alpha_s^5)$ in physical distributions and much smaller then the uncertainty from other missing higher orders in HNNLO (in the following will use $p_t^*=0.7M_H$, and $q=p_{t,H}$)

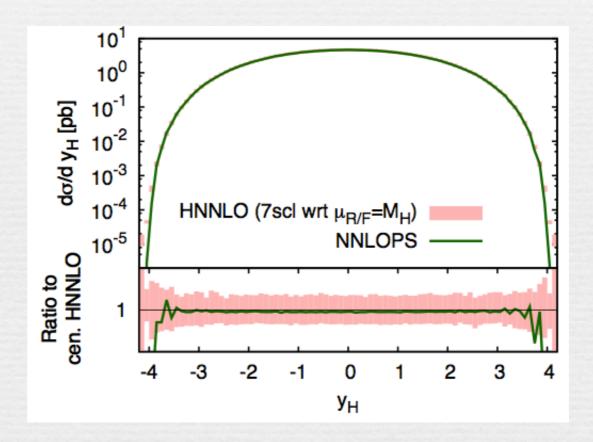
Should one rescale Les Houches events or events after parton shower?



It makes no difference. Having reweighted the hardest emission events to NNLO, the parton shower has negligible impact on rapidity spectrum

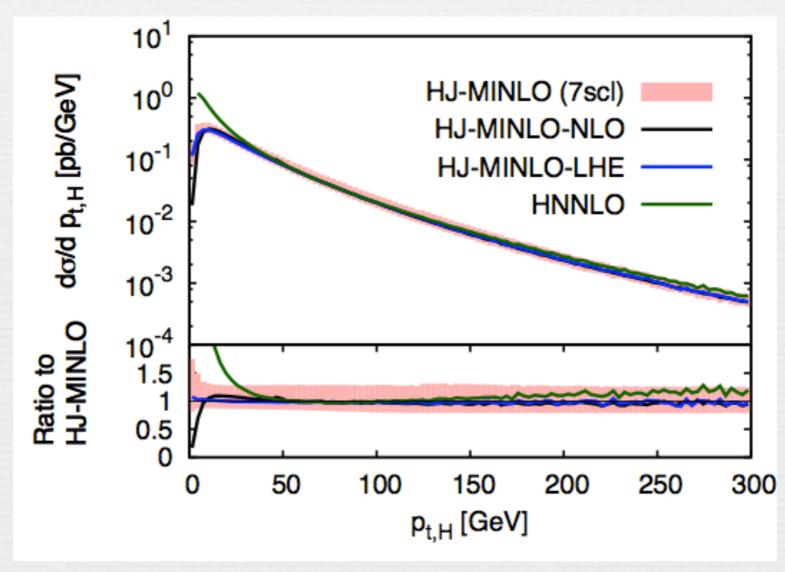
After promoting PWG+PYT to NNLO, uncertainty bands in HNNLO and MINLO-NNLOPS are comparable





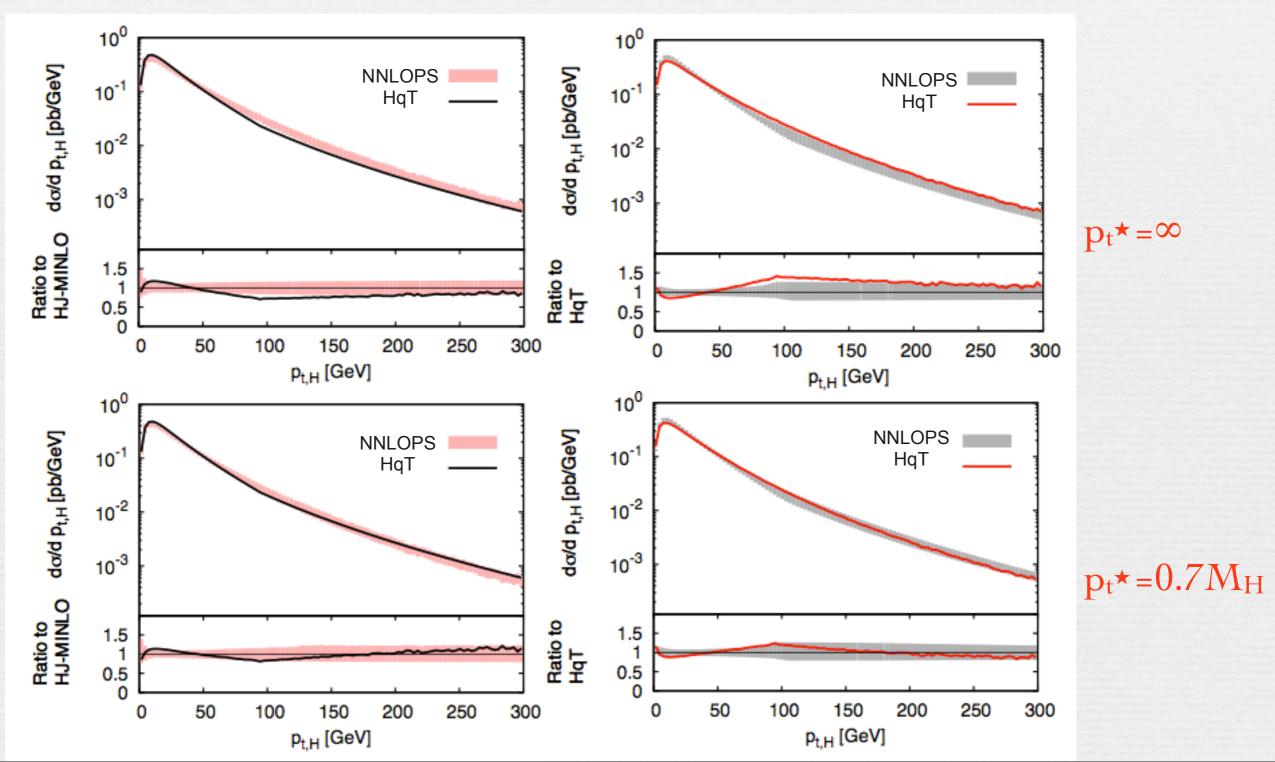
What about other distributions ...?

E.g. look at Higgs transverse momentum

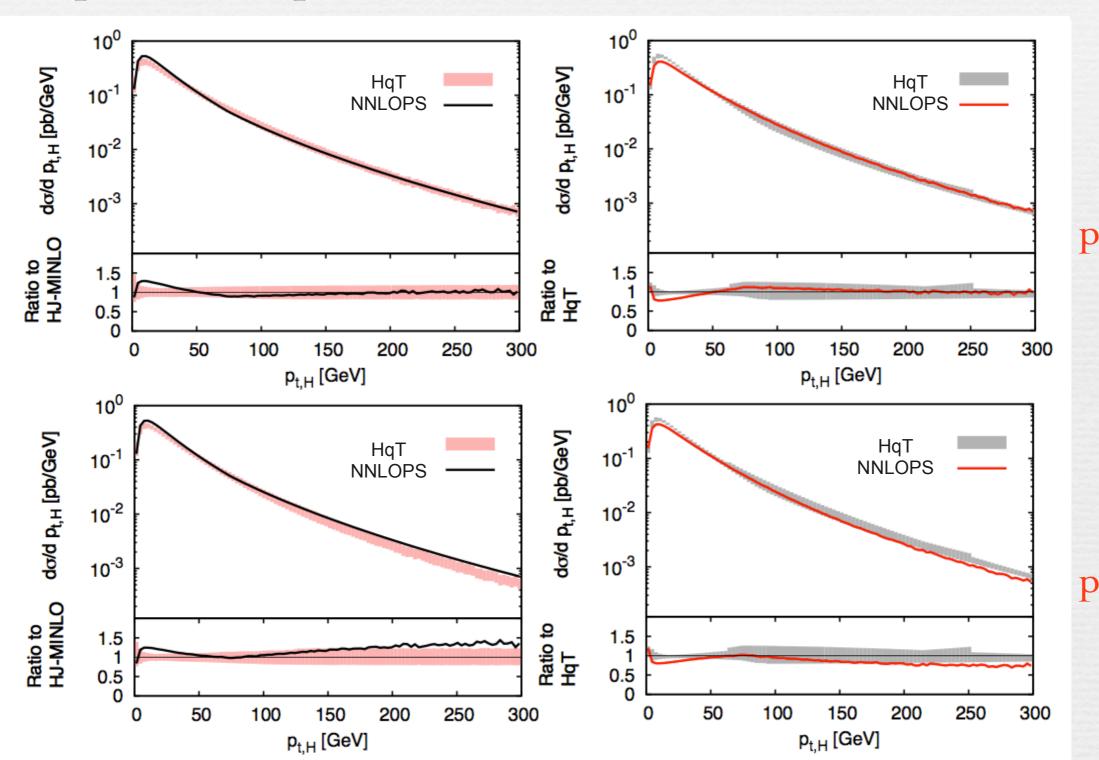


At small p_t, fixed order (HJ-MINLO NLO, or HNNLO) diverge, at high p_t agreement between all predictions

Comparison to HqT (NNLO+NNLL) at scale MH with 7-scale bands



Comparison to HqT (NNLO+NNLL) at scale $M_H/2$ with 7-scale bands



 $p_t \star = \infty$

 $p_t \star = 0.7 M_H$

Conclusions

MiNLO born as a simple procedure to assign scales and Sudakov form factors in NLO calculations to account for distinct kinematical scales.

Key features

- results well-behaved in Sudakov region, where standard NLO calculations break down
- away from the Sudakov regions, results are accurate at NLO
- procedure simple to implement in NLO calculations, just try it out ...
- FIJ, WJ, ZJ NLO calculations upgraded with (new) MiNLO reproduce NLO results also for inclusive distributions, i.e. merging achieved without doing merging
- MiNLO provides a simple way to upgrade POWHEG to NNLO
- first validation results shown here. Phenomenology in progress.

A useful integral

$$I(m,n) \equiv \int_{\Lambda^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2} \left(\log \frac{Q^2}{q^2} \right)^m \alpha_s^n \left(q^2 \right) \exp \left\{ - \int_{q^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2} A \, \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2} \right\}$$
$$\approx \left[\alpha_s(Q^2) \right]^{n - \frac{m+1}{2}}$$

i.e. each log "counts" as a square-root of $1/\alpha_s$ after integration over a transverse momentum when a Sudakov weight is present