Jet cross sections at NNLO accuracy

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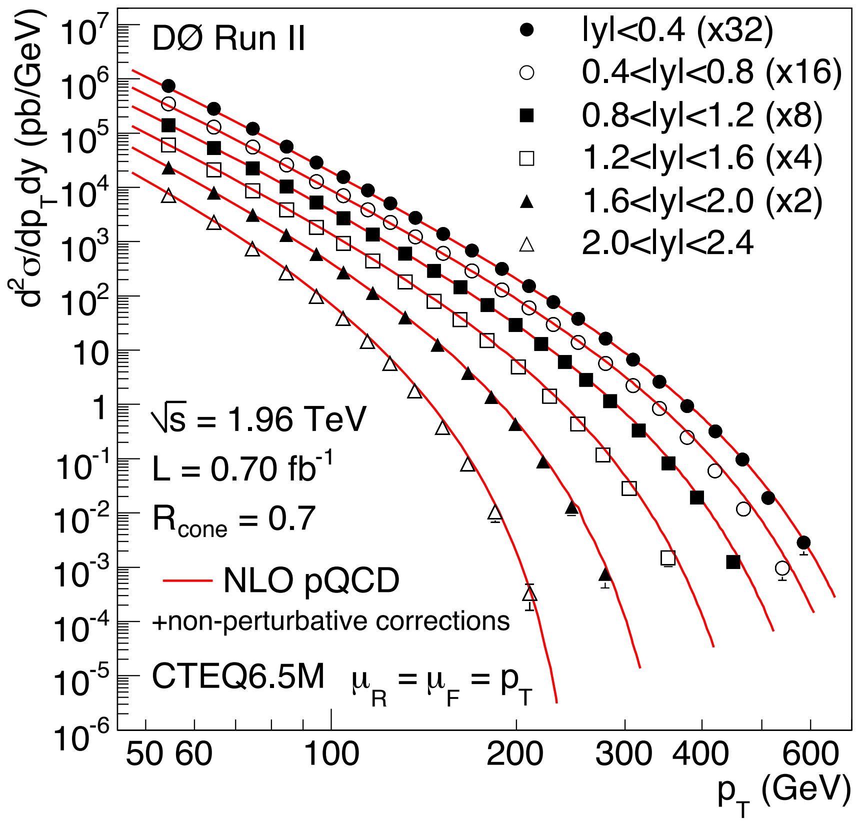
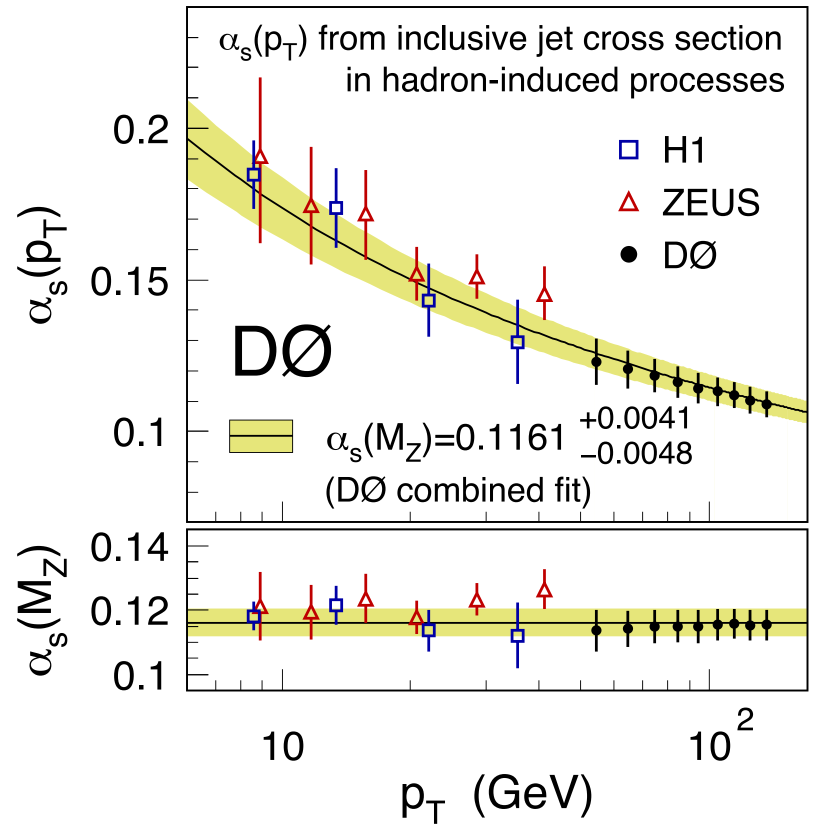
Jet cross sections at hadron colliders

Precision observable at the Tevatron

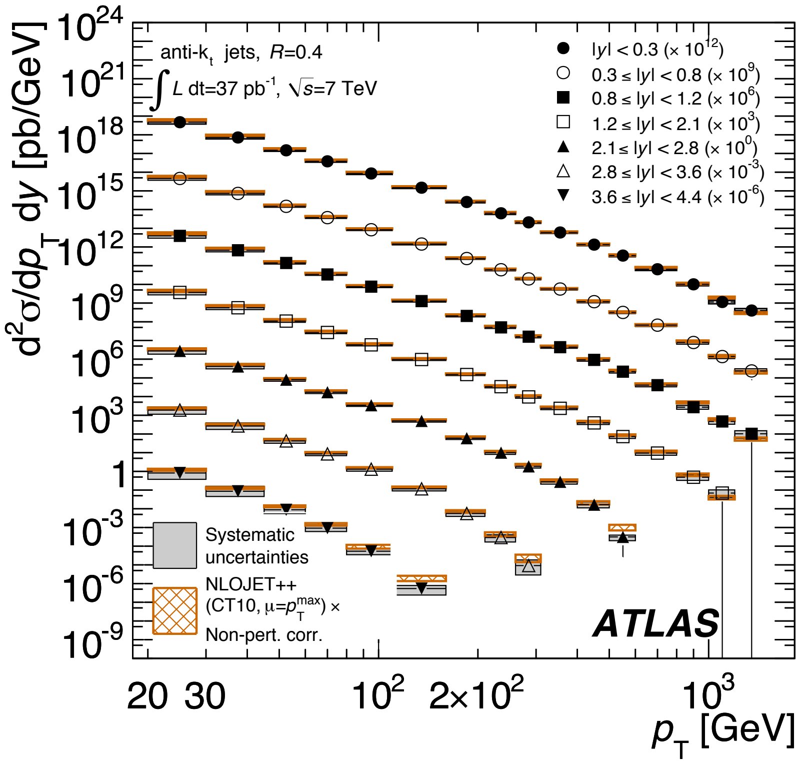


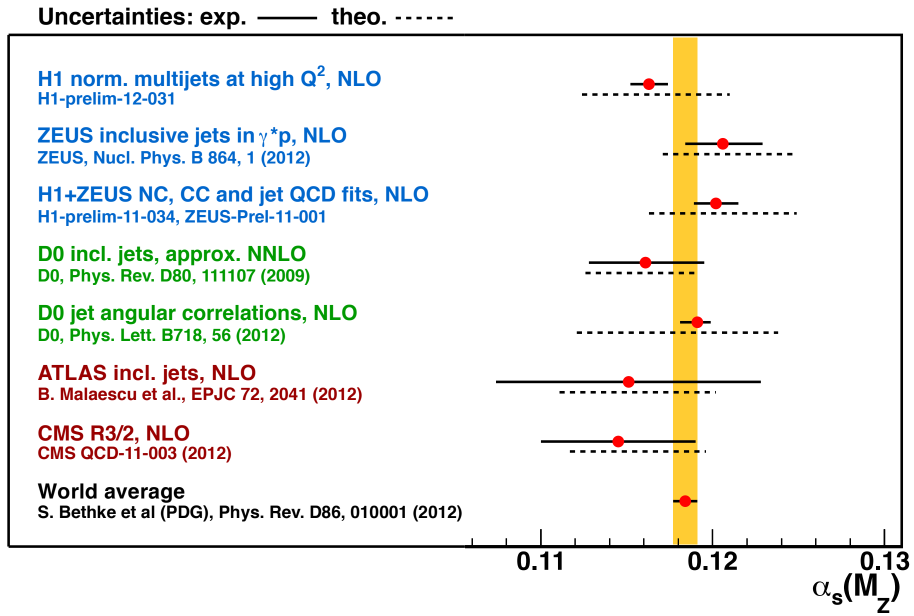


- Constrain parton distributions
- Measure strong coupling constant

Jet cross sections at hadron colliders

ATLAS 2010 results

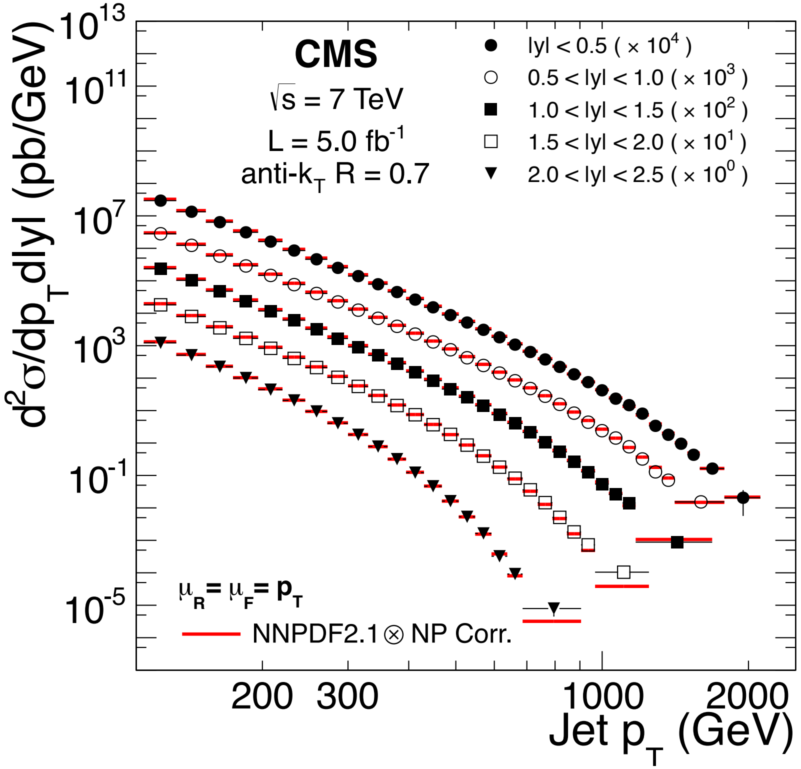


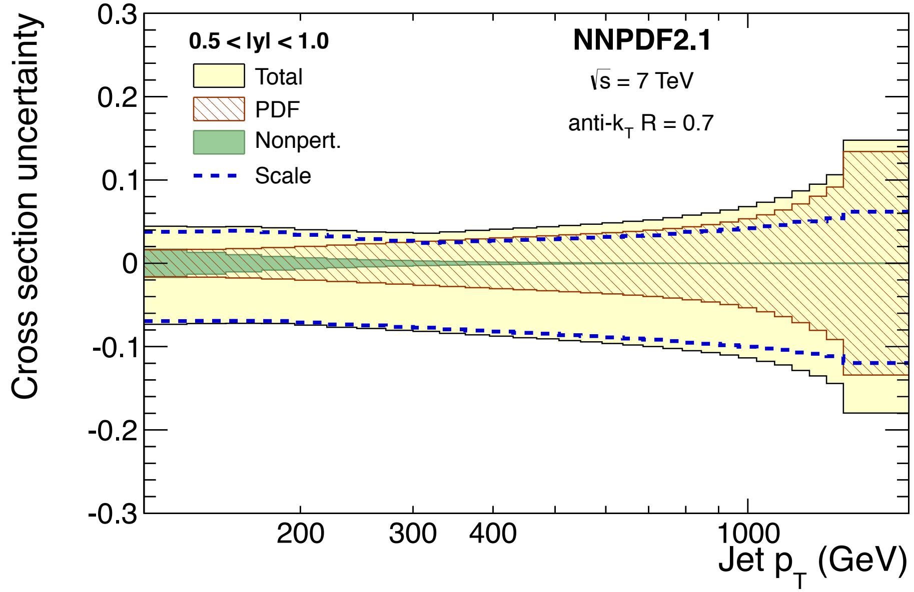


- On track to a multiple-differential high-precision measurement
- Limiting factor in interpretation will be theory accuracy

Jet cross sections at hadron colliders

CMS 2011 results

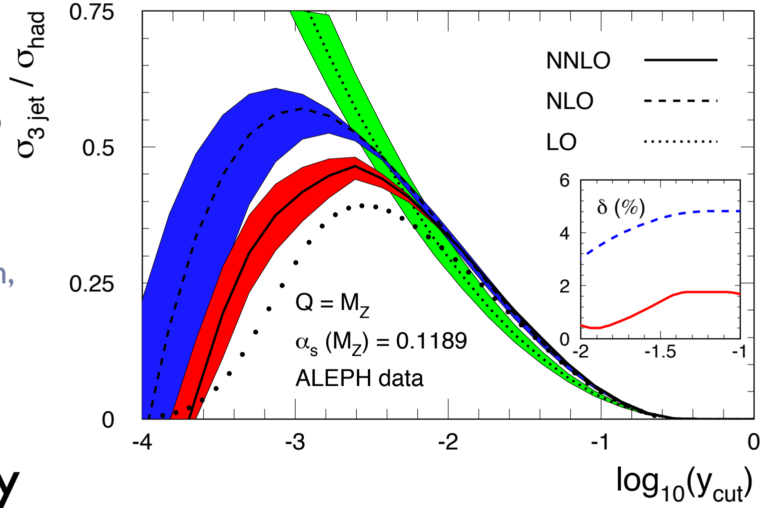




- uncertainty on NLO prediction comparable to spread from partons
- need improved theory for precise extraction of parton distributions from jets

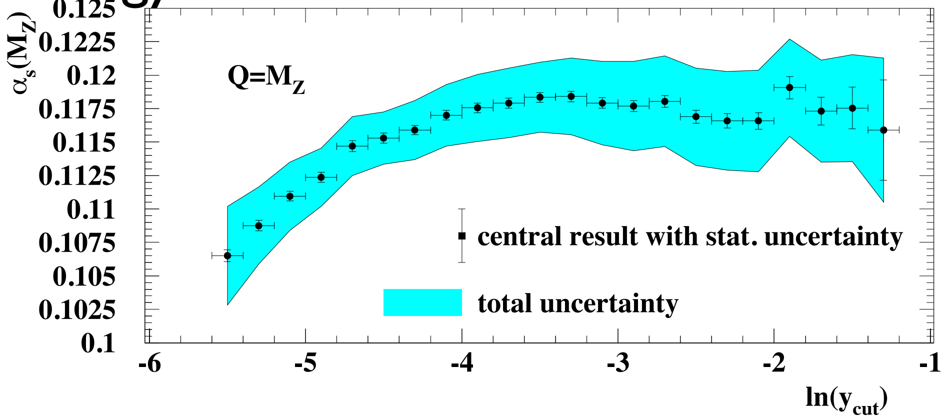
NNLO corrections to $e^+e^- \rightarrow 3$ jets

- Calculation of NNLO corrections to $e^+e^- \rightarrow 3$ jets and related event shapes (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S.Weinzierl)



- Revisit LEP precision phenomenology (with G. Dissertori, G. Luisoni, H. Stenzel)

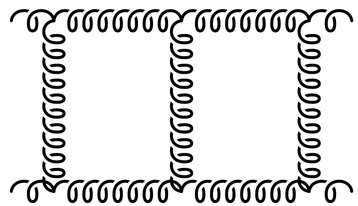
- Strong coupling from R_{3jet}



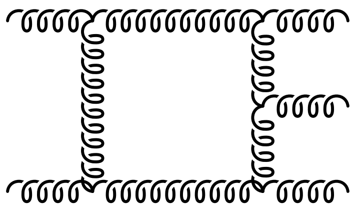
$$\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\mathrm{exp}) \pm 0.0015\,(\mathrm{theo})$$

Ingredients to jet production at NNLO

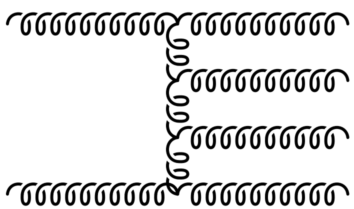
- **Two-loop matrix elements**
  (C. Anastasiou, E.W.N. Glover, C. Oleari, M. Tejeida-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
  - Explicit infrared poles from loop integrals



- **One-loop matrix elements**
  (Z. Kunszt, A. Signer, Z. Trocsanyi)
  - Explicit infrared poles from loop integral
  - Implicit infrared poles from real radiation



- **Tree-level matrix elements**
  - Implicit infrared poles from real radiation



Two-loop matrix elements

- **Generation of diagrams** (QGRAF: P. Noguiera, FORM: J. Vermaseren)
  - Expressed in terms of two-loop Feynman integrals
- **Reduction to master integrals**
  - Integration-by-parts identities

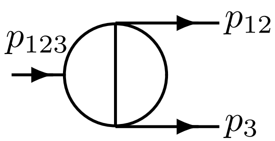
$$\int \frac{d^dk}{(2\pi)^d}\frac{d^dl}{(2\pi)^d}\frac{\partial}{\partial a^\mu}\left[b^\mu f(k,l,p_i)\right] = 0 \qquad \text{with } a^\mu = k^\mu, l^\mu; b^\mu = k^\mu, l^\mu, p_i^\mu$$

  - Complemented by Lorentz invariance and symmetry
  - Solution based on lexicographic ordering (S. Laporta)
    - AIR (C. Anastasiou, A. Lazopoulos)
    - FIRE (A. Smirnov)
    - Reduze (A. von Manteuffel, C. Studerus)

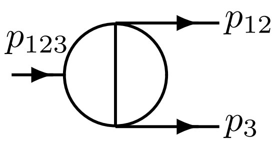
Two-loop matrix elements

- Master integrals from differential equations
  - Differentiate integrand with respect to masses and momenta
  - Apply integration-by-parts identities

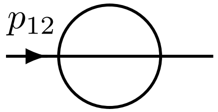
$$s_{123}\frac{\partial}{\partial s_{123}}$$



$$= \frac{d-4}{2}\,\frac{2s_{123}-s_{12}}{s_{123}-s_{12}}$$



$$-\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$



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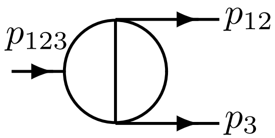
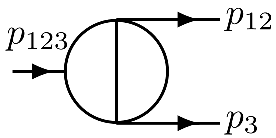
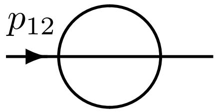
$$s_{12}\frac{\partial}{\partial s_{12}}$$



$$= -\frac{d-4}{2}\,\frac{s_{12}}{s_{123}-s_{12}}$$



$$+\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$



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  - Integrate differential equations and match boundary

Two-loop matrix elements for jet production

- Analytic 2 →2 results for processes with jets
  - Di-jet production (C. Anastasiou, E.W.N. Glover, C. Oleari, M. Tejeida-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
  - Vector-boson-plus-jet production (L. Garland, E.W.N. Glover, A. Koukoutsakis, E. Remiddi, L. Tancredi, E. Weihs, TG)
  - Higgs-boson-plus-jet production (E.W.N. Glover, M. Jaquier, A. Koukoutsakis, TG)
- Top quark pair production
  - Numerical representation (P. Bärnreuther, M. Czakon)
  - Analytical work ongoing (C. Studerus, A. von Manteuffel et al.)
- Next frontier: automation and 2 → 3
  - Unitarity-based methods (P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)
  - Classification of integral basis (H. Johansson, D. Kosower, K. Larsen)

Real radiation at NNLO: factorization

- Single unresolved radiation at one loop
  - One-loop correction to collinear splitting factors (Z. Bern, V. Del Duca, W. Kilgore, C. Schmidt)
  - One-loop correction to soft eikonal factor (S. Catani, M. Grazzini)
- Double unresolved radiation factors at tree level (J. Campbell, E.W.N. Glover; S. Catani, M. Grazzini)
  - Double soft
  - Soft/Collinear
  - Triple collinear
  - Double single collinear
- Require method to extract singular contributions

Real radiation at NNLO: methods

- Sector decompostion
  (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)
  - $pp \to H$, $pp \to V$, including decays
    (C. Anastasiou, K. Melnikov, F. Petriello; S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)
- q_T-subtraction (S. Catani, M. Grazzini)
  - $pp \to H$, $pp \to V$, $pp \to \gamma\gamma$, $pp \to VH$
    (S. Catani, L. Cieri, D. de Florian, G. Ferrera M. Grazzini, F. Tramontano)
- Sector-improved subtraction schemes
  (M. Czakon; R. Boughezal, K. Melinkov, F. Petriello)
  - $pp \to t\bar{t}$ (M. Czakon, P. Fiedler, A. Mitov)
  - $pp \to H+j$ (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
- Antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - $e^+e^- \to 3j$ (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)
  - $pp \to 2j$ (A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)

NNLO Infrared Subtraction

Structure of NNLO cross section

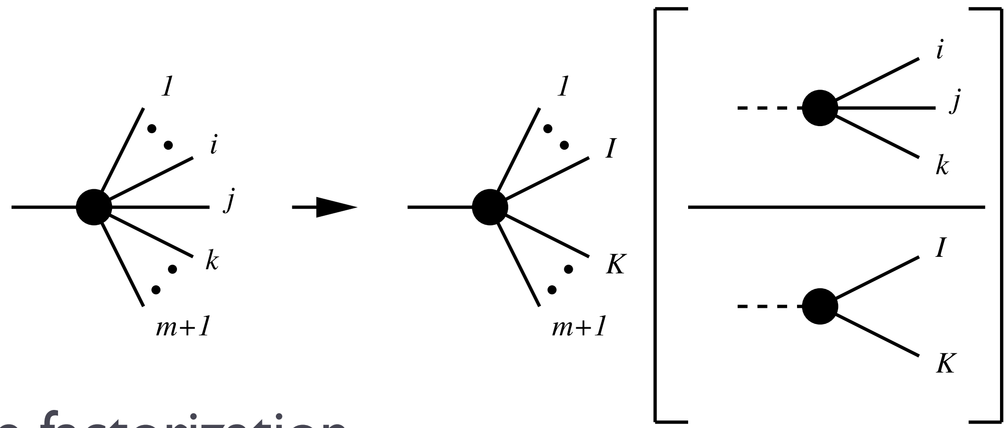
$$\mathrm{d}\sigma_{NNLO} = \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^R_{NNLO} - \mathrm{d}\sigma^S_{NNLO}\right)$$
$$+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{MF,1}_{NNLO}$$
$$+ \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{MF,2}_{NNLO}$$

- Real and virtual contributions: $\mathrm{d}\sigma^R_{NNLO}, \mathrm{d}\sigma^{V,1}_{NNLO}, \mathrm{d}\sigma^{V,2}_{NNLO}$
- Subtraction term for double real radiation: $\mathrm{d}\sigma^S_{NNLO}$
- Subtraction term for one-loop single real radiation: $\mathrm{d}\sigma^{VS,1}_{NNLO}$
- Mass factorization terms: $\mathrm{d}\sigma^{MF,1}_{NNLO}, \mathrm{d}\sigma^{MF,2}_{NNLO}$

- Each line finite and free of poles
  → numerical implementation

Antenna subtraction

- Subtraction terms constructed from antenna functions
  - Antenna function contains all emission between two partons



  - Phase space factorization

$$d\Phi_{m+1}(p_1, \ldots, p_{m+1}; q) = d\Phi_m(p_1, \ldots, \tilde{p}_I, \tilde{p}_K, \ldots, p_{m+1}; q) \cdot d\Phi_{X_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K)$$

  - Integrated subtraction term

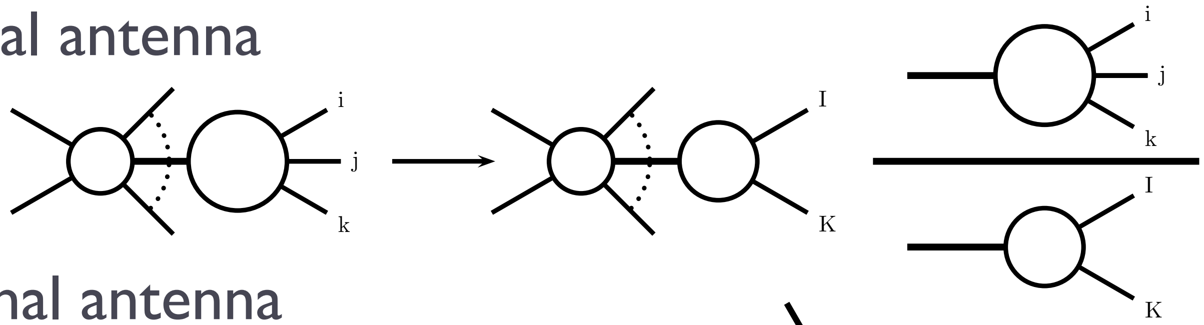
$$\mathcal{X}_{ijk} = \int d\Phi_{X_{ijk}} X_{ijk}$$

Antenna functions

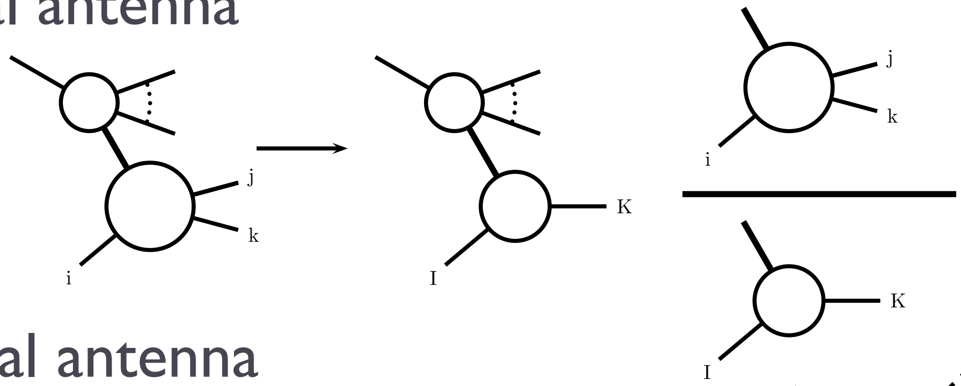
- Colour-ordered pair of hard partons (radiators)
  - Hard quark-antiquark pair
  - Hard quark-gluon pair
  - Hard gluon-gluon pair
- NLO (D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover)
  - Three-parton antenna: one unresolved parton
- NNLO (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - Four-parton antenna: two unresolved partons
  - Three-parton antenna at one loop
  - Products of NLO antenna functions
  - Soft antenna function

Antenna subtraction: incoming hadrons

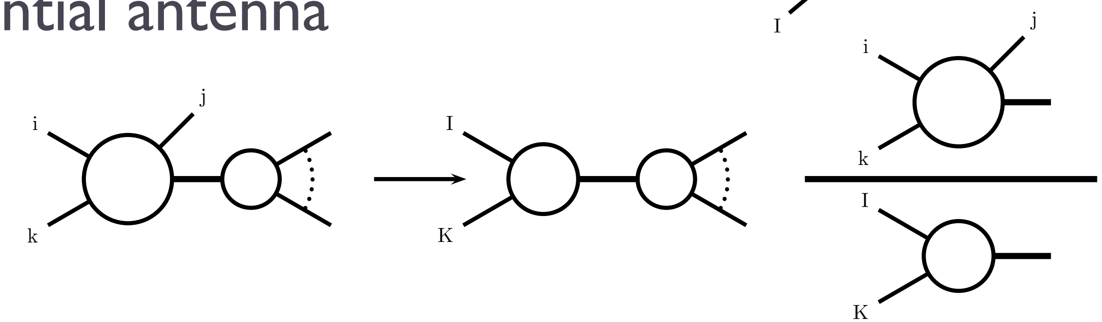
- **Three antenna types** (A. Daleo, D. Maitre, TG)
  - Final-final antenna



  - Initial-final antenna

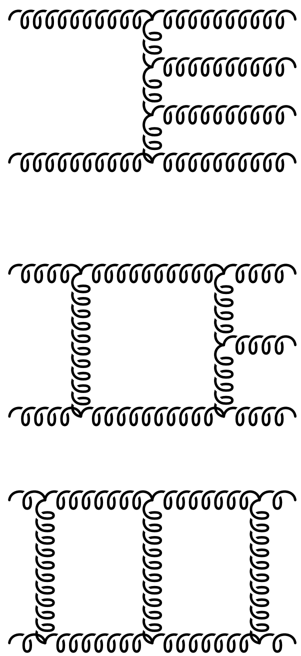


  - Initial-intial antenna



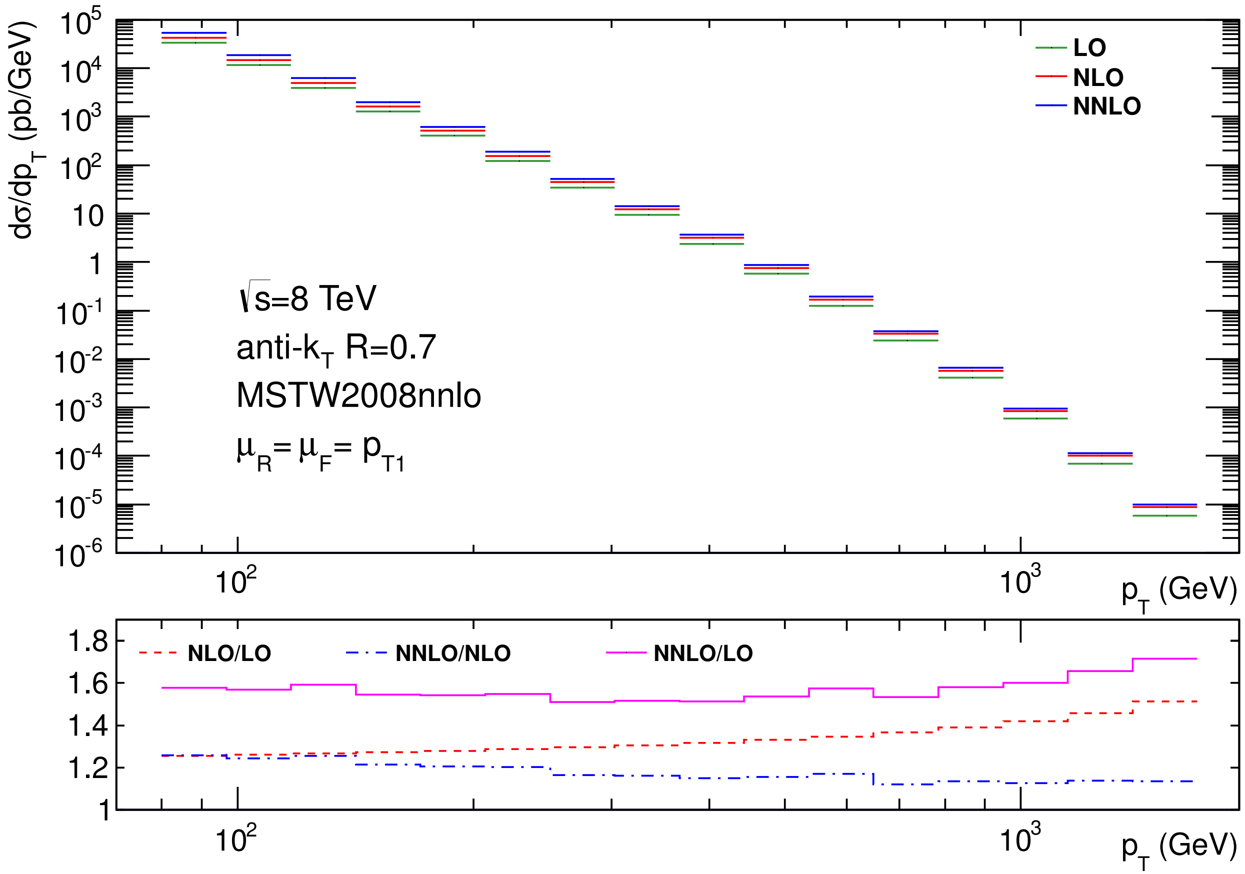
NNLO corrections to pp → 2j

- Leading colour gluons-only as proof of concept
  (A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
  - Double real radiation
    - Subtraction terms constructed and implemented
    - Azimuthal correlations from gluon splitting
  - Single real radiation at one loop
    - Subtraction terms constructed and implemented
    - Interplay of antenna functions and mass factorization
  - Two-loop contributions
    - Added integrated subtraction terms from above
    - Observe analytic cancellation of all infrared poles
- All implemented in parton-level event generator



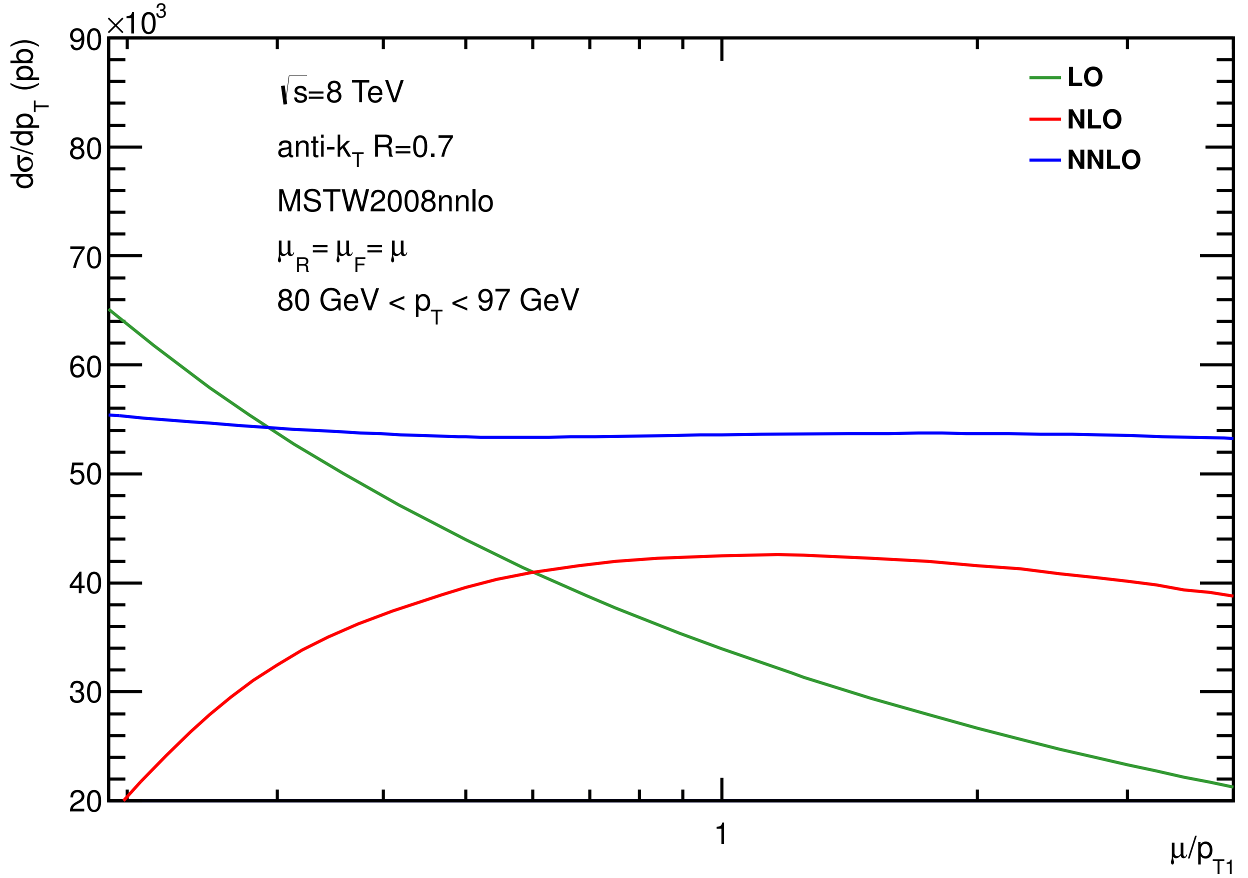
NNLO corrections to pp → 2j

Inclusive jet p_T distribution



NNLO corrections to pp → 2j

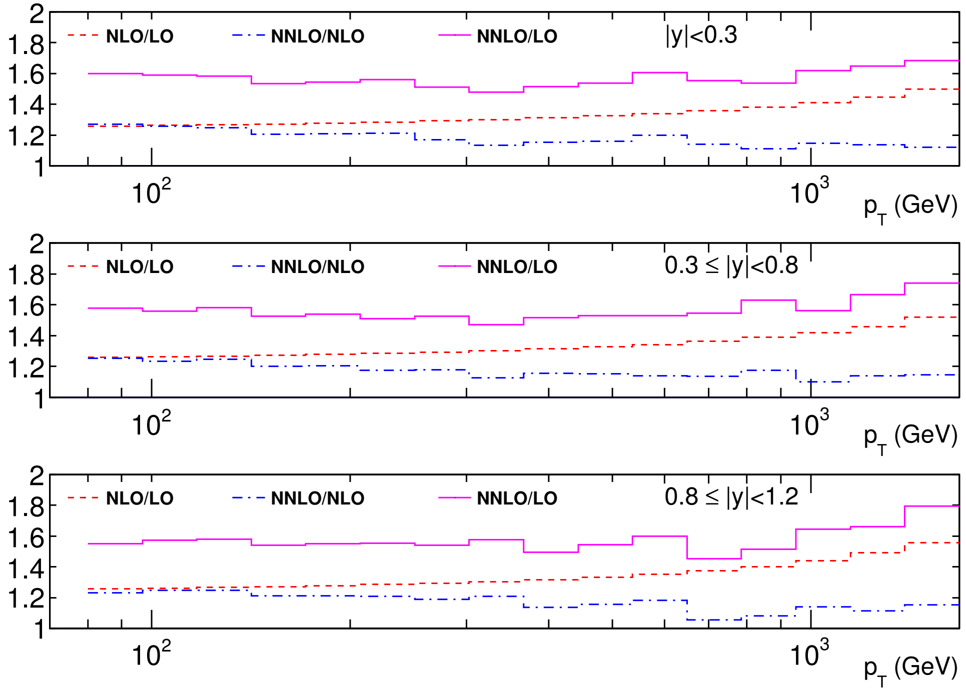
Inclusive jet p_T distribution: scale dependence

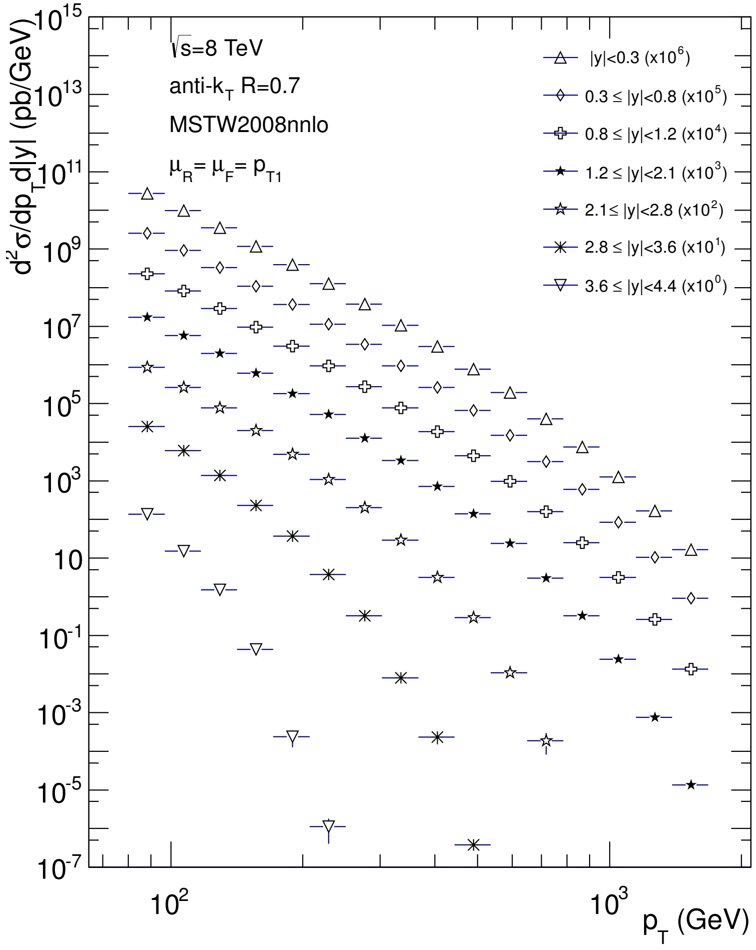


NNLO corrections to pp → 2j

Inclusive jet production:
double differential distributions

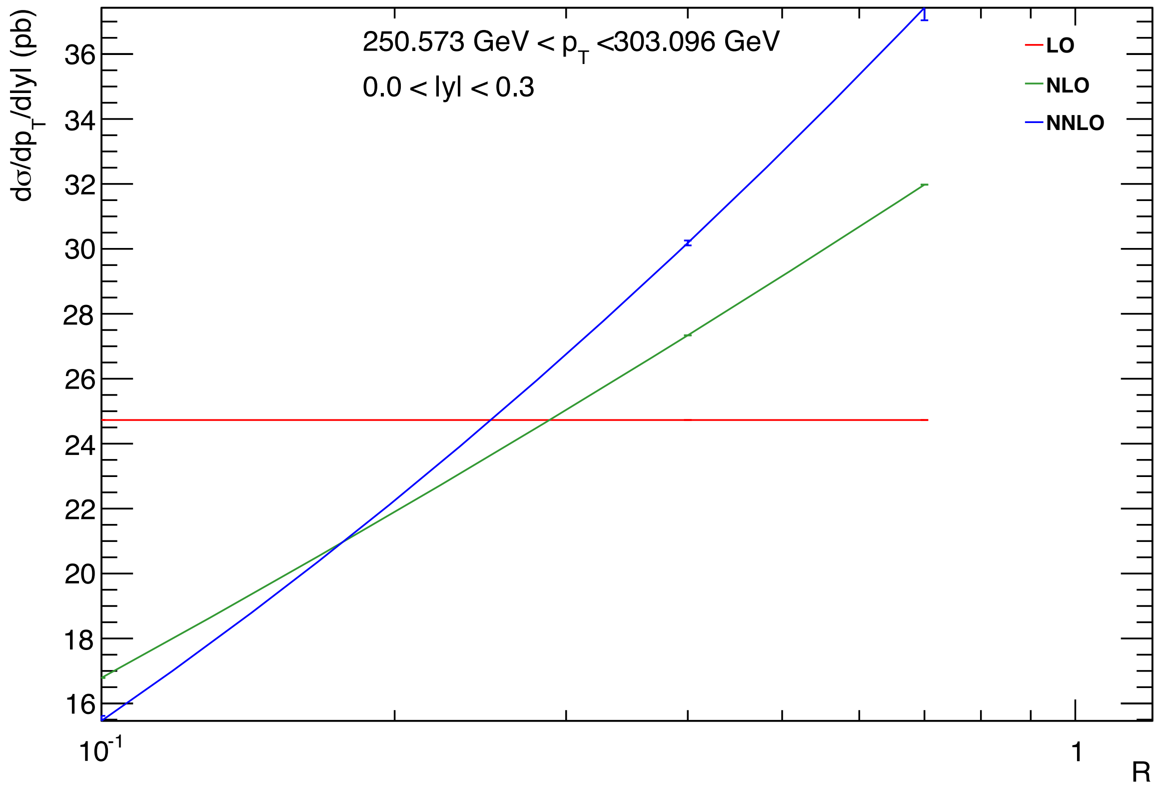
R = 0.7





NNLO corrections to pp → 2j

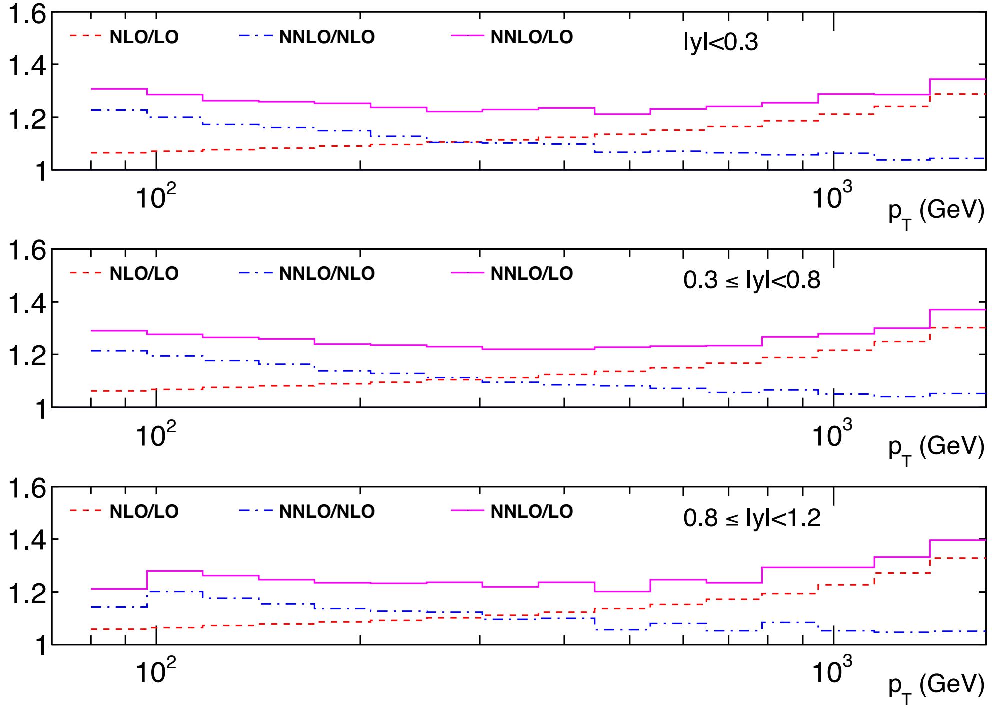
Single-jet inclusive: jet size dependence in anti-k_T algorithm

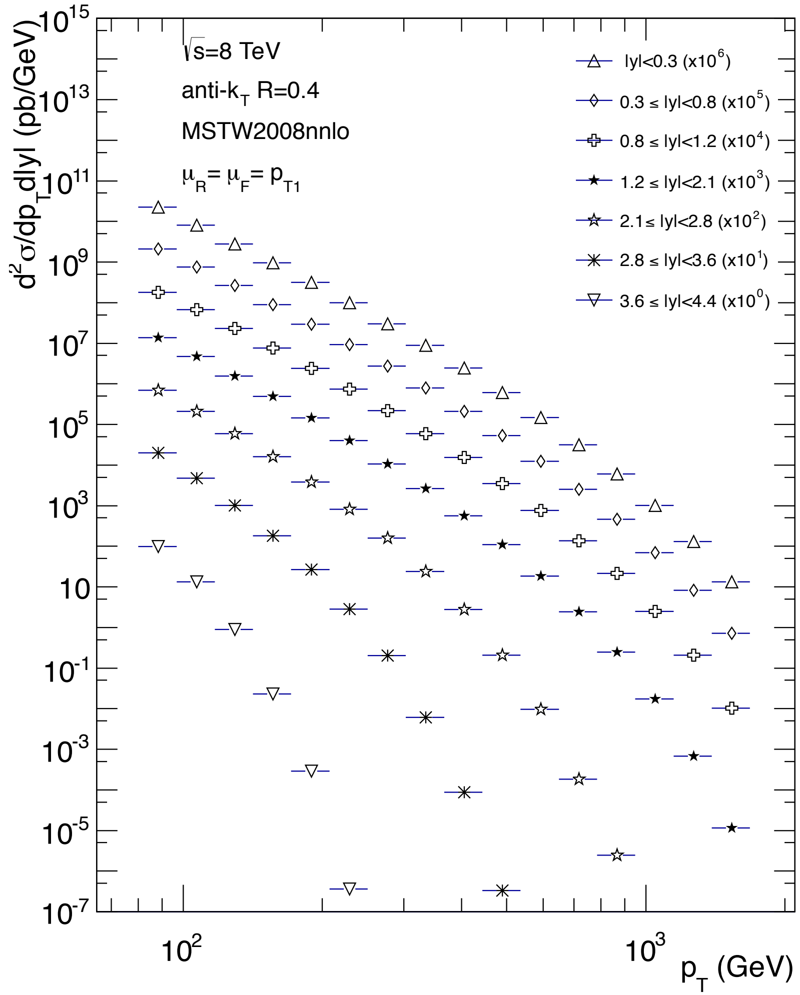


NNLO corrections to pp → 2j

Inclusive jet production:
double differential distributions

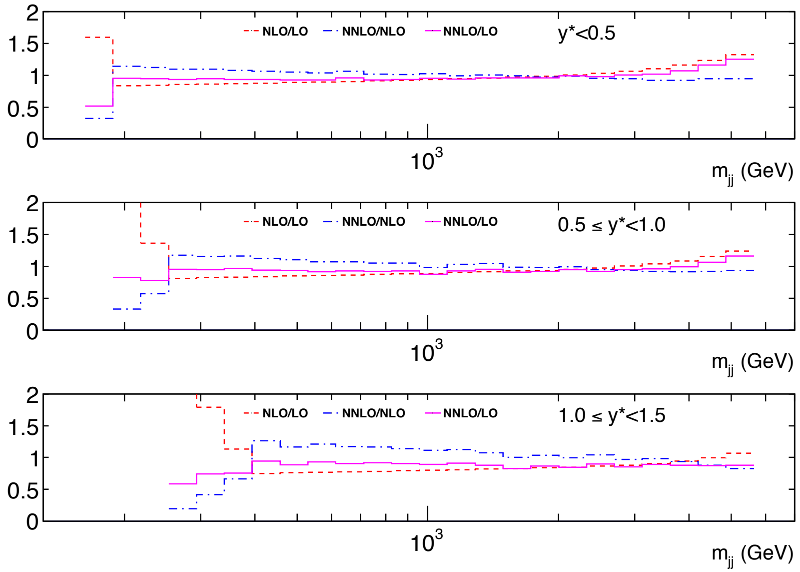
R = 0.4

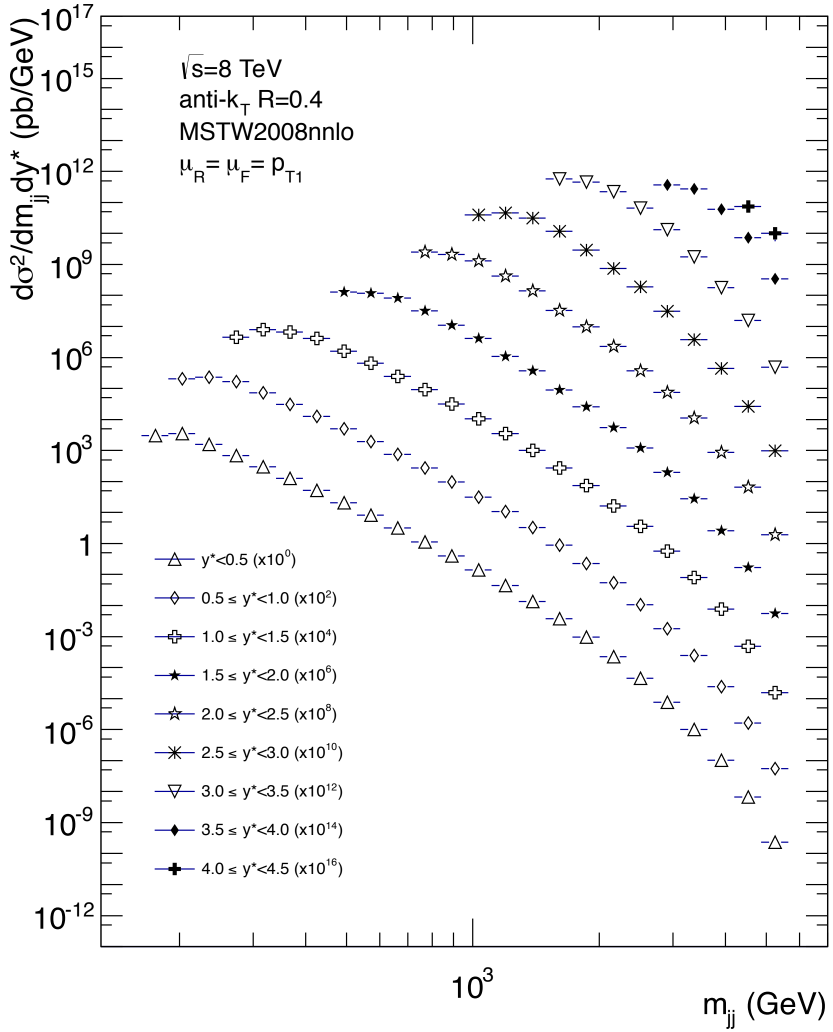




NNLO corrections to pp → 2j

Exclusive di-jet production





Outlook: next steps

- Current status of pp $\rightarrow$ 2j: leading colour, gluons only
  - Serves as proof-of-principle
  - Implementation of all parton-level processes at NNLO
    (J. Currie, E.W.N. Glover, S. Wells)
  - Towards automated generation of subtraction terms
    - Systematic understanding of infrared cancellations
      (J. Currie, E.W.N. Glover)
- Other processes of similar complexity: 2 $\rightarrow$ 2
  - pp $\rightarrow$ H+j
  - pp $\rightarrow$ V+j
- Higher-multiplicity processes: two-loop virtuals needed

Conclusions

- NNLO corrections to precision observables at LHC
  - Various methods have been applied successfully
  - Healthy competition between groups

- Current frontier: $2 \to 2$ QCD processes
  - Top quark pairs
  - Higgs-plus-jet cross section (gluons only)
  - Single-jet inclusive and di-jet cross sections (gluons only)

- Precision phenomenology with jet observables starting
  - Measurements of coupling constants
  - Determination of parton distributions