The Dilaton, the Radion, and Duality

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Introduction

Strong conformal dynamics plays an important role in many theories of electroweak symmetry breaking.

Some Examples

In theories of technicolor, and of the Higgs as a composite pseudo-Nambu-Goldstone boson (pNGB), strong conformal dynamics can help separate the electroweak scale from the flavor scale, allowing flavor constraints to be satisfied.

The AdS/CFT correspondence can be used to relate Randall-Sundrum models to theories where the hierarchy between the Planck and weak scales is realized through strong conformal dynamics.

In theories where an exact conformal symmetry is spontaneously broken, the low energy effective theory contains a massless scalar, the dilaton.

The dilaton can be thought of as the NGB asociated with the breaking of conformal invariance. (Just 1 NGB, not 5, because conformal invariance is a space-time symmetry.)

The form of the dilaton couplings is fixed by the requirement that the symmetry be realized non-linearly. Very predictive.

AdS/CFT identifies radion in Randall-Sundrum setup with dilaton.

However, in the class of theories of interest for electroweak symmetry breaking, conformal symmetry is explicitly broken by nearly marginal operators that grow in the infrared to become large at the breaking scale. Should we expect a light dilaton in the spectrum? How are its couplings modified?

In this talk, I will show that in a specific class of theories, where the operator that breaks conformal symmetry remains close to marginal until the breaking scale, the dilaton mass can naturally lie below the scale of strong dynamics. (Rattazzi)

However, in general, this condition is not satisfied in the theories most relevant for electroweak symmetry breaking. Nevertheless, a light dilaton in these theories is only associated with mild tuning.

I will also show that the results for the radion in RS models match those of the dilaton, and provide a holographic interpretation.

In this framework, corrections to the form of dilaton couplings from conformal symmetry violating effects are suppressed by the square of the ratio of the dilaton mass to the strong coupling scale, and are under good theoretical control if the dilaton is light.

The Mass of the Dilaton

Consider a theory where conformal symmetry is spontaneously broken. Then the low energy effective theory contains a dilaton field *σ(x)* .

Below the breaking scale the symmetry is realized non-linearly. Under scale transformations,

$$x^\mu \to x'^\mu = e^{-\omega} x^\mu$$

the dilaton transforms as

$$\sigma(x) \to \sigma'(x') = \sigma(x) + \omega f$$

where *f* is the symmetry breaking scale.

It is convenient to define the object $\chi(x)$, which transforms linearly under scale transformations.

$$\chi(x) = f e^{\sigma(x)/f}$$

Under the scale transformation

$$x^\mu \to x'^\mu = e^{-\omega} x^\mu$$

$$\chi(x) \to \chi'(x') = e^\omega \chi(x)$$

The low energy effective theory will in general contain all terms consistent with this transformation.

What terms does the Lagrangian contain?

The symmetry allows derivative terms of the form

$$\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{c}{\chi^4}\left(\partial_\mu\chi\partial^\mu\chi\right)^2 + \ldots$$

However, crucially, a non-derivative term is also allowed.

$$V(\chi) = \kappa_0\chi^4$$

The effect of this term is to drive *f* to zero, corresponding to unbroken conformal symmetry, if the coefficient κ_0 is positive. If κ_0 is negative, *f* is driven to infinity, and conformal symmetry is never realized.

Only if κ_0 is identically zero is the symmetry spontaneously broken. The potential then vanishes and there is a massless dilaton. However, in general setting κ_0 to zero is associated with tuning, since there is no symmetry reason for it to vanish.

The situation changes if conformal symmetry violating effects are present. Add to the theory an operator *O(x)* of dimension Δ close to 4.

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \lambda_{\mathcal{O}} \mathcal{O}(x)$$

Under scale transformations,

$$x^{\mu} \to x'^{\mu} = e^{-\omega} x^{\mu}$$

$$\mathcal{O}(x) \to \mathcal{O}'(x') = e^{\omega\Delta} \mathcal{O}(x)$$

Define a dimensionless coupling constant ,

$$\hat{\lambda}_{\mathcal{O}} = \lambda_{\mathcal{O}} \mu^{\Delta-4}$$

The operator *O(x)* is normalized such that $\lambda \sim 1$ corresponds to strong coupling. For small $\lambda \ll 1$, it satisfies the RG equation,

$$\frac{\mathrm{d}\,\log \hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\,\log \mu} = -(4 - \Delta)$$

We can determine how λ appears in the low energy theory by promoting it to a spurion. For small λ the form of the UV theory is invariant under the following transformation.

$$x^\mu \to x'^\mu = e^{-\omega} x^\mu$$

$$\lambda_{\mathcal{O}} \to \lambda'_{\mathcal{O}} = e^{(4-\Delta)\omega} \lambda_{\mathcal{O}}$$

To leading order in λ the form of the potential now becomes

$$V(\chi) = \kappa_0 \chi^4 - \kappa_1 \lambda_{\mathcal{O}} \chi^\Delta$$

(Rattazzi & Zaffaroni)

The dilaton potential admits a minimum at

$$f^{(\Delta-4)} = \frac{4\kappa_0}{\kappa_1 \lambda_{\mathcal{O}} \Delta}$$

From this, we find the dilaton mass at the minimum,

$$m_\sigma{}^2 = \kappa_1 \lambda_{\mathcal{O}} \Delta (4-\Delta) f^{\Delta-2} = 4\kappa_0 (4-\Delta) f^2$$

The dilaton mass is suppressed if the operator that breaks conformal symmetry is marginal! **(Goldberger, Grinstein & Skiba)**

Since Δ is expected to be close to 4 in theories that address the flavor or hierarchy problems, potentially a very interesting result! ➔ A new light state below the strong coupling scale.

In technicolor frameworks, the new state observed by the LHC at 125 GeV could be the dilaton!

In theories where new particle is a composite Higgs arisng from strong conformal dynamics, this result predicts the existence of an additional scalar state below the strong coupling scale.

Unfortunately, the analysis that led up to this conclusion is only valid at small λ, corresponding to weak coupling. To validate this result, must establish it at strong coupling.

Our approach will be to assume small (perturbative) λ, but work to all orders in this parameter. Check if the result survives when $\lambda \rightarrow 1$, its strong coupling value.

Working to all orders in λ involves incorporating 4 distinct effects.

- **In writing down the Lagrangian, did not take into account the breaking of scale invariance by the regulator. Must include this.**

- **In determining the vacuum structure used the potential, not the effective potential. This needs to be accounted for.**

- **Need to include terms with all powers of λ in the Lagrangian. Setting $\epsilon = 4 - \Delta$, the potential becomes**

$$V(\chi) = \kappa_0 \chi^4 - \sum_{n=1}^{\infty} \kappa_n \lambda_{\mathcal{O}}^n \, \chi^{(4-n\epsilon)}$$

- **As λ approaches strong coupling, its RG evolution is affected. The RG for λ now takes the more general form**

$$\frac{\mathrm{d}\,\log \hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\,\log \mu} = -g(\hat{\lambda}_{\mathcal{O}})$$

where g(λ) is a polynomial in λ. The constant term in this polynomial is ϵ = (4 - Δ).

Of these 4 effects, the first 3 do not alter the conclusions of the naive small λ analysis. The underlying reason is that in each of these 3 cases, the corrections are of order λ times the leading order effect and are therefore at most of the same size.

The 4th effect is qualitatively different. Consider again the RGE

$$\frac{\mathrm{d}\,\log \hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\,\log \mu} = -g(\hat{\lambda}_{\mathcal{O}}) \qquad g(\hat{\lambda}_{\mathcal{O}}) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_{\mathcal{O}}^n$$

The leading order term in the polynomial g(λ) is (4 - Δ) = $\epsilon \ll 1$, while the corrections begin at order λ. Even before strong coupling is reached the higher order terms dominate, and their effects can alter the conclusions of the naïve small λ analysis.

The form of the UV theory is invariant if λ is promoted to a spurion that transforms under scale transformations as

$$\hat{\lambda}_{\mathcal{O}}(\mu) \to \hat{\lambda}'_{\mathcal{O}}(\mu) = \hat{\lambda}_{\mathcal{O}}(\mu e^{-\omega})$$

Then the combination

$$\overline{\Omega}(\hat{\lambda}_{\mathcal{O}}, \chi/\mu) = \hat{\lambda}_{\mathcal{O}} \left(\frac{\chi}{\mu}\right)^{-g(\hat{\lambda}_{\mathcal{O}})}$$

is invariant under infinitesimal scale and RG transformations.

By requiring invariance under spurious scale transformations, we can obtain the low energy effective theory for the dilaton. To leading order in Ω, the potential takes the form

$$V(\chi) = \frac{\chi^4}{4!}\left(\kappa_0 - \kappa_1 \overline{\Omega}\right)$$

Exactly the same form as before, but with ϵ replaced by $g(\lambda)$.

The dilaton mass is given by

$$m_\sigma^2 = 4\frac{\kappa_0}{4!} g(\hat{\lambda}_{\mathcal{O}}) f^2$$

It is the scaling dimension of *O(x)* at the breaking scale that determines dilaton mass, not scaling dimension at fixed point.

$$g(\hat{\lambda}_{\mathcal{O}}) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_{\mathcal{O}}^n$$

To obtain a light dilaton, it is not sufficient that $c_0 = \epsilon \ll 1$. Require $g(\lambda) \ll 1$ at the breaking scale.

This is equivalent to requiring that not just c_0 but all the $c_n \ll 1$.

Although this can happen naturally in some cases, for example in theories with fixed lines, this criterion is not expected to be satisfied in most theories of interest for EWSB.

The presence of a light dilaton is associated with tuning!

However, there is an interesting feature! Consider the potential,

$$V(\chi) = \frac{\chi^4}{4!}\left(\kappa_0 - \kappa_1 \overline{\Omega}\right)$$

If the conformal field theory is such that the parameter κ_0 is smaller than its natural strong coupling value by some factor `Q', Q > 1, then the potential is minimized for λ ~ 1/Q .

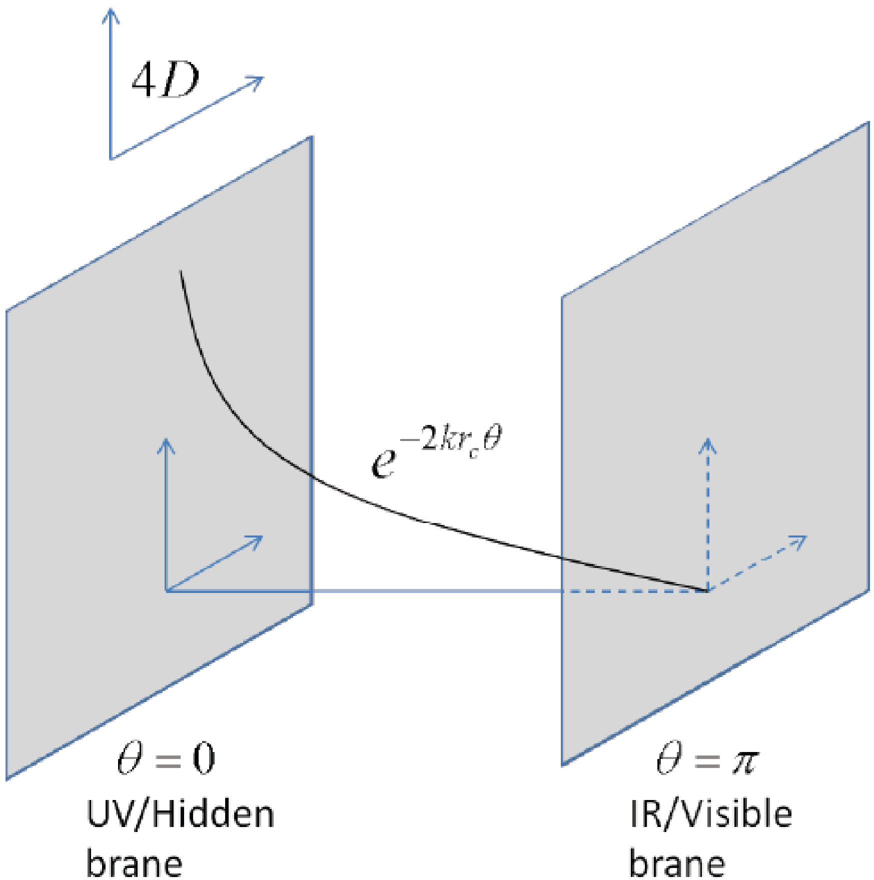
$$m_\sigma^2 = 4\frac{\kappa_0}{4!} g(\hat{\lambda}_{\mathcal{O}}) f^2$$

For $\epsilon < 1/\mathrm{Q}$, g(λ) is of order 1/Q at the breaking scale and it can be seen that the dilaton mass is suppressed by the same factor.

Now small values of κ_0 are associated with tuning (coincidence problem), the tuning scaling as Q . This analysis suggests that a dilaton mass a factor of 5 below the strong coupling scale is only tuned at the level of 1/5 (20%). Compare this to the quadratic tuning (4%) for a non-pNGB composite scalar. The tuning is mild!

The Holographic Viewpoint

The AdS/CFT correspondence allows large *N* theories where an exact or approximate conformal symmetry is spontaneously broken to be realized as Randall-Sundrum models in warped extra dimensions.



The presence of an IR brane in the RS scenario is associated with the spontaneous breaking of conformal symmetry at the IR scale.

In this framework, the dilaton is identified with the dynamical scalar field associated with fluctuations in the spacing between the two branes, the radion.

We would like to understand how our results for the dilaton emerge from the holographic viewpoint. (Seem to contradict the conventional wisdom that the radion in RS is naturally light.)

The first step is to obtain the effective potential for the radion in the absence of a mechanism that stabilizes the brane spacing.

The action for the RS model is of the form,

$$\mathcal{S}_{GR}^{5D} = \int d^4x \int_{-\pi}^{\pi} d\theta \left[\sqrt{G}\Big(- 2M_5^3\mathcal{R}[G] - \Lambda_b\Big) - \sqrt{-G_h}\delta(\theta)T_h - \sqrt{-G_v}\delta(\theta-\pi)T_v\right]$$

bulk cosmological constant **brane tensions**

The RS metric can be written as,

$$ds^2 = e^{-2kr_c|\theta|}\eta_{\mu\nu}(x)dx^\mu dx^\nu - r_c^2\, d\theta^2, \qquad -\pi \le \theta \le \pi$$

To parametrize the radion we promote the undetermined constant r_c which represents the brane spacing to a field, $r_c \rightarrow r(x)$.

Substituting this into the action we obtain the four dimensional effective theory for the radion field,

$$\int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right)$$

The canonically normalized radion field *φ(x)* is related to *r(x)* as,

$$\varphi(x) = Fe^{-k\pi r(x)} \qquad F = \sqrt{24M_5^3/k}$$

The potential *V(φ)* has the form,

$$V_{GR}(\varphi) = \frac{\varphi^4}{F^4}\left(T_v - \frac{\Lambda_b}{k}\right)$$

This has exactly the same form as the potential for the dilaton in the absence of conformal symmetry violating effects. As in the dilaton case, tuning is required to obtain a stable minimum. Here the brane and bulk cosmological constants must be balanced.

In order to stabilize the brane spacing, we introduce a scalar field sourced on the two branes. **(Goldberger & Wise)**

The action for the Goldberger-Wise (GW) scalar Φ has the form,

$$\mathcal{S}_{GW} = \int d^4x\, d\theta \left[\sqrt{G} \left(\frac{1}{2} G^{AB} \partial_A \Phi \partial_B \Phi - V_b(\Phi) \right) - \sum_{i=v,h} \delta(\theta - \theta_i) \sqrt{-G_i}\, V_i(\Phi) \right]$$

The bulk potential chosen for the GW field generally consists only of a mass term, $m^2\, \Phi^2$. The mass is kept slightly small in units of k, the inverse curvature, in order to generate a large hierarchy between the UV and IR scales.

However, in general there is no symmetry that forbids higher powers of the scalar field in the potential Φ^3 , Φ^4 etc. If the detuning of the IR brane is significant, these terms will dominate over $m^2\, \Phi^2$ in the IR since the mass parameter is small.

Since the radion wave function is localized towards the infrared, to get the correct physics it is necessary to solve the system keeping higher powers of Φ in the bulk potential.

However, the problem is then non-linear, even in the limit that the gravitational back-reaction is neglected.

Nevertheless, an approximate analytical solution can be found, using the methods of singular perturbation theory (boundary layer theory).

The solution for Φ is characterized by the formation of a boundary layer near the location of the IR brane.

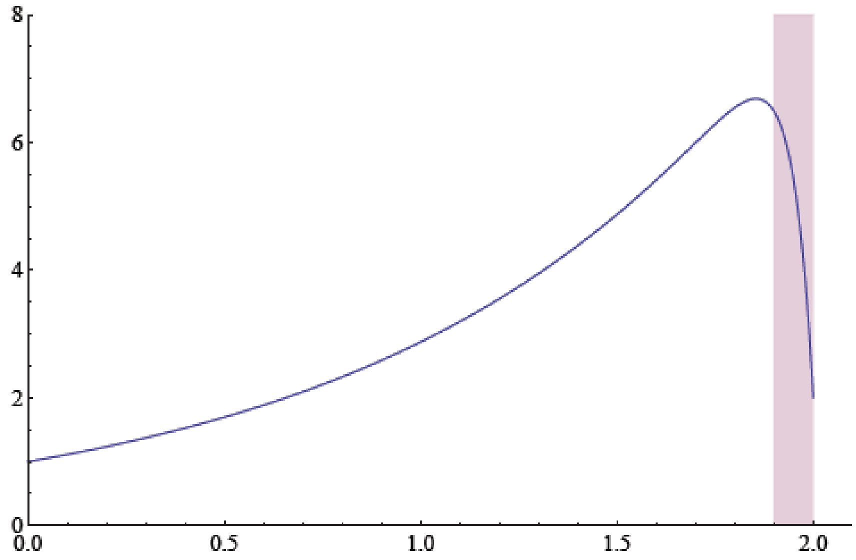
Consider a differential equation, with a small parameter ε.

$$\epsilon \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$$

This has a regular solution near $y = A\, e^{x}$, and also a singular solution $y = B\, e^{x/\varepsilon} + C$. The singular solution is needed to satisfy the two boundary conditions.

For general boundary conditions, $y(x)$ will take the form of the regular solution, except in a narrow strip of thickness ε near one of the boundaries where the singular solution dominates.

To obtain $y(x)$ combine the regular and boundary layer solutions.



Consider the differential equation for the GW scalar.

$$\partial_\theta^2 \Phi - 4kr_c \partial_\theta \Phi - r_c^2 V_b'(\Phi) = 0$$

In the case of a large hierarchy, 1/ (kr_c) ≪ 1 is a small parameter. The regular solution is valid everywhere, except in a narrow strip of width 1/ (kr_c) near the IR brane where a boundary layer forms.

$$OR: \qquad \frac{d\Phi}{d\theta} = -\frac{r_c}{4k} V_b'(\Phi) \qquad (0 \leq \theta \lesssim \pi - \epsilon)$$

$$BR: \qquad \frac{d^2\Phi}{d\theta^2} = 4kr_c \frac{d\Phi}{d\theta} \qquad (\pi - \epsilon \lesssim \theta \leq \pi)$$

The form of the boundary layer solution is independent of the bulk potential $V_b(\theta)$.

$$\Phi_{BR}(\theta) = -\frac{k^{3/2}\alpha}{4} e^{4kr_c(\theta - \pi)} + C$$

The integration constant is used to match to the regular solution at the interface.

The potential for the GW scalar in the bulk has the general form,

$$V_b(\Phi) = \frac{1}{2}m^2\Phi^2 + \frac{1}{3!}\eta\Phi^3 + \frac{1}{4!}\zeta\Phi^4 + \ldots$$

Then, away from the IR brane the differential equation for Φ is

$$\frac{d\,\log\Phi}{d\,(kr_c\theta)} = -\frac{m^2}{4k^2} - \frac{\eta}{8\sqrt{k}}\,\frac{\Phi}{k^{3/2}} - \frac{\zeta\,k}{24}\,\frac{\Phi^2}{k^3} + \ldots$$

In the AdS/CFT correspondence, introducing the GW scalar is equivalent to adding to the CFT an operator *O(x)* which grows in the IR and breaks the CFT.

In the semi-classical approximation, the value of the scalar field Φ(θ) is identified with the coefficient $\lambda(\mu)$ of the operator *O(x)*.

Comparing the differential equation for Φ(θ) with RGE for $\lambda(\mu)$, we have perfect agreement. Notice that the self-interaction terms for Φ are required to match the scaling dimension of *O(x)*.

$$\frac{d\,\log\hat{\lambda}}{d\,\log\mu} = -(4-\Delta) - c_1\hat{\lambda} - c_2\hat{\lambda}^2$$

To understand the effect of the self-interaction terms, keeping only the Φ^3 term in the potential, we solve for the GW scalar,

$$\Phi(\theta) = -\frac{k^{3/2}\alpha}{4} e^{4kr_c(\theta-\pi)} + \frac{k^{3/2}v}{1+\xi k r_c \theta}$$

Then, by integrating out the GW field in the radion background, we obtain the potential for the radion

$$V(\varphi) \approx \varphi^4 \left[\tau + \frac{z}{1+\xi k r_c \pi}\left(\frac{\varphi}{k e^{-k r_c \pi}}\right)^{\frac{\xi}{1+\xi k r_c \pi}}\right]$$

This is of the same form as the result from the CFT side of the correspondence.

$$V(\chi) = \frac{\chi^4}{4!}\left[\kappa_0 - \kappa_1 \hat{\lambda}\left(\frac{\chi}{f}\right)^{-c_1\hat{\lambda}}\right]$$

The mass of the dilaton scales as the detuning of the IR brane tension. Its natural size is of order the Kaluza-Klein mass scale.

Corrections to the Radion Couplings

Stabilizing the brane spacing leads to corrections to the radion couplings to SM fields.

These emerge from direct couplings of the GW scalar to the SM.

Focus on the case when all SM fields are on IR brane. For W and Z

$$\mathcal{L} \supseteq \delta(\theta-\pi)\,\sqrt{-G_v}\left[\left(1+\beta_W\frac{\Phi}{k^{3/2}}\right)G_v^{\mu\nu}D_\mu H^\dagger D_\nu H\right]$$

Since the VEV of the GW field is a function of the brane spacing,

$$\Phi(\pi) \rightarrow \Phi(\pi)\left(1+\frac{\partial_\varphi\Phi(\pi)}{\Phi(\pi)}\,\widetilde{\varphi}\right)$$

This leads to a correction to the coupling of the radion to *W* and *Z.* This correction is suppressed by the square of the ratio of the radion mass to the Kaluza-Klein scale. Small for a light radion.

Agrees with the result from the CFT side of the correspondence.

Conclusions

In theories where the operator that breaks conformal symmetry remains close to marginal until the breaking scale, the dilaton mass can naturally lie below the scale of strong dynamics.

However, in general, this condition is not satisfied in the theories most relevant for electroweak symmetry breaking.

Nevertheless, a light dilaton in these theories is only associated with modest tuning.

In this framework, corrections to the form of dilaton couplings from conformal symmetry violating effects are suppressed by the square of the dilaton mass over the strong coupling scale, and are under good theoretical control (if the dilaton is light).

The results for the radion in RS models match those of the dilaton, as expected from holography.