## NEW TECHNIQUE(S) FOR MASS MEASUREMENT AT HADRON COLLIDERS

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(with Doojin Kim, Roberto Franceschini, Kyle Wardlow: I 209.0772, I2 I 2.5230 and to appear)

## Basic goal

- determine mass of mother by measuring energy/momentum of (visible) decay products


TECHNICUES SO FAR (many cases)

## Fully visible I ("clean")

 ( invariant mass of decay products has Breit-Wigner peak


- have to be "lucky"!


## Fully visible II ( so clean)

- fully hadronic top decay

- problem: combinatorics (especially with jets from initial state radiation)
"Partially" visible I (can be reconstructed)
- 1 daughter fully visible, other partially
- semileptonic top decay (cleaner)

- problem: discrete ambiguity in reconstructing W; must use MET; still combinatorics (which W with which b)...


## "Partially" visible II (cannot be reconstructed)

- 1 daughter fully visible,
- R-parity conserving SUSY, top-partner in T-parity little Higgs models...

- Use transverse mass $\left(M_{T 2}\right)$ : "involved"; need MET...


## Bottomline:

## dunk!

- useful to have more techniques, especially simpler; complementary (different systematics, e.g., avoid MET and combinatorics)

NEW OBSERVATION technieue

## Basic assumptions

- 2-body decay: one daughter (fully) visible, massless:

- ...other (A) don't care (almost)!
- more assumptions later
- extensions/generalizations later


## Energy ( invariant) of daughter

- simple function of masses in rest frame of mother:

$$
E_{a}^{\mathrm{rest}}=\frac{M_{B}^{2}-M_{A}^{2}}{2 M_{B}}
$$

- determine $M_{B}$ if $M_{A}$ known and $E_{a}^{\text {rest }}$ measured


## simple to be practical/useful?!

- hadron collider: mother has varies event to event $\square$ distribution in $E_{a}^{\text {lab }}$

- lose rest-frame information


## Outline

- Peak (of lab. distribution) still retains this information...as simply and precisely!
"'Test" application (top mass): obtain approximation to theory curve Fit it to (simulated) data for extracting peak
- New physics (Cascade decay): general SUSY example (preliminary)
- Three-body decay (time permitting)
- Conclusions


# "INVARIANCE" OF TWOBODY DECAY KINEMATICS 

## Rectangle for fixed, but arbitrary boost

- In general: $E_{a}^{\text {lab }}=E_{a}^{\text {rest }} \gamma_{B}\left(1+\beta_{B} \cos \theta_{a B}\right)$
- Assume unpolarized mother: $\cos \theta_{a B}$ is flat



# Rectangle vs. 

$E_{a}^{\text {lab }}$ gets larger contribution from given boost than does $E_{a}^{\text {rest }}$

- no $E_{a}^{\text {lab }}$ is contained in every rectangle
- asymmetric on linear (symmetric on log...)



## (Generic) Boost distribution: "stacking" up rectangles

- distribution of $E_{a}^{\mathrm{lab}}$ has peak at $E_{a}^{\mathrm{rest}}$
- ...no matter what is the
- boost distribution depends on production mechanism, mother mass, PDF's...



## How to "avoid" plateau

- Boost distribution does not vanish close to $\gamma_{B}=1$



## "'Massive" daughter

- argument goes thru' (rectangle contains $E_{a}^{\text {rest } . . .) ~}$ even for massive daughter if boost distribution restricted to $\gamma_{B}<\left[2\left(\gamma_{a}^{\text {rest }}\right)^{2}-1\right]$
- This critical boost is typically large value for massive, but "light" daughter


## Formal proof

- Single Rectangle $\left(x=\frac{E^{\text {lab }}}{E_{a}^{\text {rest }}}\right)$ :

$$
\left.\frac{1}{\Gamma} \frac{d \Gamma}{d x}\right|_{\text {fixed } \gamma_{B}}=\frac{\Theta\left(x-\gamma_{B}+\sqrt{\gamma_{B}^{2}-1}\right) \Theta\left(-x+\gamma_{B}+\sqrt{\gamma_{B}^{2}-1}\right)}{2 \sqrt{\gamma_{B}^{2}-1}}
$$

- Stacking up rectangles:

$$
f(x) \equiv \frac{1}{\Gamma} \frac{d \Gamma}{d x}=\int_{\frac{1}{2}\left(x+\frac{1}{x}\right)}^{\infty} d \gamma_{B} \frac{g\left(\gamma_{B}\right)}{2 \sqrt{\gamma_{B}^{2}-1}}
$$

- Slope:

$$
f^{\prime}(x)=\frac{\operatorname{sgn}(1-x)}{2 x} g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)
$$

- Behavior at $x=1$ :
$f^{\prime}(x=1) \propto g(1)=0 \Rightarrow$ extremum or
$f^{\prime}(x)$ flips its sign at $x=1 \Rightarrow$ a cusp
$f(x)$ is positive and vanishes for both $x \rightarrow 0$ and $x \rightarrow \infty$
$\Rightarrow$ peak at $E_{a}^{\text {rest }}$


## (POSSIBLE) APPLICATIONS

## General Idea

mother
(B) a (visible, massless)

- determine $M_{B}$ (if $M_{A}$ known) using $E_{a}^{\text {rest }}$
(measured from peak in $E_{a}^{\text {lab }}$ )
$E_{a}^{\mathrm{rest}}=\frac{M_{B}^{2}-M_{A}^{2}}{2 M_{B}}$



## Measuring the peak

- peak can be wide ( to read-off value "by eye")
- extract peak by fitting to "theory curve": a la Breit-Wigner (simple, analytic function)
- ...but exact, analytic formula to obtain here (depends on boost distribution, thus PDF'...)


## APPROXIMATIONTO THEORY CURVE

## Do know (analytically) properties of distribution

- value of $f(x)$ remains the same under $x \leftrightarrow \frac{1}{x}$
- $f$ is maximized at $x=1$
- $f$ vanishes as $x$ approaches 0 or $\infty$
- $f$ becomes a $\delta$-function in some limit of its parameters


# Ansatz (based on properties) 

## width parameter <br> $$
f(x)=K_{1}^{-1}(p) \exp \left[-\frac{p}{2}\left(x+\frac{1}{x}\right)\right]
$$

Bessel function

- simple, but
unique "peak finder"...

Test on b-jet energy from top quark decay (production unpolarized...)


- bottom "'massless": $\gamma_{b}^{\text {rest }} \approx 15 \Rightarrow \gamma_{\text {top }} \lesssim 500$ suffices
- good fit for heavier "top" quark as well: different PDF's, boost distribution (width parameter encompasses this variation)


## "New" Breit-Wigner

- Based on theory fits, assume

$$
f(x)=K_{1}^{-1}(p) \exp \left[-\frac{p}{2}\left(x+\frac{1}{x}\right)\right]
$$

FURTHER TEST:FIT TO (SIMULATED)DATA

# (Again) Top quark decay: basic idea 

neglect $r r_{b}$ in $E_{b}^{\text {rest }}$

- Peak in measured b-jet energy distribution $\approx \frac{M_{t}^{2}-M_{W}^{2}}{2 M_{t}}$
- Assuming $M_{W}$ (but need to detect it at all!), get $M_{t}$


## Top mass measurement: details

- Fully leptonic with $5 / \mathrm{fb}$ at LHC7
- Madgraph $\longrightarrow$ Pythia $\longrightarrow$ Delphes/Fastjet
- 100 pseudo-experiments
- ATLAS choice of cuts
- no background


## Result

(I pseudo-experiment shown)

(use only blue dots)

- consistent with input value
- fitting not spoiled by cuts or detector effects


# Discussion 

- neglect hard radiation from bottom (3-body): suppressed by $\alpha_{s} / \pi^{+}$jet-veto

- safe from soft radiation off of bottom
- safe from ISR (include both b's)
- no combinatorics
- independent of production mechanism (single or pair) as long as unpolarized (cf. matrix element method)

> A NEW PHYSICS APPLICATION: CASCADE DECAY

## In General: Topology

- Two 2-body decays: primary (C) and secondary (B) mothers)



## Two energy peaks

- Based on new observation:

$$
E_{b}^{\text {peak }}=\frac{M_{C}^{2}-M_{B}^{2}}{2 M_{C}} \text { and } E_{a}^{\text {peak }}=\frac{M_{B}^{2}-M_{A}^{2}}{2 M_{B}}
$$



## Edge in invariant mass (old)

- On-shell intermediate particle (sharp) edge



## = 3 (independent) observables for

 determining 3 masses!
## CASCADE DECAY IN SUSY (PRELIMINARY)

## Gluino, sbottom, neutralino


(2 natural SUSY: 1st/2nd generation squarks heavy, stop/sbottom and gluino, Higgsino light

## Double (b-jet energy) peak


© mass hierarchy: $M_{\tilde{g}} \approx M_{\tilde{b}} \gg M_{\chi_{1}^{0}} \square$ 'soft"-hard b-jets

## Background

- $\bar{t} t \bar{b} b$ reducible and $Z+4 b$ irreducible
- template for background: $N_{p^{\prime}} \exp \left(-p^{\prime} \sqrt{E}\right)$

$Z+4 b$
(old plot)


## Results

- $M_{\tilde{g}}=1000 \mathrm{GeV} ; M_{\tilde{b}}=930 \mathrm{GeV}$ and $M_{\chi_{1}^{0}}=100 \mathrm{GeV}$ with $300 / \mathrm{fb}$ at LHCI4

3 (2 signal + I background) template fit (assume this model)
no sensitivity to $M_{\chi_{1}^{0}}: 2 \sqrt{E_{b}^{\text {peak } 1} E_{b}^{\text {peak } 2}} \approx M_{b b}^{\max }$



## ansatz/fitting function

 works for (boost distribution of) a "secondary" mother as well!
## Conclusions

- Two body decay of unpolarized mother at hadron colliders:
peak in energy distribution of massless daughter same as rest frame energy (simple function of masses)
- Obtain approximation to theory curve (for fitting to data to extract peak)
- Application(s):
top quark mass (test)
new particles decaying semi-invisibly: extract all masses from cascade decay (e.g., gluino to sbottom...)

BACK-UP

## Another spectrum: sensitivity to neutralino mass

- mass hierarchy: $M_{\tilde{g}} \gg M_{\tilde{b}} \gtrsim M_{\chi_{1}^{0}} \square$ both b-jets hard


## peaks



- Ansatz can extract 2 peaks separately (assume this model)


## Other/cleaner possibilities

- $a \neq b$ : peaks in different distributions (no
"pollution" between peaks)

- lepton instead of jet


## THREE-BODY DECAY

## Endpoint of distribution in rest frame

- Endpoint related simply to masses



## Peak of distribution in lab frame

$$
E_{a}^{\mathrm{lab}, \mathrm{peak}}<E_{a}^{\mathrm{rest}, \max }
$$

- Obtain inequality for masses
- used in distinguishing $Z_{3}$ vs. $Z_{2}$-stabilized dark matter

(Motivation: fundamental parameter of SM;enters calculation of other observables) Conventional methods
- Basic idea: reconstruct (full) decay of top

- can achieve $\mathrm{O}(0.6 \mathrm{GeV})$ uncertainty at LHCI4, with $300 / \mathrm{fb}$
- further gain may be possible with $3000 / \mathrm{fb}$ by using a more extended approach to constraining uncertainties using data
- Simulation (using SM matrix element in production) is used to handle combinatorics


## Latest: endpoint of $M_{b l}$



- more cleanly interpreted as measurements of the pole quark mass
- combinatorics resolved without assuming SM matrix element in production
 resulting top quark mass immune to possible contaminations from New Physics in production of top quarks
- can provide precision competitive with more conventional methods, especially using 3000/fb at LHCI4


## Using energy-peak for searches

- if background is flat or peaks elsewhere from signal
- Stops (Low: I304.049I):
for $\tilde{t} \rightarrow b \tilde{\chi}_{1}^{+}$, peak in $E_{b}^{\text {lab }}$ at $\left(M_{\hat{t}}^{2}-M_{\tilde{\chi}_{1}^{+}}^{2}\right) /\left(2 M_{\hat{t}}\right) \ldots$
can be $\gg\left(M_{t}^{2}-M_{W}^{2}\right) /\left(2 M_{t}\right)$ from $t \bar{t}$ background (from SM or from $\left.\tilde{t} \rightarrow t \tilde{\chi}_{1}^{0}\right)$

