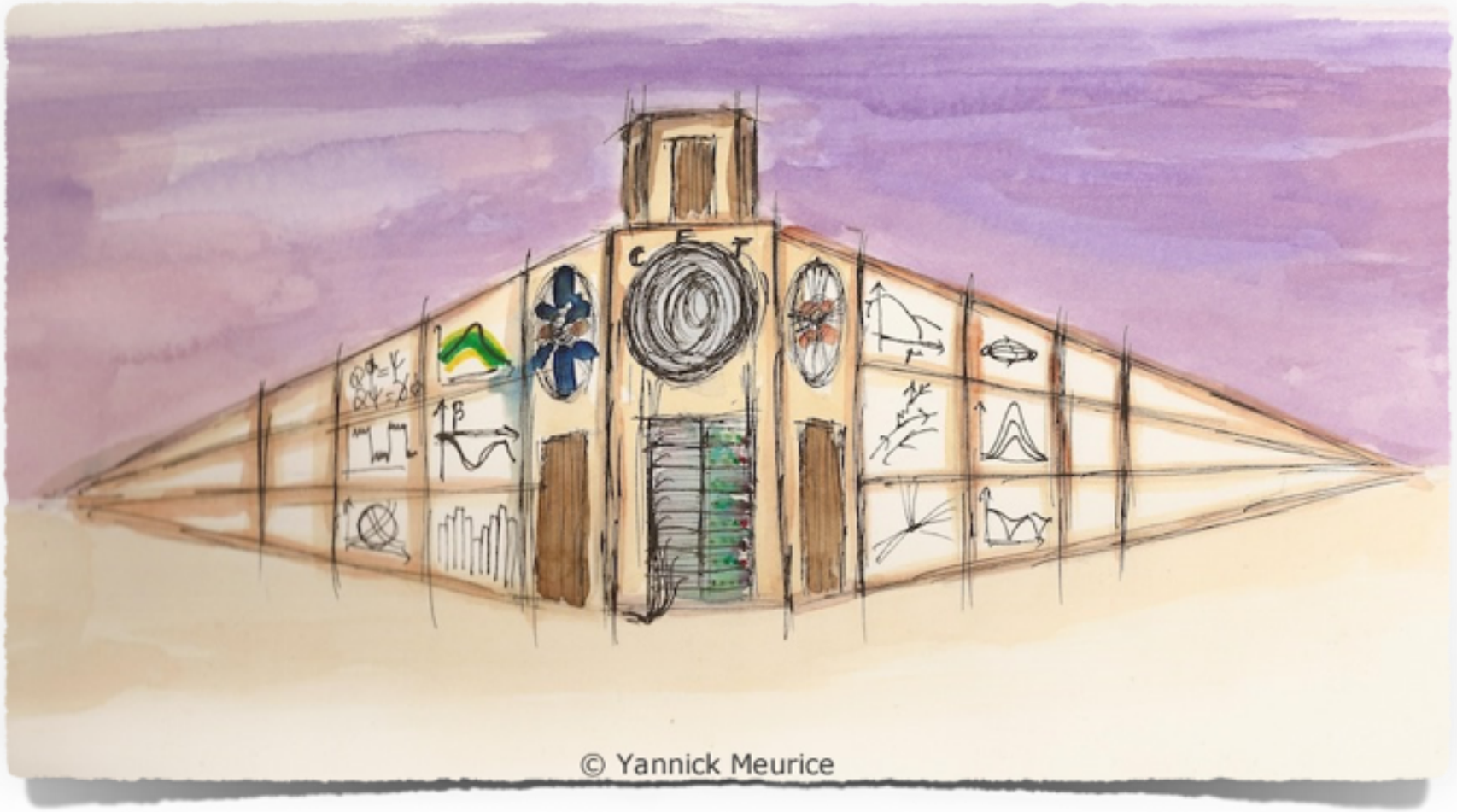

Exclusive decays of heavy baryons

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Work in collaboration with Christoph Lehner, David Lin, **Stefan Meinel**, Matt Wingate

FCNC decays: $\Lambda_b \to \Lambda\, \mu^+ \mu^-$

[Detmold, Lin, Meinel, & Wingate Phys. Rev. D 87, 074502 (2013)]

$|V_{ub}/V_{cb}|$: $\Lambda_b \to p\, \mu^- \bar{\nu}$ and $\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}$

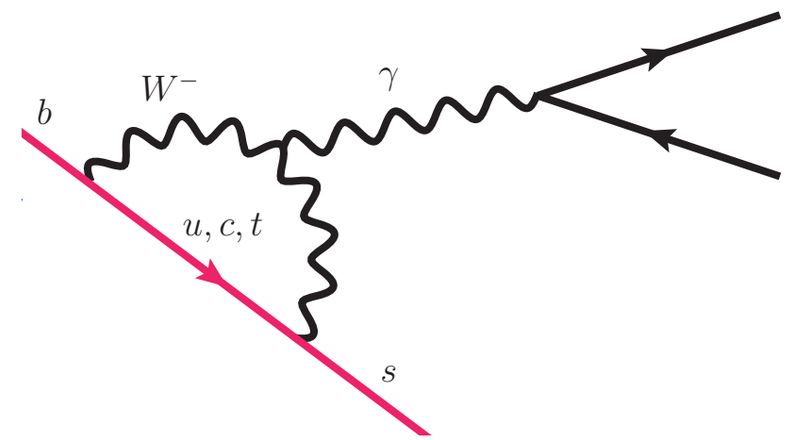
[Detmold, Lin, Meinel, & Wingate PRD 88 (2013) 014512]
[Detmold, Lehner,Meinel PRD 92 (2015) 034503]

FCNC decays: $\Lambda_b \to \Lambda\, \mu^+ \mu^-$

[Detmold, Lin, Meinel, & Wingate Phys. Rev. D 87, 074502 (2013)]

Flavour-changing neutral currents

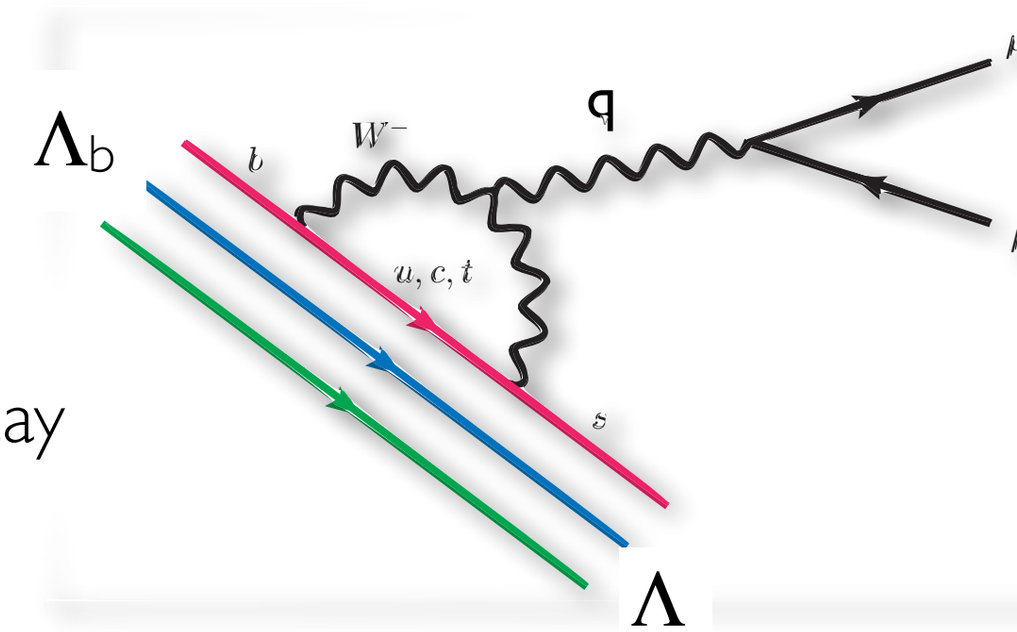
- Flavour changing neutral currents are absent in the SM at tree level
  - First occur at loop level and are generally GIM suppressed
  - Small size allows sensitivity to possible BSM contributions which may be of similar size



- Well studied in B $\rightarrow$ K decays and also more recently in studies of B $\rightarrow$ K*
  - Somewhat interesting hints for deviations from SM

Flavour-changing neutral currents

- Baryon decay modes $\Lambda_b \to \Lambda\,\gamma$, $\Lambda_b \to \Lambda\, l^+ l^-$ depend on polarisation of Λ_b and Λ so many angular observables possible
  - In principle different sensitivities to BSM physics [Mannel & Recksiegel 1997]



- Final state undergoes further weak decay $\Lambda \to$ p which is self-analysing

$$\frac{dN}{d\Omega}[\Lambda \to p\pi] \sim (1 + a\vec{s}_\Lambda \cdot \vec{p}_p), \qquad a = 0.64(1)$$

- At LHC, Λ_b is produced almost unpolarised [Aaij 1302.5578]
- First observation of baryonic decay at CDF [2012]
- LHCb Run I results published recently [LHCb JHEP 06 (2015) 115]

Effective Hamiltonian

- At hadronic scales the relevant interactions are described by the effective Hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1,\ldots,10,S,P} (C_i O_i + C_i' O_i'),$$

where the relevant b $\longrightarrow$ s operators are

$$\begin{aligned}
O_7 &= \frac{e}{16\pi^2} m_b\, \bar{s}\sigma^{\mu\nu} P_R b\, F_{\mu\nu}^{(\rm e.m.)}, & O_7' &= \frac{e}{16\pi^2} m_b\, \bar{s}\sigma^{\mu\nu} P_L b\, F_{\mu\nu}^{(\rm e.m.)}, \\
O_9 &= \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_L b\, \bar{l}\gamma_\mu l, & O_9' &= \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_R b\, \bar{l}\gamma_\mu l, \\
O_{10} &= \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_L b\, \bar{l}\gamma_\mu\gamma_5 l, & O_{10}' &= \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_R b\, \bar{l}\gamma_\mu\gamma_5 l, \\
O_S &= \frac{e^2}{16\pi^2} m_b\, \bar{s} P_R b\, \bar{l} l, & O_S' &= \frac{e^2}{16\pi^2} m_b\, \bar{s} P_L b\, \bar{l} l, \\
O_P &= \frac{e^2}{16\pi^2} m_b\, \bar{s} P_R b\, \bar{l}\gamma_5 l, & O_P' &= \frac{e^2}{16\pi^2} m_b\, \bar{s} P_L b\, \bar{l}\gamma_5 l,
\end{aligned}$$

C_i, C_i' are Wilson coefficients containing short distance physics

$\Lambda_b \to \Lambda\, \mu^+ \mu^-$

- Decay amplitude determined by matrix elements of H_{eff}

$$\mathcal{M} = -\langle \Lambda(p', s')\, \ell^+(p_+, s_+)\, \ell^-(p_-, s_-) | \mathcal{H}_{\rm eff} | \Lambda_b(p, s) \rangle$$

- Hadronic part determined by $\Lambda_b \to \Lambda$ form factors
  - In general, 10 form factors contribute
  - In static limit ($m_b \to \infty$), only two FFs ($F_{1,2}$) survive

$$\langle \Lambda(p', s')|\, \bar{s}\Gamma Q\, |\Lambda_Q(v, 0, s)\rangle = \overline{u}(p', s')\, [F_1(p' \cdot v) + v\, F_2(p' \cdot v)]\, \Gamma\, \mathcal{U}(v, s)$$

where v=4-velocity of Λ_b and the FFs are independent of the choice of Dirac matrix Γ and we will use the basis

$$F_\pm = F_1 \pm F_2$$

- Calculating FFs requires lattice QCD

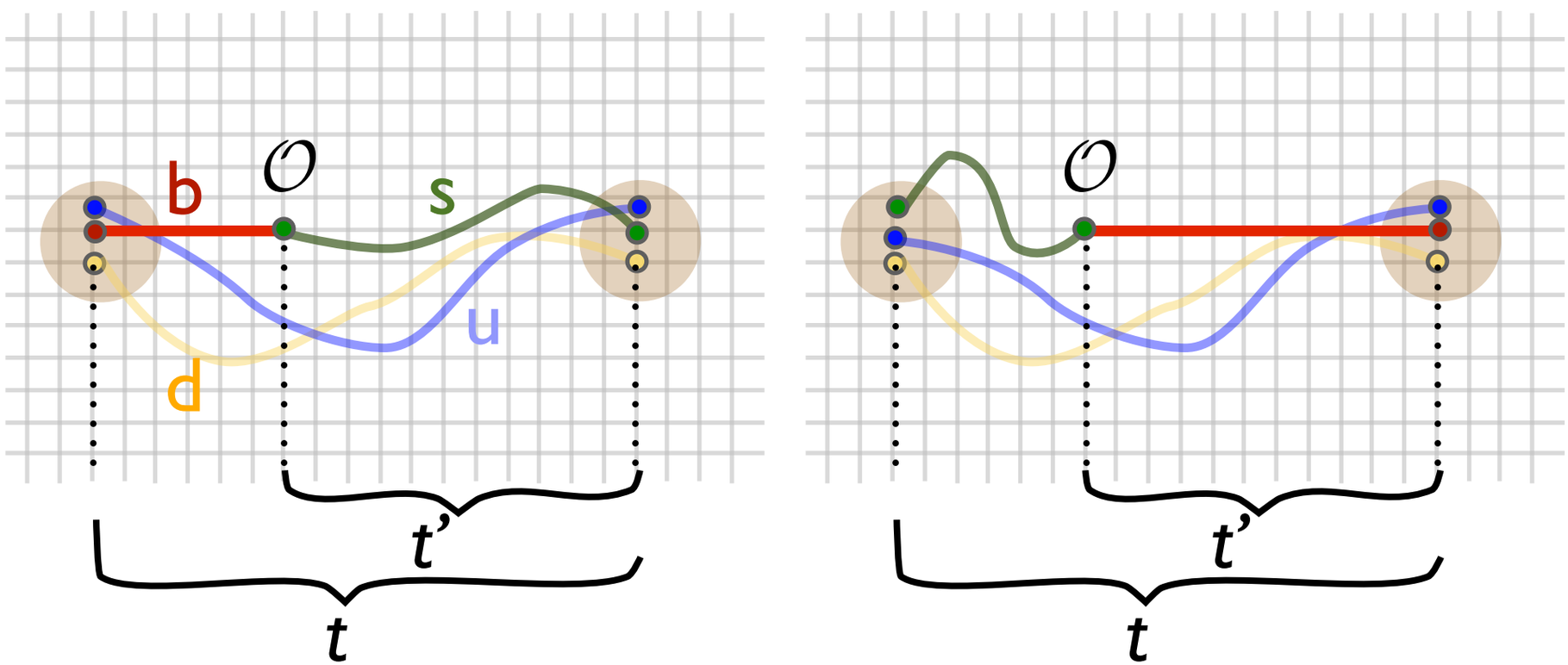
Anatomy of the QCD calculation

- Gluon configurations from RBC/UKQCD collaborations [Aoki et al. 2011]
  - Two lattice spacings with a single large volume
- Light and strange quarks: domain wall fermions with multiple quark masses (some partially quenched)
- b quarks: HQET static action [Eichten-Hill] with HYP-smearing

Set	$N_s^3 \times N_t \times N_5$	am_5	$am_s^{(\mathrm{sea})}$	$am_{u,d}^{(\mathrm{sea})}$	a (fm)	$am_s^{(\mathrm{val})}$	$am_{u,d}^{(\mathrm{val})}$	$m_\pi^{(\mathrm{vv})}$ (MeV)	$m_{\eta_s}^{(\mathrm{vv})}$ (MeV)	N_{meas}
C14	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.04	0.001	245(4)	761(12)	2705
C24	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.04	0.002	270(4)	761(12)	2683
C54	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.04	0.005	336(5)	761(12)	2780
C53	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.03	0.005	336(5)	665(10)	1192
F23	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.03	0.002	227(3)	747(10)	1918
F43	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.03	0.004	295(4)	747(10)	1919
F63	$32^3 \times 64 \times 16$	1.8	0.03	0.006	0.0848(17)	0.03	0.006	352(7)	749(14)	2785

Correlation functions

- Matrix elements extracted from ratios of two and three- point correlation functions
  - Two-point functions for Λ_b and Λ are standard
  - Forward and backward three-point functions



Correlation functions

- Matrix elements extracted from ratios of two and three- point correlation functions
  - Two-point functions for Λ_b and Λ are standard
  - Forward and backward three-point functions

$$C^{(3)}_{\delta\alpha}(\Gamma,\ \mathbf{p}',t,t') = \sum_{\mathbf{y}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} \left\langle \Lambda_\delta(x_0,\mathbf{x})\ \ J^{(\mathrm{HQET})\dagger}_\Gamma(x_0-t+t',\mathbf{y})\ \ \overline{\Lambda}_{Q\alpha}(x_0-t,\mathbf{y}) \right\rangle$$

$$C^{(3,\mathrm{bw})}_{\alpha\delta}(\Gamma,\ \mathbf{p}',t,t-t') = \sum_{\mathbf{y}} e^{-i\mathbf{p}'\cdot(\mathbf{y}-\mathbf{x})} \left\langle \Lambda_{Q\alpha}(x_0+t,\mathbf{y})\ \ J^{(\mathrm{HQET})}_\Gamma(x_0+t',\mathbf{y})\ \ \overline{\Lambda}_\delta(x_0,\mathbf{x}) \right\rangle$$

- NB: some technicalities in matching QCD current to HQET
- Spectral decomposition (ellipsis ~ excited states):

$$C^{(3)}_{\delta\alpha}(\Gamma,\ \mathbf{p}',t,t') = Z_{\Lambda_Q}\ \frac{1}{2E_\Lambda}\frac{1}{2}\ e^{-E_\Lambda(t-t')}\ e^{-E_{\Lambda_Q}t'}\ \Big[(Z^{(1)}_\Lambda + Z^{(2)}_\Lambda\gamma^0)(m_\Lambda + \not{p}')\big(F_1+\gamma^0F_2\big)\,\Gamma\,(1+\gamma^0)\Big]_{\delta\alpha} + \ldots$$

Correlator ratios

- Form ratios of correlators to cancel energy and time dependence for ground-state contribution

$$\mathcal{R}(\Gamma, \mathbf{p}', t, t') = \frac{4\,\mathrm{Tr}\left[C^{(3)}(\Gamma,\ \mathbf{p}', t, t')\ C^{(3,\mathrm{bw})}(\Gamma,\ \mathbf{p}', t, t-t')\right]}{\mathrm{Tr}[C^{(2,\Lambda,\mathrm{av})}(\mathbf{p}', t)]\ \mathrm{Tr}[C^{(2,\Lambda_Q,\mathrm{av})}(t)]}$$

- Combine for different Dirac structures

$$\mathcal{R}_+(\mathbf{p}', t, t') = \frac{1}{4}\left[\mathcal{R}(1, \mathbf{p}', t, t') + \mathcal{R}(\gamma^2\gamma^3, \mathbf{p}', t, t') + \mathcal{R}(\gamma^3\gamma^1, \mathbf{p}', t, t') + \mathcal{R}(\gamma^1\gamma^2, \mathbf{p}', t, t')\right]$$

$$\mathcal{R}_-(\mathbf{p}', t, t') = \frac{1}{4}\left[\mathcal{R}(\gamma^1, \mathbf{p}', t, t') + \mathcal{R}(\gamma^2, \mathbf{p}', t, t') + \mathcal{R}(\gamma^3, \mathbf{p}', t, t') + \mathcal{R}(\gamma_5, \mathbf{p}', t, t')\right]$$

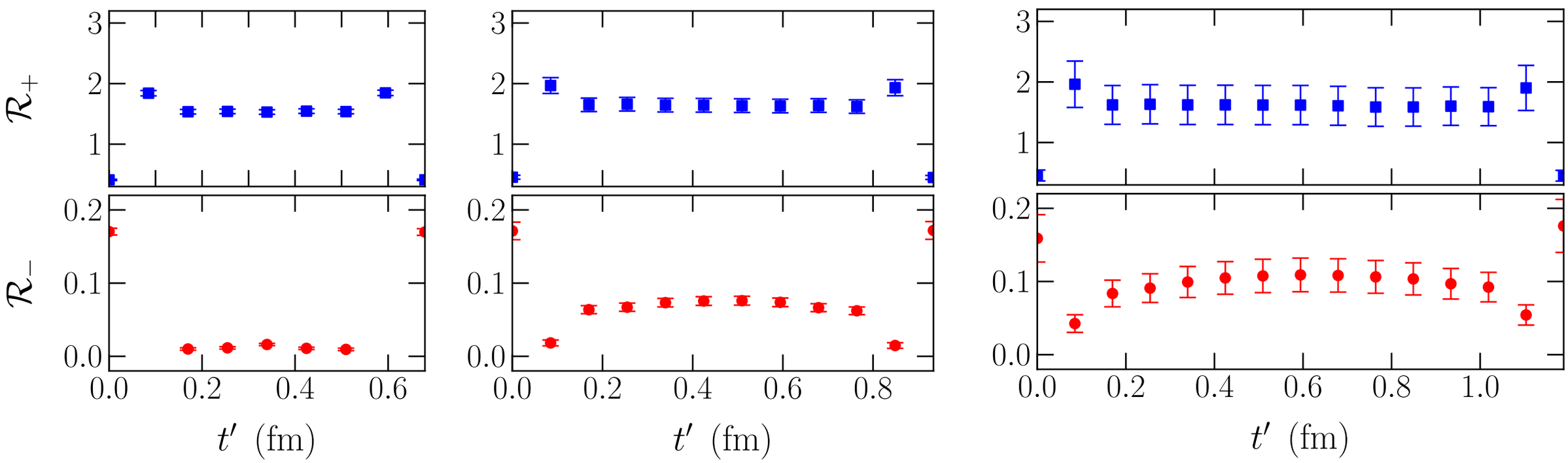
- Determine form factors (up to exponential contamination)

$$R_+(|\mathbf{p}'|^2, t) = \sqrt{\frac{E_\Lambda}{E_\Lambda + m_\Lambda}\mathcal{R}_+(|\mathbf{p}'|^2,\ t,\ t/2)} \overset{t\to\infty}{\longrightarrow} F_+(v\cdot p) + \ldots$$

$$R_-(|\mathbf{p}'|^2, t) = \sqrt{\frac{E_\Lambda}{E_\Lambda - m_\Lambda}\mathcal{R}_-(|\mathbf{p}'|^2,\ t,\ t/2)} \overset{t\to\infty}{\longrightarrow} F_-(v\cdot p) + \ldots$$

Form factor extractions

- Ratios are relatively insensitive to operator insertion time
  - Take midpoint to reduce excited state



- Strongly dependent on source-sink separation

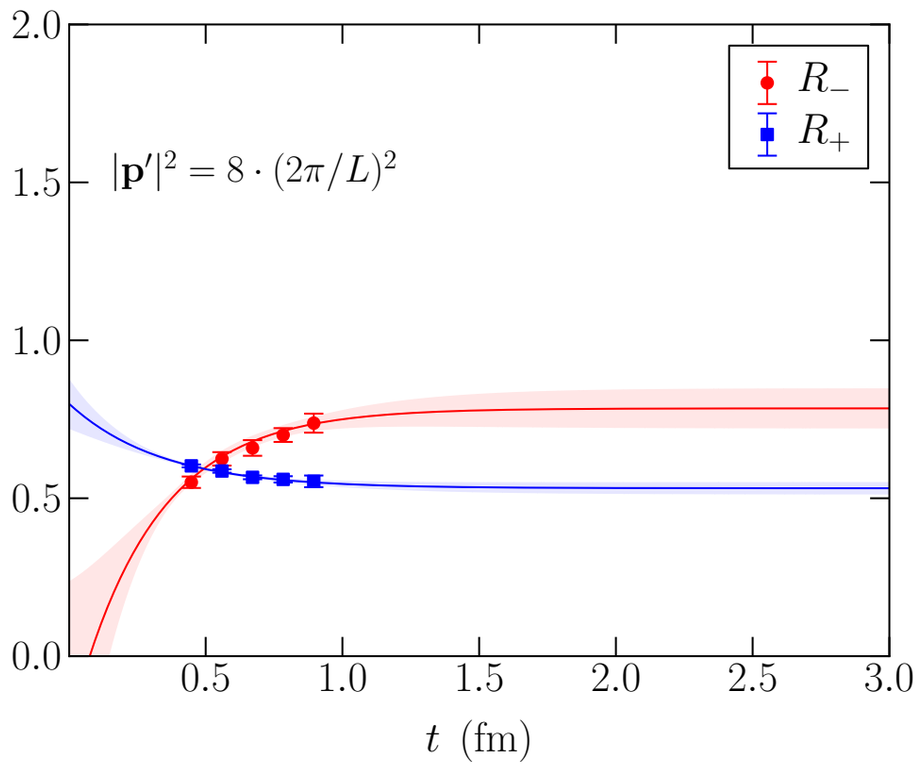
Source sink separation

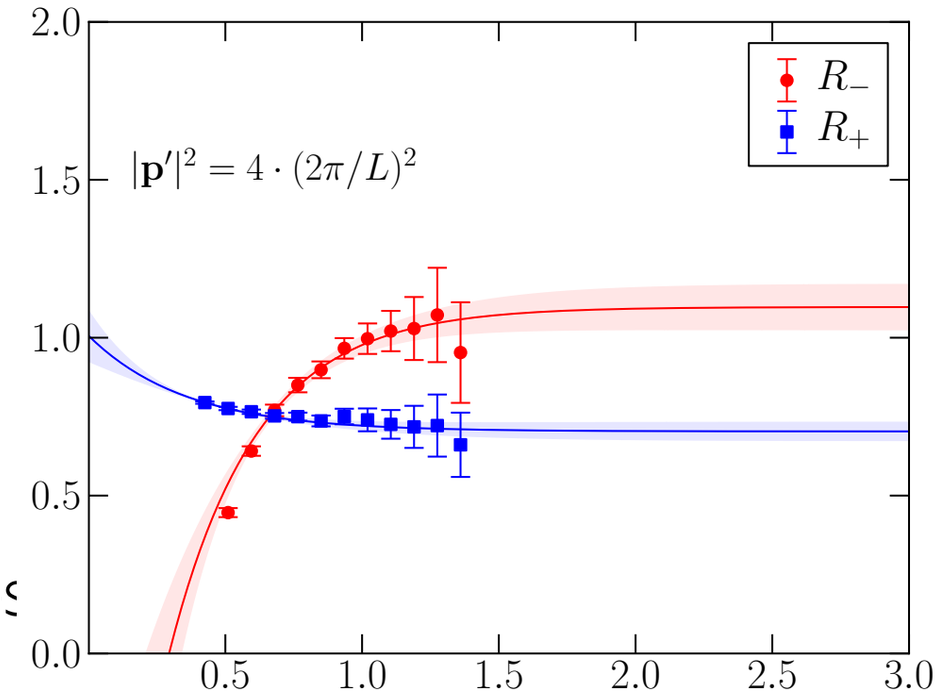
- Extrapolate to infinite source-sink separation to extract ground state matrix elements
  - Allow for single exponential contamination

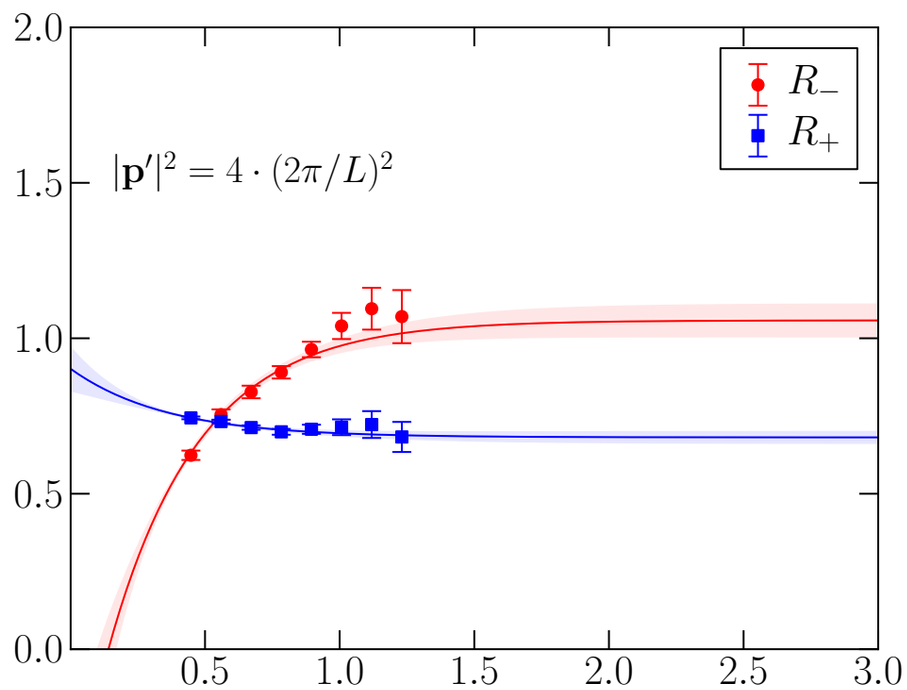
$$R_{\pm}^{i,n}(t) = F_{\pm}^{i,n} + A_{\pm}^{i,n}\ \exp[-\delta^{i,n}\ t]$$

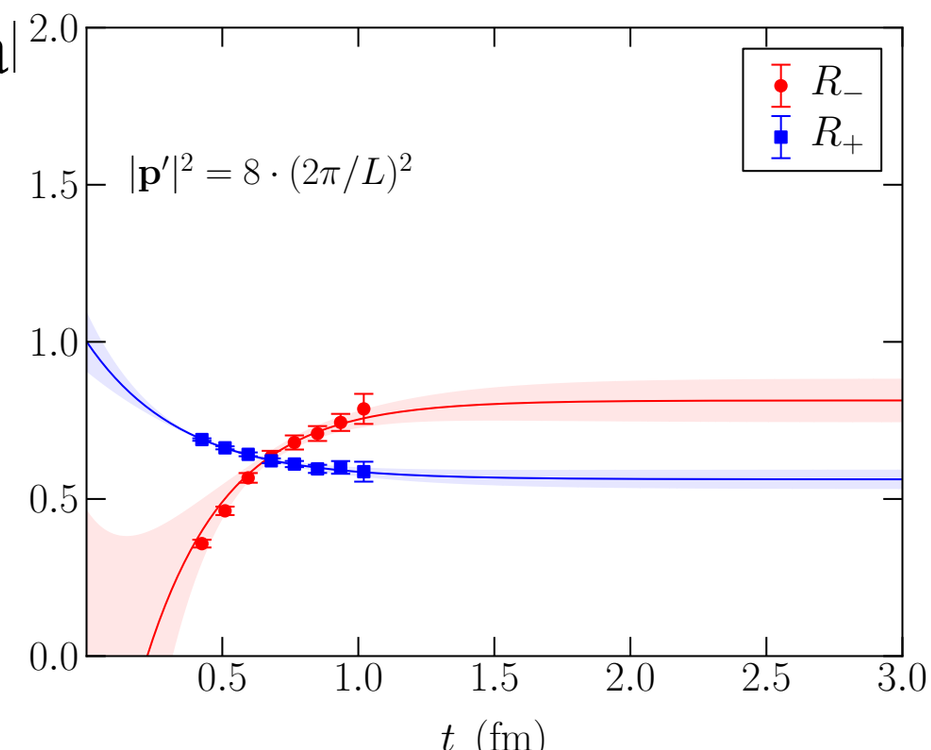
  - Constrain energy gap to be positive and to be similar between the fits to the different ensembles
- Systematic fitting uncertainty assessed by adding a second exponential contamination and by dropping data at short *t*

Source sink separation

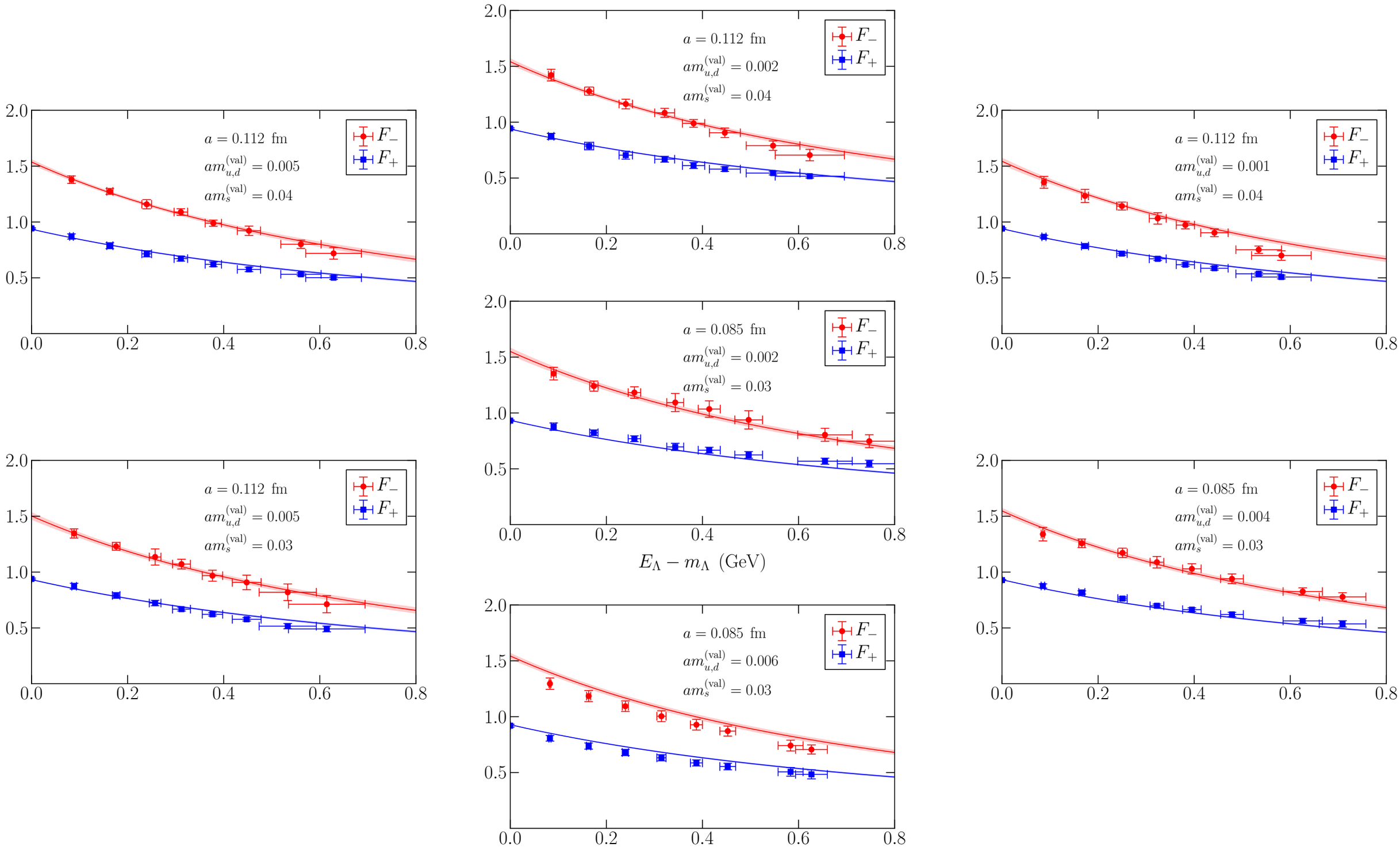








Form factors



Extrapolation of form factors

- Form factors extracted at non-zero lattice spacing, unphysical quark masses and for a limited range of momenta
- Coupled extrapolations performed using the form

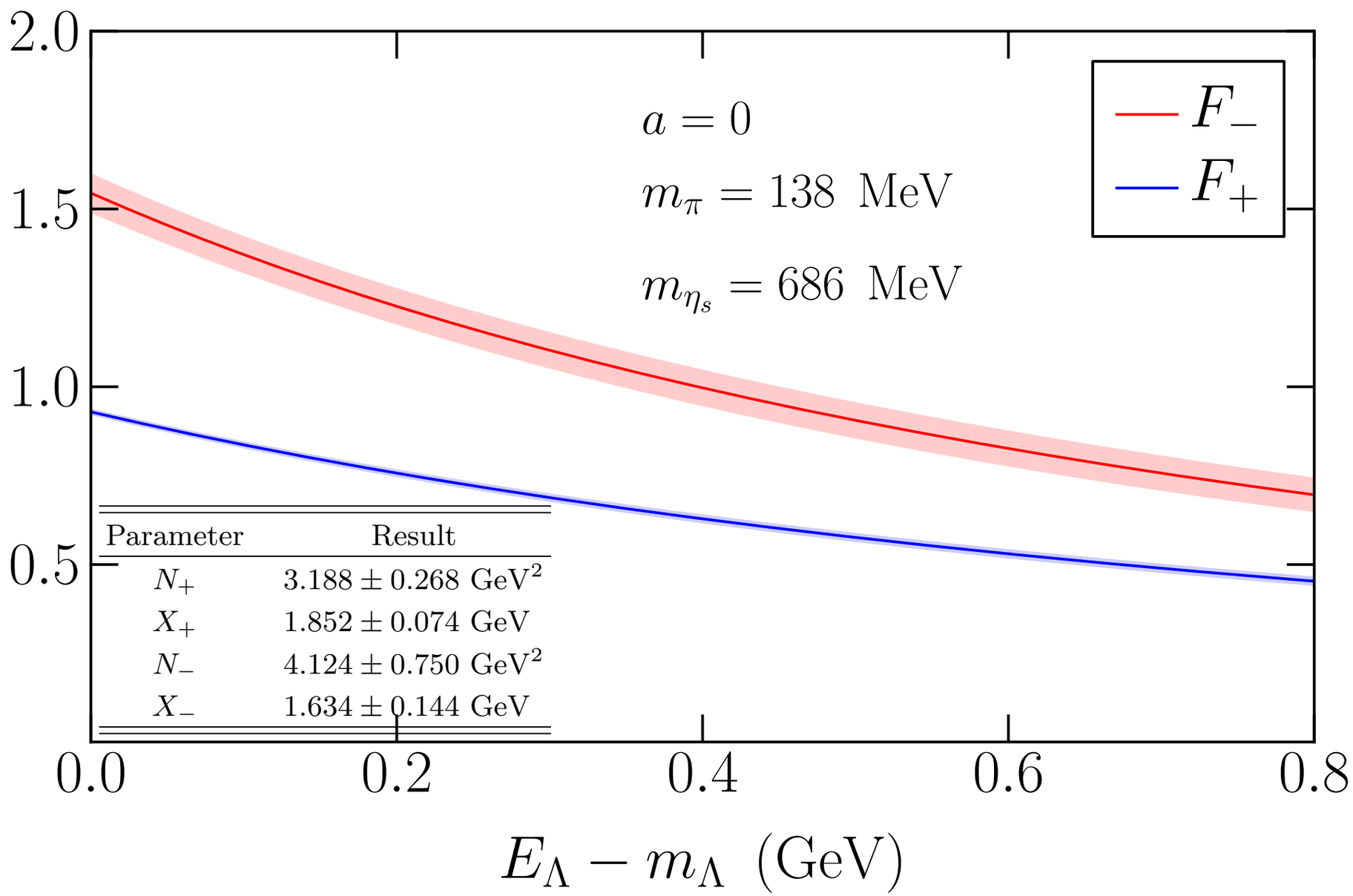
$$F_{\pm}^{i,n} = \frac{N_{\pm}}{(X_{\pm}^{i} + E_{\Lambda}^{i,n} - m_{\Lambda}^{i})^2} \cdot [1 + d_{\pm}(a^i E_{\Lambda}^{i,n})^2]$$

with $X_{\pm}^{i} = X_{\pm} + c_{l,\pm} \cdot \left[(m_{\pi}^{i})^2 - (m_{\pi}^{\mathrm{phys}})^2\right] + c_{s,\pm} \cdot \left[(m_{\eta_s}^{i})^2 - (m_{\eta_s}^{\mathrm{phys}})^2\right]$

  - Simple modified dipole form
    - Necessarily phenomenological (momenta of Λ beyond range of applicability of χPT)
    - Lattice spacing and light and strange quark mass dependence through c's and d's

Form factors

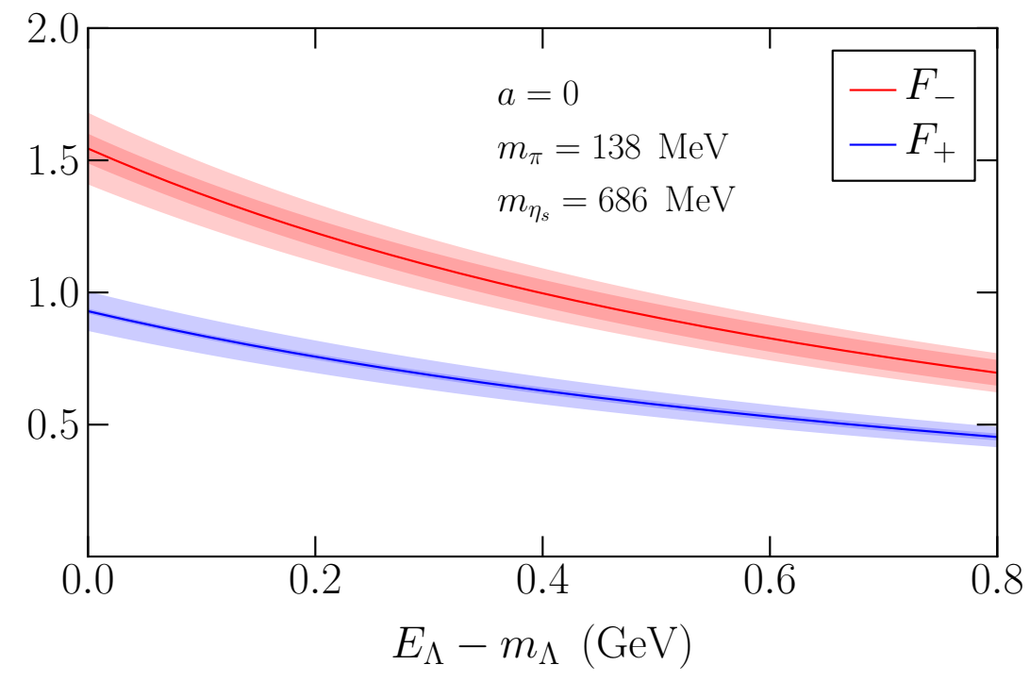
- Fit has χ^2/dof < 1 and fitted lattice spacing and quark mass parameters consistent with zero

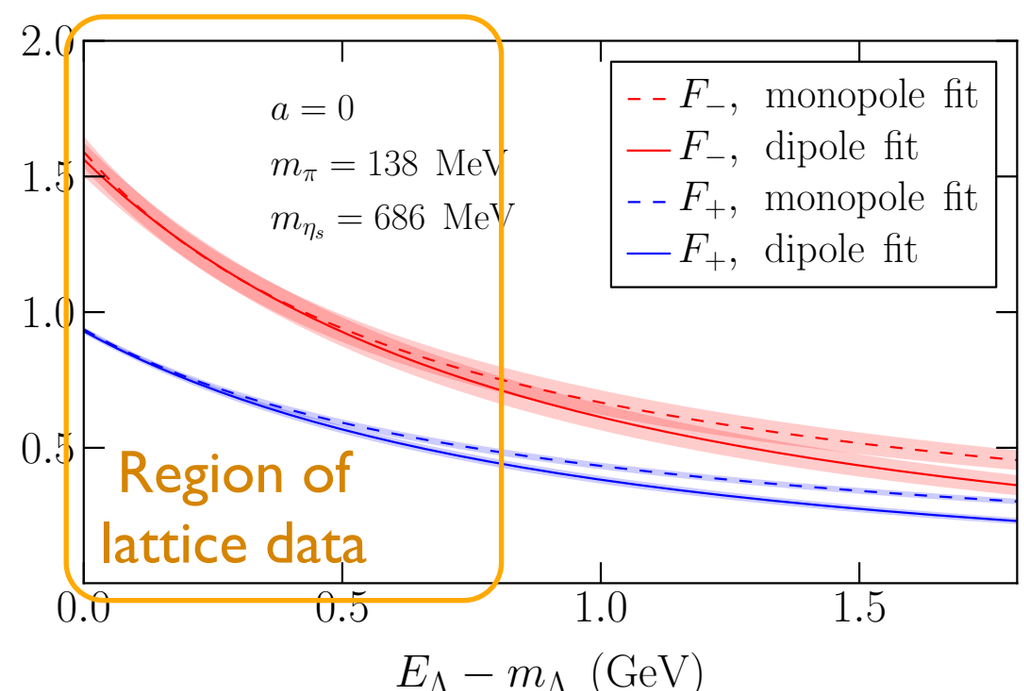


Parameter	Result
N_+	3.188 ± 0.268 GeV^2
X_+	1.852 ± 0.074 GeV
N_-	4.124 ± 0.750 GeV^2
X_-	1.634 ± 0.144 GeV

Systematic Uncertainties

- Main sources of systematic uncertainty in FFs are
  - Higher order effects in renormalisation of currents ~6%
  - Finite volume ~3%
  - Chiral extrapolation ~5%
  - Residual discretisation effects ~4%
- Extrapolation functional form
  - Dipole vs monopole vs ...
  - Agree in data region
    Uncertainty hard to quantify





Differential branching fraction

- Taking SM Wilson coefficients from the literature we can compute the SM decay rate

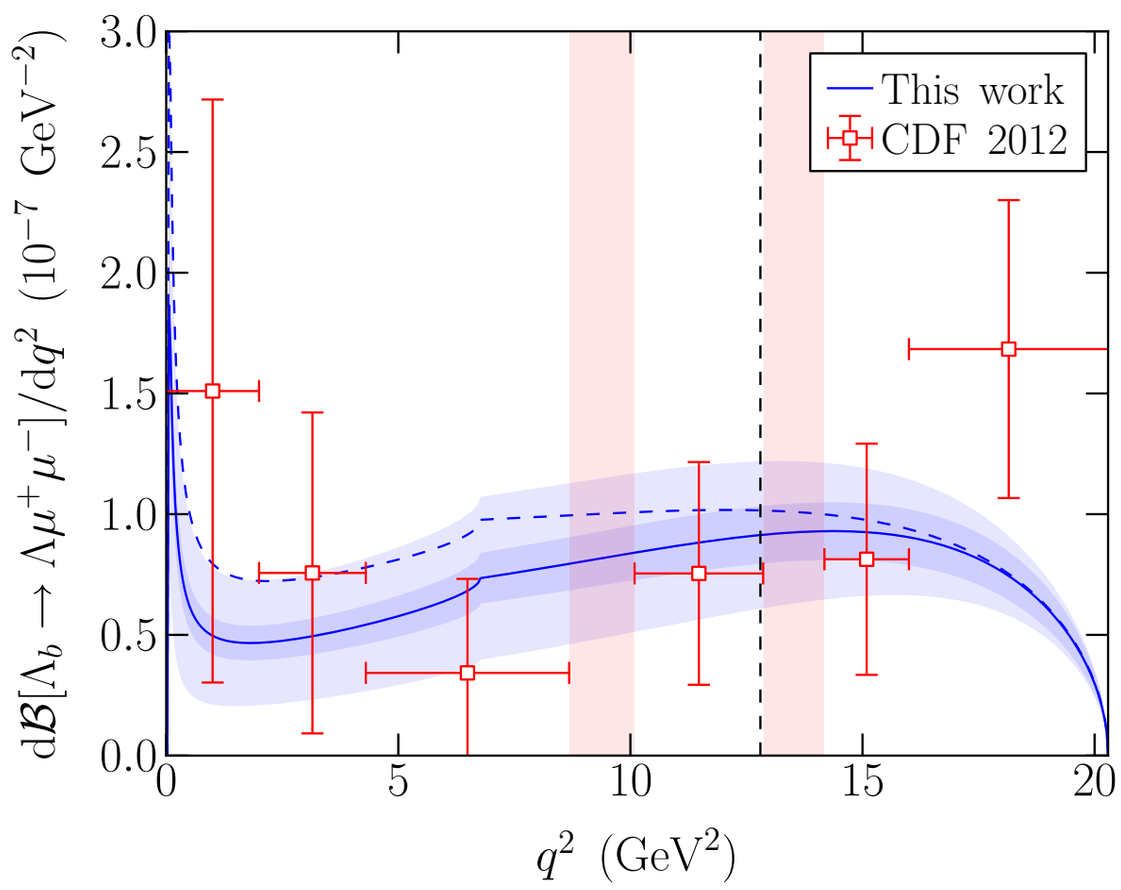
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{\alpha_{\mathrm{em}}^2 G_F^2 |V_{tb}V_{ts}^*|^2}{6144\,\pi^5\,q^4\,m_{\Lambda_b}^5}\sqrt{1-\frac{4m_l^2}{q^2}}\sqrt{((m_{\Lambda_b}-m_\Lambda)^2-q^2)((m_{\Lambda_b}+m_\Lambda)^2-q^2)}$$
$$\times\Big[q^2|C_{10,\mathrm{eff}}|^2\mathcal{A}_{10,10}+16c_\sigma^2 m_b^2(q^2+2m_l^2)|C_{7,\mathrm{eff}}|^2\mathcal{A}_{7,7}+q^2(q^2+2m_l^2)|C_{9,\mathrm{eff}}(q^2)|^2\mathcal{A}_{9,9}$$
$$+8q^2c_\sigma m_b(q^2+2m_l^2)m_{\Lambda_b}\Re[C_{7,\mathrm{eff}}\,C_{9,\mathrm{eff}}(q^2)]\mathcal{A}_{7,9}\Big],$$

$$\mathcal{A}_{10,10} = \Big[\left(2c_\gamma^2+2c_\gamma c_v+c_v^2\right)\left(2m_l^2+q^2\right)\left(m_{\Lambda_b}^4-2m_{\Lambda_b}^2m_\Lambda^2+\left(q^2-m_\Lambda^2\right)^2\right)$$
$$+2m_{\Lambda_b}^2q^2\left(4c_\gamma^2\left(q^2-4m_l^2\right)-\left(2c_\gamma c_v+c_v^2\right)\left(q^2-10m_l^2\right)\right)\Big]\mathcal{F}+4c_\gamma\left(c_\gamma+c_v\right)\left(2m_l^2+q^2\right)\mathcal{G}F_+F_-,$$
$$\mathcal{A}_{7,7} = \left(m_{\Lambda_b}^4+m_{\Lambda_b}^2\left(q^2-2m_\Lambda^2\right)+\left(q^2-m_\Lambda^2\right)^2\right)\mathcal{F}+2\mathcal{G}F_+F_-,$$
$$\mathcal{A}_{9,9} = \Big[\left(2c_\gamma^2+2c_\gamma c_v+c_v^2\right)\left(m_{\Lambda_b}^4+\left(q^2-m_\Lambda^2\right)^2\right)-2m_{\Lambda_b}^2\left(2c_\gamma^2\left(m_\Lambda^2-2q^2\right)+\left(2c_\gamma c_v+c_v^2\right)\left(m_\Lambda^2+q^2\right)\right)\Big]\mathcal{F}$$
$$+4c_\gamma\left(c_\gamma+c_v\right)\mathcal{G}F_+F_-,$$
$$\mathcal{A}_{7,9} = 3c_\gamma\left(m_{\Lambda_b}^2-m_\Lambda^2+q^2\right)\mathcal{F}\;+2\left(3c_\gamma+c_v\right)\left(m_\Lambda^4-2m_\Lambda^2\left(m_{\Lambda_b}^2+q^2\right)+\left(q^2-m_{\Lambda_b}^2\right)^2\right)F_+F_-,$$

$$\mathcal{F} = ((m_{\Lambda_b}-m_\Lambda)^2-q^2)F_-^2+((m_{\Lambda_b}+m_\Lambda)^2-q^2)F_+^2,$$
$$\mathcal{G} = m_{\Lambda_b}^6-m_{\Lambda_b}^4\left(3m_\Lambda^2+q^2\right)-m_{\Lambda_b}^2\left(q^2-m_\Lambda^2\right)\left(3m_\Lambda^2+q^2\right)+\left(q^2-m_\Lambda^2\right)^3$$

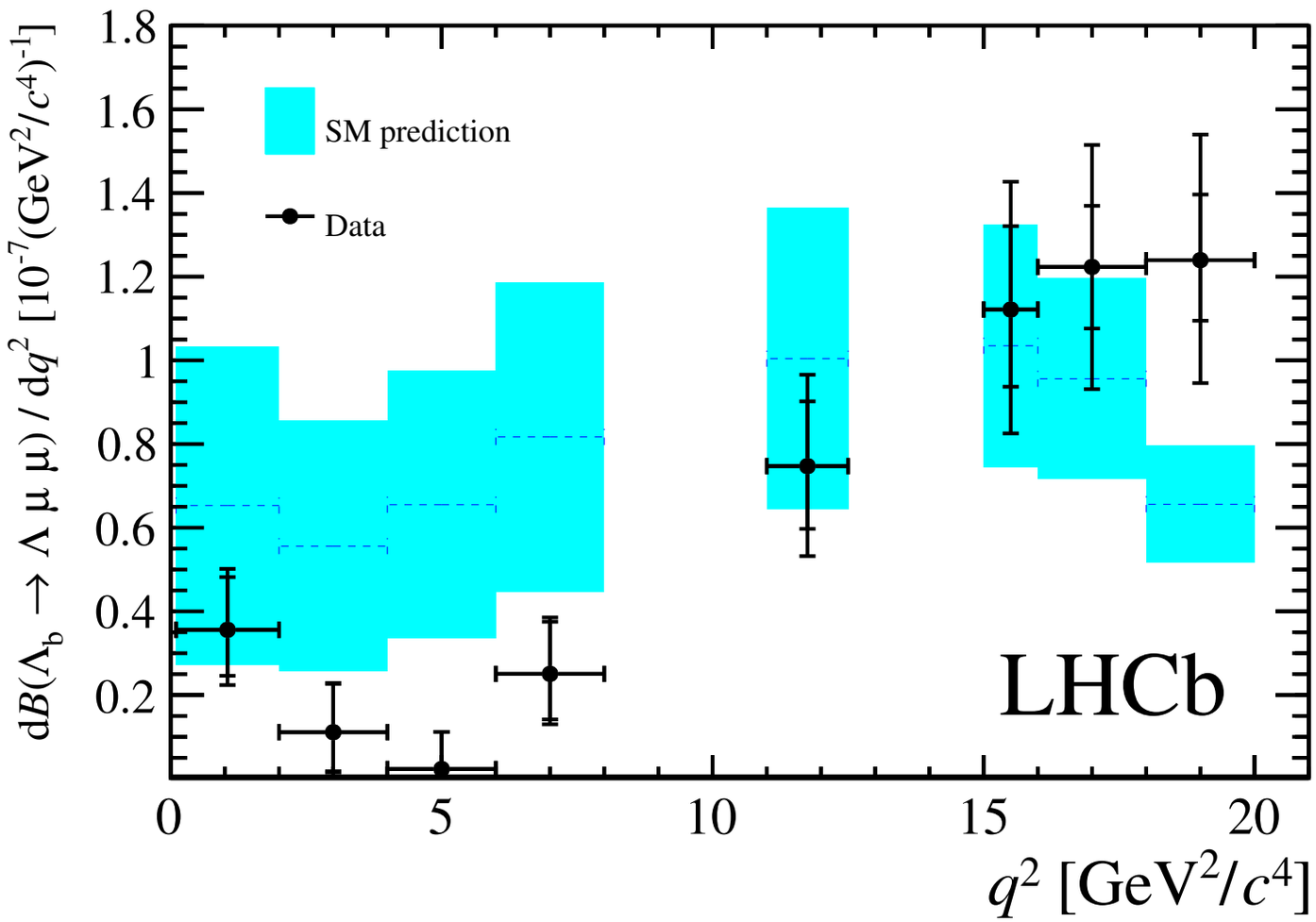
Differential branching fraction

- Evaluate using lattice FFs
- Additional systematic uncertainty from using static limit FFs taken as $\sqrt{|\vec{p}|^2 + \Lambda_{\mathrm{QCD}}^2} \Big/ m_b$
- Comparison to CDF measurements



Differential branching fraction

- New LHCb data are much more precise



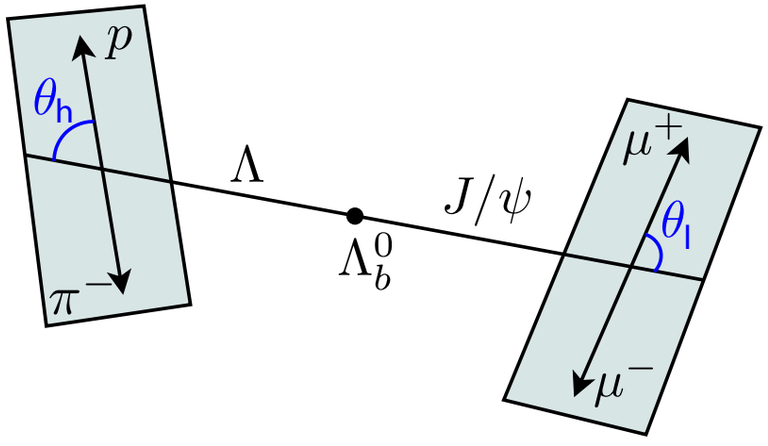
- LQCD calculation will improve soon (relativistic heavy quarks)

[LHCb JHEP 06 (2015) 115]

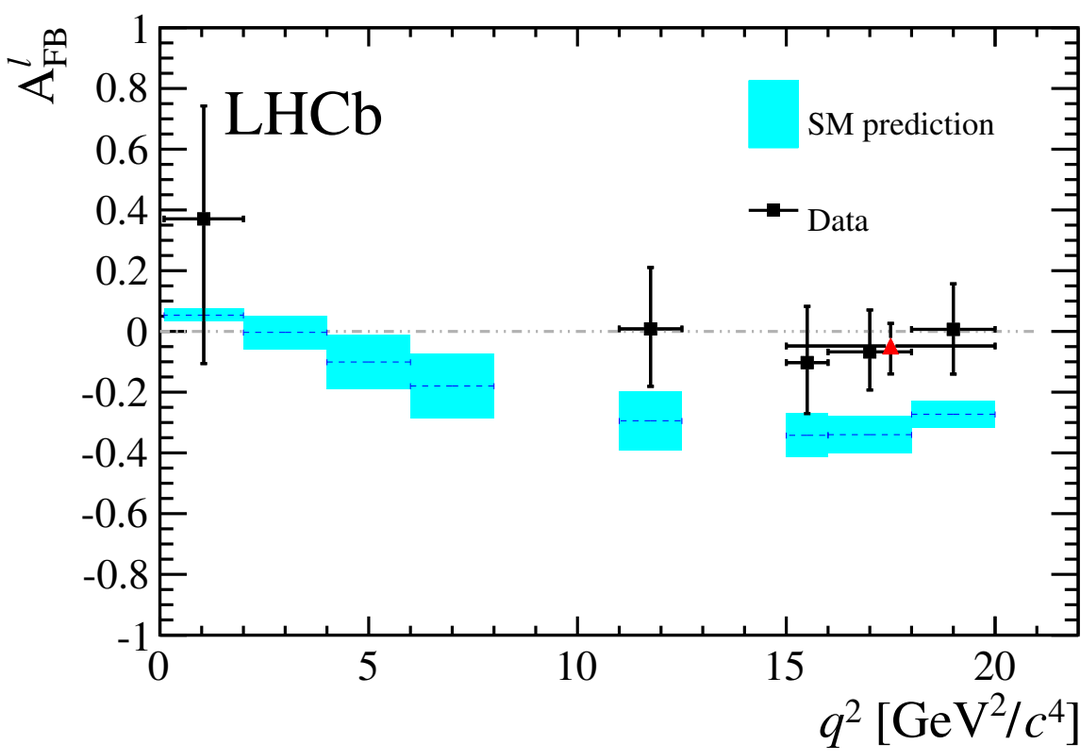
Asymmetries

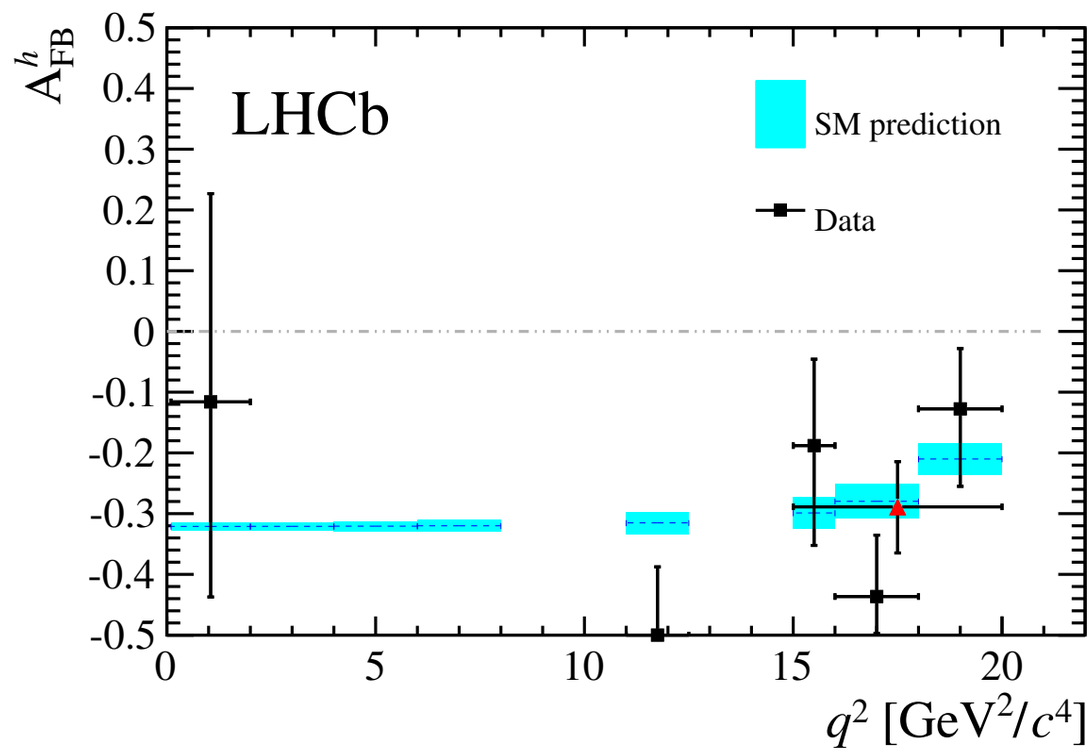
- Leptonic and Hadronic FB asymmetries

$$A^i_{\mathrm{FB}}(q^2) = \frac{\int_0^1 \frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_i}\mathrm{d}\cos\theta_i - \int_{-1}^0 \frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_i}\mathrm{d}\cos\theta_i}{\mathrm{d}\Gamma/\mathrm{d}q^2}$$



- Leptonic above SM in controlled region





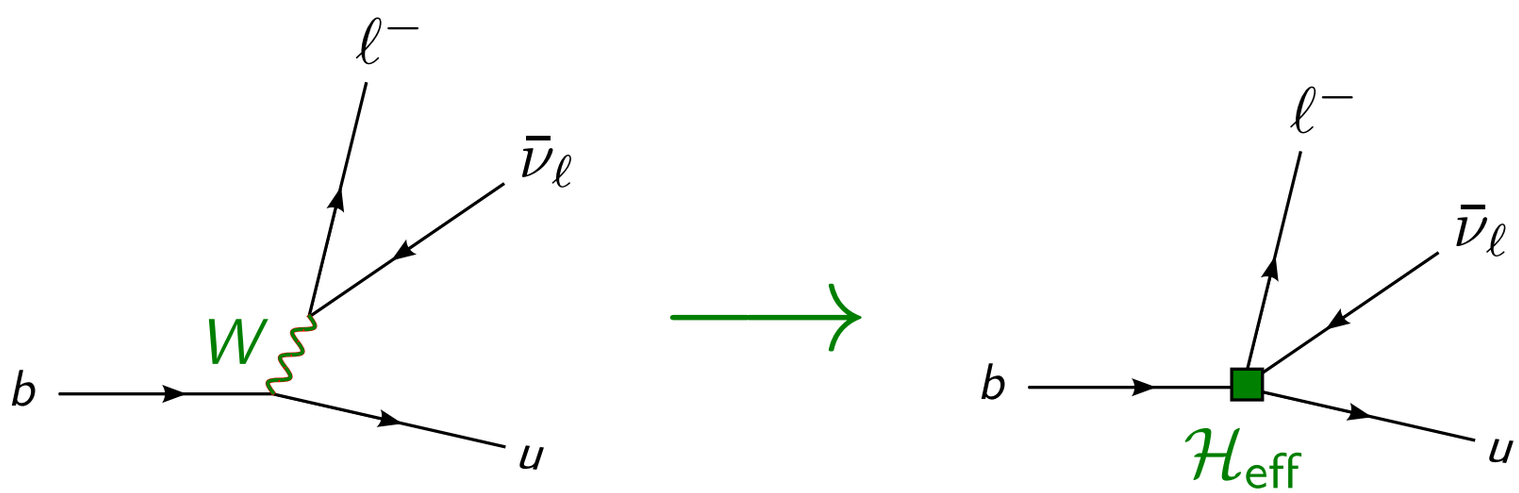
[LHCb JHEP 06 (2015) 115]

$|V_{ub}/V_{cb}|$: $\Lambda_b \to p\, \mu^- \bar{\nu}$ and $\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}$

[Detmold, Lin, Meinel, & Wingate PRD 88 (2013) 014512]
[Detmold, Lehner,Meinel PRD 92 (2015) 034503]

Inclusive vs exclusive V_{ub} & V_{cb}

- Long running tension between V_{ub} (and V_{cb}) extractions from inclusive $B \to X_u$ ($B \to X_c$) and exclusive decays $B \to \pi$ ($B \to D$)



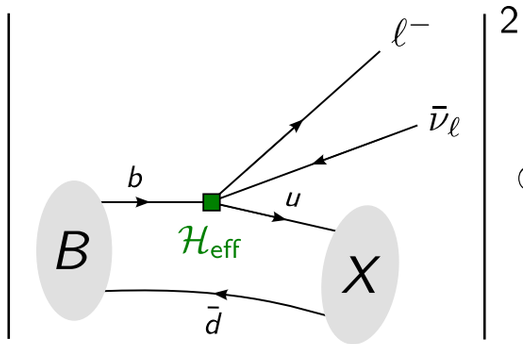
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} \underbrace{\bar{u}\gamma^\mu(1-\gamma_5)b}_{\equiv J^\mu} \; \bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

Inclusive vs exclusive V_{ub} & V_{cb}

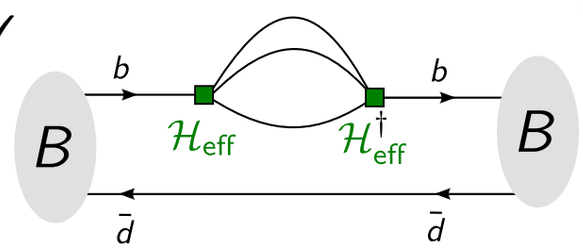
- Long running tension between V_{ub} (and V_{cb}) extractions from inclusive $B \to X_u$ ($B \to X_c$) and exclusive decays $B \to \pi$ ($B \to D$)

Inclusive

decay rate $\propto \sum_X$



$\propto \mathrm{Im}\Bigg($



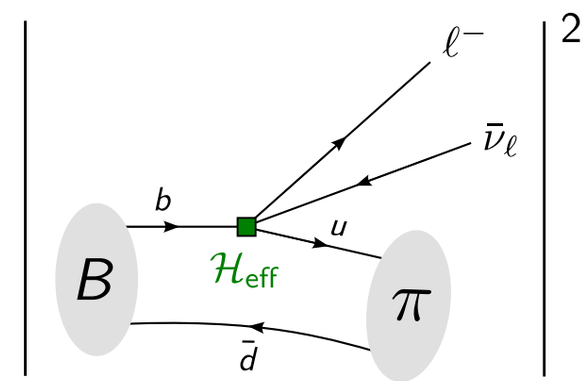
$\Bigg)$

$$\frac{d\Gamma}{dq^2 dE_\ell} \propto |V_{ub}|^2 (\ldots)_{\mu\nu}$$

$$\times\ \mathrm{Im}\Bigg(\underbrace{-i\int d^4x\, e^{-iq\cdot x}\, \langle B|\, \mathbf{T}\, J^{\mu\dagger}(x)\, J^\nu(0)\, |B\rangle}_{\text{OPE, HQET}}\Bigg)$$

Exclusive

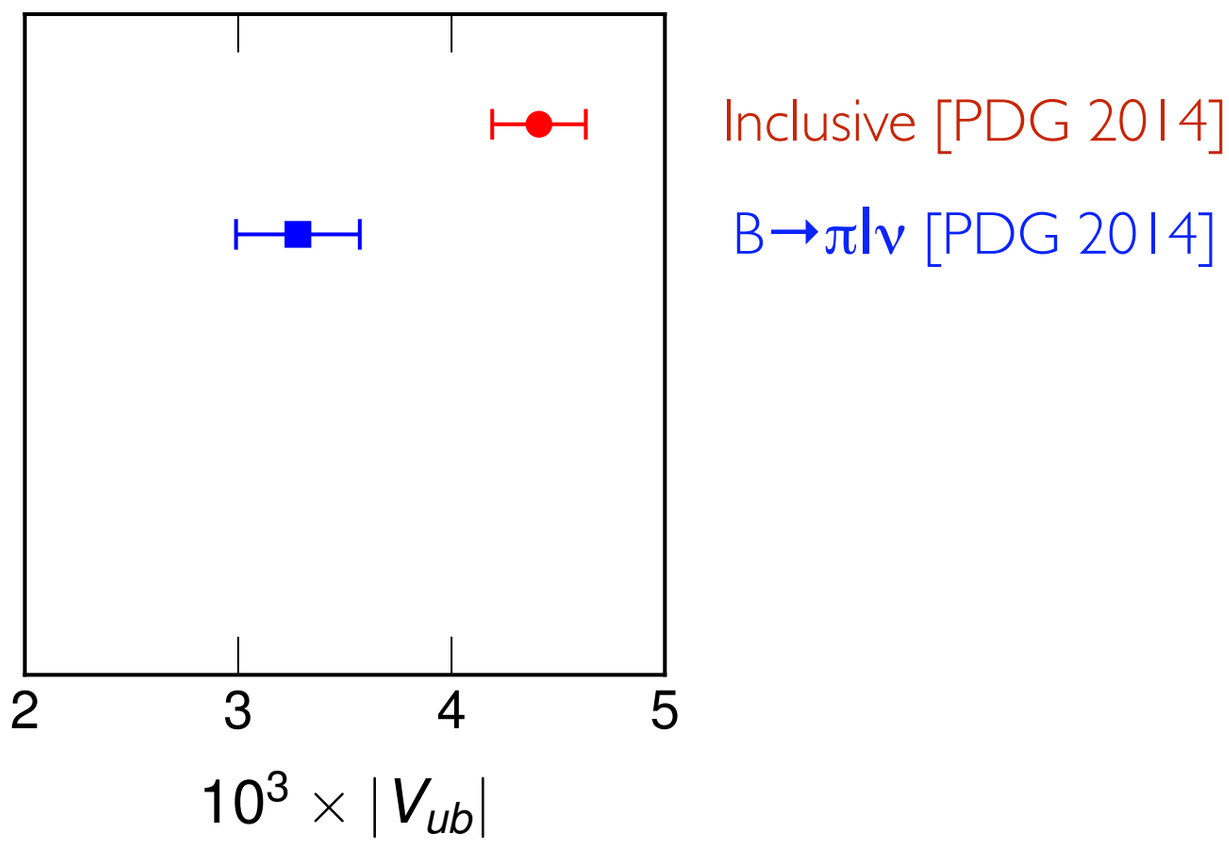
decay rate $\propto$



$$\frac{d\Gamma}{dq^2} \propto |V_{ub}|^2 \left|(\ldots)_\mu \underbrace{\langle \pi | J^\mu | B \rangle}_{\text{lattice QCD}}\right|^2$$

Inclusive vs exclusive V_{ub} & V_{cb}

- Long running tension between V_{ub} (and V_{cb}) extractions from inclusive $B \to X_u$ ($B \to X_c$) and exclusive decays $B \to \pi$ ($B \to D$)



Inclusive vs exclusive V_{ub} & V_{cb}

- Possible to reconcile through BSM scenarios that produce RH currents at low energy

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^L \left[(1+\epsilon_R)\bar{u}\gamma^\mu b - (1-\epsilon_R)\,\bar{u}\gamma^\mu\gamma_5 b\right]\,\bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

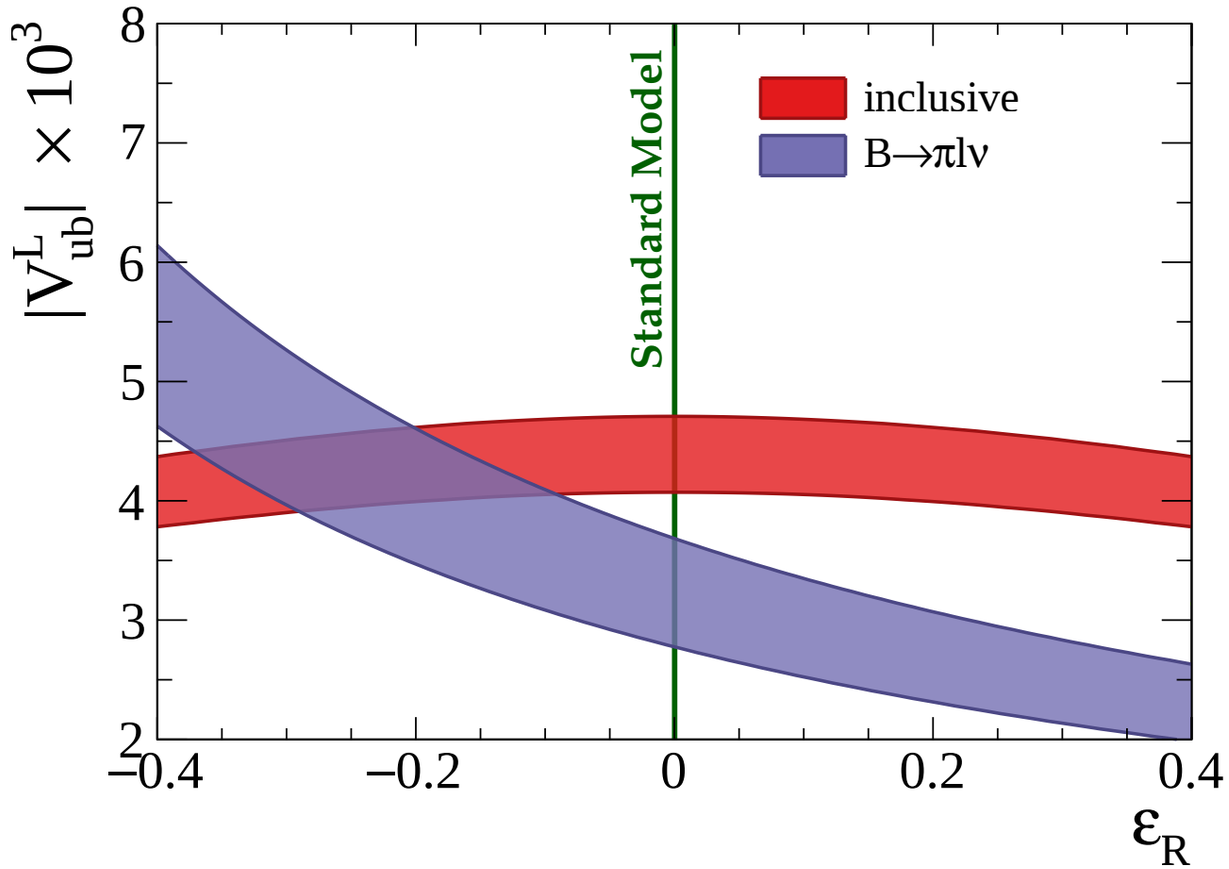
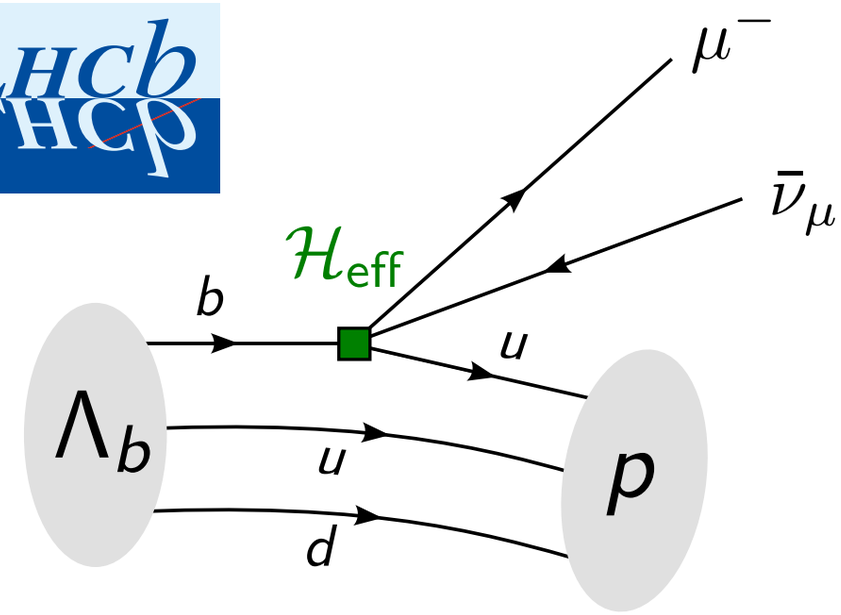


figure modified from LHCb 1504.01568

Λ_b decays

- Bottom baryons provide another exclusive decay channel: $\Lambda_b \to p l \nu$



- LHCb: branching fraction ratio measured

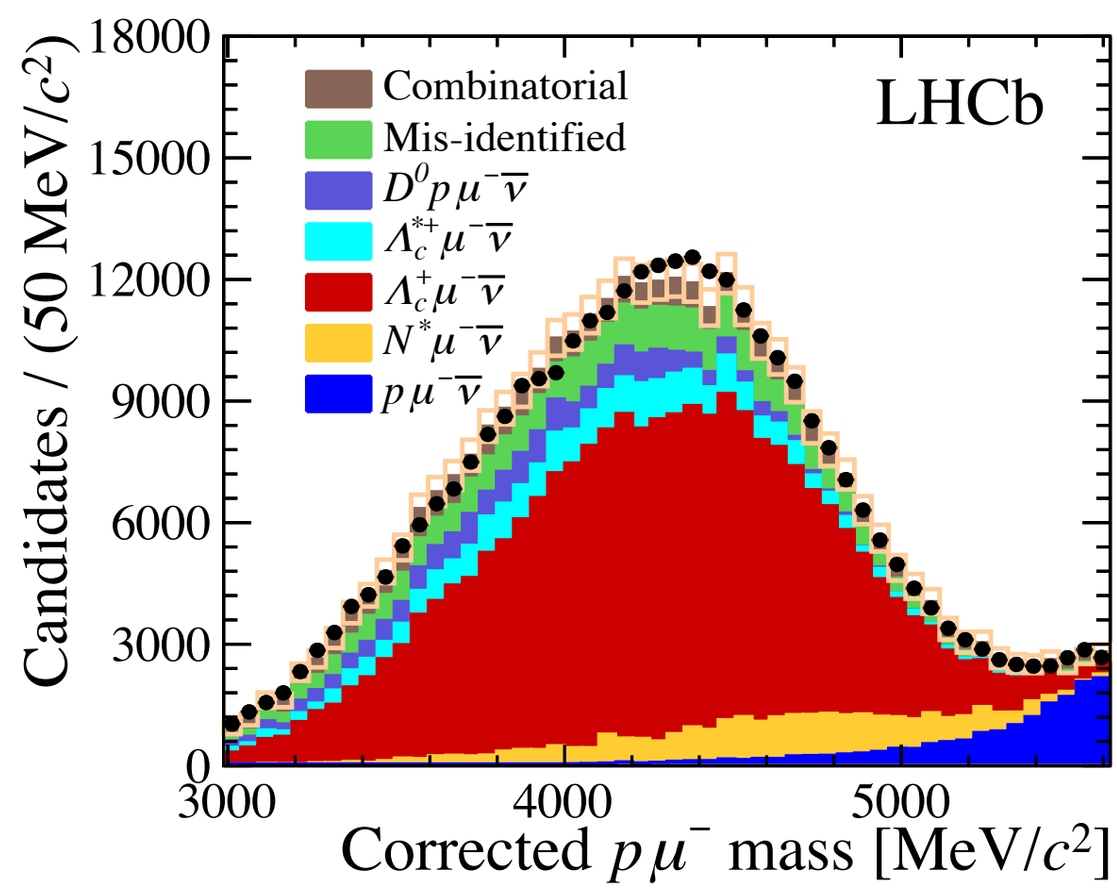
$$\frac{\displaystyle\int_{15\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{d\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{dq^2} dq^2}{\displaystyle\int_{7\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{d\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{dq^2} dq^2} = (1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$$

[1504.01568=Nature Phys. 11 (2015)]

- Extraction of $|V_{ub}/V_{cb}|$ requires hadronic matrix elements

$$\langle p\,|\,\bar{u}\gamma^\mu b\,|\Lambda_b\rangle,\ \ \langle p\,|\,\bar{u}\gamma^\mu\gamma_5 b\,|\Lambda_b\rangle,$$
$$\langle \Lambda_c|\,\bar{c}\gamma^\mu b\,|\Lambda_b\rangle,\ \ \langle \Lambda_c|\,\bar{c}\gamma^\mu\gamma_5 b\,|\Lambda_b\rangle$$

from LQCD



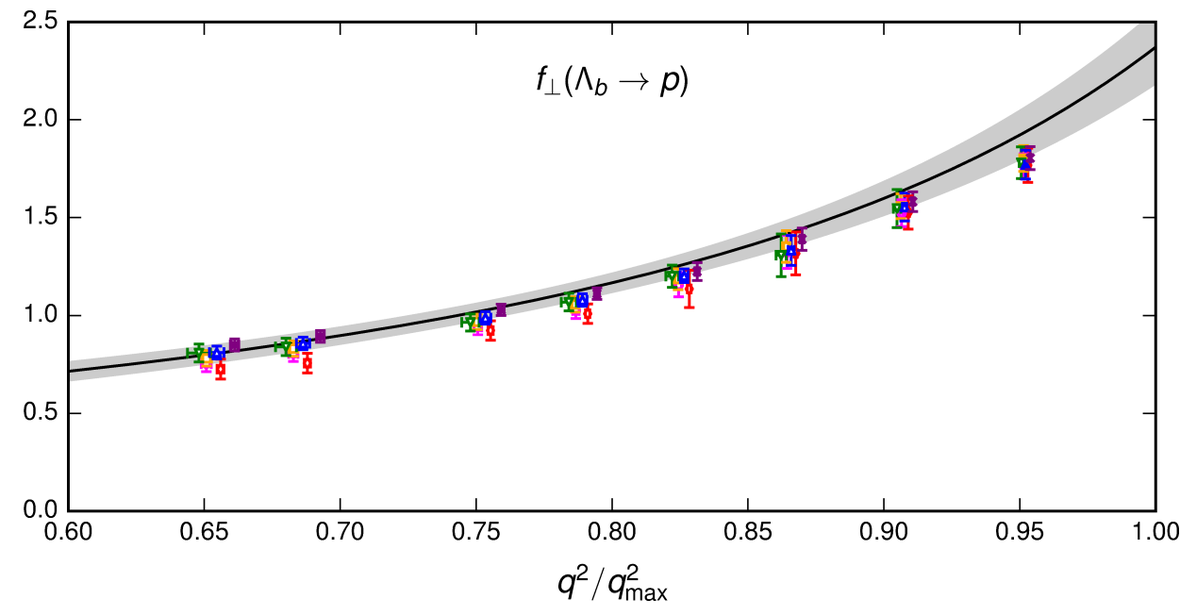
LQCD calculation

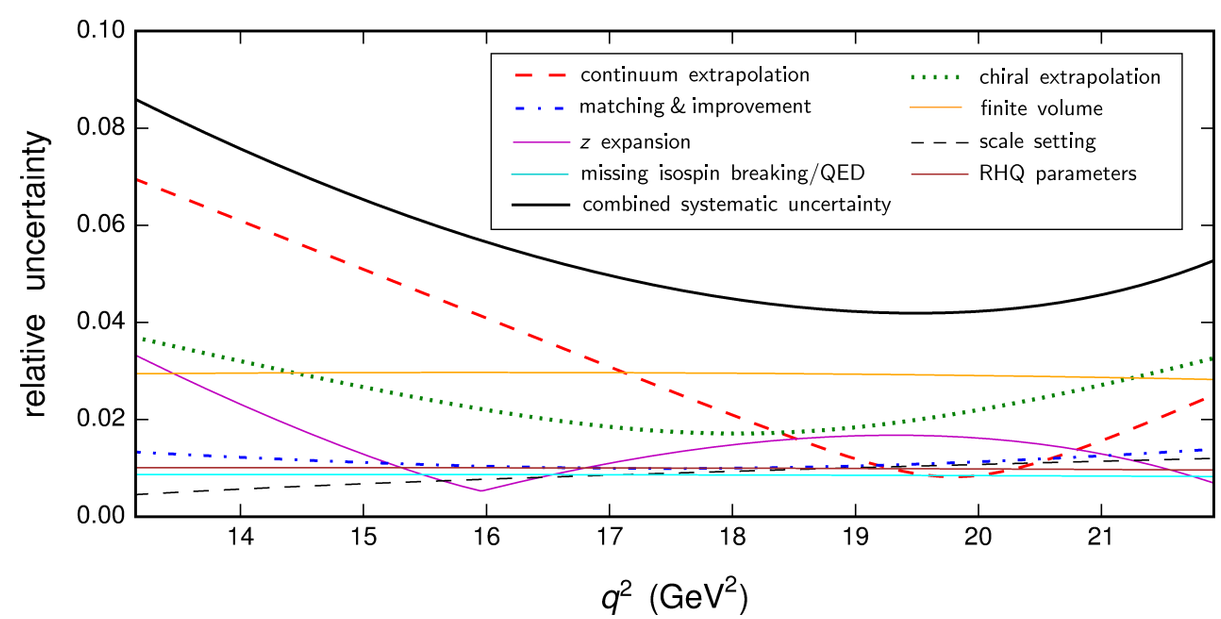
[WD, C Lehner, S Meinel PRD 92 (2015) 034503]

- Extends previous calculation that used static quarks [WD,Lin,Meinel,Wingate]
  - RHQ, z-expansion,….
- 12 form factors needed
- Compare partial integrals

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.083(4)_{\rm expt}(4)_{\rm latt}$$

- Combine with exclusive V_{cb} to get $|V_{ub}|$





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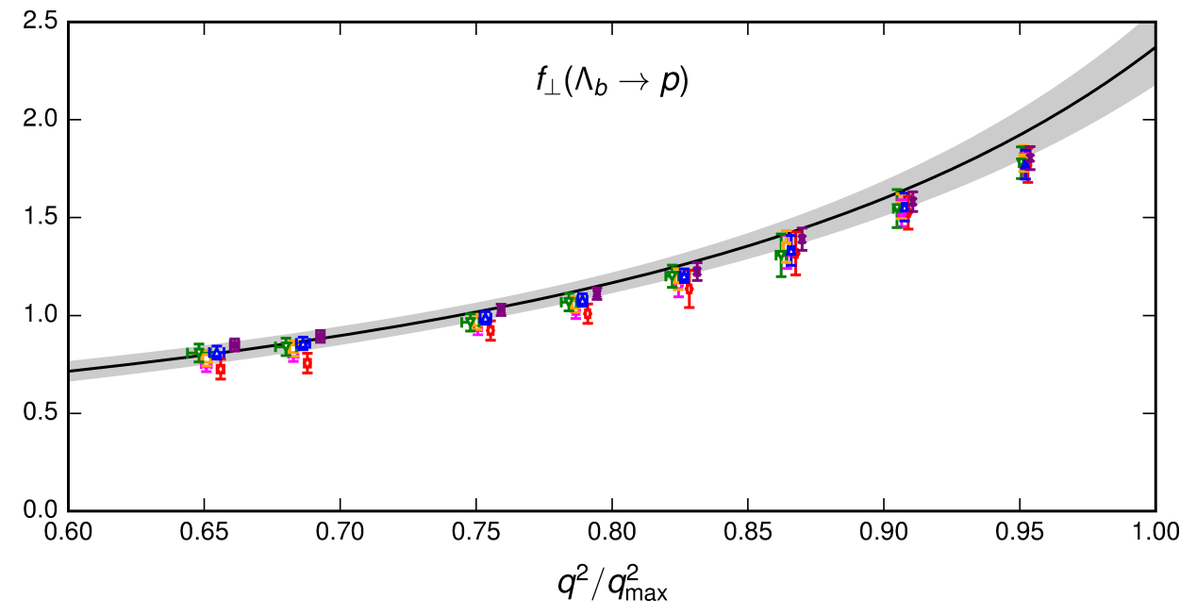
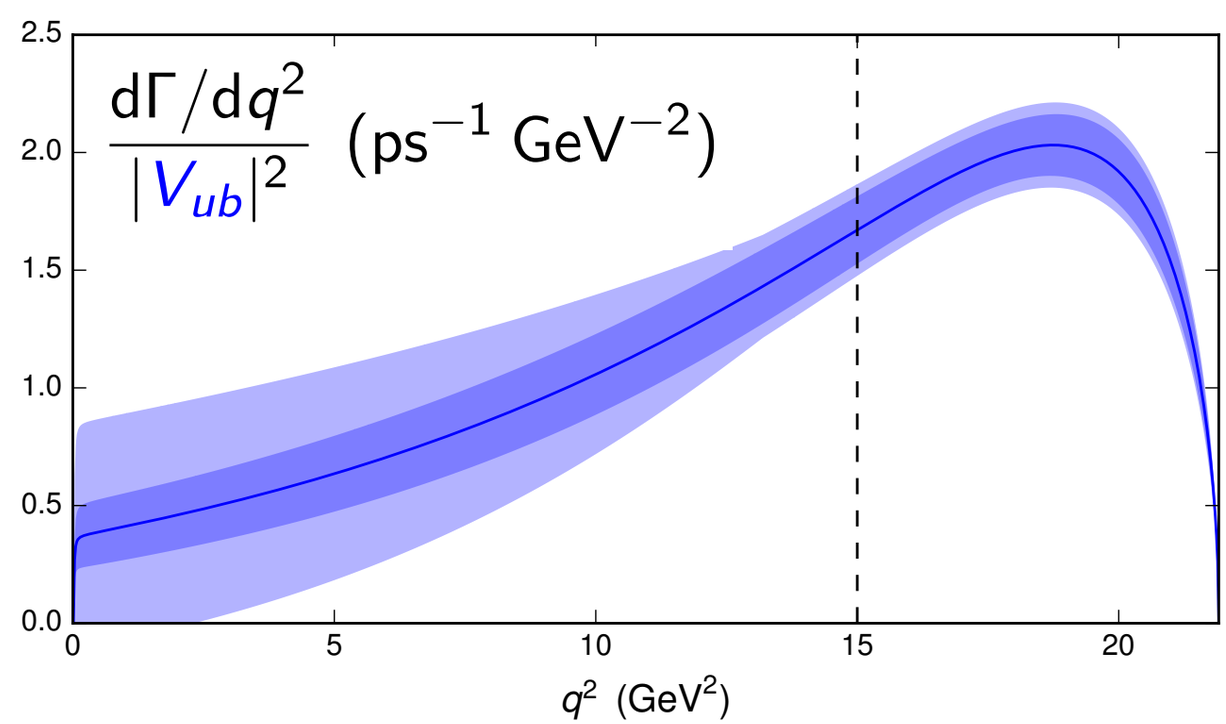
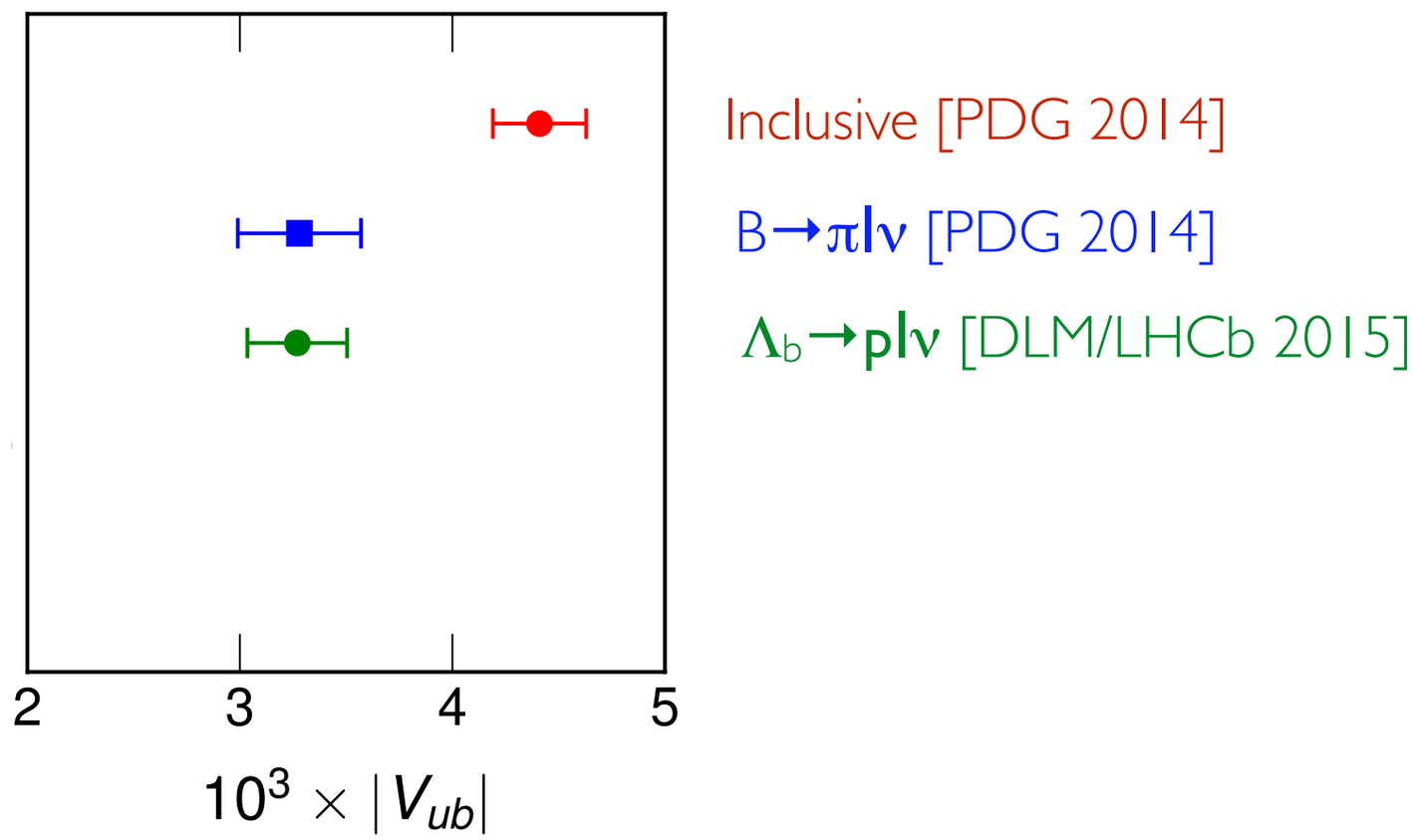




Inclusive vs exclusive V_{ub}

- Consistent with mesonic exclusive measurement

$$|V_{ub}| = 3.27(0.15)_{\text{expt}}(0.16)_{\text{latt}}(0.06)_{V_{cb}} \times 10^{-3}$$



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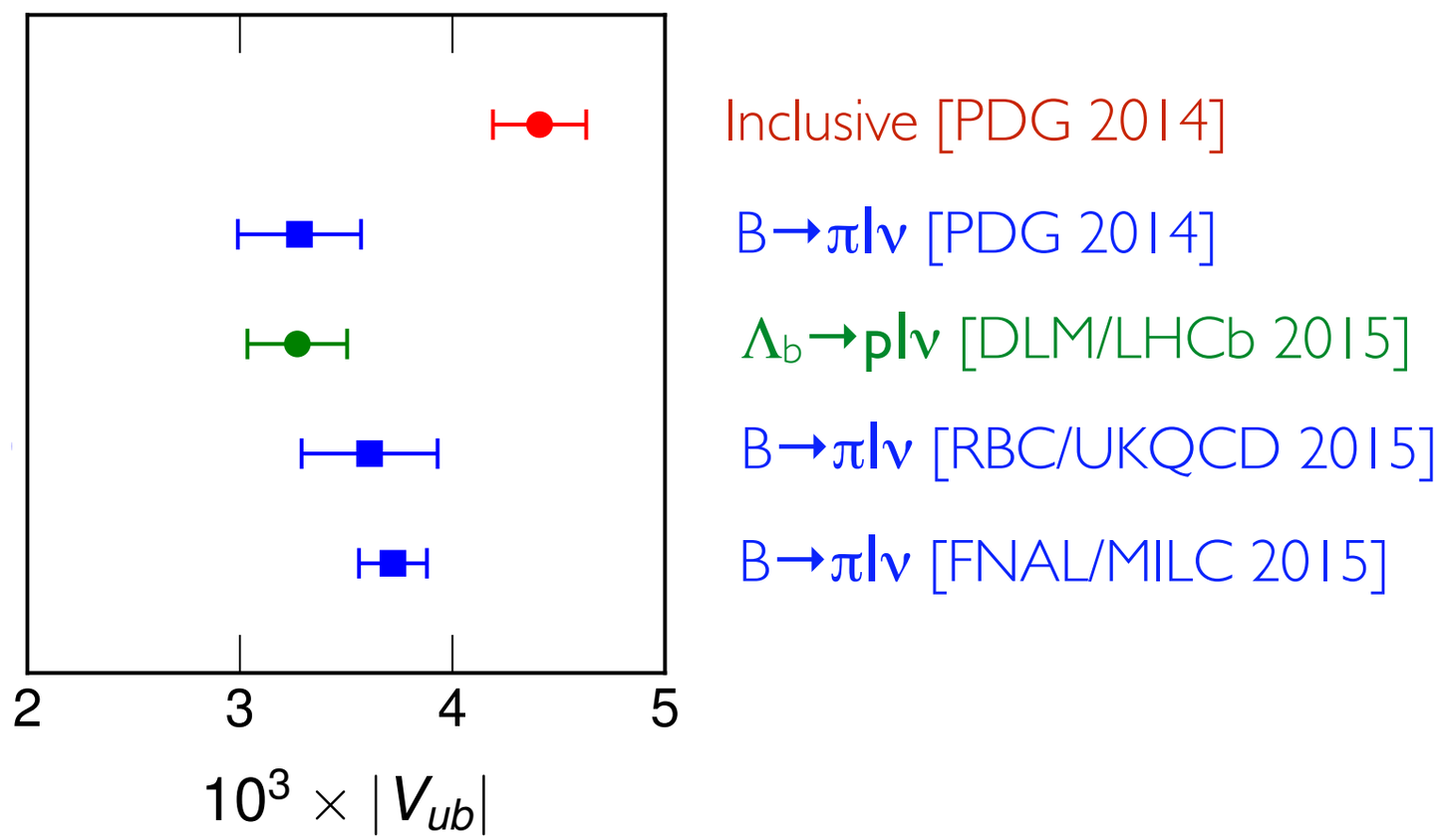


- New LQCD calculations for B→π decays too!

Inclusive vs exclusive V_{ub}

- Different dependence of baryon decay disfavours RH currents as a solution to inclusive/exclusive tension

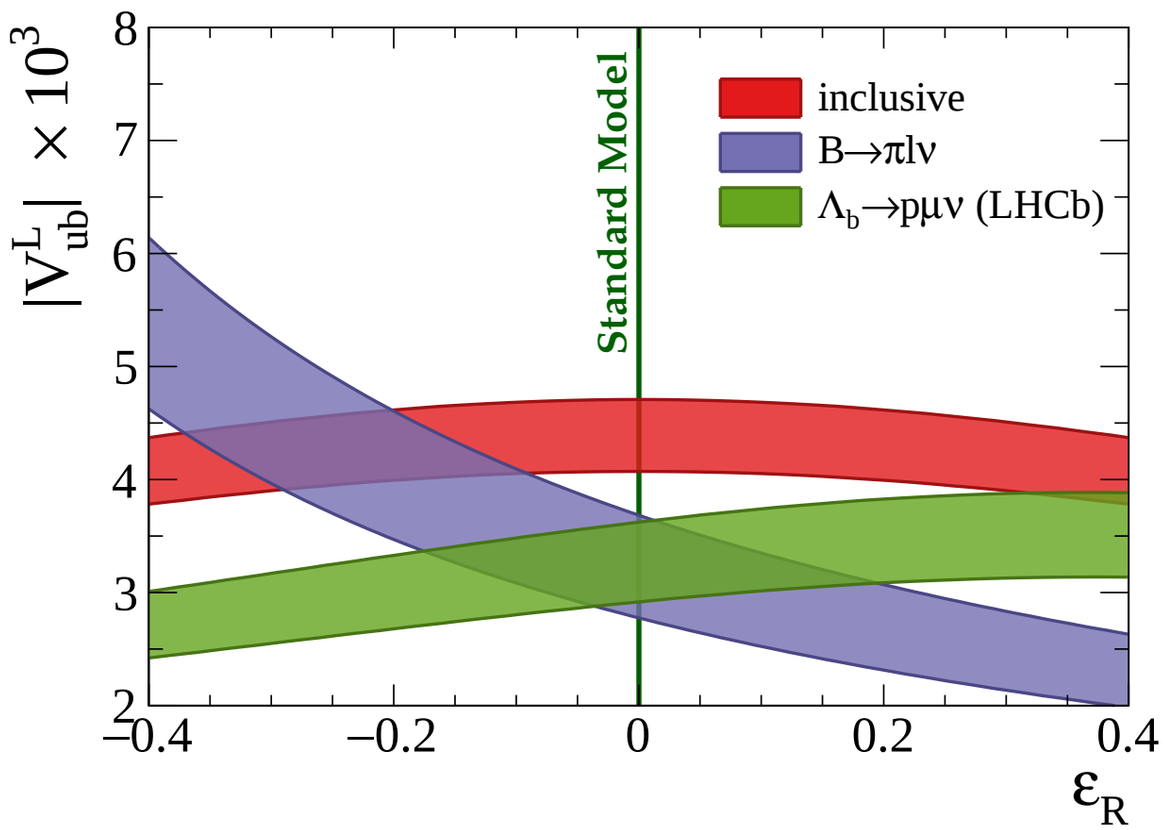
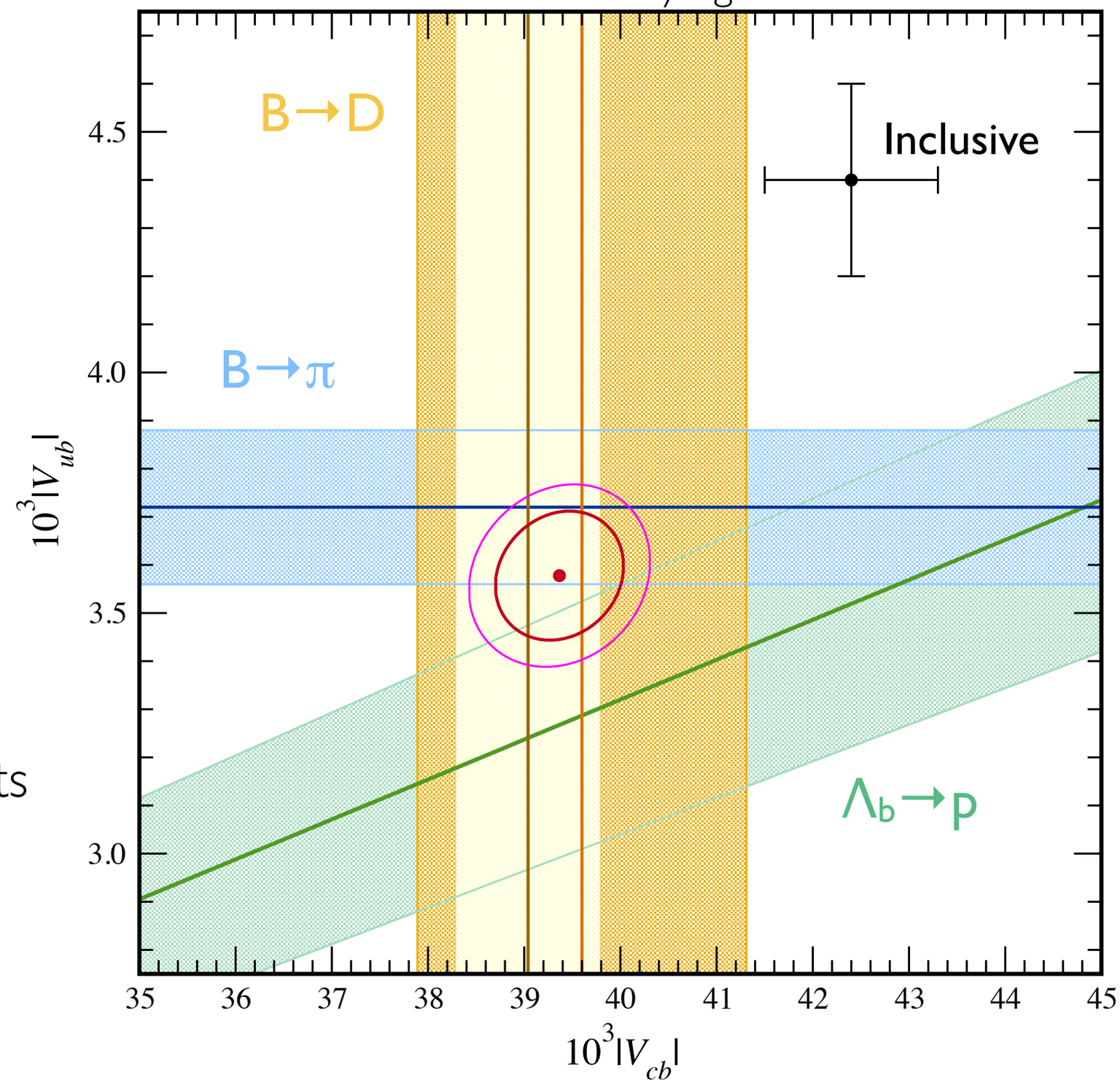


figure modified from LHCb 1504.01568

Inclusive vs exclusive V_{ub} & V_{cb}

- Exclusive extractions:
  - very different experimental and theoretical systematics
  - Mutual consistency (p=0.26)
- Inclusive extractions creates significant tension
- Solution from RH currents disfavoured by baryonic extraction

Summary figure from A Kronfeld



Extensions

- Other baryonic semi-leptonic decays
  - Strange spectators: $\Xi_b \to \Sigma l \nu, \Lambda l \nu$, $\Omega_b \to \Xi l \nu$??
    Nice from LQCD perspective as final state is strongly stable
- Shape, angular observables?

Technical slides follow

Lattice actions

- RBC/UKQCD 2+1 flavour gauge configs
  Light and strange quarks are DWF using standard parameters

Set	β	$N_s^3 \times N_t \times N_5$	am_5	$am_s^{(\mathrm{sea})}$	$am_{u,d}^{(\mathrm{sea})}$	a (fm)	$am_{u,d}^{(\mathrm{val})}$	$m_\pi^{(\mathrm{val})}$ (MeV)	N_{meas}
C14	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.001	245(4)	2672
C24	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.002	270(4)	2676
C54	2.13	$24^3 \times 64 \times 16$	1.8	0.04	0.005	0.1119(17)	0.005	336(5)	2782
F23	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.002	227(3)	1907
F43	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.004	0.0849(12)	0.004	295(4)	1917
F63	2.25	$32^3 \times 64 \times 16$	1.8	0.03	0.006	0.0848(17)	0.006	352(7)	2782

- Heavy quarks: RHQ action a la Fermilab/Columbia/Tsukuba

$$S_Q = a^4 \sum_x \bar{Q} \left[m_Q + \gamma_0 \nabla_0 - \frac{a}{2}\nabla_0^{(2)} + \nu \sum_{i=1}^{3} \left(\gamma_i \nabla_i - \frac{a}{2}\nabla_i^{(2)} \right) - c_E \frac{a}{2} \sum_{i=1}^{3} \sigma_{0i} F_{0i} - c_B \frac{a}{4} \sum_{i,\,j=1}^{3} \sigma_{ij} F_{ij} \right] Q$$

- Tuning of b quark from RBC/UKQCD
  Tuning of c quark [Brown,WD,Meinel,Orginos 2014]
  m_Q and $\boldsymbol{\nu}$ tuned to give spin averaged charmonium mass and dispersion relation;
  $c_{E,B}$ fixed to mean-field tree-level improved values

Parameter	coarse	fine
$am_Q^{(b)}$	8.45	3.99
$\xi^{(b)}$	3.1	1.93
$c_{E,\,B}^{(b)}$	5.8	3.57
$am_Q^{(c)}$	0.1214	−0.0045
$\xi^{(c)}$	1.2362	1.1281
$c_E^{(c)}$	1.6650	1.5311
$c_B^{(c)}$	1.8409	1.6232

Lattice current renormalisation

- Lattice versions of vector and axial currents

$$V_0 = \sqrt{Z_V^{(qq)} Z_V^{(bb)}} \rho_{V_0} \left[\bar{q}\gamma_0 b + 2a \left(c_{V_0}^R \, \bar{q}\gamma_0\gamma_j \overrightarrow{\nabla}_j b + c_{V_0}^L \, \bar{q} \overleftarrow{\nabla}_j \gamma_0\gamma_j b \right) \right],$$

$$A_0 = \sqrt{Z_V^{(qq)} Z_V^{(bb)}} \rho_{A_0} \left[\bar{q}\gamma_0\gamma_5 b + 2a \left(c_{A_0}^R \, \bar{q}\gamma_0\gamma_5\gamma_j \overrightarrow{\nabla}_j b + c_{A_0}^L \, \bar{q} \overleftarrow{\nabla}_j \gamma_0\gamma_5\gamma_j b \right) \right],$$

$$V_i = \sqrt{Z_V^{(qq)} Z_V^{(bb)}} \rho_{V_i} \left[\bar{q}\gamma_i b + 2a \left(c_{V_i}^R \, \bar{q}\gamma_i\gamma_j \overrightarrow{\nabla}_j b + c_{V_i}^L \, \bar{q} \overleftarrow{\nabla}_j \gamma_i\gamma_j b + d_{V_i}^R \, \bar{q} \overrightarrow{\nabla}_i b + d_{V_i}^L \, \bar{q} \overleftarrow{\nabla}_i b \right) \right],$$

$$A_i = \sqrt{Z_V^{(qq)} Z_V^{(bb)}} \rho_{A_i} \left[\bar{q}\gamma_i\gamma_5 b + 2a \left(c_{A_i}^R \, \bar{q}\gamma_i\gamma_5\gamma_j \overrightarrow{\nabla}_j b + c_{A_i}^L \, \bar{q} \overleftarrow{\nabla}_j \gamma_i\gamma_5\gamma_j b + d_{A_i}^R \, \bar{q}\gamma_5 \overrightarrow{\nabla}_i b + d_{A_i}^L \, \bar{q} \overleftarrow{\nabla}_i \gamma_5 b \right) \right],$$

- Mostly non-perturbative renormalisation
  - $Z^{(qq)}$'s fixed non-perturbativly from current conservation of flavour conserving current (q = l,c,b)
  - ρ's and c's calculated at one loop in mean-field improved lattice perturbation theory using PhySyHCAl [C Lehner]

Form factor definitions

$$s_\pm = (m_{\Lambda_b} \pm m_X)^2 - q^2$$

$$q = p - p'$$

- Form factors

$$\begin{aligned}\langle X(p',s')|\overline{q}\,\gamma^\mu\, b|\Lambda_b(p,s)\rangle = \overline{u}_X(p',s')\Bigg[& f_0(q^2)\,(m_{\Lambda_b} - m_X)\frac{q^\mu}{q^2} \\ & + f_+(q^2)\frac{m_{\Lambda_b}+m_X}{s_+}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_X^2)\frac{q^\mu}{q^2}\right) \\ & + f_\perp(q^2)\left(\gamma^\mu - \frac{2m_X}{s_+}p^\mu - \frac{2m_{\Lambda_b}}{s_+}p'^\mu\right)\Bigg] u_{\Lambda_b}(p,s),\end{aligned}$$

$$\begin{aligned}\langle X(p',s')|\overline{q}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p,s)\rangle = -\overline{u}_X(p',s')\,\gamma_5\Bigg[& g_0(q^2)\,(m_{\Lambda_b} + m_X)\frac{q^\mu}{q^2} \\ & + g_+(q^2)\frac{m_{\Lambda_b}-m_X}{s_-}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_X^2)\frac{q^\mu}{q^2}\right) \\ & + g_\perp(q^2)\left(\gamma^\mu + \frac{2m_X}{s_-}p^\mu - \frac{2m_{\Lambda_b}}{s_-}p'^\mu\right)\Bigg] u_{\Lambda_b}(p,s).\end{aligned}$$

- Alternate notation

$$\langle X(p',s')|\overline{q}\,\gamma^\mu\, b|\Lambda_b(p)\rangle = \overline{u}_X(p',s')\left[f_1^V(q^2)\,\gamma^\mu - \frac{f_2^V(q^2)}{m_{\Lambda_b}}i\sigma^{\mu\nu}q_\nu + \frac{f_3^V(q^2)}{m_{\Lambda_b}}q^\mu\right] u_{\Lambda_b}(p,s),$$

$$\langle X(p',s')|\overline{q}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p)\rangle = \overline{u}_X(p',s')\left[f_1^A(q^2)\,\gamma^\mu - \frac{f_2^A(q^2)}{m_{\Lambda_b}}i\sigma^{\mu\nu}q_\nu + \frac{f_3^A(q^2)}{m_{\Lambda_b}}q^\mu\right]\gamma_5\, u_{\Lambda_b}(p,s),$$

Form factor extraction

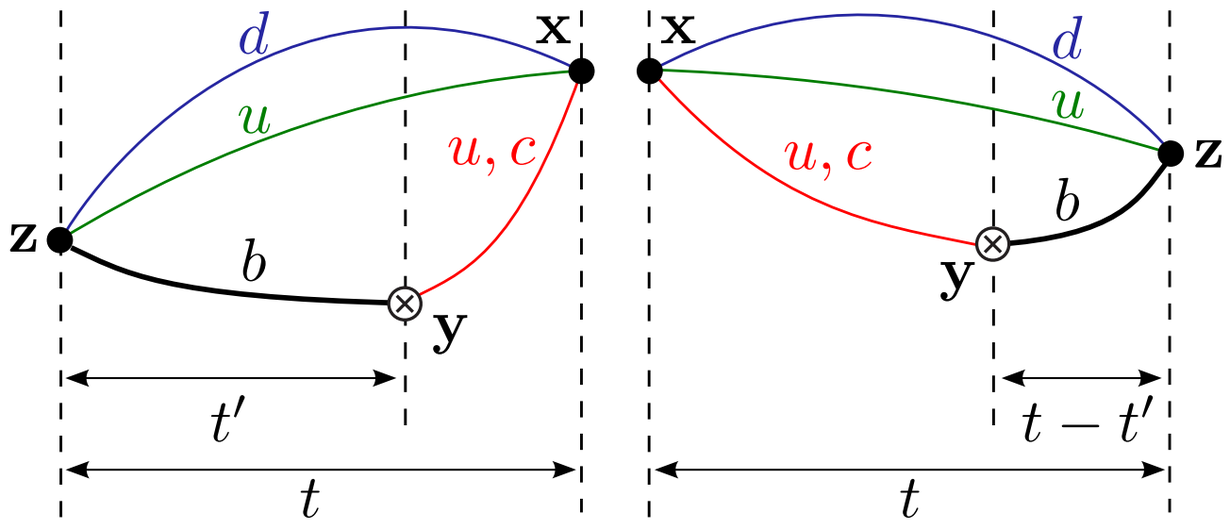
- Measure two point and three point functions (simple interpolating operators)

$$C^{(2,X,\mathrm{fw})}_{\delta\alpha}(\mathbf{p}',t) = \sum_{\mathbf{y}} e^{-i\mathbf{p}'\cdot(\mathbf{y}-\mathbf{x})} \left\langle X_\delta(x_0+t,\mathbf{y})\, \overline{X}_\alpha(x_0,\mathbf{x})\right\rangle,$$

$$C^{(2,X,\mathrm{bw})}_{\delta\alpha}(\mathbf{p}',t) = \sum_{\mathbf{y}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} \left\langle X_\delta(x_0,\mathbf{x})\, \overline{X}_\alpha(x_0-t,\mathbf{y})\right\rangle,$$

$$C^{(2,\Lambda_b,\mathrm{fw})}_{\delta\alpha}(t) = \sum_{\mathbf{y}} \left\langle \Lambda_{b\delta}(x_0+t,\mathbf{y})\, \overline{\Lambda}_{b\alpha}(x_0,\mathbf{x})\right\rangle,$$

$$C^{(2,\Lambda_b,\mathrm{bw})}_{\delta\alpha}(t) = \sum \left\langle \Lambda_{b\delta}(x_0,\mathbf{x})\, \overline{\Lambda}_{b\alpha}(x_0-t,\mathbf{y})\right\rangle.$$



$$C^{(3,\mathrm{fw})}_{\delta\alpha}(\Gamma,\ \mathbf{p}',t,t') = \sum_{\mathbf{y},\mathbf{z}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} \Big\langle X_\delta(x_0,\mathbf{x}) \quad J^\dagger_\Gamma(x_0-t+t',\mathbf{y}) \quad \bar{\Lambda}_{b\alpha}(x_0-t,\mathbf{z})\Big\rangle,$$

$$C^{(3,\mathrm{bw})}_{\alpha\delta}(\Gamma,\ \mathbf{p}',t,t-t') = \sum_{\mathbf{y},\mathbf{z}} e^{-i\mathbf{p}'\cdot(\mathbf{y}-\mathbf{x})} \Big\langle \Lambda_{b\alpha}(x_0+t,\mathbf{z}) \quad J_\Gamma(x_0+t',\mathbf{y}) \quad \bar{X}_\delta(x_0,\mathbf{x})\Big\rangle,$$

Form factor extraction

- Ratios to pull out form factors (axial-vector case similar)

$$\mathscr{R}_+^V(\mathbf{p}',t,t') = \frac{r_\mu[(1,\mathbf{0})]\, r_\nu[(1,\mathbf{0})]\, \mathrm{Tr}\Big[C^{(3,\mathrm{fw})}(\mathbf{p}',\ \gamma^\mu,t,t')\, C^{(3,\mathrm{bw})}(\mathbf{p}',\ \gamma^\nu,t,t-t')\Big]}{\mathrm{Tr}\Big[C^{(2,X,\mathrm{av})}(\mathbf{p}',t)\Big]\mathrm{Tr}\Big[C^{(2,\Lambda_b,\mathrm{av})}(t)\Big]}$$

$$\mathscr{R}_\perp^V(\mathbf{p}',t,t') = \frac{r_\mu[(0,\mathbf{e}_j\times\mathbf{p}')]\, r_\nu[(0,\mathbf{e}_k\times\mathbf{p}')]\, \mathrm{Tr}\Big[C^{(3,\mathrm{fw})}(\mathbf{p}',\ \gamma^\mu,t,t')\gamma_5\gamma^j\, C^{(3,\mathrm{bw})}(\mathbf{p}',\ \gamma^\nu,t,t-t')\gamma_5\gamma^k\Big]}{\mathrm{Tr}\Big[C^{(2,X,\mathrm{av})}(\mathbf{p}',t)\Big]\mathrm{Tr}\Big[C^{(2,\Lambda_b,\mathrm{av})}(t)\Big]}$$

$$\mathscr{R}_0^V(\mathbf{p}',t,t') = \frac{q_\mu\, q_\nu\, \mathrm{Tr}\Big[C^{(3,\mathrm{fw})}(\mathbf{p}',\ \gamma^\mu,t,t')\, C^{(3,\mathrm{bw})}(\mathbf{p}',\ \gamma^\nu,t,t-t')\Big]}{\mathrm{Tr}\Big[C^{(2,X,\mathrm{av})}(\mathbf{p}',t)\Big]\mathrm{Tr}\Big[C^{(2,\Lambda_b,\mathrm{av})}(t)\Big]}$$

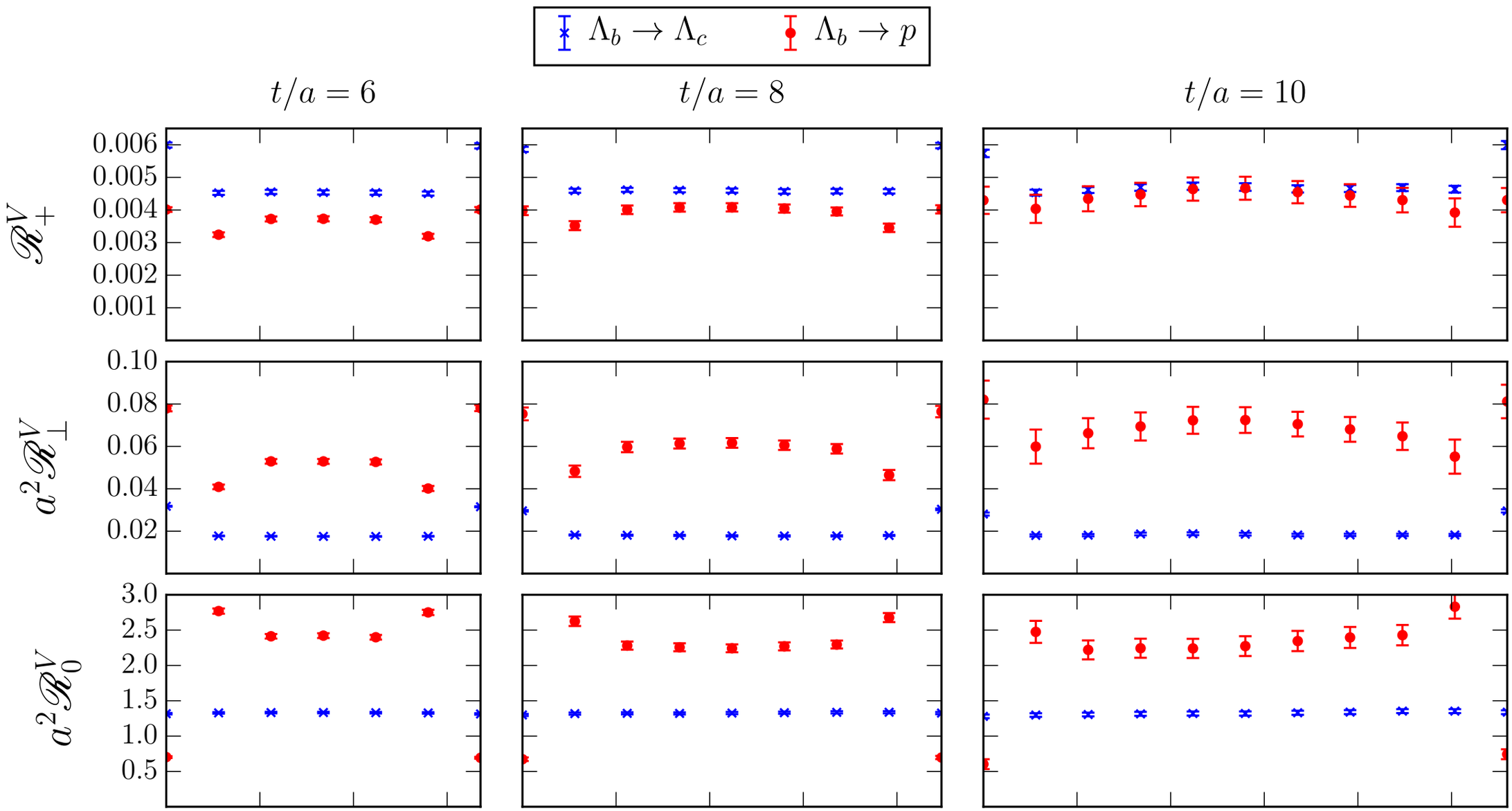
$$R_{f_+}(|\mathbf{p}'|,t) = \frac{2\,q^2}{(E_X-m_X)(m_{\Lambda_b}+m_X)}\sqrt{\frac{E_X}{E_X+m_X}\mathscr{R}_+^V(|\mathbf{p}'|,t,t/2)} \quad = f_+ + (\text{excited-state contributions})$$

$$R_{f_\perp}(|\mathbf{p}'|,t) = \frac{1}{E_X-m_X}\sqrt{\frac{E_X}{E_X+m_X}\mathscr{R}_\perp^V(|\mathbf{p}'|,t,t/2)} \quad = f_\perp + (\text{excited-state contributions})$$

$$R_{f_0}(|\mathbf{p}'|,t) = \frac{2}{m_{\Lambda_b}-m_X}\sqrt{\frac{E_X}{E_X+m_X}\mathscr{R}_0^V(|\mathbf{p}'|,t,t/2)} \quad = f_0 + (\text{excited-state contributions})$$

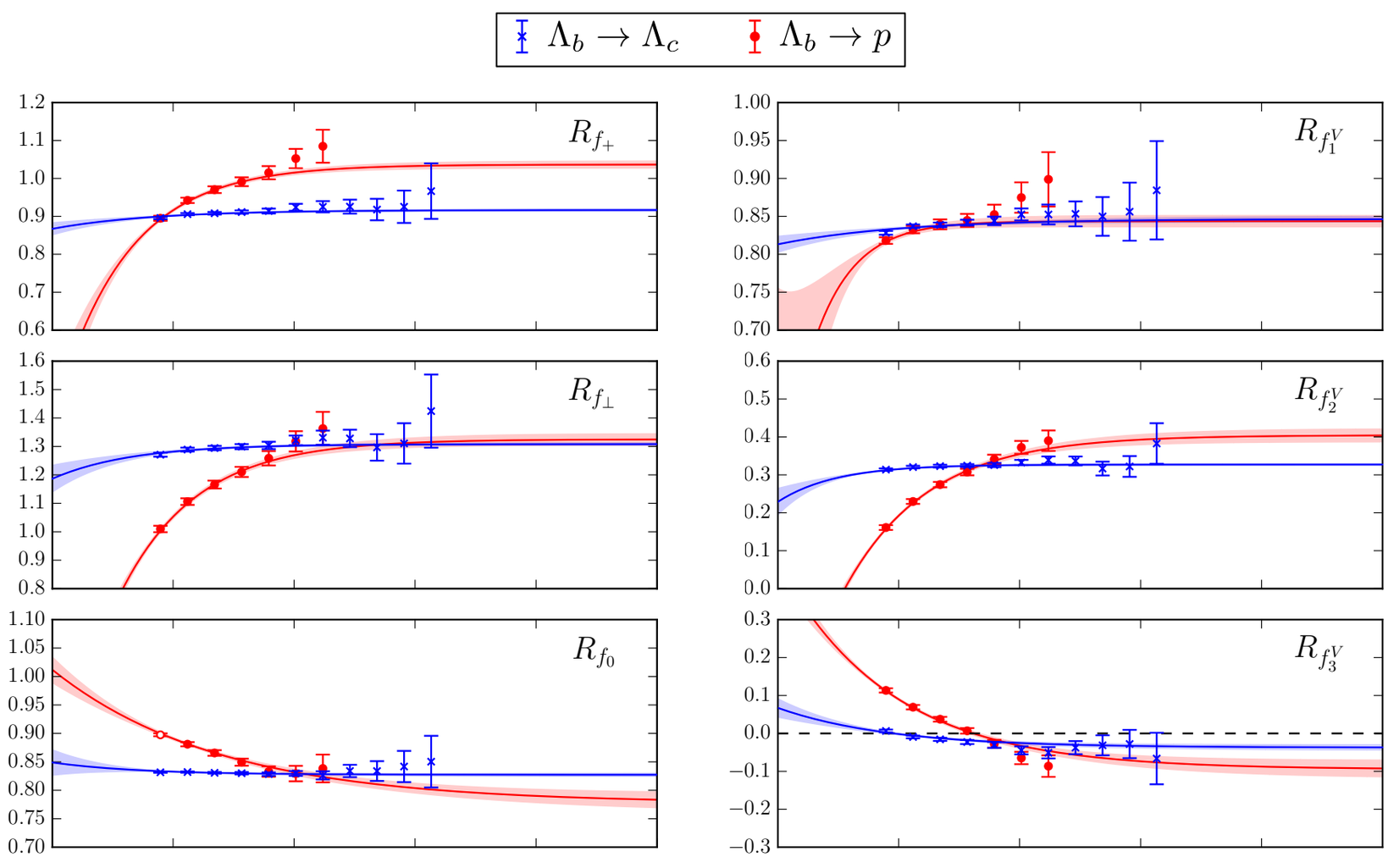
Form factor extraction

- Ratios to pull out form factors (axial-vector case similar)



Form factor extraction

- Extrapolation: $R_{f,i,n}(t) = f_{i,n} + A_{f,i,n}\, e^{-\delta_{f,i,n}\, t}, \quad \delta_{f,i,n} = \delta_{\min} + e^{l_{f,i,n}}\ \mathrm{GeV}$
  (use augmented chi-sq to impose expected relations)
- Fits correlated between different ensembles, FFs



Form factor extraction

- Chiral/continuum extrapolation using z-expansion after factoring leading pole

$$f(q^2) = \frac{1}{1 - q^2/(m^f_{\rm pole})^2}\left[a^f_0\left(1 + c^f_0\frac{m^2_\pi - m^2_{\pi,\rm phys}}{\Lambda^2_\chi}\right) + a^f_1\, z^f(q^2)\right]$$
$$\times\left[1 + b^f\,\frac{|\mathbf{p}'|^2}{(\pi/a)^2} + d^f\,\frac{\Lambda^2_{\rm QCD}}{(\pi/a)^2}\right],$$

with $am^f_{\rm pole} = am_{\rm PS} + a\Delta^f$

$$z^f(q^2) = \frac{\sqrt{t^f_+ - q^2} - \sqrt{t^f_+ - t_0}}{\sqrt{t^f_+ - q^2} + \sqrt{t^f_+ - t_0}} \qquad t_0 = (m_{\Lambda_b} - m_X)^2$$

t^+ below any singularities (based on Q# of FF channel)

$$a^2 t^f_+ = (am_{\rm PS} + am_{\pi,\rm phys})^2 \qquad (\text{for } \Lambda_b \to p)$$
$$a^2 t^f_+ = (am_{\rm PS} + a\Delta^f)^2 \qquad (\text{for } \Lambda_b \to \Lambda_c)$$

f	J^P	$t^f_+(\Lambda_b \to p)$	$m^f_{\rm pole}(\Lambda_b \to p)$	$\Delta^f(\Lambda_b \to p)$	$t^f_+(\Lambda_b \to \Lambda_c)$	$m^f_{\rm pole}(\Lambda_b \to \Lambda_c)$	$\Delta^f(\Lambda_b \to \Lambda_c)$
$f_+,\, f_\perp$	1^-	$(m_B + m_\pi)^2$	$m_B + \Delta^f$	46 MeV	$(m^f_{\rm pole})^2$	$m_{B_c} + \Delta^f$	56 MeV
f_0	0^+	$(m_B + m_\pi)^2$	$m_B + \Delta^f$	377 MeV	$(m^f_{\rm pole})^2$	$m_{B_c} + \Delta^f$	449 MeV
$g_+,\, g_\perp$	1^+	$(m_B + m_\pi)^2$	$m_B + \Delta^f$	427 MeV	$(m^f_{\rm pole})^2$	$m_{B_c} + \Delta^f$	492 MeV
g_0	0^-	$(m_B + m_\pi)^2$	$m_B + \Delta^f$	0	$(m^f_{\rm pole})^2$	$m_{B_c} + \Delta^f$	0

Fitting systematics

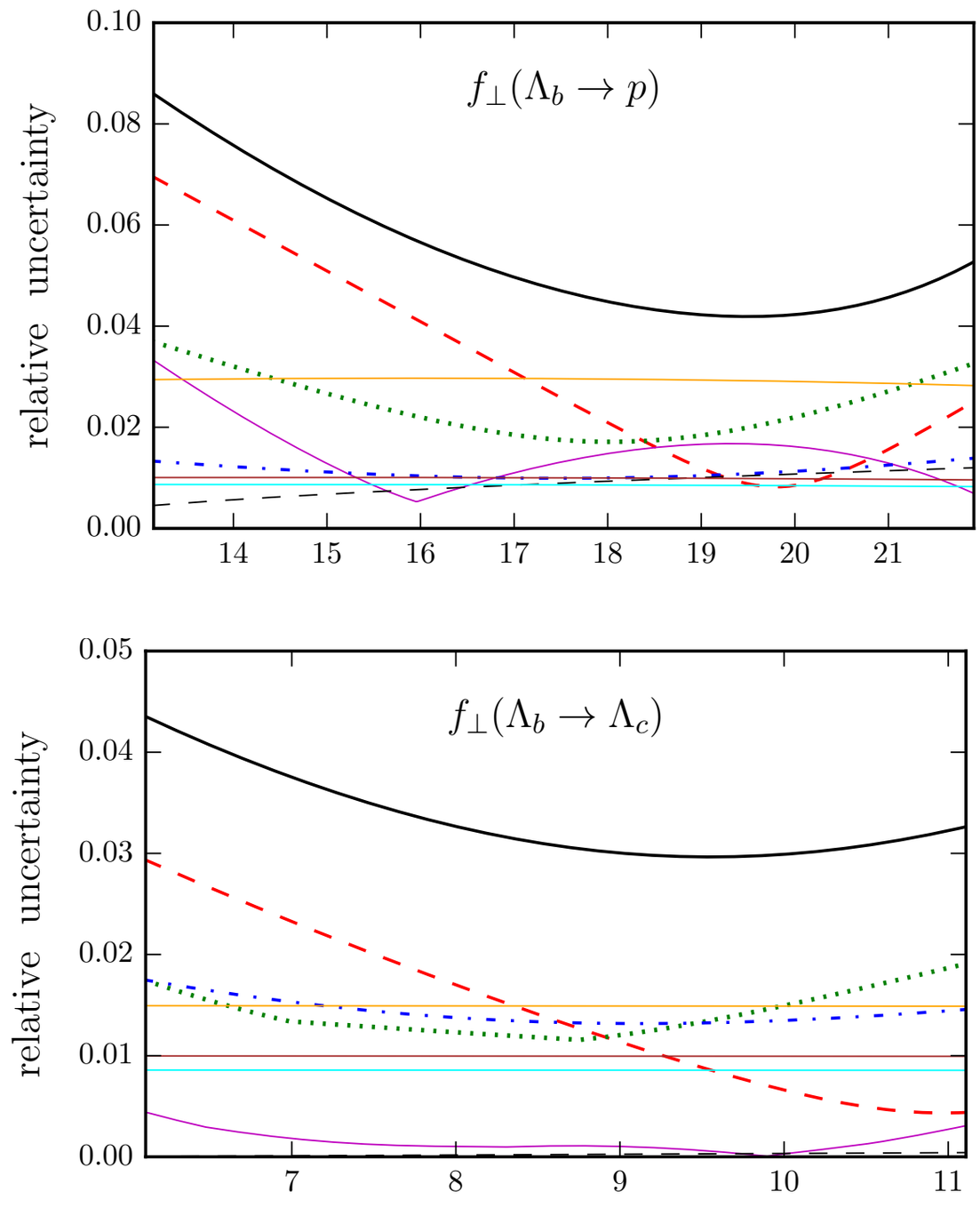
- Generally assess by adding higher order terms to fit
  - Chiral, continuum, z-dependence

$$O \pm \underbrace{\sigma_O}_{\text{stat.}} \pm \underbrace{\max\left(|O_{\text{HO}} - O|,\ \sqrt{|\sigma^2_{O,\text{HO}} - \sigma^2_O|}\right)}_{\text{syst.}}$$

- Matching and improvement coeffs, lattice spacing sampled within uncertainties in each bootstrap sample
- Finite volume, isospin breaking, EM, heavy quark parameter tuning uncertainties estimated

Uncertainties

- Dominated by z-expansion and continuum extrapolation at low q^2
  - Limits precision of shape calculations
- Chiral extrapolation important at large q^2
  - Address with physical mass calculations



Decay rate

- In terms of form factors, differential decay rate given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2 |V_{qb}^L|^2 \sqrt{s_+ s_-}}{768\pi^3 m_{\Lambda_b}^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2$$
$$\times \Bigg\{ 4\left(m_\ell^2 + 2q^2\right)\left(s_+ \left[(1-\epsilon_q^R) g_\perp\right]^2 + s_- \left[(1+\epsilon_q^R) f_\perp\right]^2\right)$$
$$+ 2\frac{m_\ell^2 + 2q^2}{q^2}\left(s_+ \left[(m_{\Lambda_b} - m_X)(1-\epsilon_q^R) g_+\right]^2 + s_- \left[(m_{\Lambda_b} + m_X)(1+\epsilon_q^R) f_+\right]^2\right)$$
$$+ \frac{6m_\ell^2}{q^2}\left(s_+ \left[(m_{\Lambda_b} - m_X)(1+\epsilon_q^R) f_0\right]^2 + s_- \left[(m_{\Lambda_b} + m_X)(1-\epsilon_q^R) g_0\right]^2\right) \Bigg\},$$

- Since LQCD less precise at low q^2, partly integrated decay rates

$$\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2) \equiv \frac{1}{|V_{ub}|^2} \int_{15\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2 = (12.31 \pm 0.76 \pm 0.77)\ \mathrm{ps}^{-1},$$
$$\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2) \equiv \frac{1}{|V_{cb}|^2} \int_{7\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2 = (8.37 \pm 0.16 \pm 0.34)\ \mathrm{ps}^{-1},$$

$$\frac{\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2)}{\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2)} = 1.471 \pm 0.095 \pm 0.109$$

- Lepton non-universality??

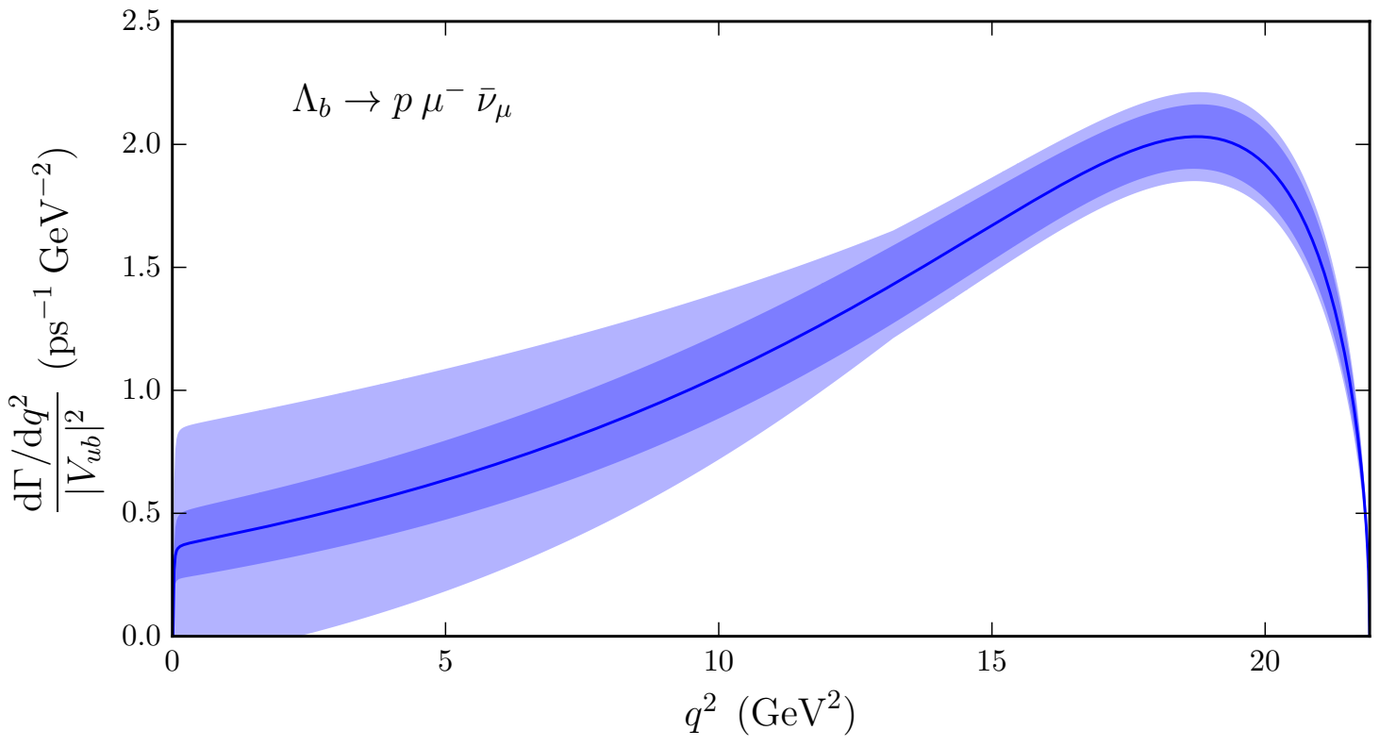
$$\frac{\Gamma(\Lambda_b \to \Lambda_c\,\tau^-\bar{\nu}_\mu)}{\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)} = 0.3328 \pm 0.0074 \pm 0.0070$$

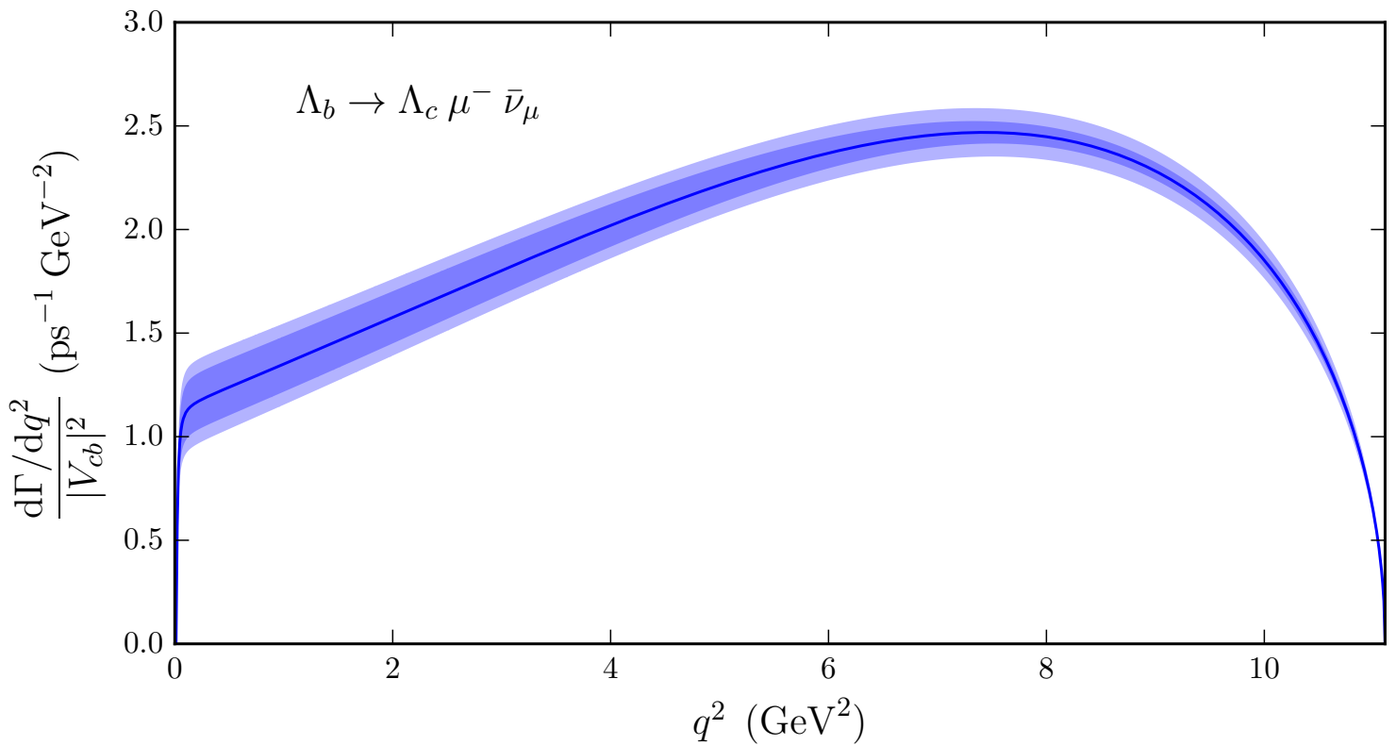
Uncertainties

- Uncertainty budget

	$\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2)$	$\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2)$	$\frac{\zeta_{p\mu\bar{\nu}}(15\ \mathrm{GeV}^2)}{\zeta_{\Lambda_c\mu\bar{\nu}}(7\ \mathrm{GeV}^2)}$
Statistics	6.2	1.9	6.5
Finite volume	5.0	2.5	4.9
Continuum extrapolation	3.0	1.4	2.8
Chiral extrapolation	2.6	1.8	2.6
RHQ parameters	1.4	1.7	2.3
Matching & improvement	1.7	0.9	2.1
Missing isospin breaking/QED	1.2	1.4	2.0
Scale setting	1.7	0.3	1.8
z expansion	1.2	0.2	1.3
Total	8.8	4.5	9.8

Decay rate


$\Lambda_b \to p\,\mu^-\,\bar{\nu}_\mu$
$\frac{d\Gamma/dq^2}{|V_{ub}|^2}$ (ps^{-1} GeV^{-2})
q^2 (GeV^2)
0.0
0.5
1.0
1.5
2.0
2.5
0
5
10
15
20


$\Lambda_b \to \Lambda_c\,\mu^-\,\bar{\nu}_\mu$
$\frac{d\Gamma/dq^2}{|V_{cb}|^2}$ (ps^{-1} GeV^{-2})
q^2 (GeV^2)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10

Extra

News on inclusive extraction

- Need to look carefully at the inclusive extraction