Aspects of gauge/gravity duality

Original AdS/CFT correspondence:

Maldacena
Gubser, Klebanov, Polyakov
Witten

between $\mathcal{N}=4$ SYM

and type IIB string theory on $AdS_5 \times S^5$

$$ds^2 = \underbrace{\frac{R^2}{z^2}(dx^2 + dz^2)}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{S_5}$$

(this is a solution to Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$$

$$\| \quad F_\mu{}^{\alpha_1\alpha_2\alpha_3\alpha_4} F_{\nu\alpha_1\alpha_2\alpha_3\alpha_4} \quad)$$

Large 't Hooft limit $\longleftrightarrow$ small curvature

$$g^2 N_c \to \infty \quad \longleftrightarrow \quad \sqrt{\alpha'} \ll R$$

$\downarrow$ correlation functions are computable

string theory $\to$ supergravity

The dictionary of gauge/gravity duality:

gauge theory	gravity
operator $\hat{O}$	field ϕ
$T_{\mu\nu}$	graviton $h_{\mu\nu}$
dimension of operator	mass of field
global symmetry	gauge symmetry
conserved current	gauge field
anomaly	Chern-Simons term
......	....

$$\int e^{iS_{4D} + \phi_0 O} = \int_{\substack{\phi \to \phi_0 \\ z \to 0}} e^{iS_{5D}}$$

$$\lim_{z \to 0} \phi(x,z) = \phi_0(x)$$

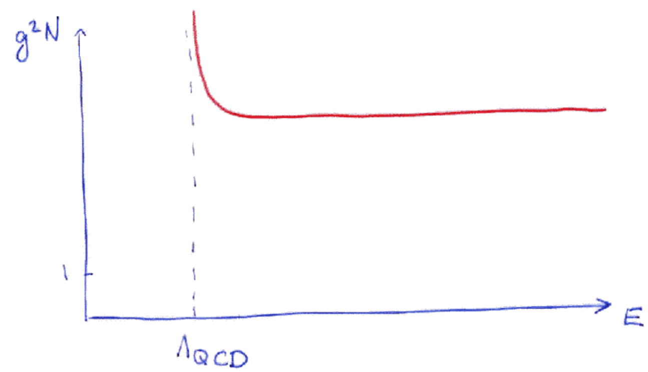
partition fn of 4D theory with source

partition fn of 5D theory with nontrivial boundary condition

Many more examples of gauge/gravity duality have been discovered

including theories with confinement and chiral symmetry breaking

To be able to compute using string theory: strong gauge coupling



consequence : 2 different scales

$$\Lambda_{QCD} \ll \frac{1}{\sqrt{\alpha'}}$$

masses of lowest hadrons
spin ≤ 2

string excitations
arbitrary spin

"Hadronic physics"

4D		5D
hadron		normalizable mode $\psi(z)$
hadron mass		eigenvalues of 5D operator
decay constant	$\langle 0 \vert J_\mu \vert \rho \rangle$	$\lim\limits_{z\to 0} \dfrac{\psi'(z)}{z^{\#}}$
meson coupling		overlap integral $\int \psi_\rho(z)\psi_\pi^2(z)$

Qualitative, sometimes ~~qua~~ semi-quantitative agreement with low-energy hadron phenomenology.

Low-energy QCD has a useful string representation?

Finite-temperature AdS/CFT correspondence:

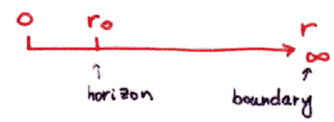
black 3-brane solution:

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2 f(r)}dr^2 + R^2 d\Omega_5^2$$

$$f(r) = 1 - \frac{r_0^4}{r^4}$$

corresponds to

$\mathcal{N}=4$ SYM at temperature



$$T = T_H = \frac{r_0}{\pi R^2}$$

$$\text{Entropy} = \frac{A}{4G}$$

A ← area of event horizon $r = r_0$

G ← 10d Newton constant

$$A = \int dx\,dy\,dz \sqrt{g_{xx}g_{yy}g_{zz}} \times \pi^3 R^5$$

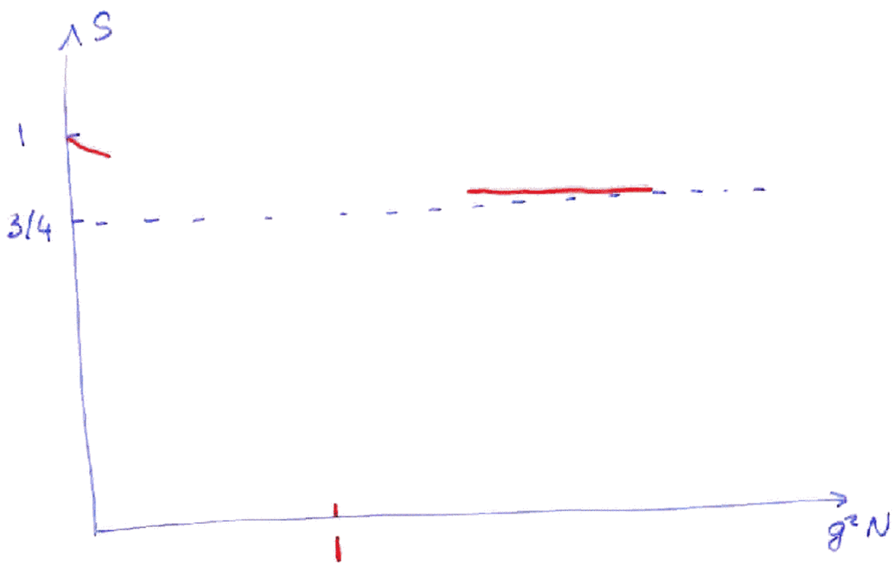
$\pi^3 R^5$ ← area of S^5

$$= V_{3D}\,\pi^3 r_0^3 R^2 \sim T^3$$

$$G \sim \frac{R^8}{N^2}$$

$$\Rightarrow S = \# N^2 T^3$$

$\#$ ↑ coefficient $= \frac{3}{4} \times$ (free gas)



Compare with QCD thermodynamics

$$\frac{\varepsilon}{\varepsilon_{SB}}\,(2-3T_c) \sim 0.8$$

pure coincidence ?

Next:

- Viscosity in thermal field theories from AdS/CFT
- The viscosity/entropy density ratio

Kovtun, Son, Starinets 2004

Policastro, Son, Starinets 2001

Herzog

Buchel, Liu

Viscosity : introduced by C. L. M. H. NAVIER (1822)

(Navier – Stokes equation)

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v}.\vec{\nabla})\vec{v} = -\vec{\nabla} P + \underbrace{\eta}_{\text{viscosity}} \nabla^2 \vec{v} \qquad \vec{\nabla}\cdot\vec{v} = 0$$

From real-world experience : ANY finite-temperature interacting system can be described, at largest time and length scales, by hydrodynamic equations :

- local
- few variables

Finite-T QFT also behave hydrodynamically

Idea : use gauge/gravity duality to investigate the hydrodynamic regime of field theory

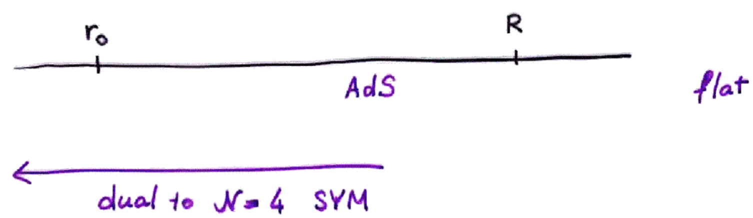
finite-T QFT $\Longleftrightarrow$ black hole with translationally invariant horizon

"black brane"

Example :

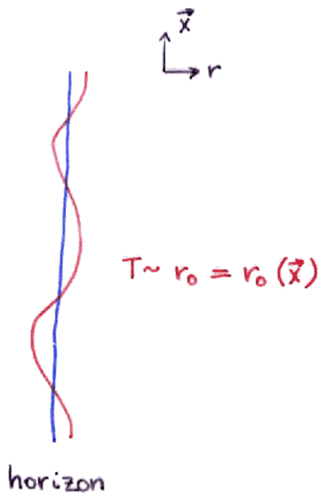
$$ds^2 = H^{-1/2}(-f dt^2 + dx^2 + dy^2 + dz^2) + \frac{1}{H^{1/2}}\left(\frac{dr^2}{f} + r^2 d\Omega_5^2\right)$$

$H = 1 + \frac{R^4}{r^4}$ $\quad f = 1 - \frac{r_0^4}{r^4}$ $\quad r_0 \ll R$



Hawking temperature $T = \frac{r_0}{\pi R^2}$

Dynamics of flat horizons:



Generalizing black hole thermodynamics $M, Q \ldots$

to black brane <u>hydrodynamics</u>

$T = T_H(\vec{x})$, $\mu = \mu(\vec{x})$ ....

Event horizons behave as vicous fluids

$$S = \frac{\text{Area of horizon}}{4G}$$ Bekenstein Hawking

$\mathcal{N}=4$ SYM: $S(g^2N \to \infty) = \frac{3}{4} S(g^2N \to 0)$

What is viscosity from the point of view of gravity?

Viscosity : Kubo's formula

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega}\int dt\, d\vec{x}\; e^{i\omega t}\langle [T_{xy}(t,\vec{x}),\ T_{xy}(0,\vec{0})]\rangle$$

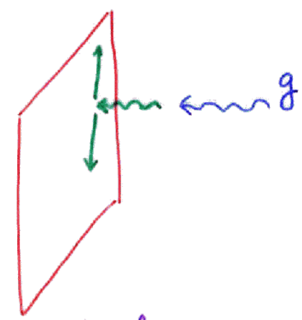
$$= -\lim_{\omega\to 0}\lim_{\vec{q}\to 0}\frac{1}{\omega}\,\mathrm{Im}\, G^R_{xy,xy}(\omega,\vec{q})$$

retarded Green's function of T_{xy}

Similar relations exist for other kinetic coefficients (diffusion constants, conductivities ...)

Gravity counterpart of Kubo's formula:

AdS/CFT "dictionary"



Coupling: $h_{\mu\nu}\, T_{\mu\nu}$

bulk graviton

boundary stress energy

1997 Klebanov: absorption of a graviton falling at right angle to the black brane

$\kappa = \sqrt{8\pi G}$

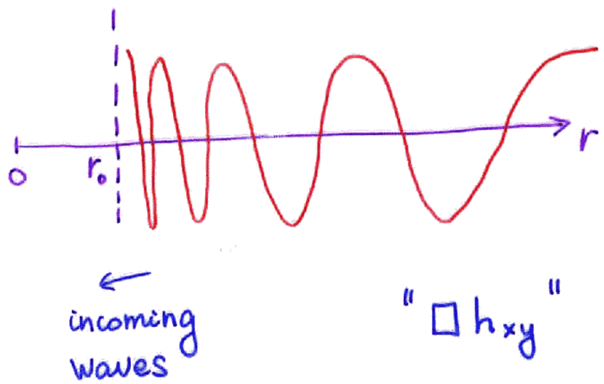
$$\sigma_{abs} = -\frac{2\kappa^2}{\omega}\,\mathrm{Im}\, G^R(\omega)$$

$$= \frac{\kappa^2}{\omega}\int d^4x\; e^{i\omega t}\langle [T_{xy}(x),\ T_{xy}(0)]\rangle$$

Viscosity = absorption cross section of low-energy gravitons

$$\eta = \frac{\sigma_{abs}(0)}{2\kappa^2} = \frac{\sigma_{abs}(0)}{16\pi G}$$

Absorption cross section can be found classically



$$\text{"}\Box h_{xy}\text{"} = 0$$

$$h''_{xy} + \frac{5r^4 - r_0^4}{r(r^4 - r_0^4)} h'_{xy} + \omega^2 \frac{r^4(r^4 + R^4)}{(r^4 - r_0^4)^2} h_{xy} = 0$$

The computation of σ_{abs} is made easy by 2 theorems, valid for a wide class of back ground:

- Equation for h_{xy} is the same as of a minimally coupled scalar
- For a minimally coupled scalar

$$\lim_{\omega \to 0} \sigma_{abs}(0) = \text{Area of event horizon}$$

Das, Gibbons, Mathur

Consequences of 2 theorems:

$$\eta = \frac{\sigma_{abs}(\omega \to 0)}{16\pi G} = \frac{A}{16\pi G}$$

$$s = \frac{A}{4G}$$

$$\Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}$$

Restoring $\hbar$ and c :

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

in theories with gravity dual

This is a very small value of η/s

weakly coupled theories $\frac{\eta}{s} \gg \hbar$

Water (1 bar, 25°C) $\frac{\eta}{s} \approx 380 \frac{\hbar}{4\pi}$

Liquid helium (including superfluid) $\left(\frac{\eta}{s}\right)_{min} \approx 9 \frac{\hbar}{4\pi}$

A viscosity bound conjecture

$$\frac{\eta}{s} \geqslant \frac{\hbar}{4\pi k_B}$$

does not contain c, G

c.f. Entropy bound $S \leqslant \frac{c^3}{\hbar G} \frac{A}{4}$

Bekenstein bound $S \leqslant \frac{c}{\hbar} 2\pi R M$

Applications :

- Quark – Gluon Plasma
- Trapped atomic gases

Finding transport coefficient on the lattice is hard

– analytic continuation from a discrete set of frequencies $i\omega_n = i\,2\pi n T$ to real frequencies

or: restoring the spectral density $\rho(\omega)$ from

$$G_E(\omega_n) = \int \frac{d\omega'}{2\pi}\,\frac{\rho(\omega')}{\omega' - i\omega_n}$$

need apriori information on $\rho(\omega)$ (Ansatz)

$$\text{Transport coefficients} \sim \underbrace{\lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}}_{\text{sensitive on the Ansatz}} = \rho'(0)$$

We now argue that $\rho'(0)$ is easier to find in strongly coupled theories than in weakly coupled theories

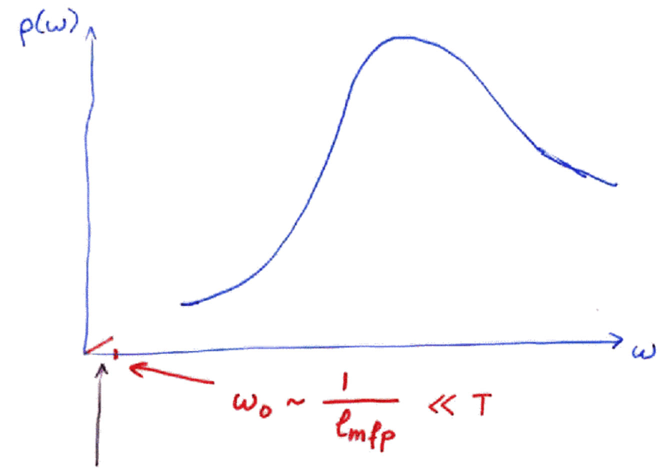
Difficulties at weak coupling:

$$\text{viscosity} \sim \underbrace{\text{mean free path}}_{\text{diverges at weak coupling}}$$

In gauge theories

$$\ell_{mfp} \sim \frac{1}{T}\,\frac{1}{g^4 \ln\frac{1}{g}}$$

$\Rightarrow$ the slope of $\rho(\omega)$ sets at very low values of ω



extremely difficult to find

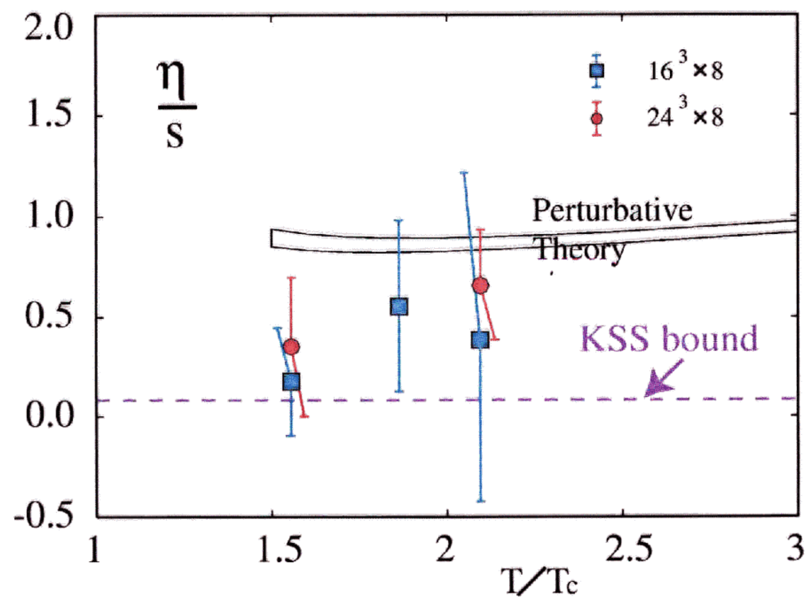
On the other hand, at strong coupling

$$l_{mfp} \sim \frac{1}{T}$$

so $\rho'(\omega)$ should not behave too violently as ω changes from 0 to $2\pi T$

$\Rightarrow$ one can hope to be able to find trasport coefficients with some reasonable accuracy

AdS/CFT calculations can suggest an Ansatz for $\rho(\omega)$



Nakamura, Sakai

Conclusion :

- Gauge/gravity duality allows solution of QCD-like theories at strong coupling.

- can address wide range of problems within these theories

  - high energy scattering Polchinski Strassler
  - vector meson dominance
  - ...

- Unexpected suggestion of a lower bound on $\frac{\eta}{s}$