AdS/CFT
Past, Present, Future?

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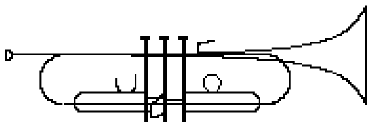
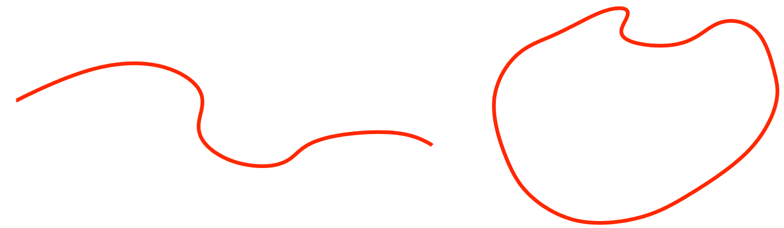
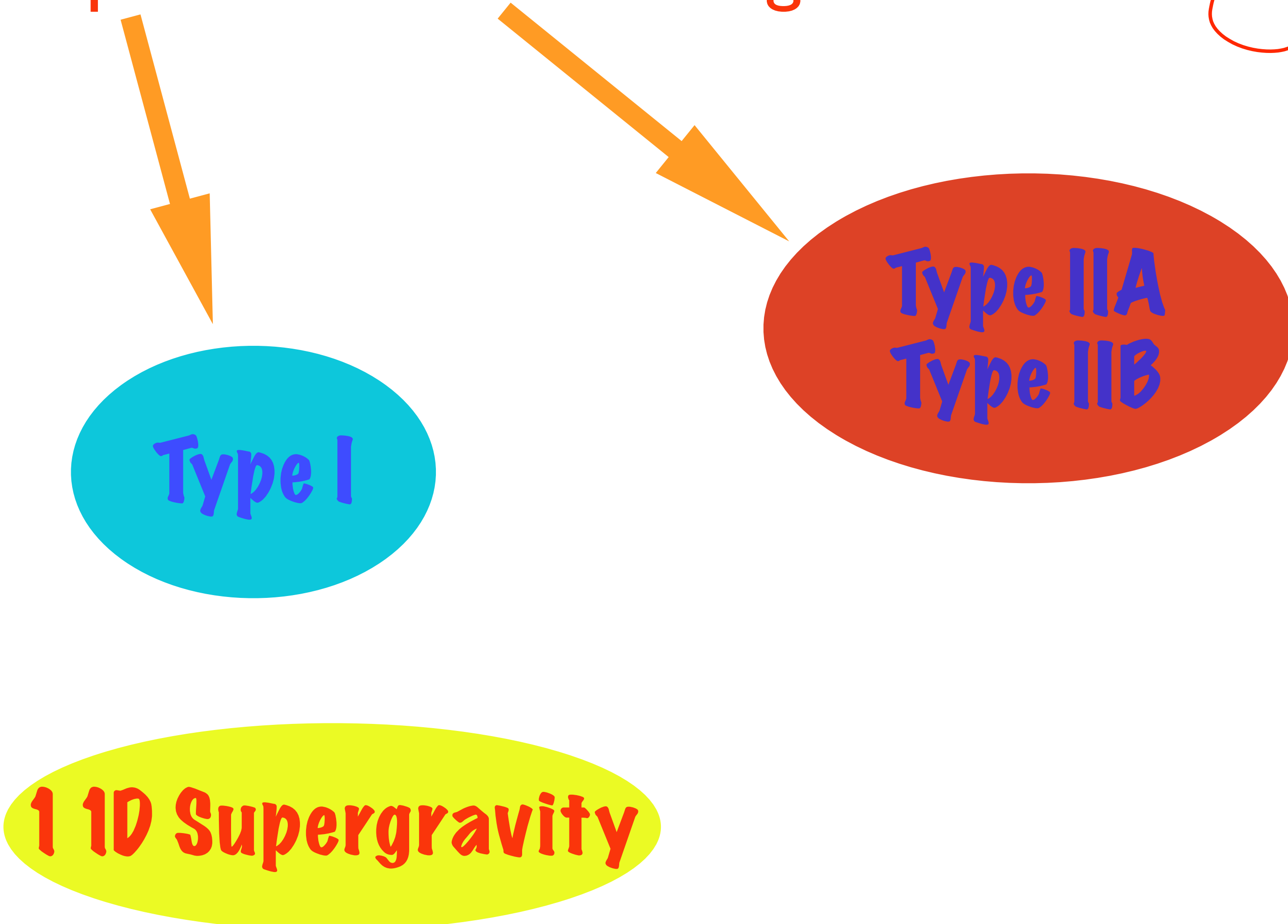
Modern Challenges for Lattice Field Theory
ITP, UCSB, 31st March 2005



The Tools (I)

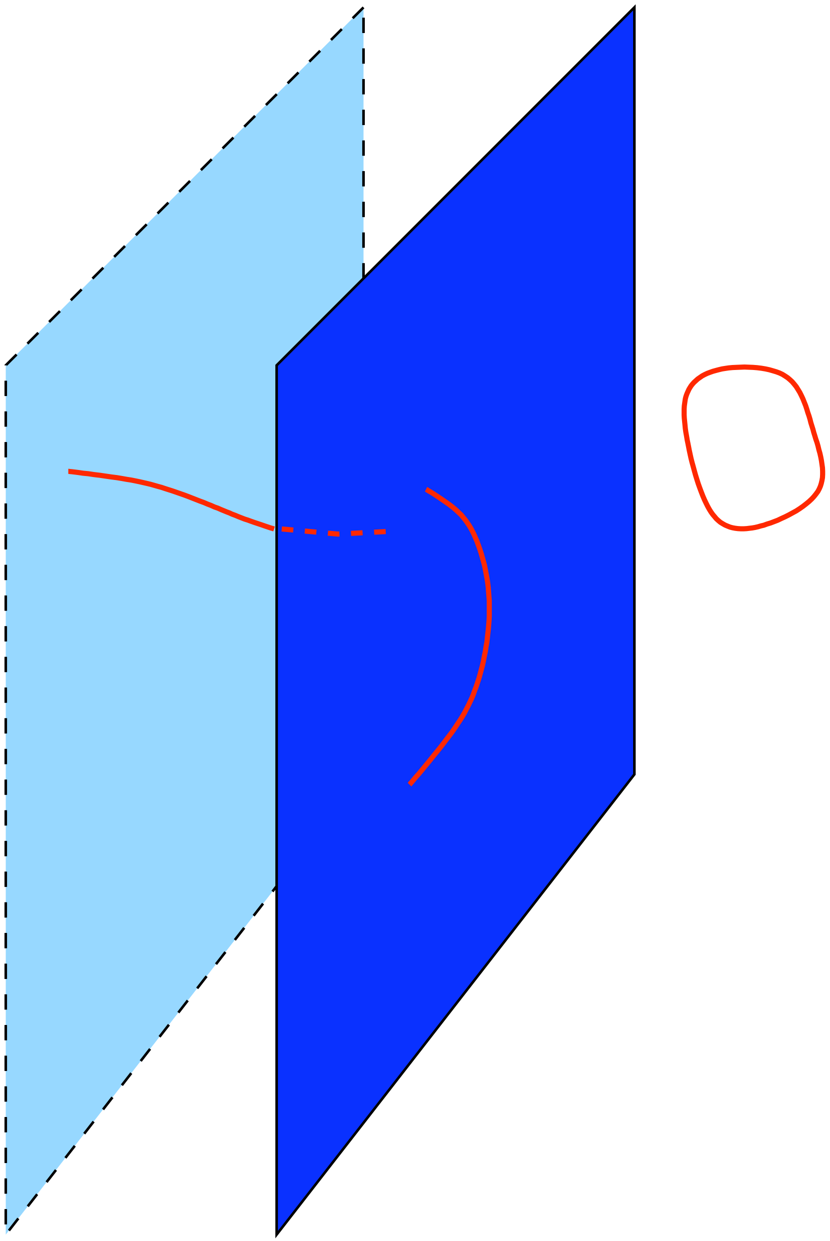
Open and Closed Strings:





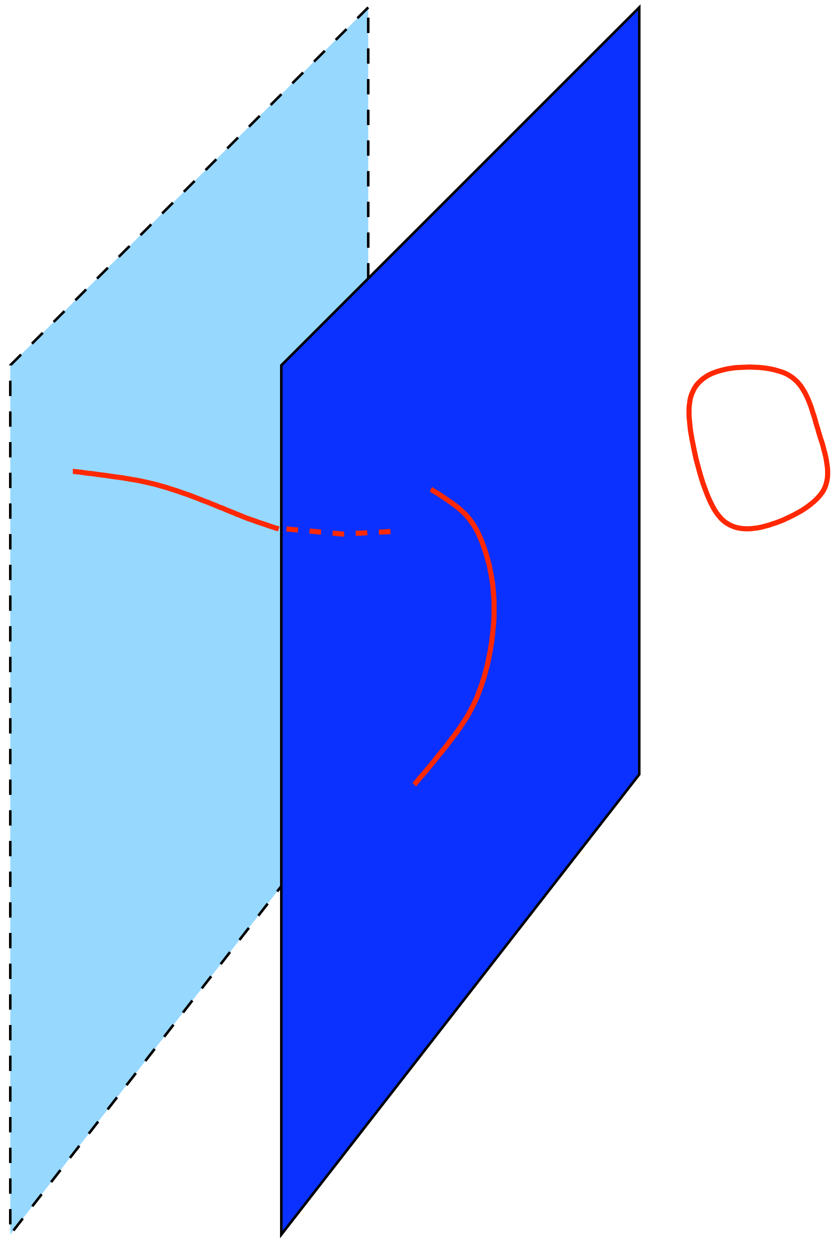
The Tools (II)

Think of open string sectors as existing within a type II (closed) string theory, as D-branes, where the endpoints lie.



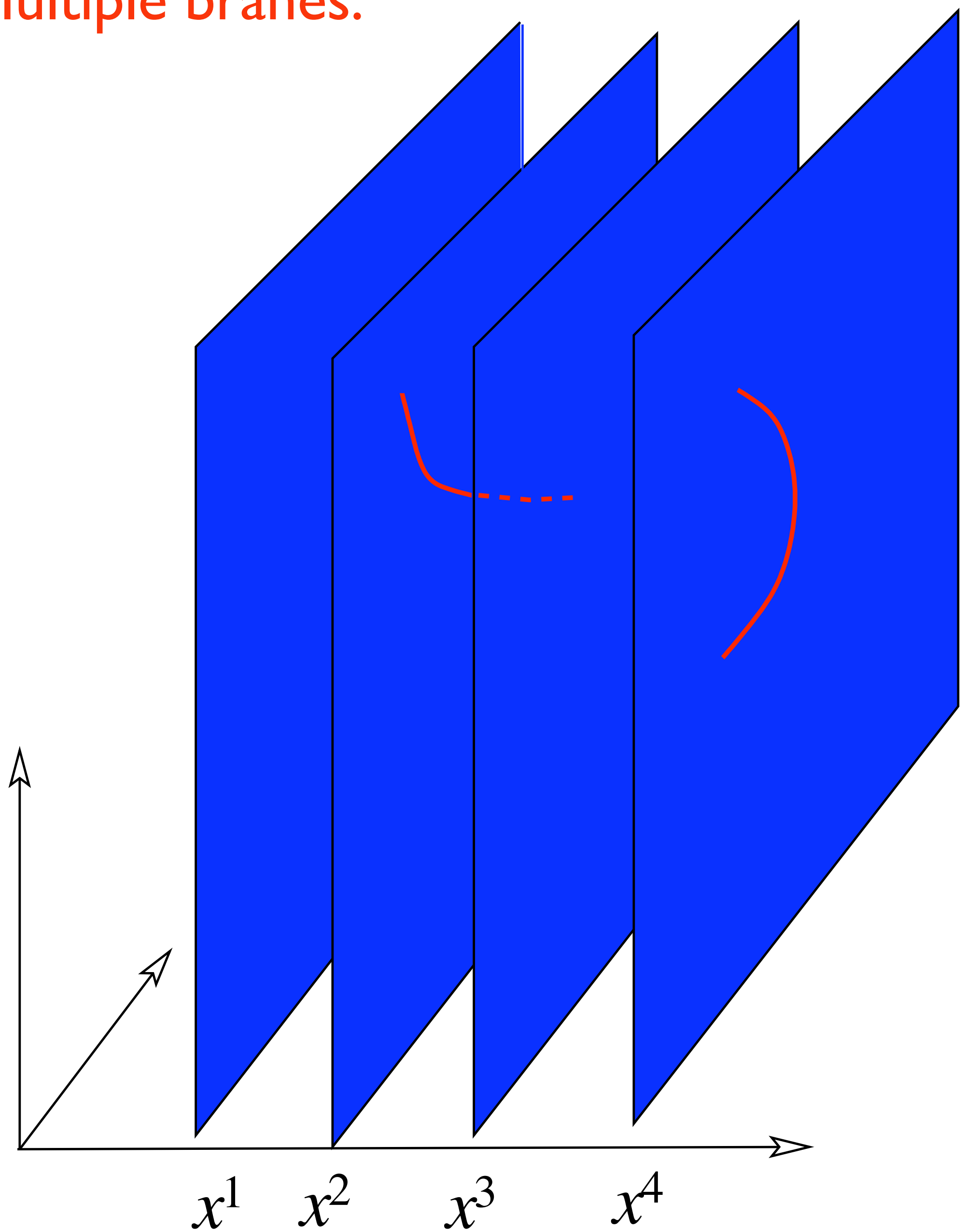
p extended directions: **Dp-brane**

Open string degrees of freedom give a $U(1)$ gauge theory in the $(p+1)-$dimensions of its "worldvolume"

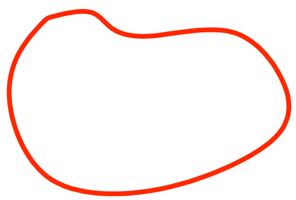


p extended directions: **Dp-brane**

Multiple branes:

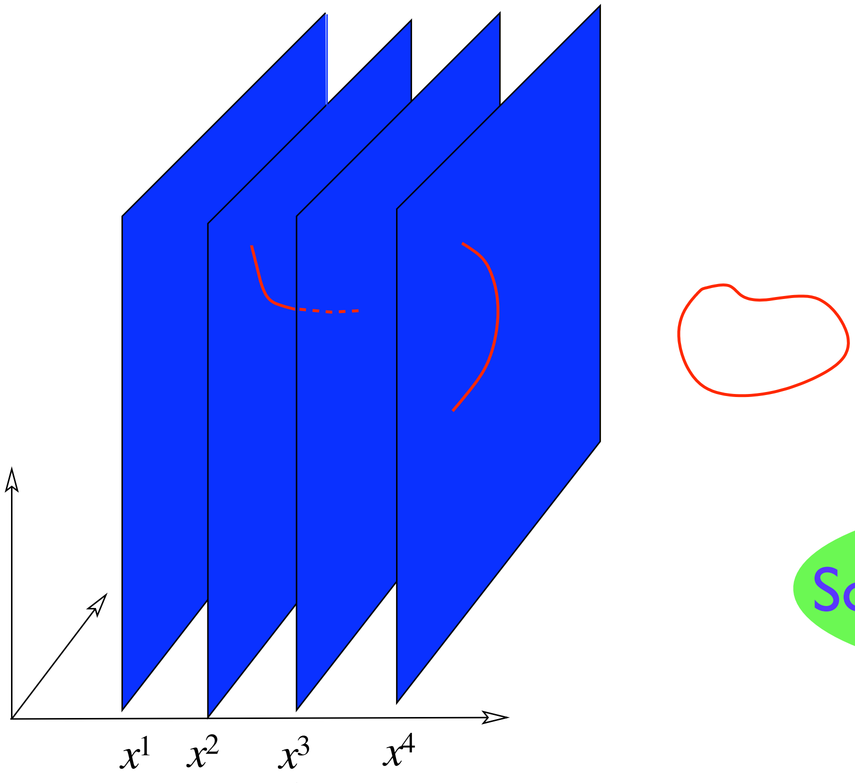


$SU(N)$ gauge theory



Witten '95, Polchinski ' 89-95

Multiple branes:



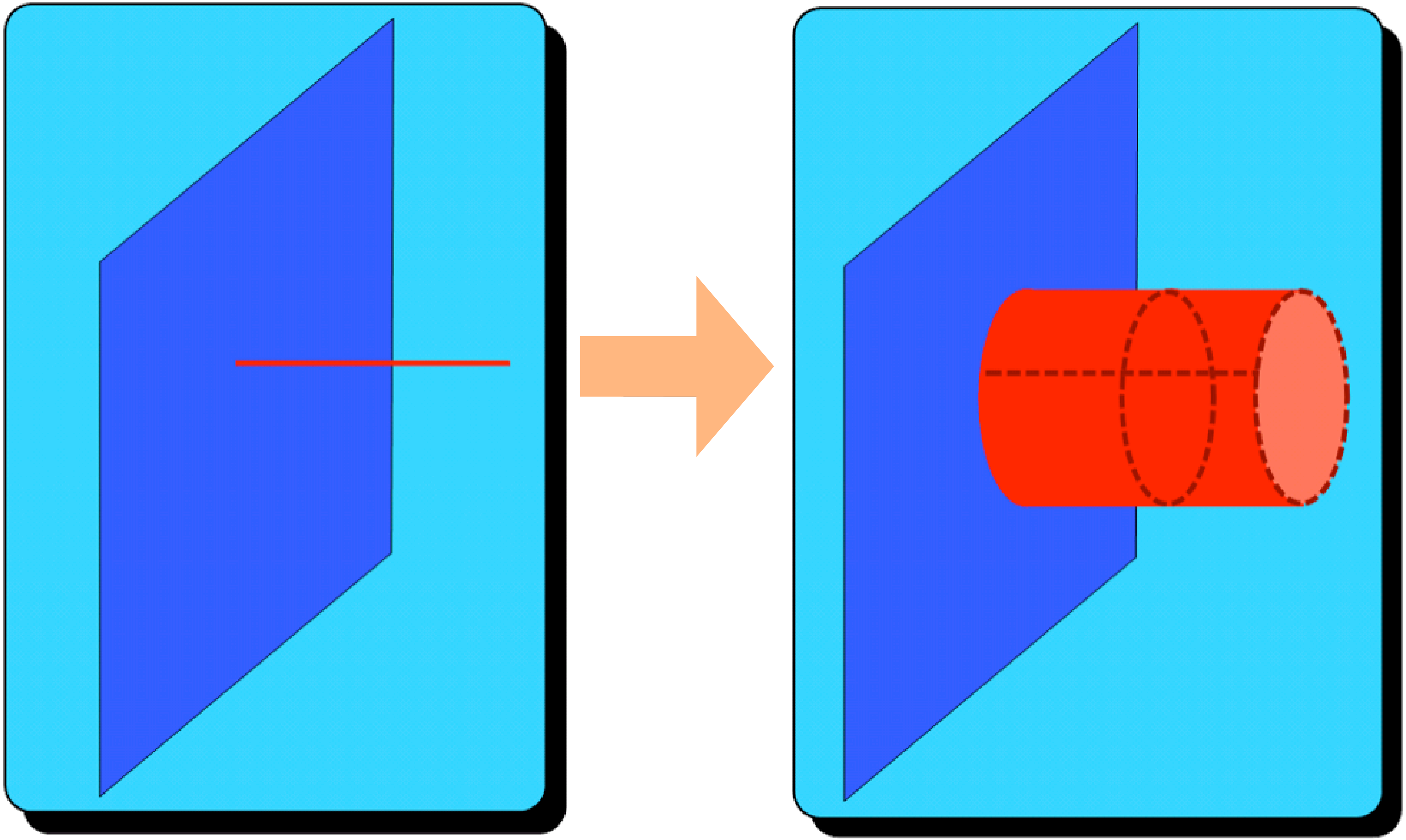
Pulling branes apart is a Higgs mechanism, from worldvolume perspective:

Some fields get vevs

Transverse flucts of strings with both ends on same brane

Some fields get masses

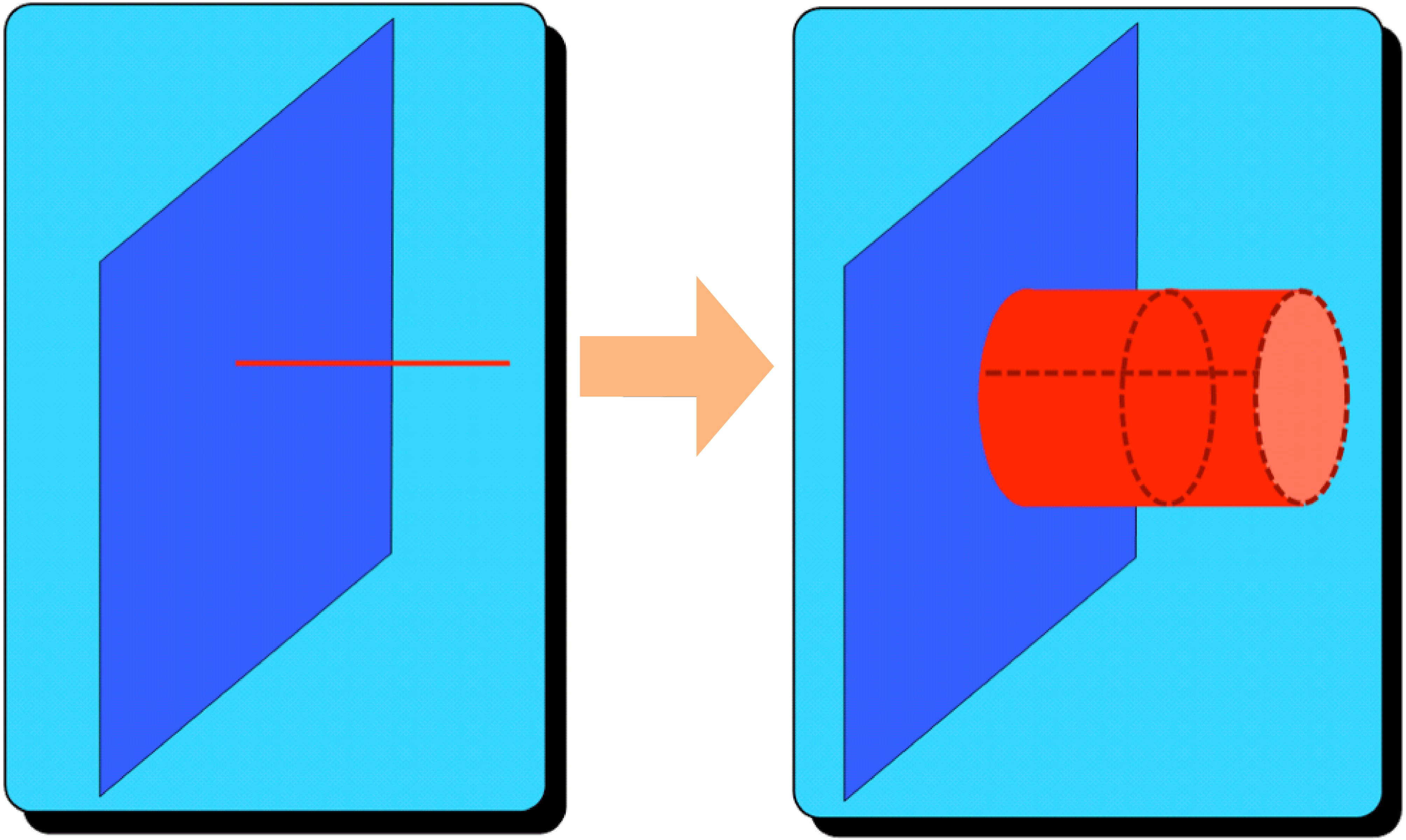
Transverse flucts of strings with ends on different branes



NS-NS: → $G_{\mu\nu}, \Phi$

Within the open/closed description, one can describe the D-branes' natural sourcing of closed string fields

R-R: → $C_{\mu_0\mu_1\cdots\mu_p}$



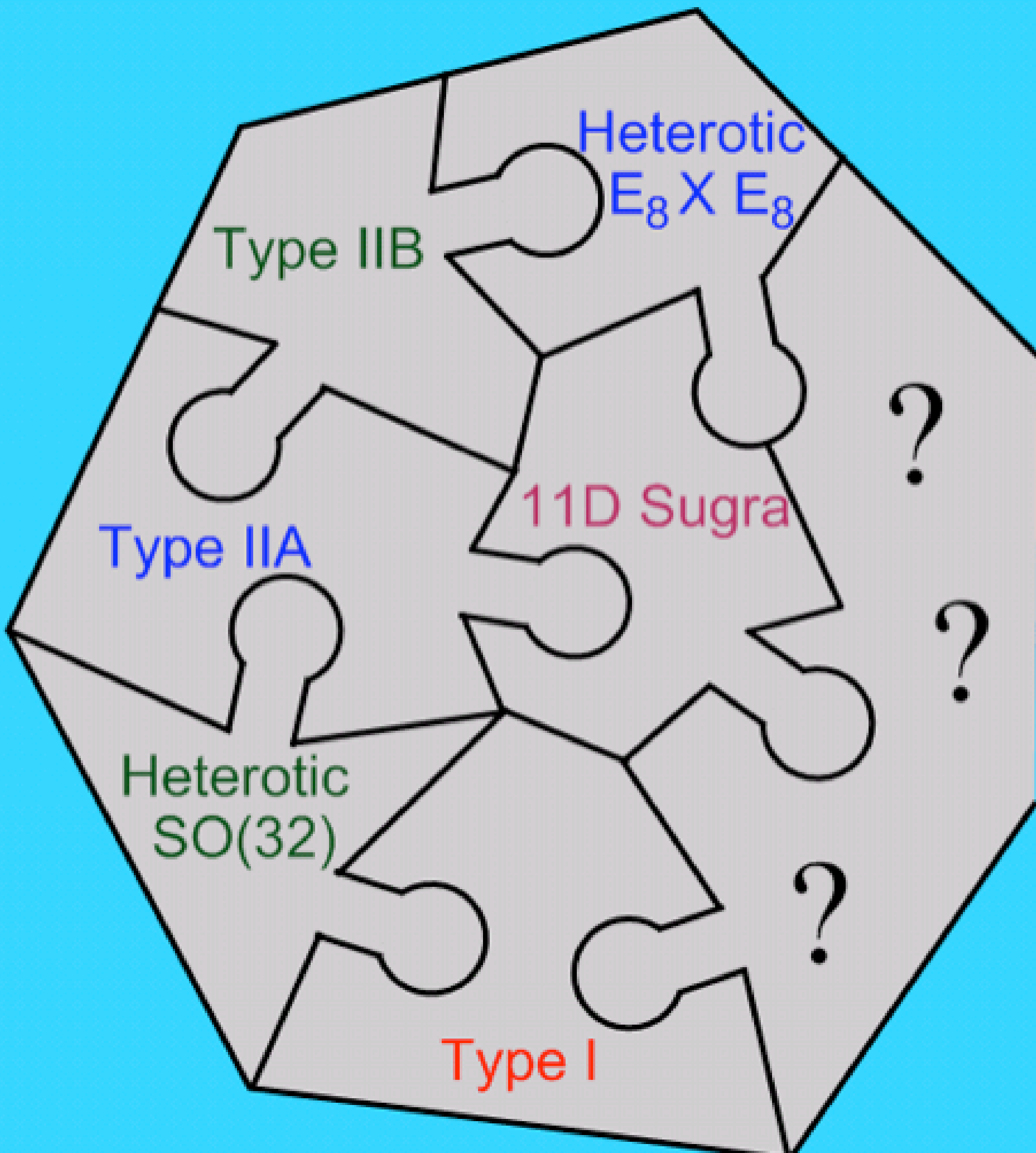
NS-NS: $\longrightarrow G_{\mu\nu}, \Phi$

This is how D-branes interact with the closed string sector.

R-R: $\longrightarrow C_{\mu_0\mu_1\cdots\mu_p}$

Polchinksi 89-95

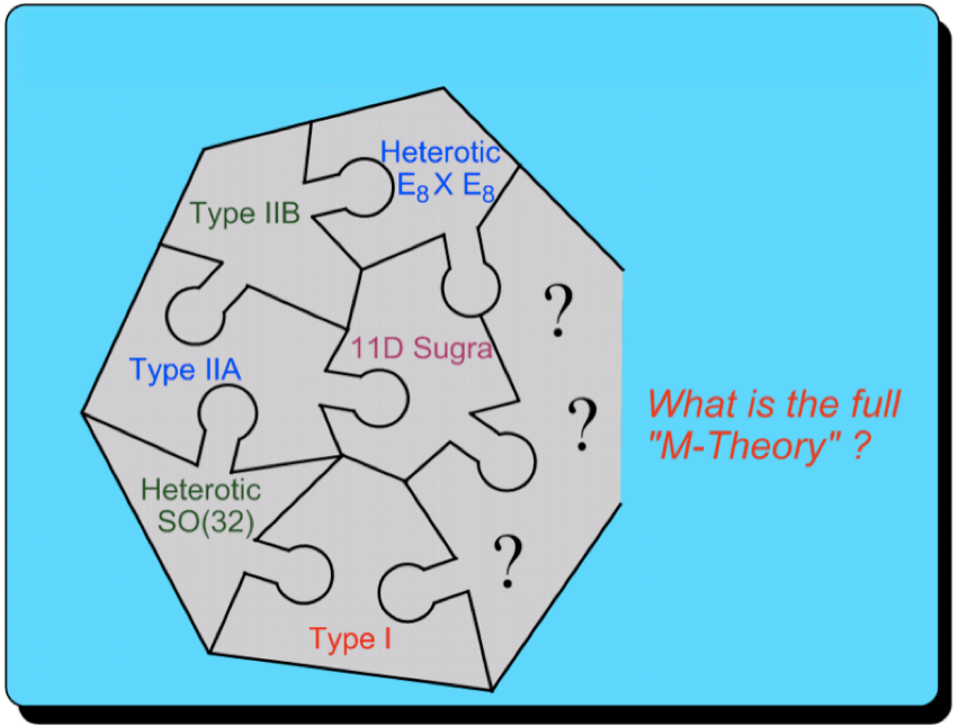
The various theories all fit together into a larger picture.



What is the full "M-Theory" ?

Witten '95 Hull-Townsend '94.....

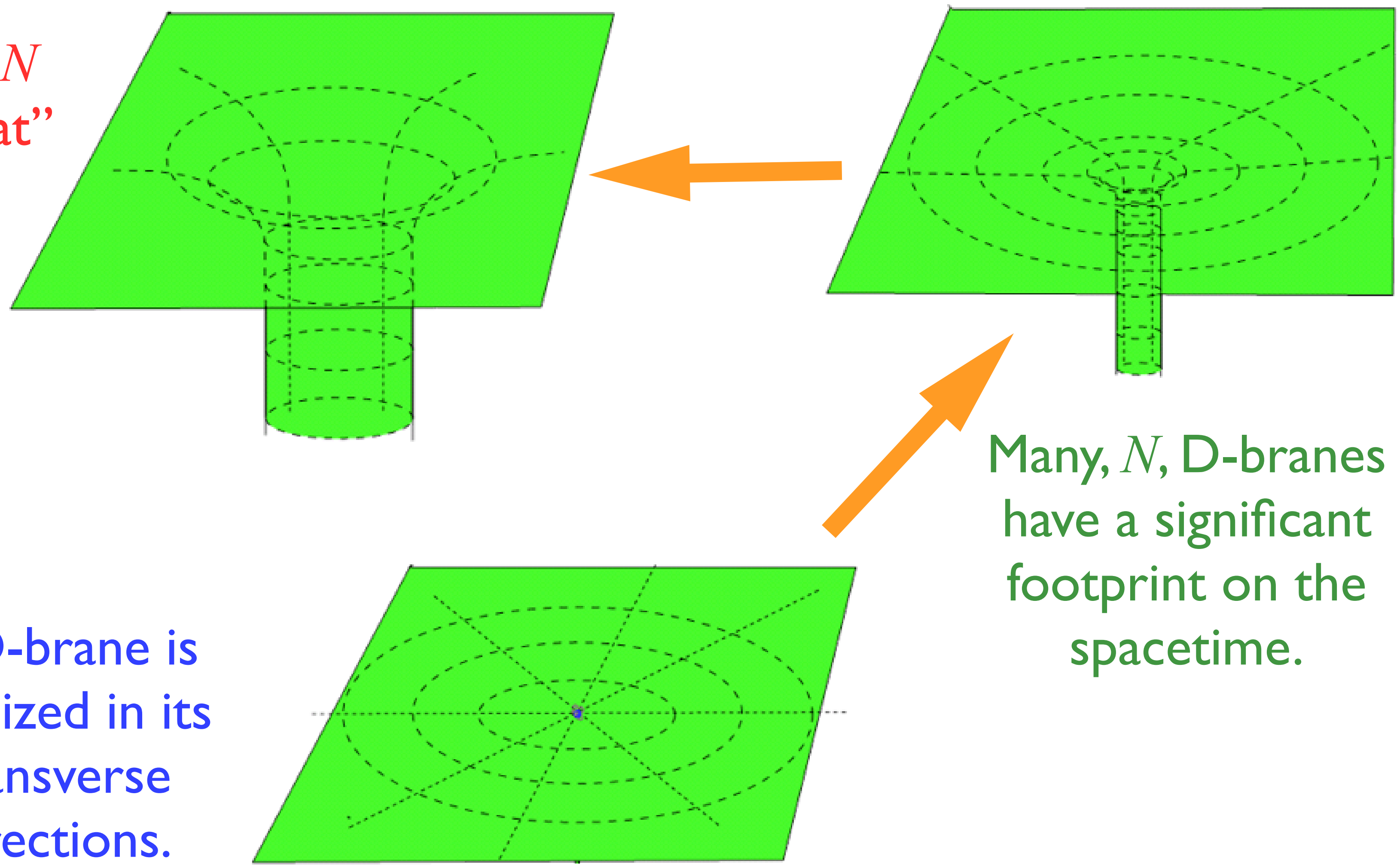
It is entirely possible that we will not fully complete the quest to understand stringy QCD until we get more to grips with M-theory.....



But exciting progress has been made by working with what we have so far......

The Journey to AdS/CFT

In the large N limit, a “throat” opens up.



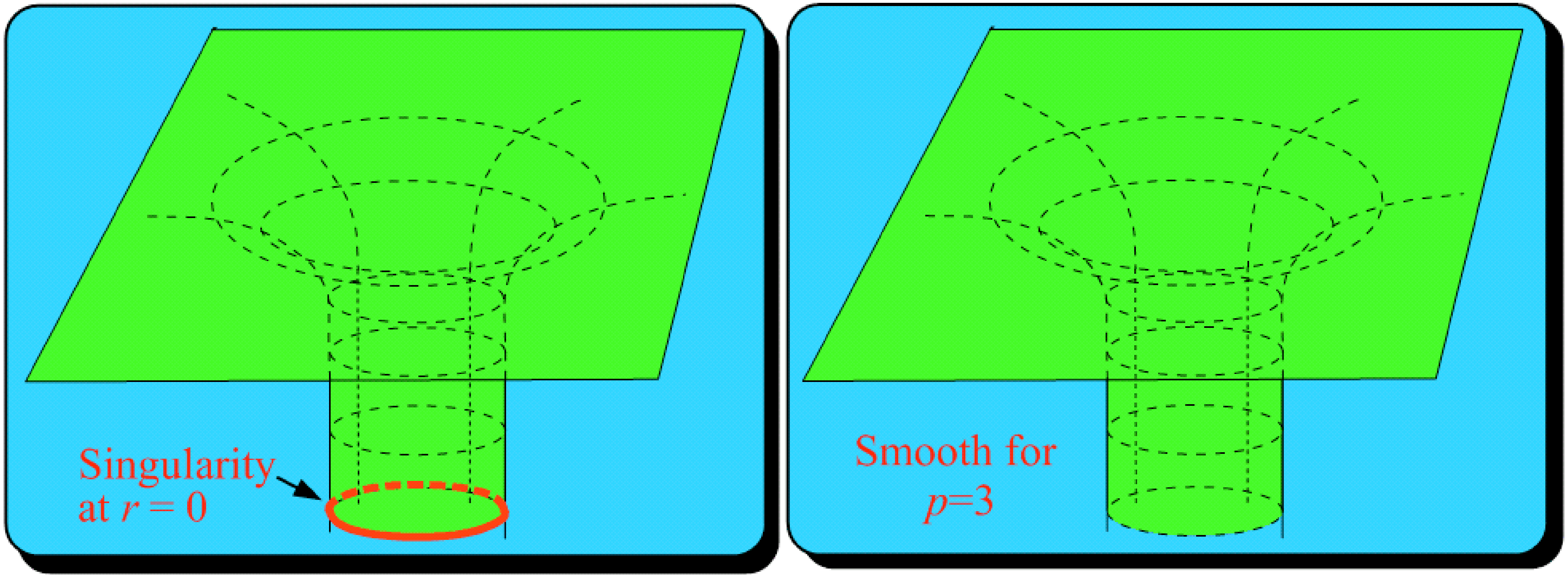
Many, N, D-branes have a significant footprint on the spacetime.

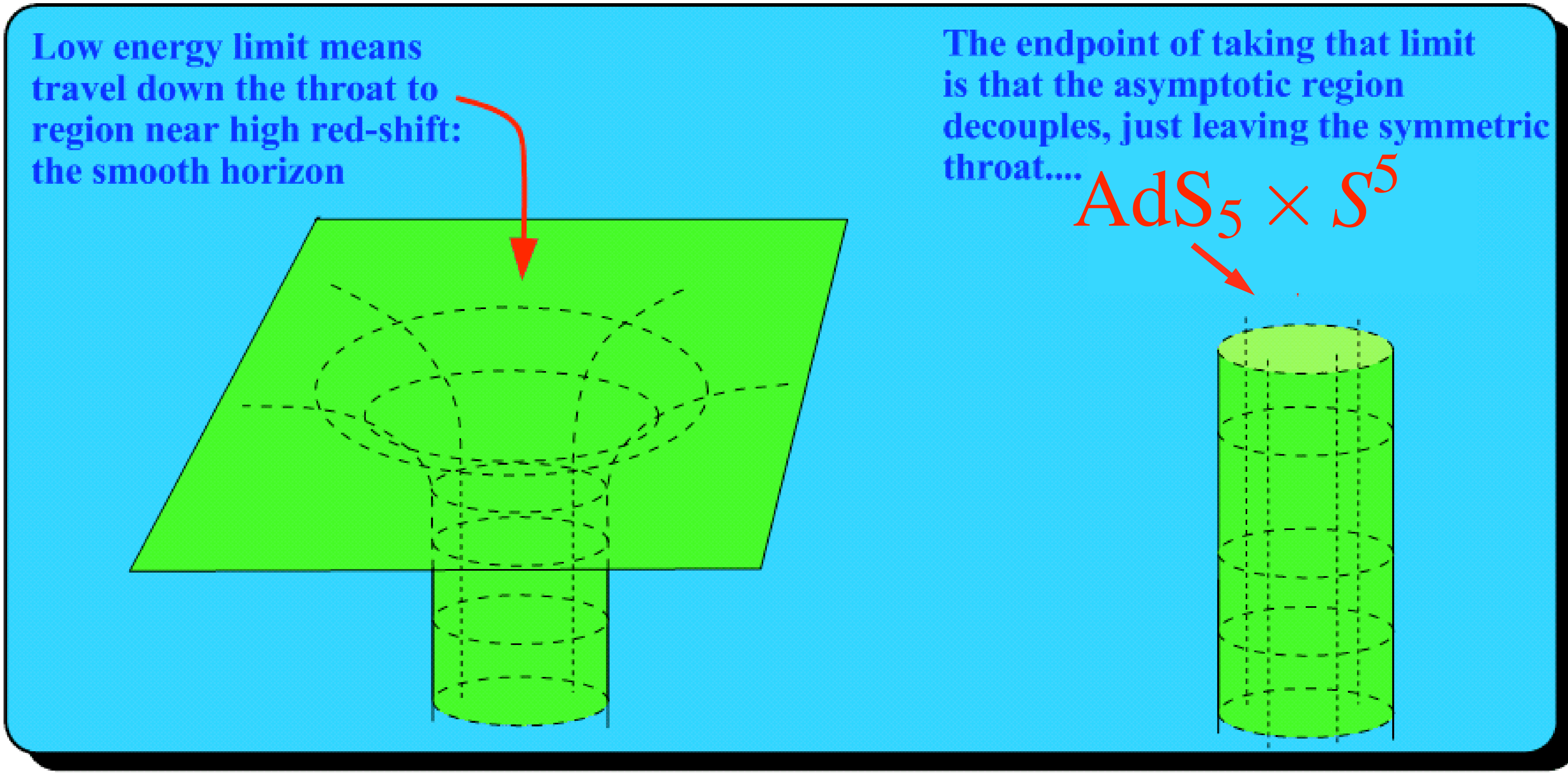
A D-brane is localized in its transverse directions.

Too many authors to mention.....

D3-branes are special.

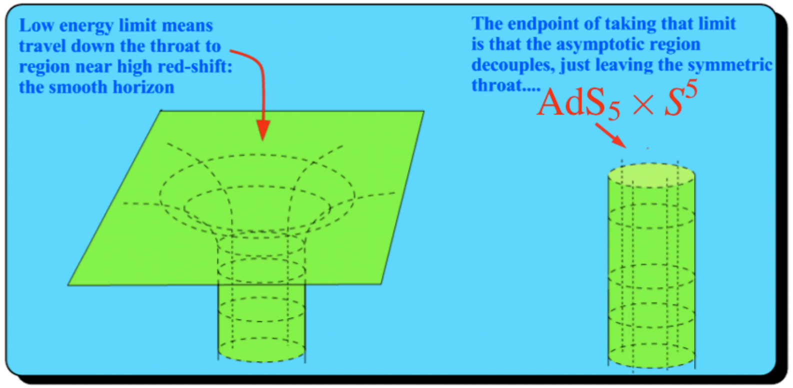
They naturally fill a *D=4* spacetime...





This is the same limit in which the *D=4* theory is *SU(N)* Yang-Mills Theory

Maldacena '97, Gubser, Klebanov, Polyakov '97-98



The solution for the D3-branes:

Horowitz-Strominger '89

$$ds^2 = \left(1+\frac{R^4}{r^4}\right)^{-1/2}(-dt^2+dx_1^2+dx_2^2+dx_3^2)+\left(1+\frac{R^4}{r^4}\right)^{1/2}(dr^2+r^2d\Omega_5^2)$$

$$R^4 = 4\pi g_s N(\alpha')^2 = \ell^4(\alpha')^2 \qquad e^{2\Phi} = g_s^2$$

$$C_{(4)} = -\left(\frac{R^4 g_s^{-1}}{R^4+r^4}\right) dx_0\wedge\cdots\wedge dx_3$$

Take the limit

$$r\to 0\ ,\ \alpha'\to 0$$

Hold fixed:

$$u = \frac{r}{\alpha'}$$

(With N units of 5-form flux on sphere)

Result:

$$ds^2 = \frac{\ell^2}{u^2}du^2+\frac{u^2}{\ell^2}(-dt^2+dx_1^2+dx_2^2+dx_3^2)+\ell^2 d\Omega_5^2$$

The Correspondence

$$g_{\mathrm{YM}}^2 = 2\pi g_s$$ g_s small

$$g_{\mathrm{YM}}^2 N = \lambda$$ N large

λ large — $g_s N$ large

$$G_5 = \pi \ell^3 / (2N^2)$$

We are studying strongly coupled gauge theory using gravity.

The Correspondence

So D=4, N=4, large N $SU(N)$ gauge theory ⟷ Type IIB Supergravity on $\mathrm{AdS}_5 \times S^5$

conformal group $SO(4,2)$ — isometry group $SO(4,2)$

R-symmetry group $SO(6) \subset SU(4)$ — isometry group $SO(6)$

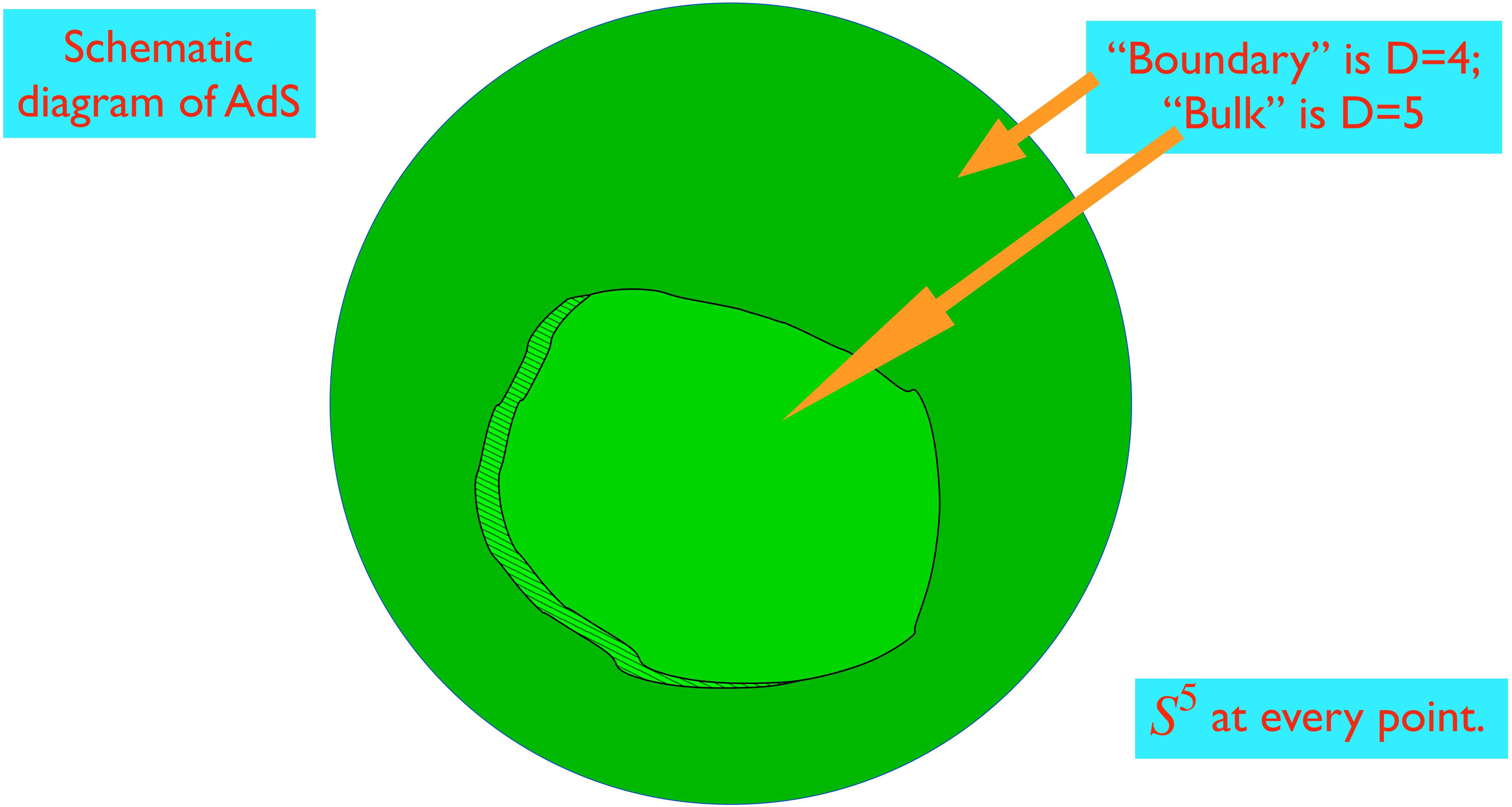
$$L = \frac{N}{4g_{\mathrm{YM}}^2} \mathrm{Tr}\left[-F_{\mu\nu}F^{\mu\nu} - 2\sum_{i=1}^{6}(D_\mu \varphi^i)^2 + \sum_{i<j}[\varphi^i,\varphi^j]^2 \right]$$

$$-\frac{iN}{2g_{\mathrm{YM}}^2}\mathrm{Tr}\left[\bar{\lambda}\Gamma^\mu D_\mu \lambda + i\bar{\lambda}\Gamma_i[\varphi^i,\lambda]\right]$$

φ in $\mathbf{6}$

λ in $\mathbf{4}$

$\bar{\lambda}$ in $\bar{\mathbf{4}}$



$$ds^2 = -\left(1+\frac{u^2}{\ell^2}\right)dt^2 + \left(1+\frac{u^2}{\ell^2}\right)^{-1} du^2 + u^2 d\Omega_3^2 + \ell^2 d\Omega_5^2 \qquad \text{global}$$

$$ds^2 = \frac{\ell^2}{u^2}du^2 + \frac{u^2}{\ell^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \ell^2 d\Omega_5^2 \qquad \text{local}$$

conformal group $SO(4,2)$

isometry group $SO(4,2)$

R-symmetry group $SO(6) \subset SU(4)$

isometry group $SO(6)$

$$Z_{\mathrm{FT}}(\partial M, \phi_{0,k}) = Z_{\mathrm{grav}}(M, \phi)$$

$$I_{\mathrm{FT}} \to I_{\mathrm{FT}} + \int_{\partial M} d^4y\, \phi_{0,k}(y) O_k(y)$$

Maldacena '97, Witten '98

So D=4, N=4, large N $SU(N)$ gauge theory ⟷ Type IIB Supergravity on $\mathrm{AdS}_5 \times S^5$

Asymptotics of bulk fields determine properties of boundary insertions

$$\phi(u,y) \rightarrow e^{\frac{u}{\ell}(\Delta-4)}\phi_m(y) + e^{-\frac{u}{\ell}\Delta}\phi_v(y)$$

non-normalisable — deformation (e.g. mass)

normalisable — vev

This is the origin of, e.g., the glueball technology....

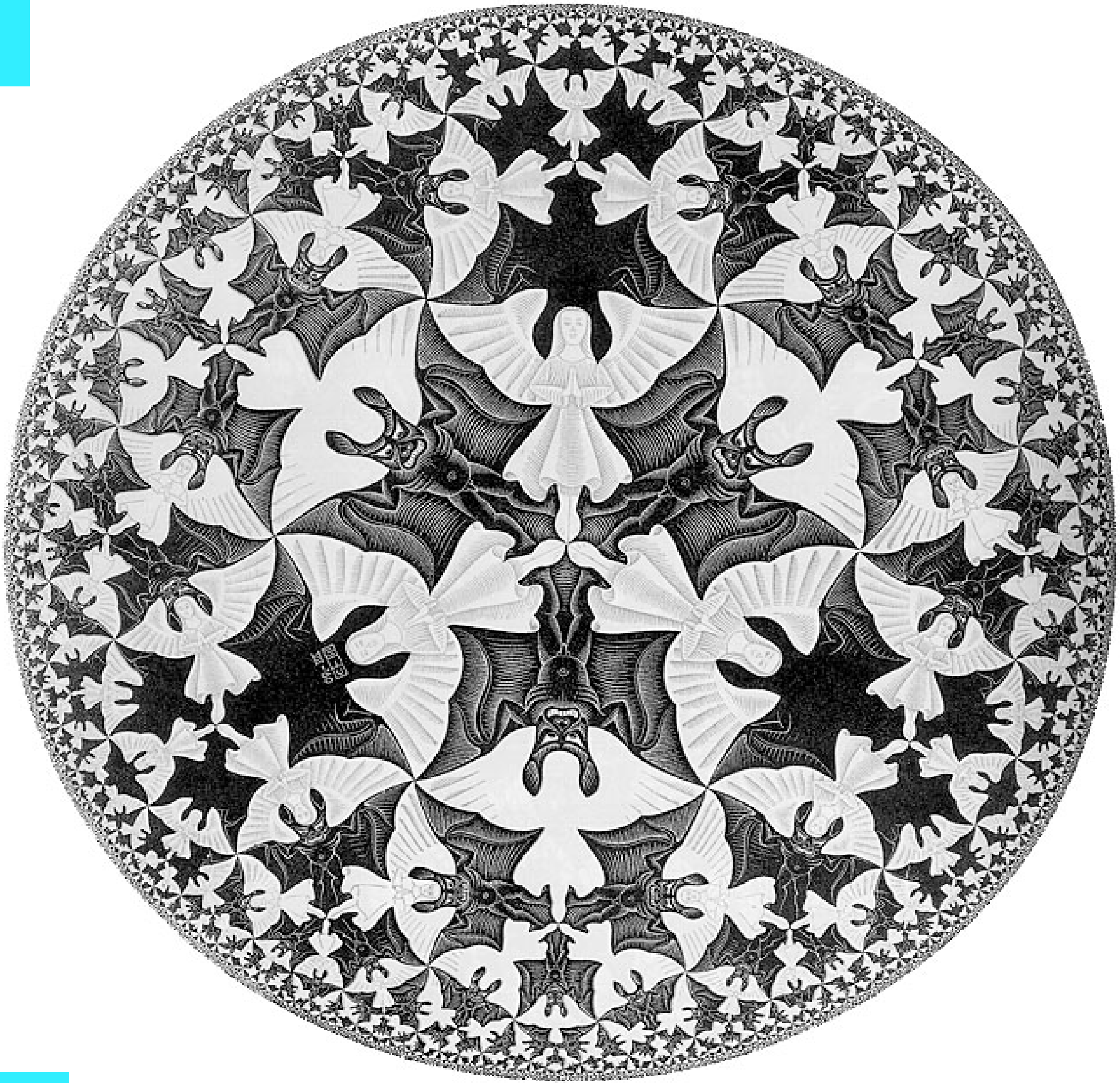
$$\Delta = 2 + \sqrt{4 + m^2\ell^2}$$

GKP'98, Witten '98

Geometry of AdS

Metric diverges at boundary.

large u = UV
small u = IR

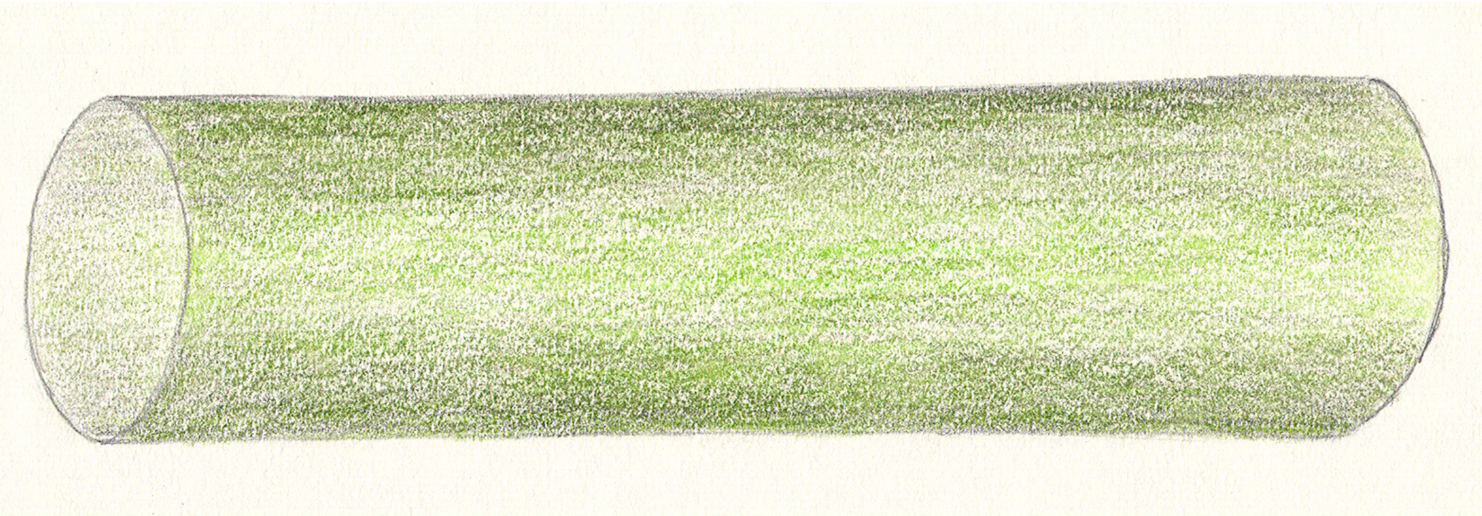


Can only choose metric on it up to a conformal factor.

$$ds^2 = \frac{\ell^2}{u^2}du^2 + \frac{u^2}{\ell^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \ell^2 d\Omega_5^2$$

The Usefulness of Throats

"Throat" structures turn up a lot in string physics now. AdS is just one example.

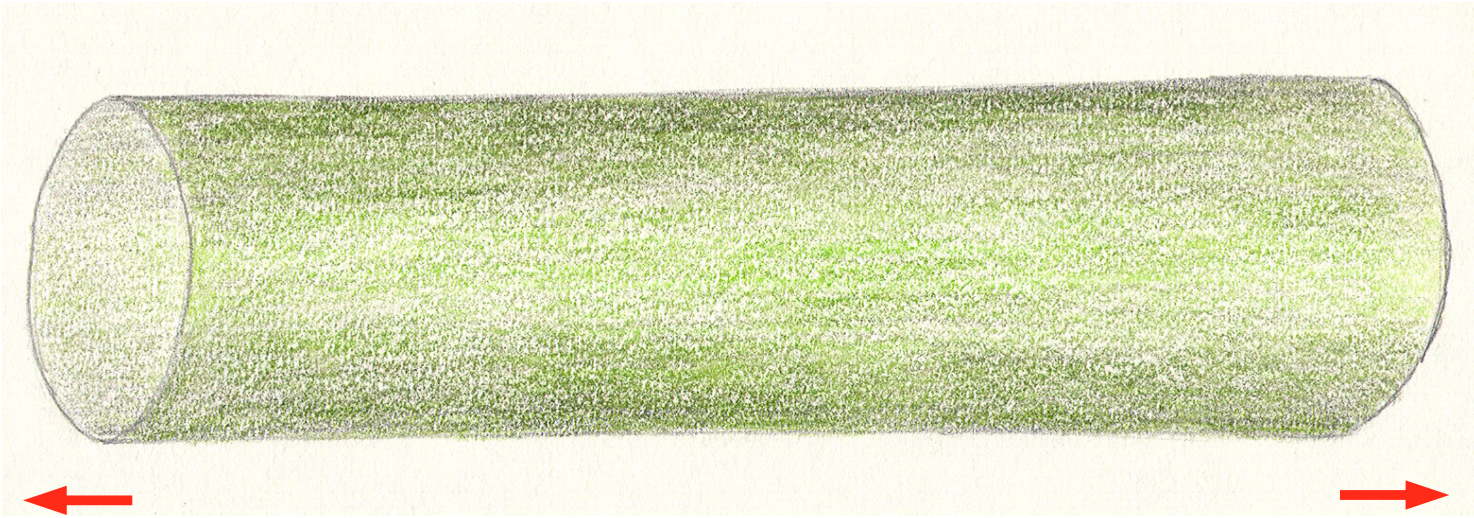


Another representation:

Metric of a cross-section

$$ds^2 = e^{2A(\perp)} \overbrace{g_{\mu\nu} dx^\mu dx^\nu} + ds_\perp^2$$

"warp factor"

IR  UV

decreasing r ← → increasing r

$$ds^2 = e^{2A(\perp)} g_{\mu\nu} dx^\mu dx^\nu + ds_\perp^2$$

Our previous coordinate:

$$u = \frac{\ell}{\alpha'} e^{r/\ell}$$

AdS_5 :

$$A(\perp) = \frac{r}{\ell}$$

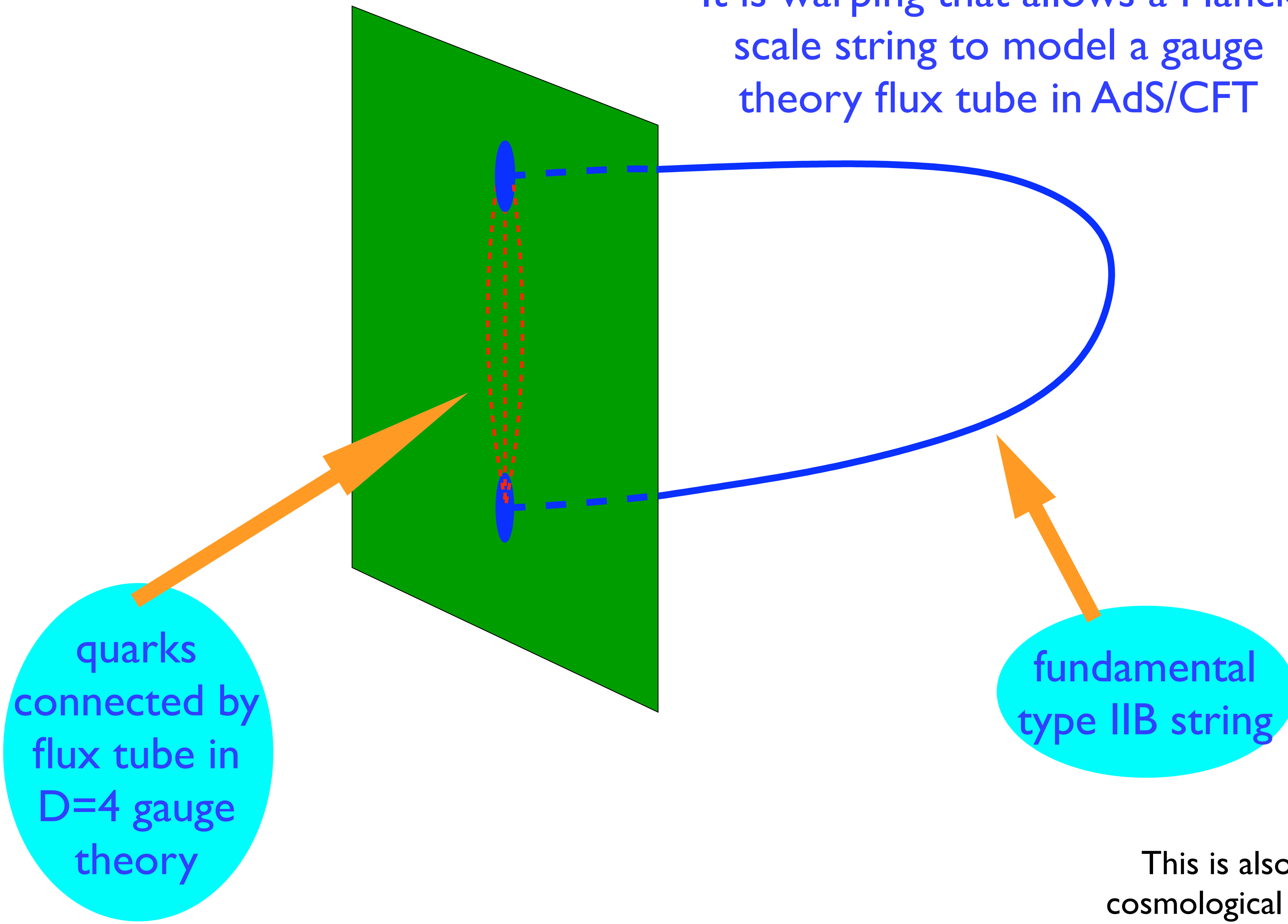
$$ds_\perp^2 = dr^2$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

At some value of r: $L_{10}^2 = e^{2A(r)} L_4^2$

So the warp factor gives small D=4 scales for large r and vice-versa!

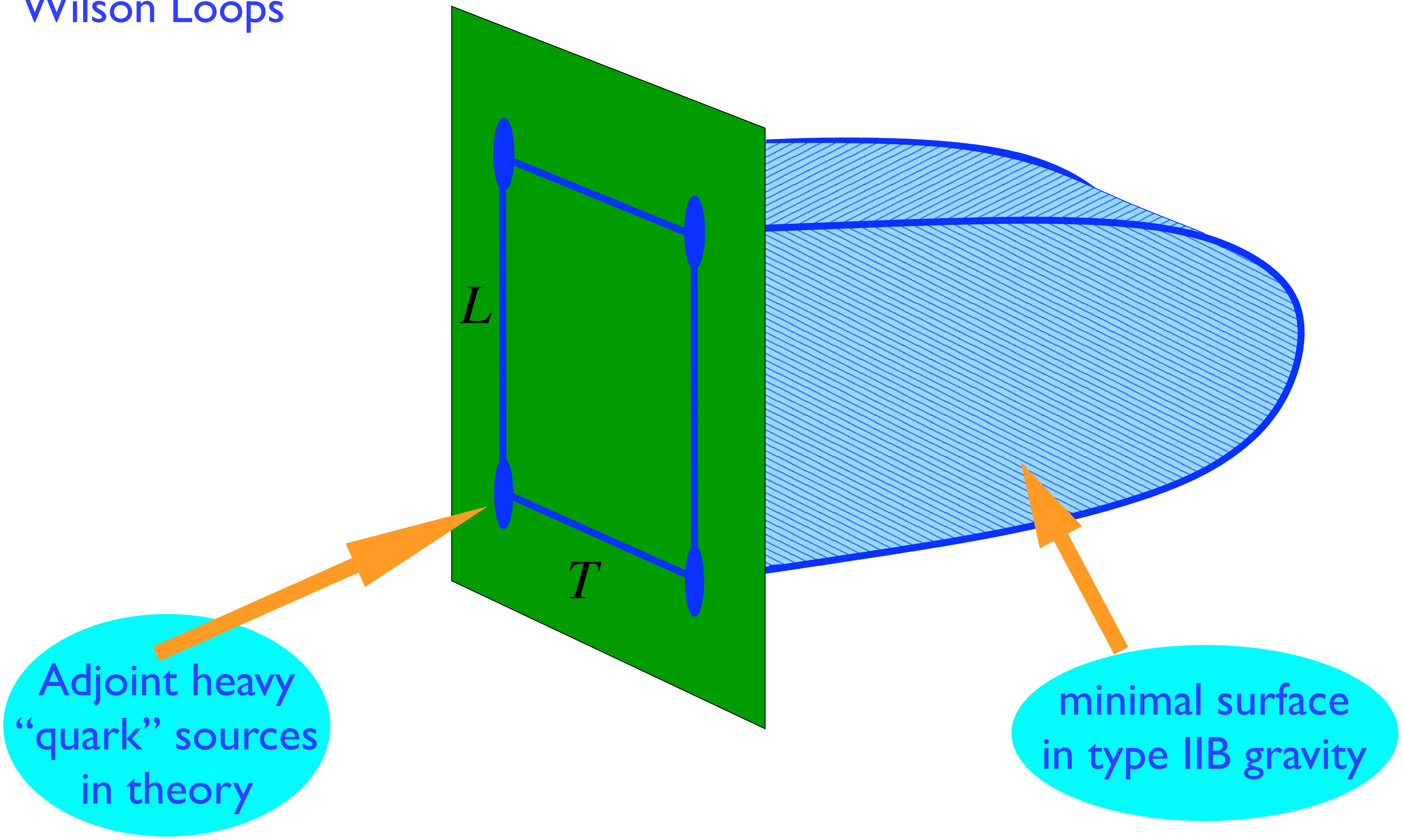
It is warping that allows a Planck scale string to model a gauge theory flux tube in AdS/CFT



This is also useful in cosmological scenarios....

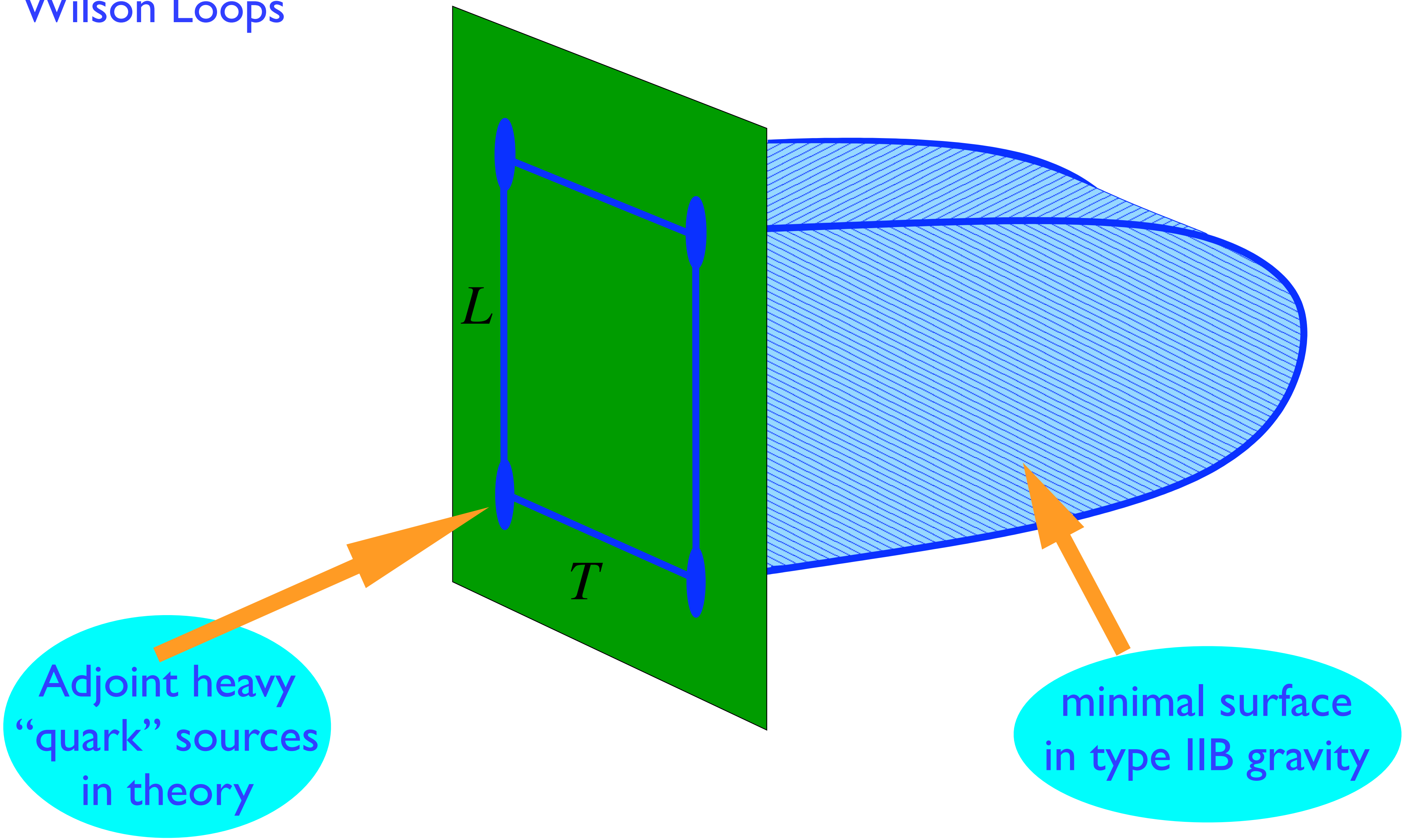
Probing the Correspondence I

Wilson Loops



Maldacena ‘98, Witten ‘98, Rey et al ‘98

Wilson Loops



Quark-anti quark potential $E \sim 1/L$, which follows from conformal invariance....

Probing the Correspondence II

For finite temperature, use "black brane" metric:

$$ds^2 = \frac{u^2}{l^2}\left(-\left(1-\frac{\mu}{u^4}\right)dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \left(1-\frac{\mu}{u^4}\right)^{-1}\frac{l^2}{u^2}du^2$$

Horizon located at: $u = \mu^{\frac{1}{4}}$

Carry out the Euclidean calculus:

$$\beta \left.\frac{dG_{tt}}{du}\right|_{u=\mu^{1/4}} = 4\pi \quad \longrightarrow \quad T = \frac{\mu^{1/4}}{\pi \ell^2}$$

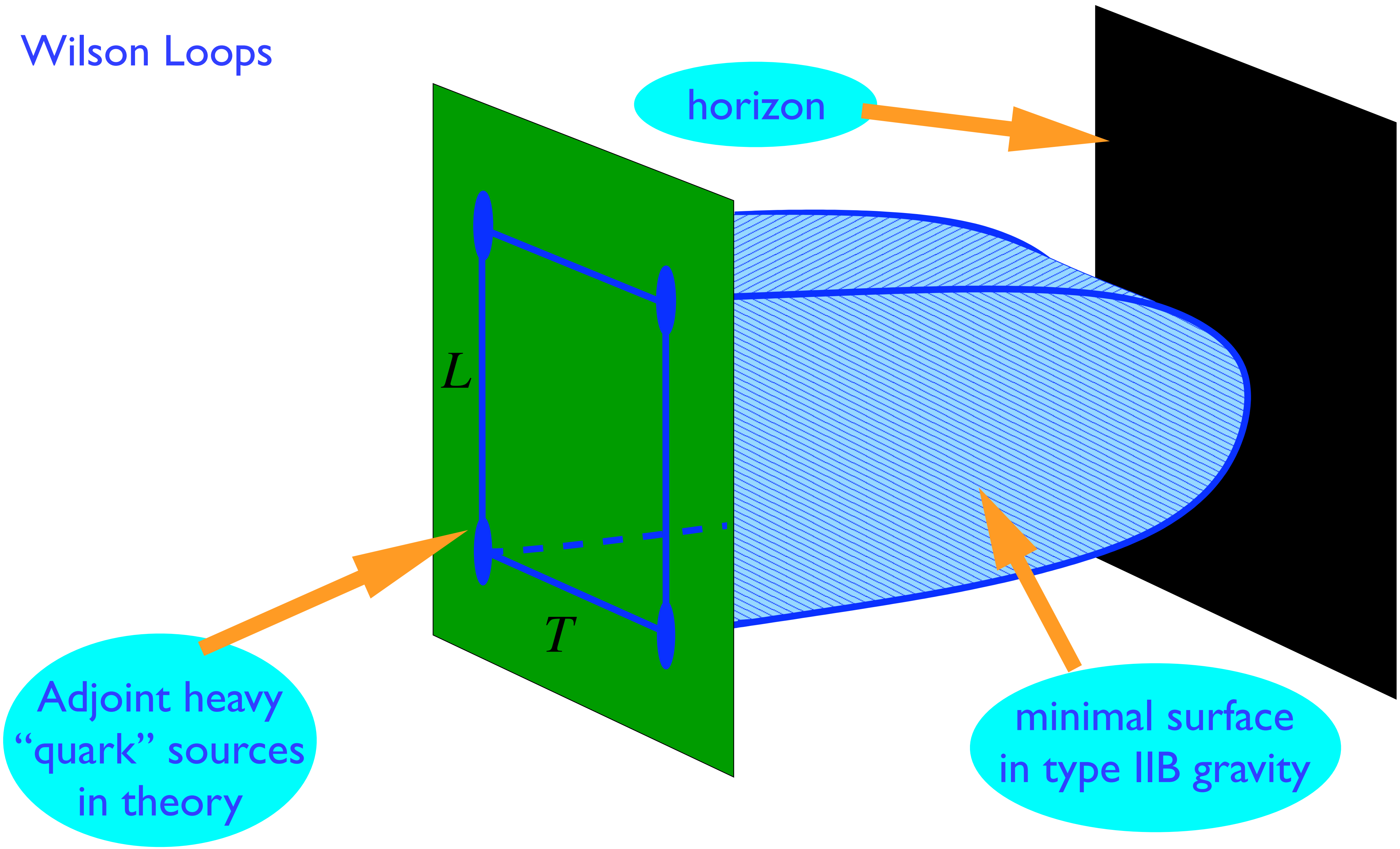
$$\frac{<E>}{V} = \frac{3\mu N^2}{8\pi^2 \ell^8} = \frac{3}{8}\pi^2 T^4 N^2$$

$$S = \frac{A}{4G_5} = \frac{\pi^2}{2} T^3 V N^2$$

Gubser Klebanov Peet '96, Strominger '96

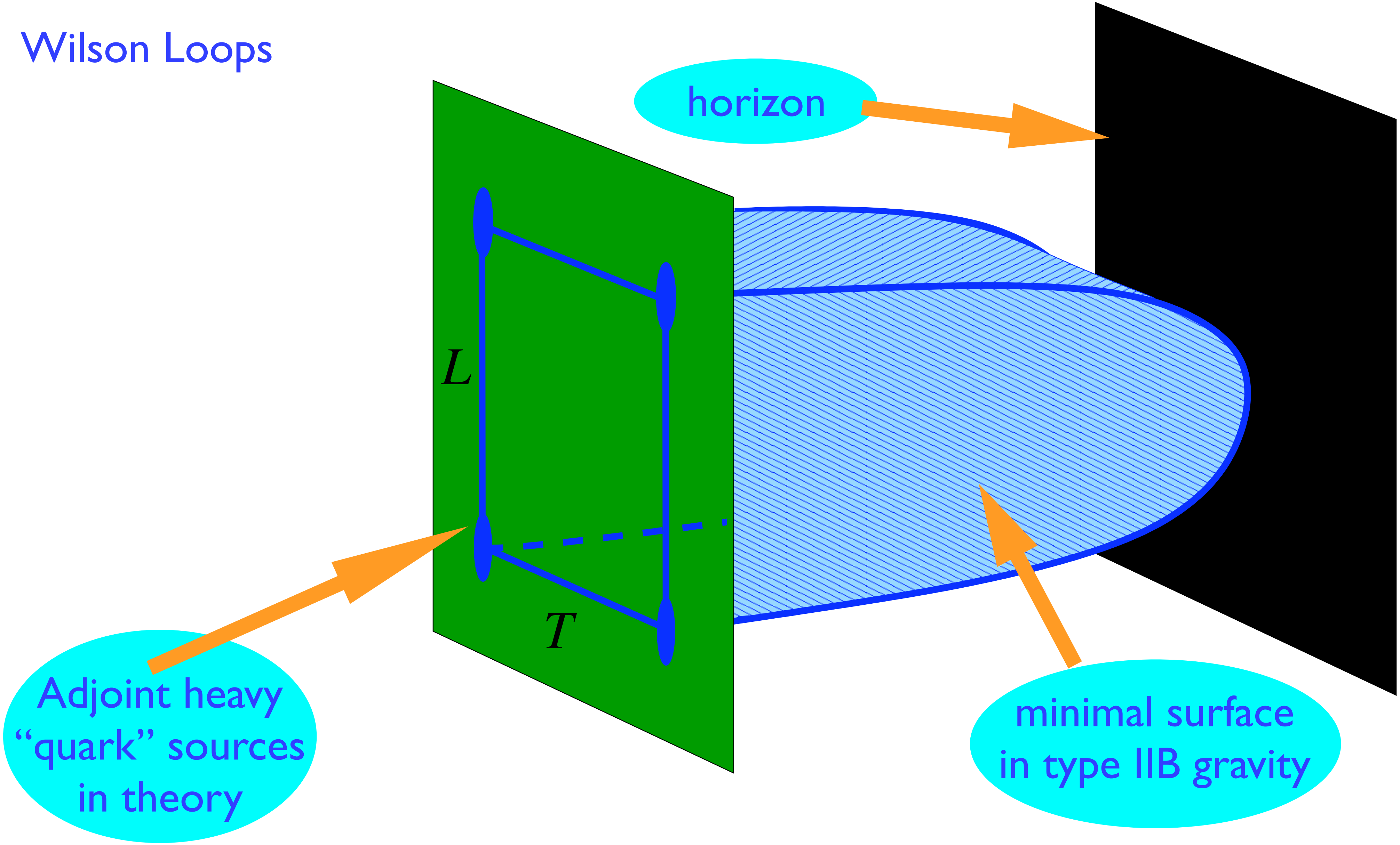
Probing the Correspondence III

Wilson Loops



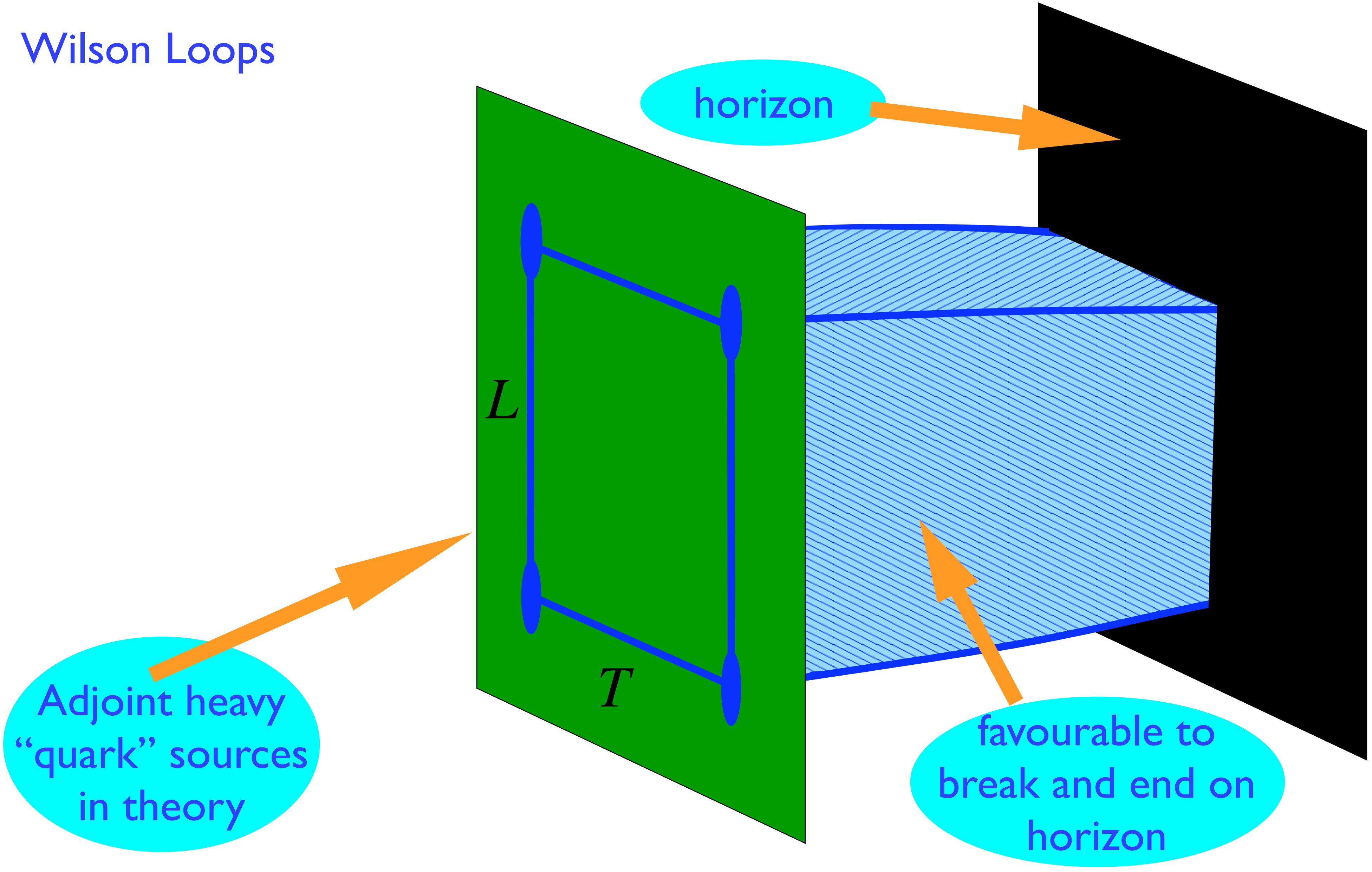
Maldacena '98, Witten '98, Rey et al '98, Branhuber et,al '98 ....

Wilson Loops



Below scale set by T, $E \sim 1/L$, (Coulomb)
But (well) above that scale, $E \sim 0$ (No force): "Deconfinement"!

Wilson Loops



Probing the Correspondence IV

Phase transitions.

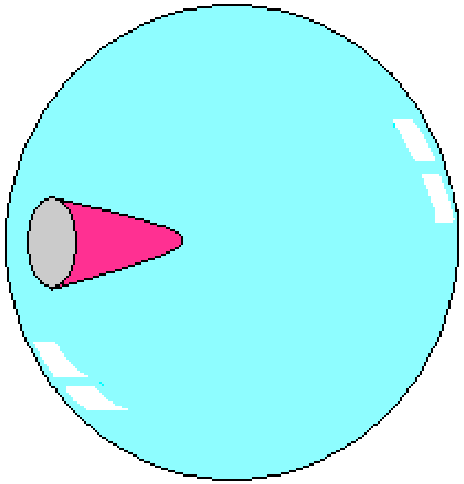
Can't have phase transition in conformal field theory.

Introduce scale by putting theory in a box.....But still have phase transitions since N large.

$$ds^2 = -\left(1+\frac{u^2}{\ell^2}\right)dt^2 + \left(1+\frac{u^2}{\ell^2}\right)^{-1} du^2 + u^2 d\Omega_3^2$$

Theory is now on $R \times S^3$

Behaviour of Wilson loops still same....



$$ds^2 = -\left(1+\frac{u^2}{\ell^2}\right)dt^2 + \left(1+\frac{u^2}{\ell^2}\right)^{-1}du^2 + u^2 d\Omega_3^2$$

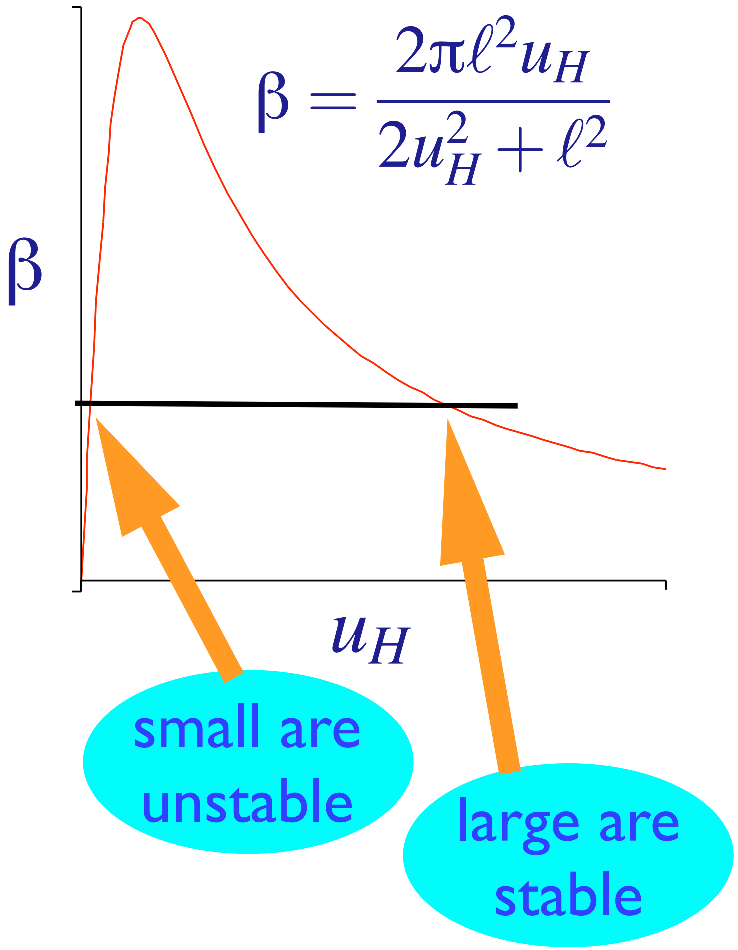
At finite temperature,
must consider also
AdS-Schwarzschild:

$$\beta = \frac{2\pi\ell^2 u_H}{2u_H^2 + \ell^2}$$

$$ds^2 = -\left(1 - \frac{\mu}{u^2} + \frac{u^2}{\ell^2}\right) dt^2 + \left(1 - \frac{\mu}{u^2} + \frac{u^2}{\ell^2}\right)^{-1} du^2 + u^2 d\Omega_3^2$$

Theory is now on $R \times S^3$

Probing the Correspondence IV

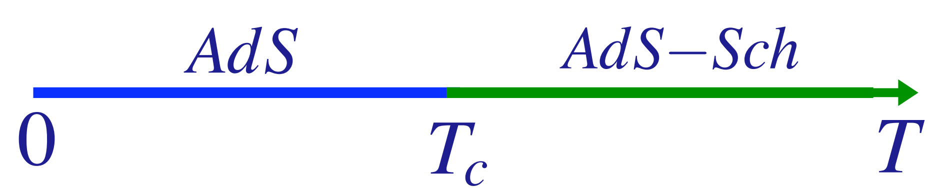


Minimum temperature, $T_{\min}$, below which the holes do not exist.

Both "small" and "large" holes exist for $T > T_{\min}$

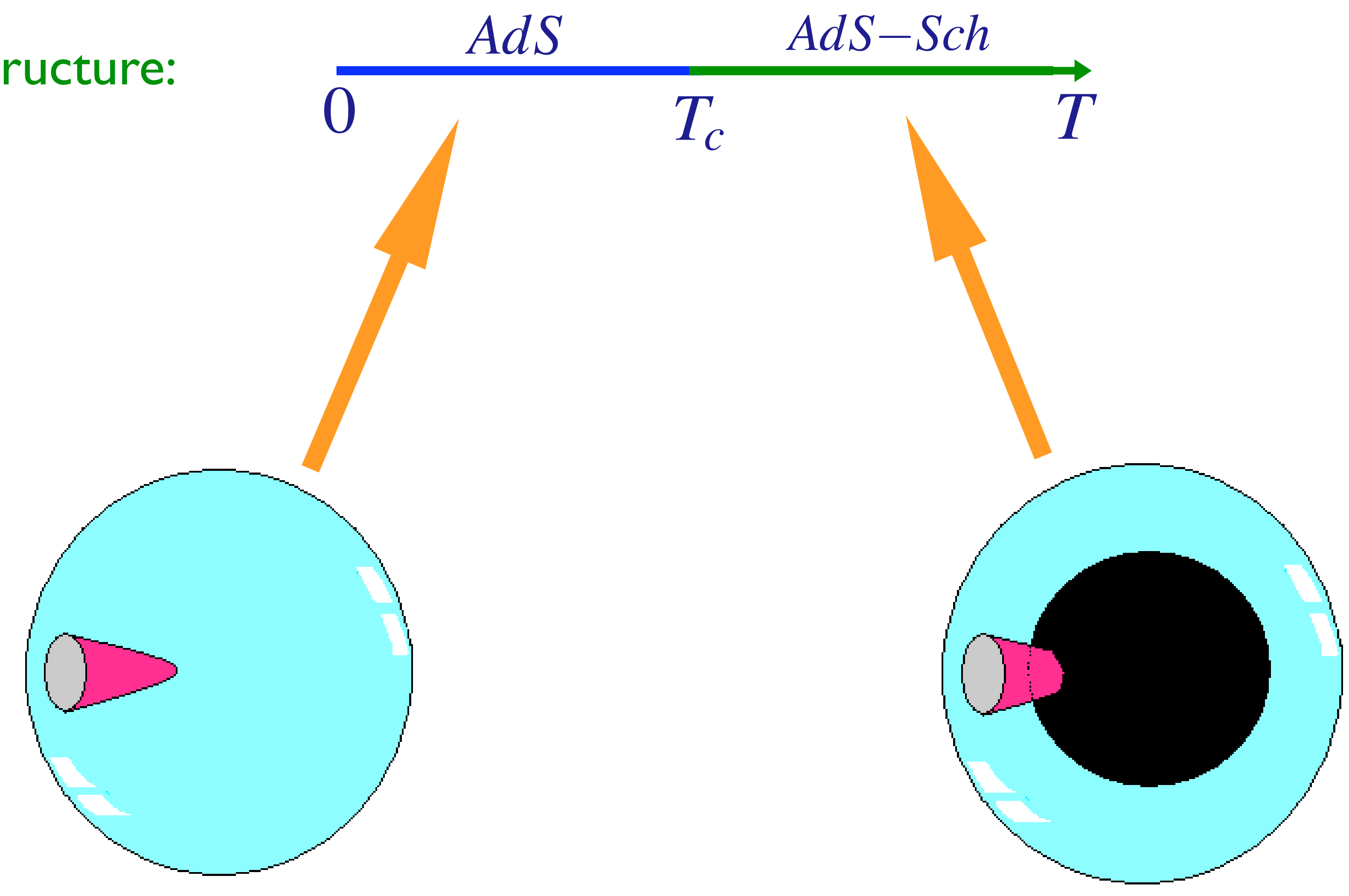
There is a critical temperature, T_c where $F_{\rm BH} < F_{\rm AdS}$

Phase structure:



Witten'98; Hawking-Page '83

Phase structure:



Probing the Correspondence V

Toward finite density

Need to study phase structure in presence of "baryon number"

Looking for a $U(1)$ which plays this role.

Use a subgroup of the R-symmetry $SO(6)$

Diagonal $U(1)$ of Cartan subalgebra is one such choice.

There are other choices

Cvetic-Gubser '99

Chamblin-Emparan-Johnson-Myers '99

For diagonal choice, the relevant solution to consider is a Reissner-Nordstrom black hole in AdS.

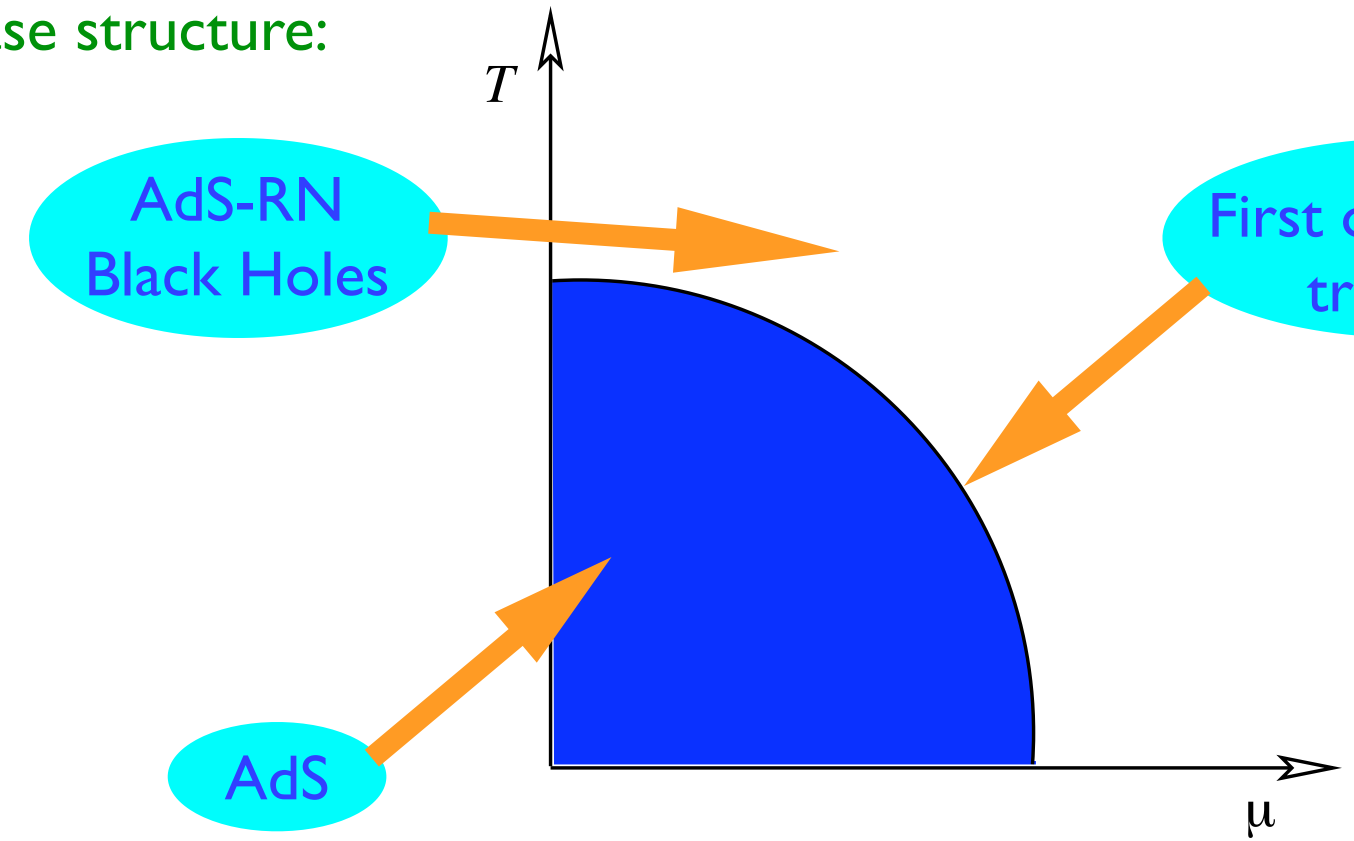
$$ds^2 = -V(u)dt^2 + V(u)^{-1}du^2 + u^2 d\Omega_3^2$$

$$V(u) = 1 - \frac{\mu}{u^2} + \frac{q^2}{u^4} + \frac{u^2}{\ell^2}$$

$$A_t = -\sqrt{\frac{3}{4}}\frac{q}{u^2} + \Phi$$

This couples to chemical potential in the gauge theory.

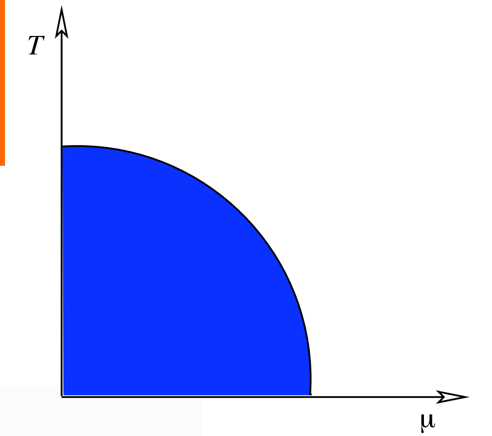
Resulting phase structure:



Chamblin-Emparan-Johnson-Myers '99

Promising step towards capturing some universal behaviour of finite temp/density QCD with black hole physics?

Johnson '99



More work needed though; need to put in more features of QCD.

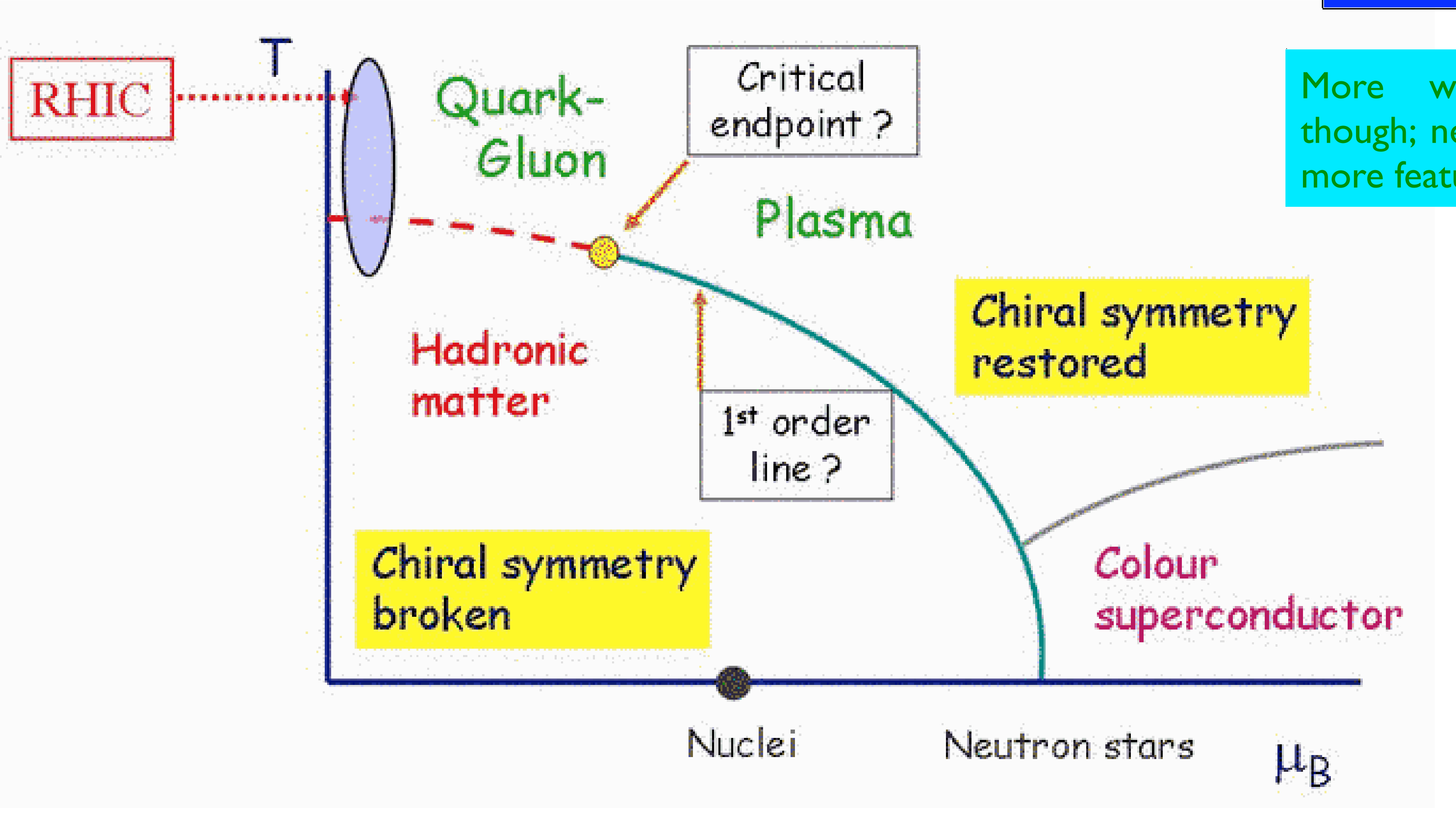


Diagram ripped from yesterday's proceedings on web. Berndt Müller's talk!

Probing the Correspondence VI

But maybe we are on the right track....

Shear Viscosity

Shuryak review '04

$$\frac{\eta}{s}$$ is implied to be unusually small by RHIC data.

The quark-gluon plasma is rather strongly coupled!

It turns out that gauge theories with gravity duals naturally achieve this!

Kubo's formula gives the viscosity in terms of a correlation function of the stress tensor.

Area of horizon of finite temperature solution is A

The bulk field which couples to this operator is the graviton.

Klebanov, '97
Das, Gibbons, Mathur '96
....
.....

The viscosity ends up being related to absorption cross sections of the graviton

$$\eta = \frac{A}{16\pi G}$$

$$S = \frac{A}{4G}$$

$$\frac{\eta}{S} = \frac{1}{4\pi}$$

Conjectured to be a lower bound (Kovtun, Son, Starinets , '03'04)

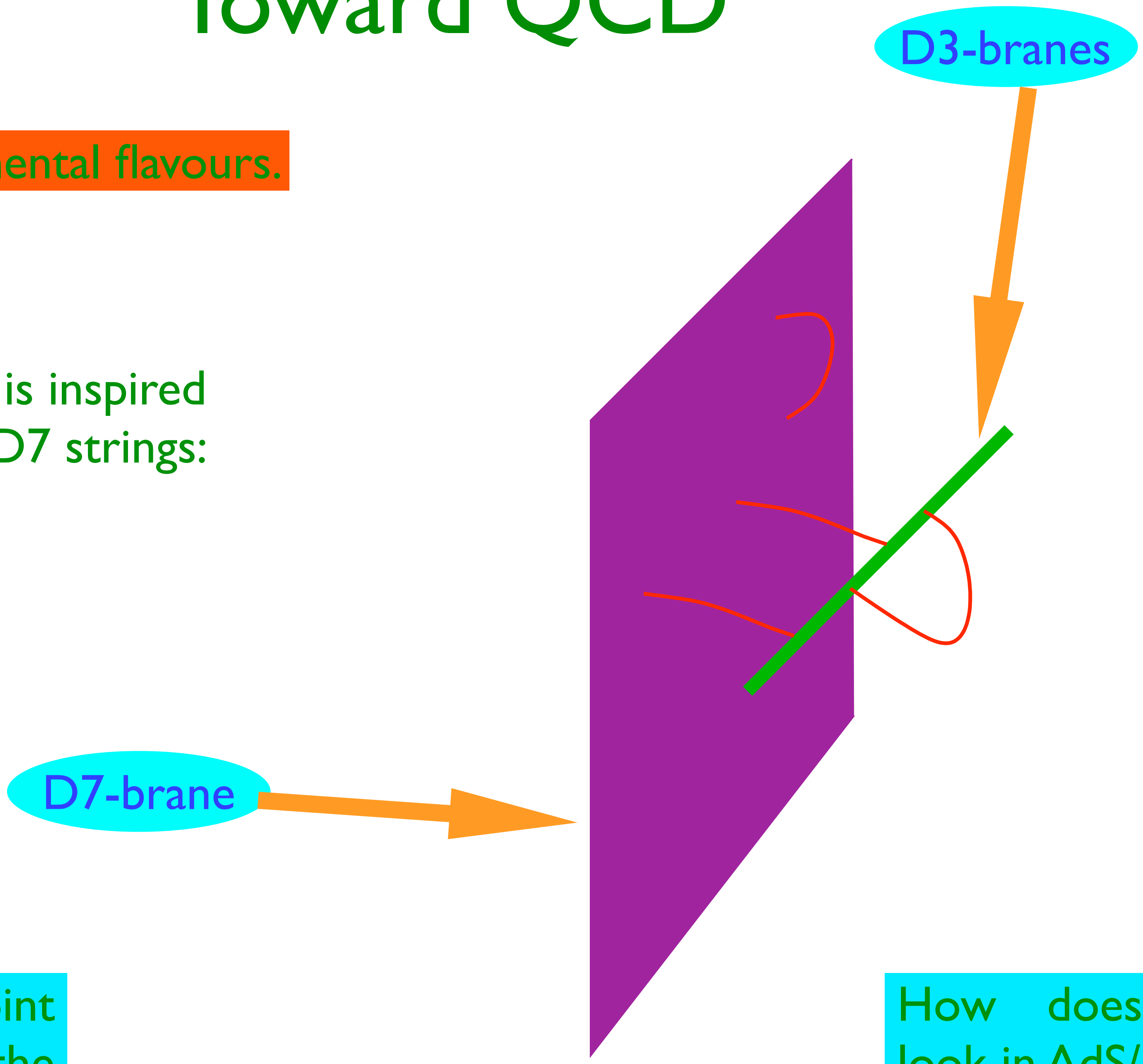
This is encouraging, but we need to do more work to get to more "QCD-like" theories.

We can anticipate some of the features we'll need with this final construction......

Toward QCD

Adding fundamental flavours.

One approach is inspired by role of D3-D7 strings:



A string endpoint transforms in the fundamental.

How does this look in AdS/CFT?

Take near horizon limit of N D3-branes

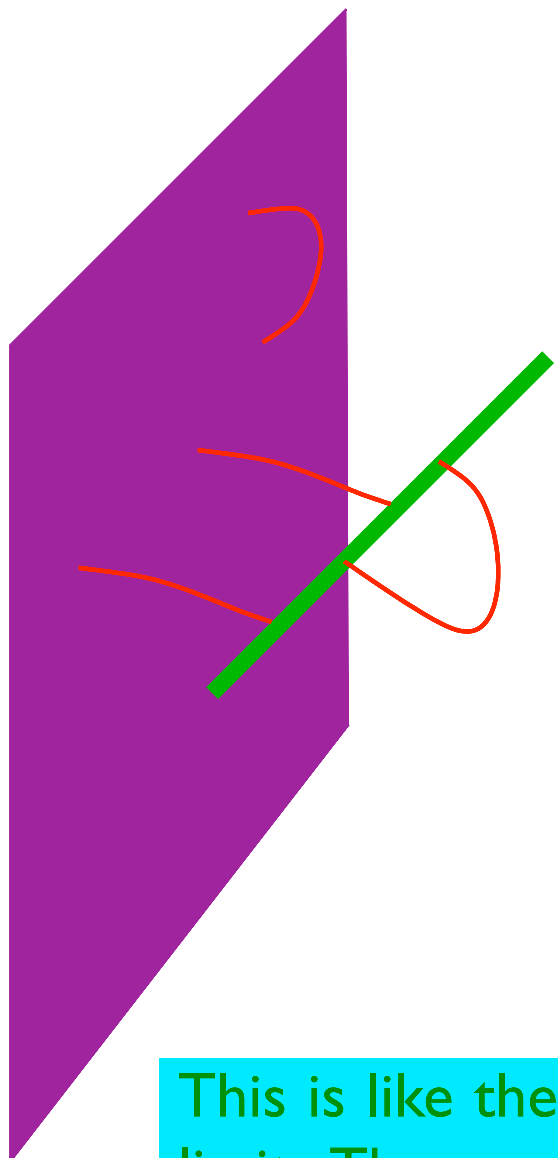
N_f D7-branes

Get a sensible, controllable geometry when:

$$N \text{ large}$$

$$g_{YM}^2 N = \lambda = \text{finite}$$

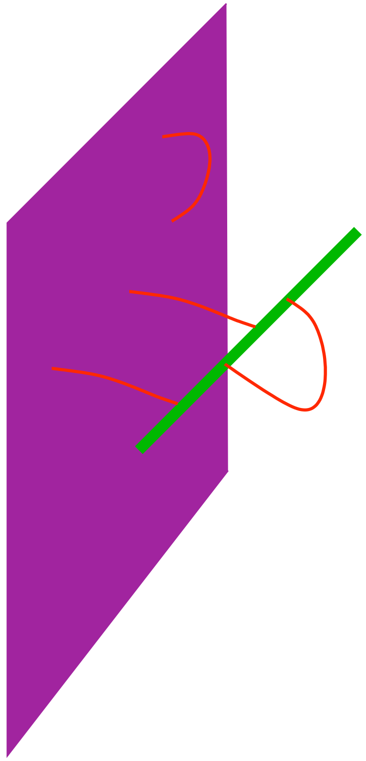
$$g_{YM}^2 N_f = 0$$



This gives limit in which D7s are simply probes of the AdS geometry.

This is like the "quenched" limit. The quarks do not back react on the physics.

Karch-Katz '02, Kruczenski et al '03

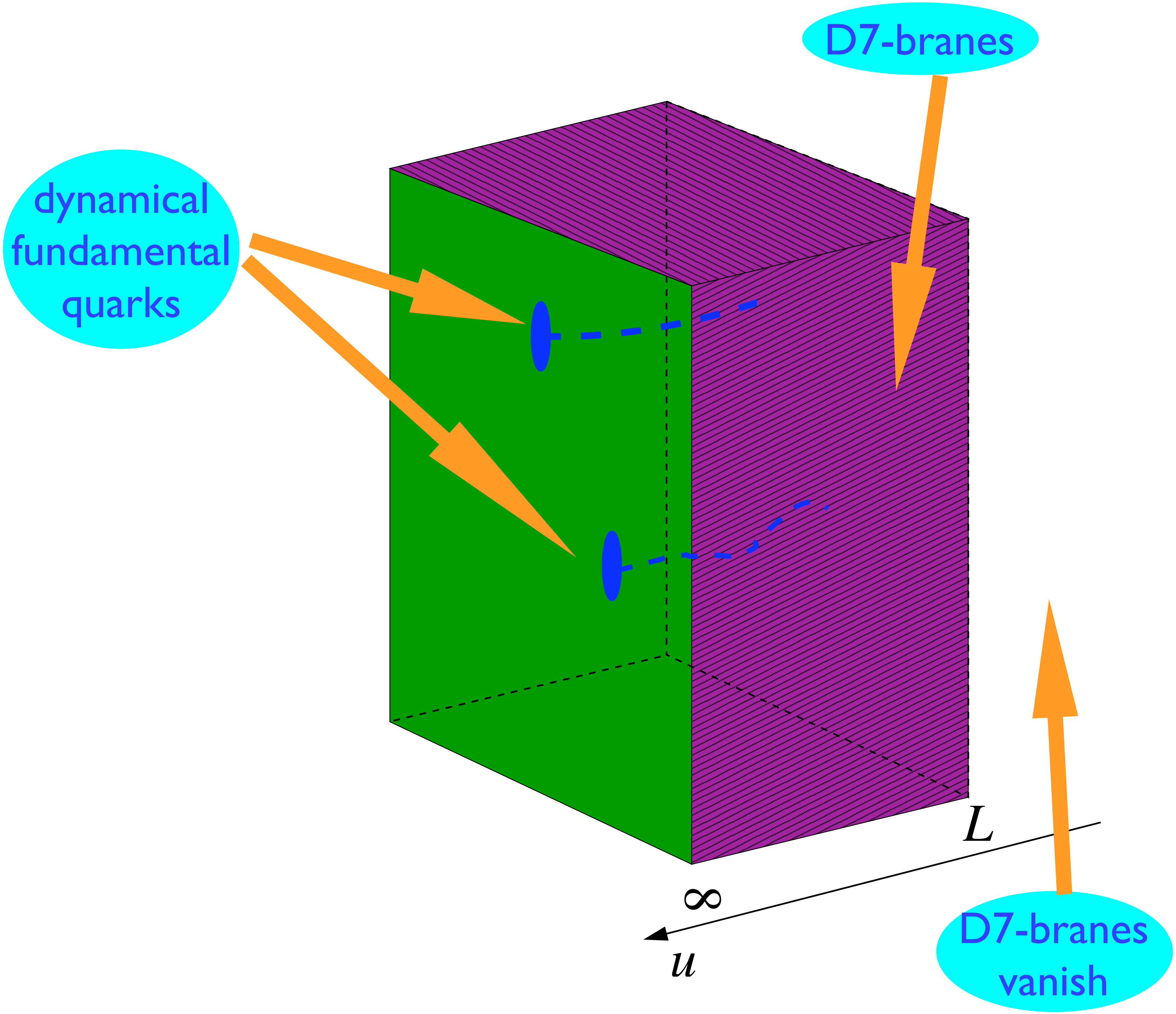


A D7-brane in the probe limit sees on its worldvolume:

$$ds^2 = \frac{u^2}{\ell^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{\ell^2}{u^2}du^2 + \frac{\ell^2(u^2 - L^2)}{u^2}d\Omega_3^2$$

Large u is just $\mathrm{AdS}_5 \times S^3$ → The D7-brane fills AdS and wraps $S^3 \subset S^5$

But when u=L, only have AdS part. → The D7-brane dissolves away!


D7-branes
dynamical
fundamental
quarks
L
∞
u
D7-branes
vanish

Toward QCD

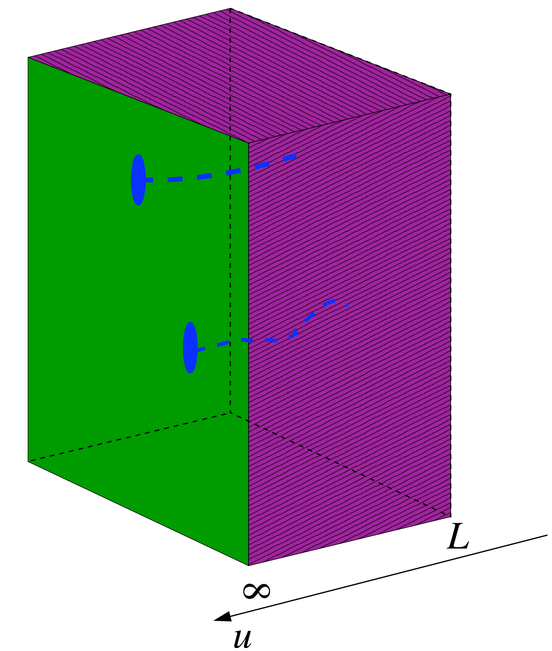
Lessons:

Extra structures, such as other D-branes (also fluxes, etc) can enrich the structure.

These structures can appear and disappear as one moves in u (changes energy scale in gauge theory).

This is how Λ_{QCD} arises in gravity picture. There is a geometrically induced cutoff at some radius.

We need more control over this sort of feature to better approach QCD.



Actually expect full sugra solution to be singular when back-reaction is included.

The full stringy physics then resolves it. (Myers '99, Johnson, Peet Polchinski' 99....)Several examples work like this.
Klebanov-Strassler,
Polchinski-Strassler, etc.

The Present

- There is no doubt that we have a powerful tool for studying strongly coupled gauge theories.
- As you know, many models have been constructed, exhibiting confinement, chiral symmetry breaking, *etc*.
- Glueball and Meson spectroscopy is (almost unreasonably) promising, comparing very well to lattice studies!
- High Energy Scattering in QCD-like models also very promising.

Maldacena '97,Witten '98, Gubser, Klebanov Polyakov,'98

Klebanov-Strassler; Polchinski-Strassler; several QCD-like models since then...Stephanov in this conference...

Csaki et al '98, de Mello Koch et al '98, Minahan '99, Ooguri' 99,Constable and Myers '99, Myers group '03,'04, Evans et al '03...

Polchinski-Strassler '01; Giddings '02, Kang-Nastase '04 .... Nastase '05

The Future?

- In order to get QCD, we need to continue improving supergravity and string techniques to handle those complicated backgrounds. Prospects are good. Not out of tricks yet.

- Will this help us get away from being strongly coupled in UV? (Big obstacle to several phenomena being cleanly studied.)

- Thermodynamics of finite temperature and density is tantalizing. Study black holes in more complicated geometries to improve corners of phase diagram? (Couple their charge to the appropriate $U(1)$, etc...) Much to do there. Exciting.

- More...More...More.... This could well be where string theory really gets its first real confrontation with Nature.