Heavy Quark Physics in 2+1-Flavor Lattice QCD

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KITP Program
“Modern Challenges for Lattice Field Theory”
February 9, 2005
Outline

• Semileptonic $D$ decays
  • Chiral extrapolation with and without BK
  • Estimation of discretization effects
• $D$ Meson Decay Constant
  • Chiral extrapolation with stagPQ$\chi$PT
• Mass of the $B_c$ Meson
  • Estimation of discretization effects
Preliminaries

• 2+1 flavor calculations with improved staggered quarks have reproduced PDG values of a wide variety of masses, mass splittings, and decay constants.

• Results assume (and suggest!?)
  • $[\det_4 M]^{1/4} \approx \det_1 (\slashed{D} + m)$

• staggered (partially quenched) chiral PT

• effective field theories for heavy quarks
• Dots at 0.04 are experimental.

• Error bars are lattice QCD.

• Linear extrap (by eye).

• Gasser-Leutwyler $\chi$log gets closer (solid).

• Sharpe-Shoresh $\chi$log even closer (dashed).
• Thus encouraged, HPQCD, MILC, and Fermilab Lattice Collaborations are using these methods to calculate matrix elements relevant to *flavor physics*.

• The stakes are high: “Are non-Standard phenomena visible in $B$ decays?”
Proofs

• We need physicists’ proofs that the methods are sound.

• For heavy quarks, using HQET/NRQCD as a theory of cutoff effects suffices.

• For staggered quarks, the fourth-root trick could benefit from a better foundation, but (I think) most of the simple arguments against it are lame.
Tests

• As a complement to (quasi)-mathematical proofs, other tests are desirable.

• Experimenters suggest making predictions.

• $D$ meson decay properties and $B_c$ mass are being improved by ongoing experiments.
$f_+^{D \rightarrow \pi}(q^2) \& f_+^{D \rightarrow K}(q^2)$
Semileptonic Decay

\[
q^2 = m_B^2 + m_\pi^2 - 2m_B E
\]

\[
\frac{d\Gamma}{dE} = \frac{G^2 m_B}{12\pi^3} |V_{ub}|^2 p^3 |f_+(E)|^2
\]

\[
\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = f_+(E) \left[ p_B + p_\pi - \frac{m_B^2 - m_\pi^2}{q^2} q \right]^\mu + f_0(E) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu
\]
Polology

• For $E < 0$, there are poles and cuts, and so on, from real states in $\nu$ scattering.
  
  • vector mesons for $f_+$ at

  • scalar mesons for $f_0$

• Their effects spill into physical region $E > 0$.

• For $D$ and $B$ mesons, the vector is nearby.
BK Ansatz

• With this in mind Becirevic and Kaidalov proposed the parametrization

\[ f_+ (q^2) = \frac{f(0)}{(1-q^2/m_{D*}^2)(1-\alpha q^2/m_{D*}^2)} \]

\[ f_0 (q^2) = \frac{f(0)}{(1-q^2/m_{D*}^2/\beta)} \]

• Builds in the closest pole, and has parameters for the slop.
• Advantages
  • builds in pole, & also heavy-quark scaling laws
  • fit to BK is most sensitive to low energy, yet $f_0$ influences $f_+$ through $f(0)$.

• Disadvantages
  • parametrization deteriorates with $E$
  • fit to BK is sensitive mostly to low energy, and $f_0$ determines $f(0)$. 
• Analysis method
  • calculate matrix elements for various \((m_q, \slashed{p})\).
  • use BK to interpolate to fiducial values of \(E\), same for each ensemble.
  • use staggered \(\chi PT\) for chiral extrapolation
  • use BK to extrapolate to full kinematic range
and D results (hep-ph/0408306, accepted for PRL)

$V_{\mu} \not{D}_f q^2 p \not{D}_p m^2 \not{D}_m q^2 | q^2 | [GeV^2]$

$0.5 \ 1 \ 1.5$

$f_0 \ f_+$

experiment

$D \rightarrow \pi$

$q^2 [GeV^2]$

$D \rightarrow K$
• An alternative is to avoid BK altogether, and use $\chi$PT to extrapolate jointly in $(m_q, E)$:

• Consistent, but no-BK has larger error in low $q^2$ (high $E$) region.
\( \bullet \ D \rightarrow K \nu \)

\[
\begin{align*}
    f_+^{D \rightarrow K} (0) &= 0.73(3)(7) \\
    f_+^{D \rightarrow K} (0) &= 0.78(5) \ [\text{BES, hep-ex/0406028}] \\
\end{align*}
\]

\( \bullet \ D \rightarrow \pi \nu \):

\[
\begin{align*}
    f_+^{D \rightarrow \pi} (0) &= 0.64(3)(6) \\
    f_+^{D \rightarrow \pi} (0) &= 0.87(3)(9) f_+^{D \rightarrow K} \\
    f_+^{D \rightarrow \pi} (0) &= 0.86(9) f_+^{D \rightarrow K} \ [\text{CLEO, hep-ex/0407035}] \\
\end{align*}
\]
$D \rightarrow Kl\nu$ vs. $q^2$

hep-ex/0410037

Okamoto et al. [Fermilab/MILC]
Discretization Effects

• Dominant error, but only one sentence!
• Both QCD and LGT can be described by
  \[ \mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{HQET}} = \sum_i C_i^{\text{cont}}(m_Q)O_i \]
  \[ \mathcal{L}_{\text{LGT}} = \mathcal{L}_{\text{HQET}}(m_0 a) = \sum_i C_i^{\text{lat}}(m_Q, m_0 a)O_i \]
• Discretization error is in mismatch of coefficients.
• In general,
\[ \text{error}_i = \left| \left[ C_i^{\text{lat}}(m_Q, m_0a) - C_i^{\text{cont}}(m_Q) \right] O_i \right| \]

• For Wilson(-like) quarks write
\[ C_i^{\text{lat}}(m_Q, m_0a) - C_i^{\text{cont}}(m_Q) = a^{\dim O_i - 4} f_i(m_0a) \]

• For heavy-light use HQET to order and estimate
\[ \text{error}_i = f_i(m_0a)(a\Lambda_{QCD})^{\dim O_i - 4} \]
• What would you use for $\Lambda_{\text{QCD}}$?

• Based on estimates of the $\Lambda$ that appears in the heavy-quark expansion—from lattice, sum rules, and experiment—the sensible range is

• $\Lambda_{\text{QCD}} = 500–700$ MeV
<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{error}_B [O(\alpha_s a) \text{ Lagrangian}]$</td>
<td>1.8</td>
<td>2.2</td>
<td>2.6</td>
<td>3.1</td>
<td>3.5</td>
<td>3.9</td>
<td>4.4</td>
</tr>
<tr>
<td>$\text{error}_3 [O(\alpha_s a) \text{ current}]$</td>
<td>1.1</td>
<td>1.4</td>
<td>1.7</td>
<td>2.0</td>
<td>2.2</td>
<td>2.5</td>
<td>2.8</td>
</tr>
<tr>
<td>$\text{error}_E [O(a^2) \text{ Lagrangian}]$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.9</td>
<td>1.2</td>
<td>1.5</td>
<td>1.9</td>
<td>2.4</td>
</tr>
<tr>
<td>($c_E = 0$)</td>
<td>1.1</td>
<td>1.6</td>
<td>2.4</td>
<td>3.2</td>
<td>4.2</td>
<td>5.3</td>
<td>6.6</td>
</tr>
<tr>
<td>$\text{error}_X [O(a^2) \text{ current}]$</td>
<td>0.9</td>
<td>1.3</td>
<td>1.9</td>
<td>2.6</td>
<td>3.4</td>
<td>4.3</td>
<td>5.4</td>
</tr>
<tr>
<td>($d_1$ off)</td>
<td>1.3</td>
<td>2.0</td>
<td>2.8</td>
<td>3.9</td>
<td>5.0</td>
<td>6.4</td>
<td>7.9</td>
</tr>
<tr>
<td>$\text{error}_Y [O(a^2) \text{ current}]$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
<td>2.3</td>
</tr>
<tr>
<td>temporal total</td>
<td>2.8</td>
<td>3.6</td>
<td>4.7</td>
<td>5.9</td>
<td>7.2</td>
<td>8.7</td>
<td>10.5</td>
</tr>
<tr>
<td>spatial total</td>
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<td>4.1</td>
<td>5.3</td>
<td>6.6</td>
<td>7.8</td>
<td>9.4</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Pending studies on finer lattices, we quoted sum in quadrature of both currents, at $\Lambda_{QCD} = 700$ MeV
All of CKM

• Okamoto has combined our (i.e., his) calculations of $D \to \pi$ and $D \to K$ with preliminary calculations of $B \to D$ to obtain the middle row of the CKM matrix.

• add $B \to \pi, K \to \pi$ and unitarity to get the top row, the right column and $|V_{ts}|$.

• add $\sin(2\beta)$ to get the last element

hep-lat/0412044
\( f_{D_s} \ & \ f_D \)
\[ f_{D_s} \text{ & } f_D \]

- \( D \) meson decay constants either
  - determine \(|V_{cs}|\) and \(|V_{cd}|\)
  - check QCD (with \(|V_{cs}|\) and \(|V_{cd}|\) from CKM unitarity).
- CLEO-c is measuring them.
- A test of light quarks and (staggered) PQ\(\chi\)PT.
Staggered PQ\(\chi\)PT

- In the case of decay constants, chiral logs are important.
- In staggered PQ\(\chi\)PT, Aubin & Bernard find

\[
m^2_{uu} \ln m^2_{qq} \rightarrow \begin{cases} 
m^2_{uu} \ln m^2_{\text{average}} \\
m^2_{uu} \ln m^2_{\text{taste singlet}}
\end{cases}
\]

so singularity of PQ\(\chi\)PT softened.
**Chiral Extrapolation** $f_D$

- Extrapolate in sea $m_u$ and valence $m_q$ to get down to real $m_f$.
- Single fit to all data constrains $\chi$PT better.
- Staggered PQ$\chi$PT treats all $a$ in same fit.
Fit all 60 combinations of \((am_u, am_q)\). Quality of the fit is obvious, right?
$am_u = 0.03$ part of stagPQ$\chi$PT fit.

With $O(a^2)$ bits turned off.
$am_u = 0.02$ part of stagPQ$\chi$PT fit.

With $O(a^2)$ bits turned off.
$am_u = 0.01$ part of stagPQ$\chi$PT fit. With $O(a^2)$ bits turned off.
\( a m_u = 0.007 \) part of stagPQ\( \chi \)PT fit.

With \( O(a^2) \) bits turned off.
$a m_u = 0.005$ part of stagPQ$\chi$PT fit.

With $O(a^2)$ bits turned off.
$am_u = am_q$ part
of stagPQ$\chi$PT fit (dotted)
with $O(a^2)$ bits turned off (solid)
Chiral Extrapolation $f_{Ds}$

- Interpolate in valence $m_q$ to get down to real $m_s$.
- Extrapolate in sea $m_u$ to get down to real $m_l$. 
Currently obtained in a separate linear fit.
Preliminary Results

- J. Simone et al., hep-lat/0410030 (Lattice ’04)

\[
\frac{f_{D_s} \sqrt{m_{D_s}}}{f_D \sqrt{m_D}} = 1.20 \pm 0.06 \pm 0.06 ,
\]

\[
f_{D_s} = 263^{+5}_{-9} \pm 24 \text{ MeV} ,
\]

\[
f_D = 225^{+11}_{-13} \pm 21 \text{ MeV} .
\]

\[
f_D = 202 \pm 41 \pm 17 \text{ MeV}
\]

CLEO-c, hep-ex/0411050

discretization uncertainty as in form factors.
Soft pion theorem

\[ f_0^{D \to \pi} (q_{\text{max}}^2) = \frac{f_{D}}{f_{\pi}} \]

\[ q^2 \geq q_{\text{max}}^2 \]

\[ D \to \pi (m_{\pi} = 0) \]
Outlook

• We will combine form factors and decay constants to obtain combinations that can be compared directly to experiment, with no CKM input:

\[
\frac{1}{\Gamma_{D \rightarrow l\nu}} \frac{d\Gamma_{D \rightarrow \pi l\nu}}{dq^2} \propto \left| \frac{f_{+ \rightarrow \pi}(q^2)}{f_D} \right|^2
\]

\[
\frac{1}{\Gamma_{D_s \rightarrow l\nu}} \frac{d\Gamma_{D \rightarrow K l\nu}}{dq^2} \propto \left| \frac{f_{+ \rightarrow K}(q^2)}{f_{D_s}} \right|^2
\]
$B_c$

- Meson composed of a beautiful anti-quark and a charmed quark.
- Unusual beast
  - contrast with $B_s$ & $D_s$, $\psi$ & $\Upsilon$: $\nu_c = 0.7$.
- no annihilation to gluons
Thursday & Friday, December 2 & 3, 2004

DØ

CDF

4:00 p.m. One West
Joint Experimental Theoretical Physics Seminar
Saverio D'Auria, University of Glasgow
*Bc*: Fully Reconstructed Decays and Mass Measurement at CDF
QCD Theory & $B_c$

- Three main tools
  - potential models
  - potential NRQCD
  - lattice QCD
- All treat both quarks as non-relativistic
  - charmed quark is pushing it, $v_c^2 = 0.5$. 
Energy Scales

- Several energy scales in (this) quarkonium
  - $2m_b, 2m_c > 2 \text{ GeV}$
  - $m_b v_b = m_c v_c \approx 1000 \text{ MeV}$
  - $\frac{1}{2}m_c v_c^2 \approx 350 \text{ MeV}, \quad \frac{1}{2}m_b v_b^2 \approx 50 \text{ MeV}$
  - $\Lambda_{QCD} \sim 500 \text{ MeV}$
NRQCD

\[ \mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{HQ}} \]

\[ \mathcal{L}_{\text{HQ}} = \mathcal{L}_{\text{light}} - \bar{h}_v (m_1 + iv \cdot D) h_v + \frac{\bar{h}_v D^2 \perp h_v}{2m_2} + \frac{\bar{h}_v B^\mu \nu h_v}{2m_2} + \frac{\bar{h}_v (D^2 \perp)^2 h_v}{8m_2^3} + \frac{\bar{h}_v D \perp \cdot E h_v}{4m_2^2} + \frac{\bar{h}_v s_{\mu \nu} D^\mu \nu h_v}{4m_2^2} + \ldots \]

\[ \mathcal{L}_{\text{HQ}} = \sum_i C_i (m_Q, m_Q/\mu) \mathcal{O}_i (\mu/m_Q v^n) \]

(Lattice errors)

(integrate out scale \( m_Q \))

(Same Lagrangian as HQET, but different power counting.)
Potential NRQCD

- Integrate out scale $m_Q v_Q$
- Hamiltonian contains kinetic terms, potentials, and their radiative corrections
- Radiative corrections from $m_Q v_Q$ in pQCD
- Bound-state solved a la positronium: assumes small shifts from scales $\Lambda, m_c v_c^2$
\[ H = \frac{p_c^2}{2m_c} + \frac{p_b^2}{2m_b} - \frac{(p_c^2)^2}{8m_c^3} - \frac{(p_b^2)^2}{8m_b^3} + \cdots + V(r) \]

\[ V(r) = -\frac{C_F\alpha_s}{r} + C_F\alpha_s(1+\alpha_s+\cdots) \left( \frac{1}{4m_c^2} + \frac{1}{4m_b^2} \right) 4\pi\delta(r) + \cdots \]
Potential Models

- Truncate at leading order (in $\alpha_s, \nu^2$).
- Linear confining potential added by hand.
- Potential model $\alpha_s, m_Q$ not connected to QCD Lagrangian $\alpha_s, m_Q$.
- Provide excellent empirical understanding.
Lattice Calculation

- **Ian Allison**, Christine Davies, Alan Gray, ASK, Paul Mackenzie, & James Simone

- conference: hep-lat/0409090
- publication: hep-lat/0411027

- Prediction: $\alpha_s, m_b, m_c$ taken from bottomonium and charmonium

- Use latNRQCD for $b$ and Fermilab for $c$. 
Essentials

• We calculate two mass splittings

\[ \Delta_{\psi \Upsilon} = m_{B_c} - \frac{1}{2}(\bar{m}_\psi + m_{\Upsilon}) \] quarkonium baseline
\[ \Delta_{D_s B_s} = m_{B_c} - (m_{D_s} + m_{B_s}) \] heavy-light baseline

• Everything is gold-plated, in the sense that the mesons are all stable, and far from threshold.

• Chiral extrapolations mild.
Isolating Lowest State

Correlator is sum of exponentials, lowest exponent is $m_{B_c}$
Error Cancellation

• Correlated statistics
• Unphysical shift in rest mass $m_1$
• Contributions from higher-in-$\nu^2$ operators, at least from quarkonium baseline.
Chiral Extrapolation

\[ \Delta_{\psi Y} \text{ (MeV)} \]

\[ -\Delta_{D_sB_s} \text{ (MeV)} \]

\[ m_l/m_s \]
Lattice Spacing Dependence

at lighter of the two sea quark masses
Error Analysis

• Statistical error is straightforward & small.
• Uncertainty from $a^{-1}, m_b, m_c$ easy to propagate: latter two are ±10, ±5 MeV.
• Main problem is to estimate the discretization effect for the heavy quarks.
Discretization Effects

(short distance mismatch) • (matrix element)

- Use calculations of tree-level mismatches
- Wave hands for one-loop mismatches
- Estimate matrix elements in potential models
- Check framework with other calculations
Hyperfine \( i\Sigma \cdot B \)

- The mismatch of the hyperfine interaction is

\[
\alpha_s a b_B (m_0 a) \times \bar{h} i \Sigma \cdot B h
\]

in both NRQCD and Fermilab Lagrangians.

- Estimate coefficient by comparing the simulation hyperfine splitting with experiment, where latter is known.

- Propagate to \( m_{B_c} \) and \( m_\Upsilon \).
The mismatch of the Darwin interaction is

\[ \{\alpha_s, 1\} a^2 b_{\text{Darwin}}(m_0a) \times \bar{h} D \cdot E h \]

for \{NRQCD, Fermilab\}.

Latter dominates; use known form of the coefficient and (Richardson) potential model estimate of matrix element.

Matrix element is small.
Relativistic \((p^2)^2 \& p_i^4\)

- The mismatch of the Darwin interaction is

\[
\{\alpha_s, 1\} a^3 b_4 (m_0 a) \times \{\bar{h}(p^2)^2 h, \bar{h} \sum_i p_i^4 h\}
\]

for \{NRQCD, Fermilab\}.

- Latter dominates; use known form of the coefficient and (Richardson) potential model estimate of matrix element.

- Matrix element is not small, but check total estimate with charmonium 1P-1S.
TABLE I: Estimated shifts in masses and the splittings $\Delta_{\psi \Upsilon}$ and $\Delta_{D_s B_s}$. Entries in MeV. Dashes (—) imply the entry is negligible.

<table>
<thead>
<tr>
<th>operator</th>
<th>$m_{B_c}$</th>
<th>$\frac{1}{2} \bar{m}_\psi$</th>
<th>$\frac{1}{2} \bar{m}_\Upsilon$</th>
<th>$\Delta_{\psi \Upsilon}$</th>
<th>$\bar{m}_D$</th>
<th>$\bar{m}_B$</th>
<th>$\Delta_{D_s B_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma \cdot B$</td>
<td>$-14$</td>
<td>$0$</td>
<td>$+3$</td>
<td>$-17$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-14$</td>
</tr>
<tr>
<td>Darwin</td>
<td>$-3$</td>
<td>$-3$</td>
<td>$\mp 1$</td>
<td>$\pm 1$</td>
<td>$-4$</td>
<td>$-$</td>
<td>$+1$</td>
</tr>
<tr>
<td>$(D^2)^2$</td>
<td>$+34$</td>
<td>$+10$</td>
<td>$\pm 3$</td>
<td>$+24$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+34$</td>
</tr>
<tr>
<td>$D_i^4$</td>
<td>$+16$</td>
<td>$+5$</td>
<td>$\pm 2$</td>
<td>$+11$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+16$</td>
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<tr>
<td>total</td>
<td>$a = \frac{1}{8}$ fm</td>
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<table>
<thead>
<tr>
<th>operator</th>
<th>$m_{B_c}$</th>
<th>$\frac{1}{2} \bar{m}_\psi$</th>
<th>$\frac{1}{2} \bar{m}_\Upsilon$</th>
<th>$\Delta_{\psi \Upsilon}$</th>
<th>$\bar{m}_D$</th>
<th>$\bar{m}_B$</th>
<th>$\Delta_{D_s B_s}$</th>
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<tr>
<td>$\Sigma \cdot B$</td>
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<td>$0$</td>
<td>$+3$</td>
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<td>$0$</td>
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<td>Darwin</td>
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<td>$\pm 1$</td>
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<td>$-$</td>
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<tr>
<td>$(D^2)^2$</td>
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<td>$D_i^4$</td>
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Results

• Splittings:

\[ \Delta_{\psi\Upsilon} = 39.8 \pm 3.8 \pm 11.2^{+18}_{-0} \text{ MeV}, \]
\[ \Delta_{D_sB_s} = -[1238 \pm 30 \pm 11^{+0}_{-37}] \text{ MeV}, \]

• Meson mass:

\[ m_{B_c} = 6304 \pm 4 \pm 11^{+18}_{-0} \text{ MeV}, \]
\[ m_{B_c} = 6243 \pm 30 \pm 11^{+37}_{-0} \text{ MeV}, \]

• More checks on quarkonium baseline, so it is our main result.
Compare with Models

\[ \frac{\bar{m}_\psi + m_\gamma}{2} \]

- potential models
- lattice QCD
- potential NRQCD
Compare with CDF

$$m_{B_c} = 6287 \pm 5 \text{ MeV}$$
CDF, W&C seminar, 12/03/04

$$m_{B_c} = 6304 \pm 12^{+18}_{-0} \text{ MeV}$$
[hep-lat/0411027]
Summary

• Results for leptonic and semi-leptonic $D$ decays and the mass of the $B_c$ meson.

• Estimates of uncertainties.

• Agreement with BES, CLEO, FOCUS, and CDF with similar time-scale and error, including predictions.

*pre-* pref.
1. a. Earlier; before; prior to: prehistoric.
   b. Preparatory; preliminary: premedical.
   c. In advance: prepay.
2. Anterior; in front of: preaxial.

[Middle English, from Old French, from Latin prae-, from prae, before, in front. See *perl* in Indo-European Roots.]