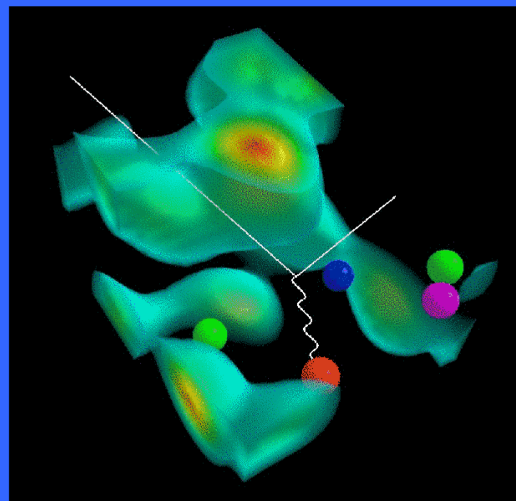
Understanding the Strong Interaction: Lattice QCD & Chiral Extrapolation



Anthony W. Thomas

Kavli Institute for Theoretical Physics

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Outline

- Quantum Chromodynamics within the Standard Model
- Lattice QCD :
  there are problems) new opportunities!

  (and, by the way, some things CAN be calculated ACCURATELY)
- M_N , M_Δ , QQCD ⟺ QCD ⟺ pQQCD, M_ρ ; μ_N , G_M^s
- Modeling Hadron Structure
- Tests of Physics Beyond the Standard Model

QCD and the Origin of Mass

mass:	u	+	u	+	d	=	proton
	0.003	+	0.003	+	0.006	≠	0.938

HOW does the rest of the proton mass arise?

Unavoidable Problem in Numerical Solution of QCD…

- Lattice QCD currently limited to m_q > 30-50 MeV

Time to decrease m_π by factor of 2 : 2^7 » 100

- NEED perhaps 500 Teraflops to get to 5 MeV !

Furthermore EFT implies ALL hadronic properties are non-analytic functions of m_q ....

HENCE: NO simple power series expansion about m_q = 0

: NO simple chiral extrapolation

) need EFT (χ PT)

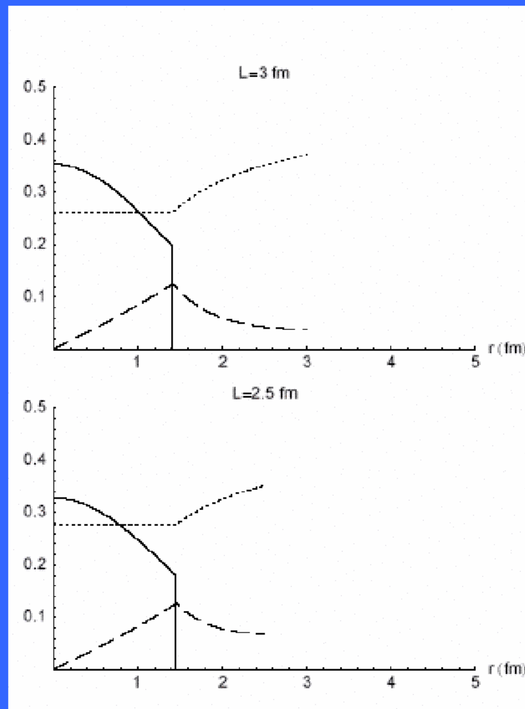
By the way….

Its NOT exp(- m_π L) that matters!

We must have

m_π (L/2 – R)>>1

with R » 0.8 fm



Thomas et al.,
hep-lat/0502002

Formal Chiral Expansion

Formal expansion of Hadron mass:

$$M_N = c_0 + c_2\, m_\pi^2 + c_{LNA}\, m_\pi^3 + c_4\, m_\pi^4 + c_{NLNA}\, m_\pi^4 \ln m_\pi + c_6\, m_\pi^6 + ..$$

Mass in chiral limit

No term linear in m_π (in FULL QCD…… there is in QQCD)

First (hence "leading") non-analytic term ~ $m_q^{3/2}$ (LNA)
Source: N ! N π ! N

Another branch cut from N ! Δ π ! N
- higher order in m_π
- hence "next-to-leading" non-analytic (NLNA)

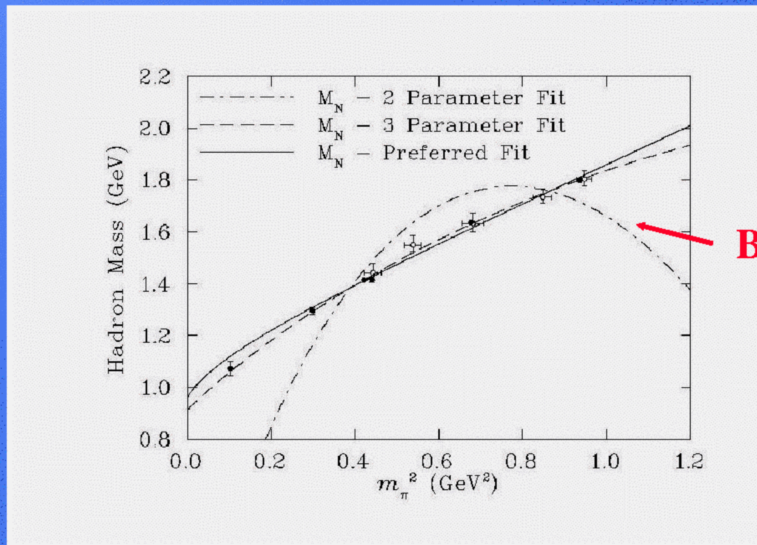
c_{NLNA} MODEL INEPENDENT

c_{LNA} MODEL INDEPENDENT

Convergence?

Relevance for Lattice data

Knowing χ PT , fit with: $\alpha + \beta\, m_\pi^2 + \gamma\, m_\pi^3$ (dashed curve)



Best fit with γ as in χ P T

Problem: γ = - 0.76 c.f. model independent value -5.6 !!

(From: Leinweber *et al*., Phys. Rev., D61 (2000) 074502)

The Solution

There is another SCALE in the problem
- not natural in (e.g.) dim-regulated χPT

Λ ~ 1 / Size of Source of Goldstone Bosons
~ 400 – 500 MeV

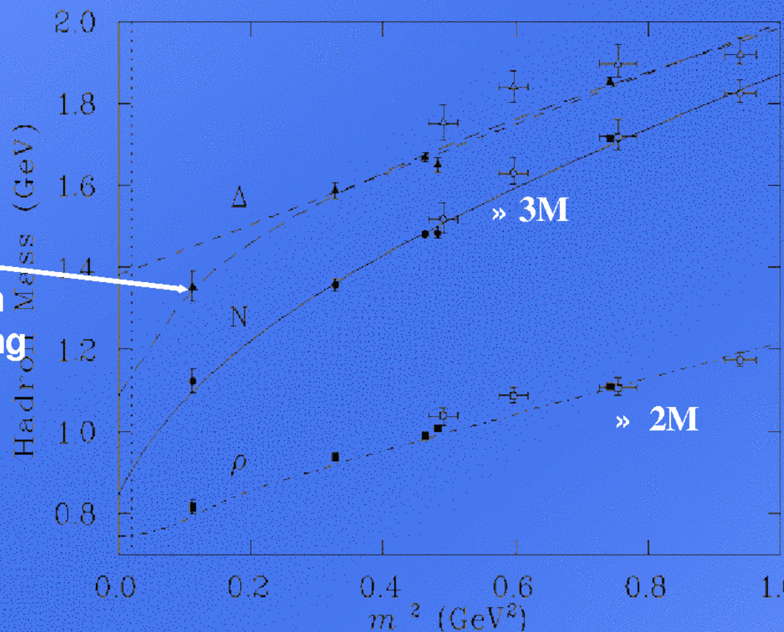
IF Pion Compton wavelength is smaller than source…..
($m_\pi \geq 0.4 - 0.5$ GeV ; $m_q \geq 50\text{-}60$ MeV)
ALL hadron properties are smooth, slowly varying (with m_q) and Constituent Quark like !
(Pion loops suppressed like $(\Lambda / m_\pi)^n$)

WHERE EXPANSION FAILS: NEW, EFFECTIVE DEGREE OF FREEDOM TAKES OVER

Behavior of Hadron Masses with m_π

From: Leinweber *et al.*, Phys. Rev., D61 (2000) 074502

Point of inflection at opening of N π channel



Lattice data from CP-PACS & UKQCD

BUT how <u>model dependent</u> is the extrapolation to the physical point?

Acceptable Extrapolation Procedure

1. Must respect non-analytic behaviour of χ P T in region $m_\pi < 400 - 500$ MeV ($m_q \le 50$ MeV) ....with correct coefficients!

2. Must suppress chiral behaviour as inverse power of m_π in region $m_\pi > 400 - 500$ MeV

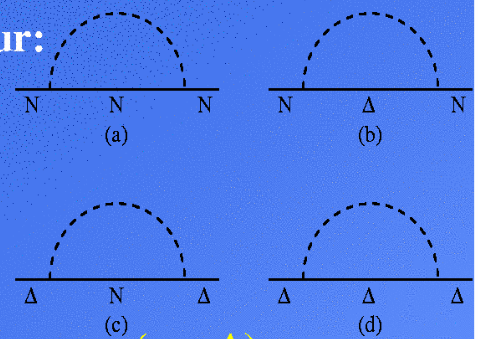
3. Finite range regularization (FRR) does both : completely equivalent to DR χ PT at small m_π BUT re-sums series at large mass!

Extrapolation of Masses

At "large m_π" preserve observed linear (constituent-quark-like) behaviour: $M_H \sim m_\pi^2$

As $m_\pi \sim 0$: ensure LNA & NLNA behaviour:

(BUT must die as $(\Lambda / m_\pi)^2$ for $m_\pi > \Lambda$)

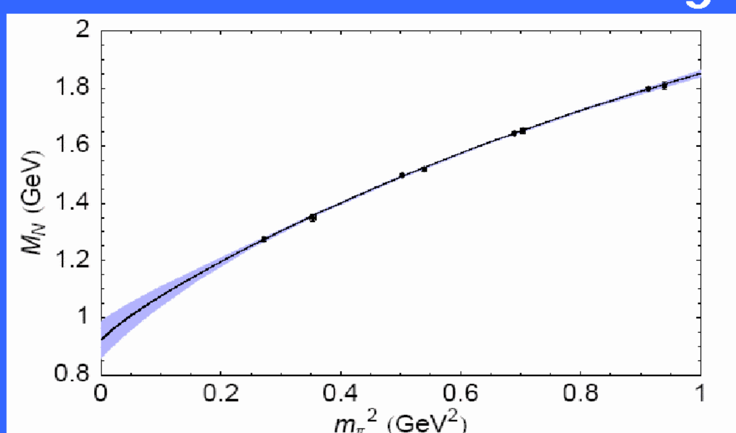


Hence use:

$$M_H = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \sigma_{LNA}(m_\pi,\Lambda) + \sigma_{NLNA}(m_\pi,\Lambda)$$

- Evaluate self-energies with form factor , "finite range regulator", FRR, with Λ » 1/Size of Hadron

χ'al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon



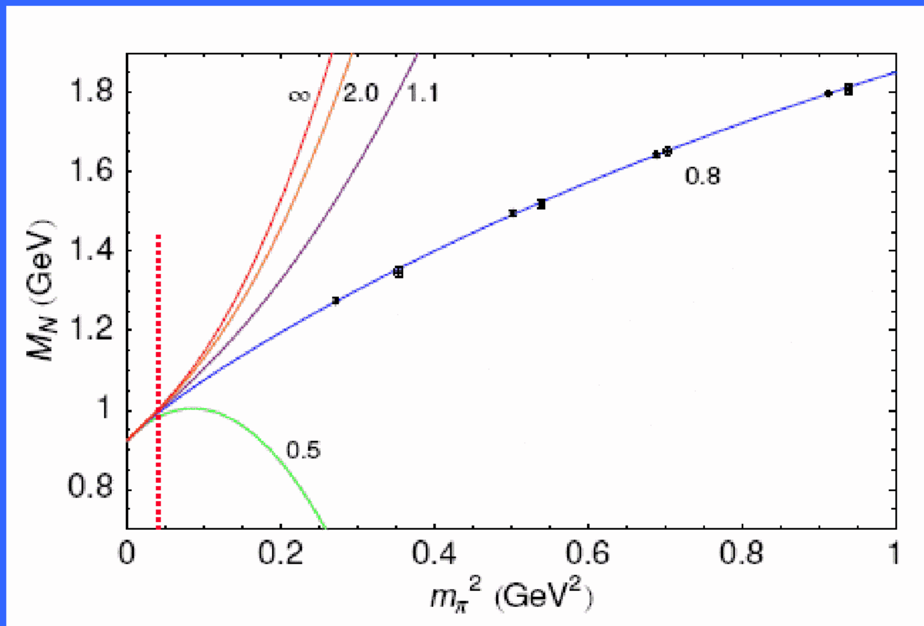
FRR give same answer to <<1% systematic error!

Regulator	Bare Coefficients a_0^Λ	a_2^Λ	a_4^Λ	Λ	Renormalized Coefficients c_0	c_2	c_4	m_N
Monopole	1.74	1.64	−0.49	0.5	0.923(65)	2.45(33)	20.5(15)	0.960(58)
Dipole	1.30	1.54	−0.49	0.8	0.922(65)	2.49(33)	18.9(15)	0.959(58)
Gaussian	1.17	1.48	−0.50	0.6	0.923(65)	2.48(33)	18.3(15)	0.960(58)
Sharp cutoff	1.06	1.47	−0.55	0.4	0.923(65)	2.61(33)	15.3(8)	0.961(58)
Dim. Reg. (BP)	0.79	4.15	+8.92	–	0.875(56)	3.14(25)	7.2(8)	0.923(51)

Leinweber et al., PRL 92 (2004) 242002

Power Counting Regime

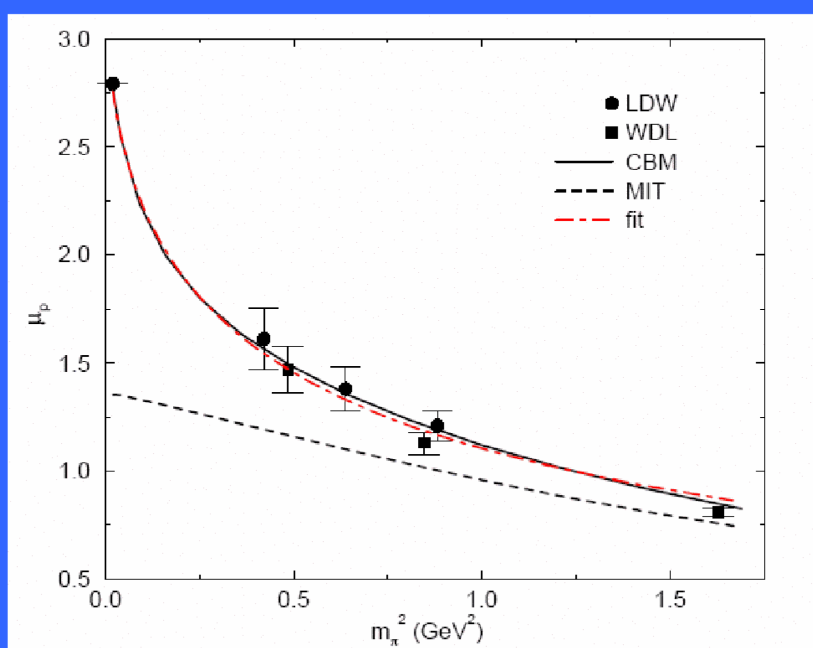
Ensure coefficients c_0, c_2, c_4 all identical to 0.8 GeV fit



Leinweber, Thomas & Young, hep-lat/0501028

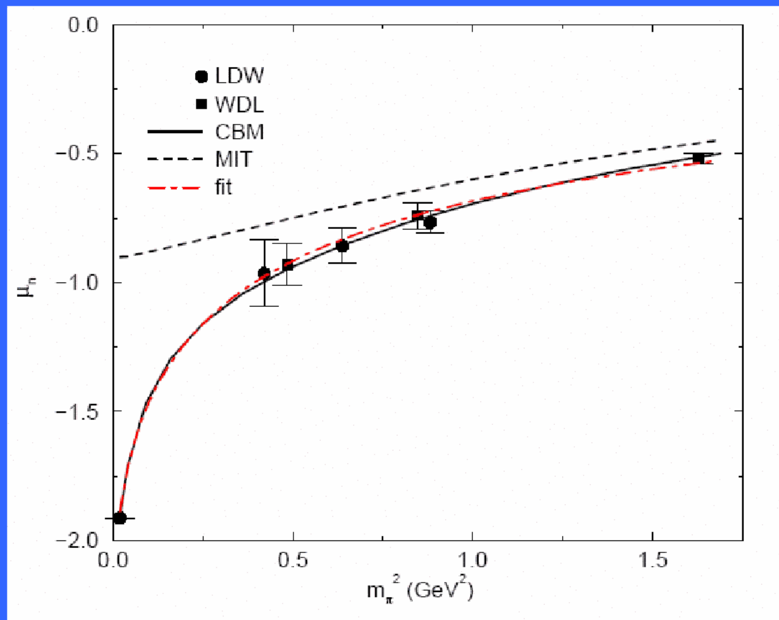
Initial Study 1998: Used Cloudy Bag Model

- m_q = 6 MeV at physical point
- scales with m_π^2
- CBM created in 1979 to restore chiral symmetry to MIT bag….



Leinweber, Lu & Thomas, Phys Rev D60 (1999) 034014

Similar Quality Fit for Neutron



CBM fit includes both LNA and NLNA terms in χ 'al expansion

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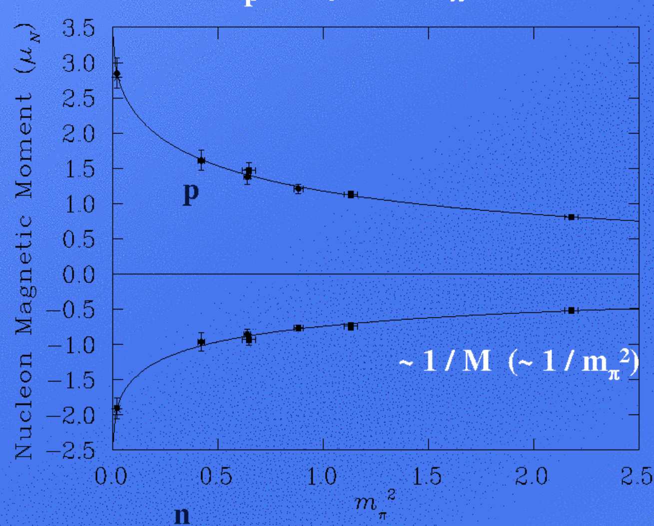
Early Fit to Lattice Data for μ_p and μ_n

$\mu_{p(n)} = \mu_0 / (1 \pm \alpha / \mu_0 \, m_\pi + \beta \, m_\pi^2)$: fit μ_0 and β to lattice data.

Thus: $\mu_p = \mu_0 - \alpha \, m_\pi + \ldots\ldots$; $\alpha = 4.4 \, \mu_N$ -GeV^{-1} (from χ PT)



At physical quark mass:

$\mu_p = 2.85 \pm 0.22 \, \mu_N$

$\mu_n = -1.90 \pm 0.15 \, \mu_N$

(purely statistical errors)

(Leinweber et al., Phys. Rev. D60 (1999) 034014;
/// Hemmert & Weise: nucl-th/0204005 and Pascalutsa, Holstein… 2004)

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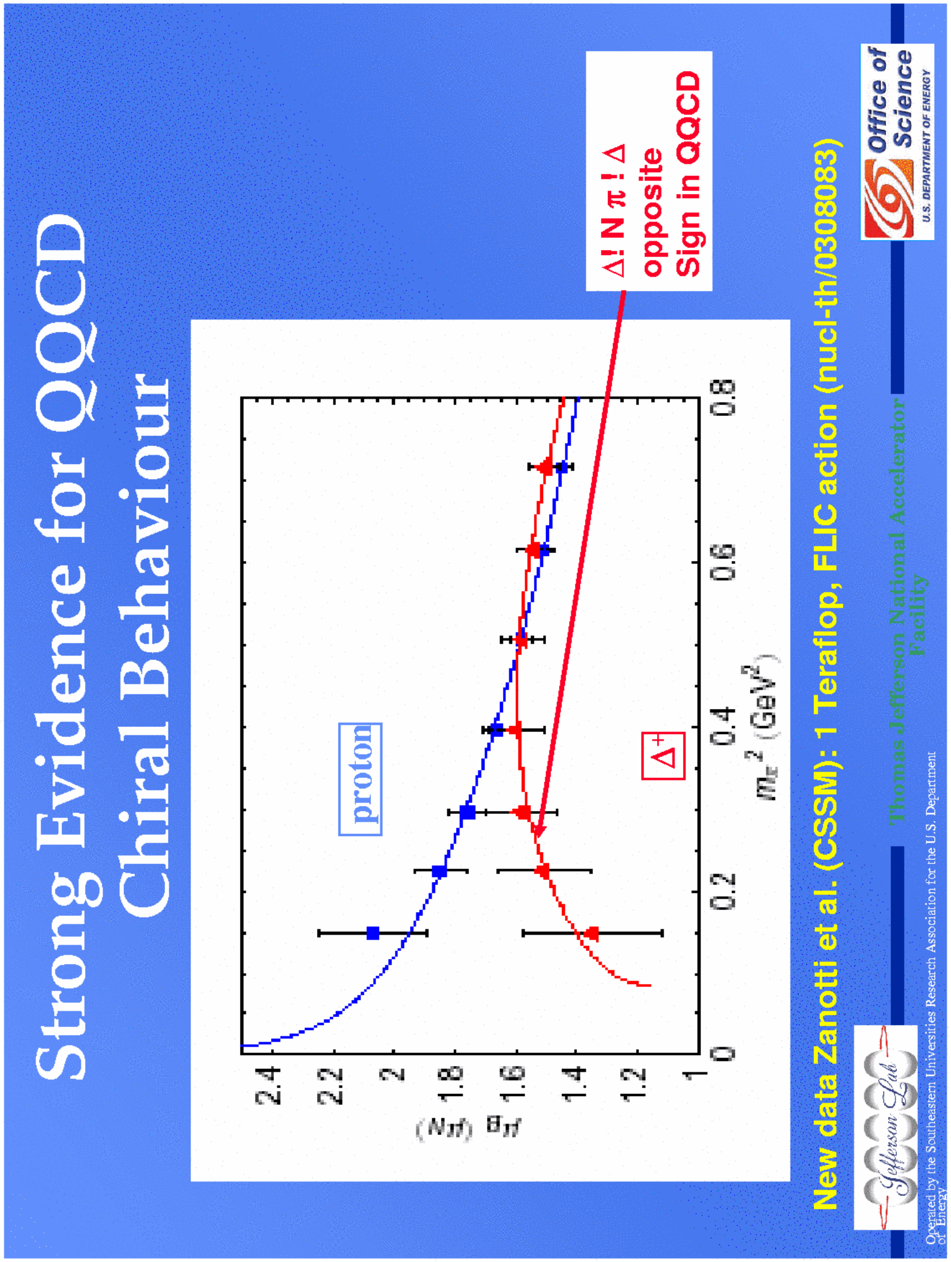

Strong Evidence for QQCD Chiral Behaviour
Δ! N π ! Δ opposite Sign in QQCD
proton
Δ+
μ_p (μ_N)
m_π^2 (GeV^2)
New data Zanotti et al. (CSSM): 1 Teraflop, FLIC action (nucl-th/0308083)
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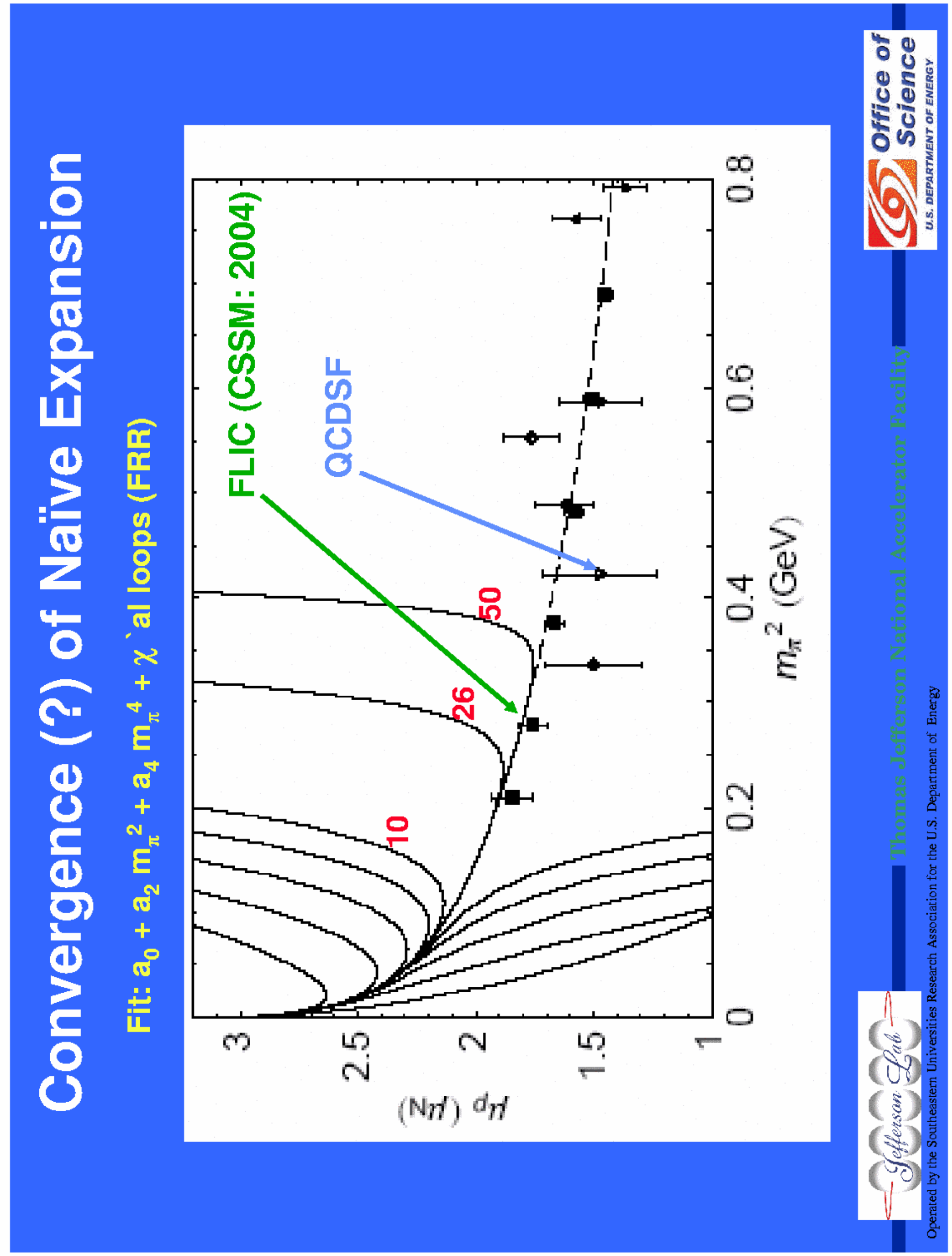

Convergence (?) of Naïve Expansion
Fit: $a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \chi$`al loops (FRR)
FLIC (CSSM: 2004)
QCDSF
10
26
50
μ_p (μ_N)
m_π^2 (GeV)
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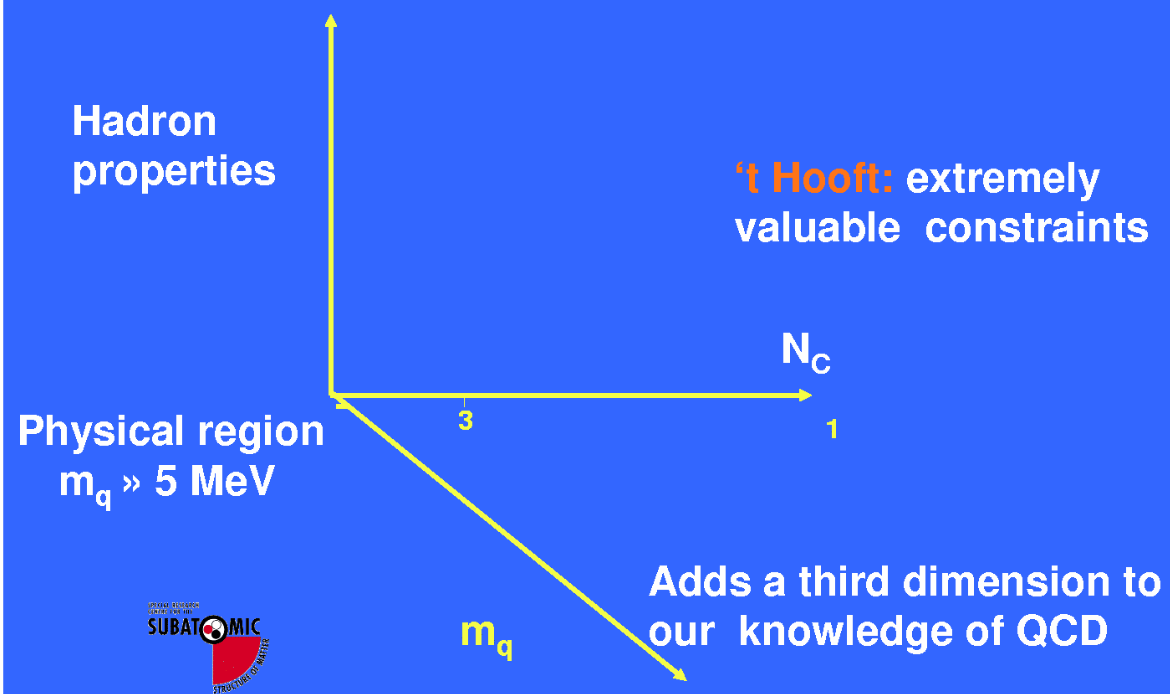
Not a Problem but a Bonus!

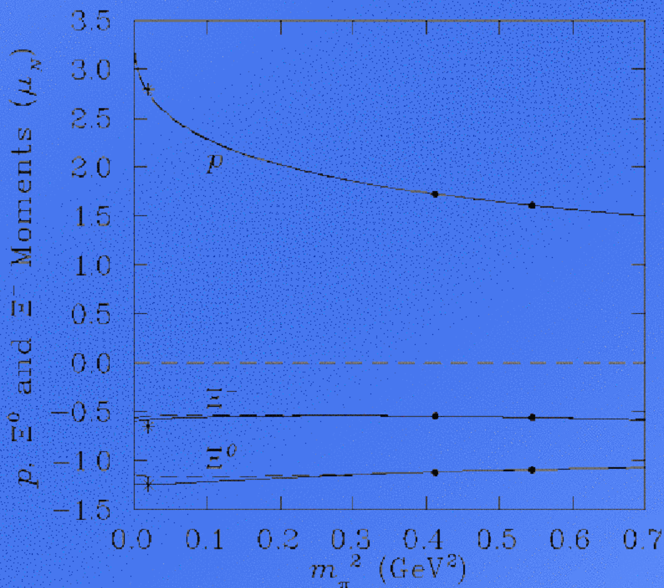


Towards a New Quark Model

- Traditionally Constituent Quark Models <u>for light quarks</u> OMIT effects of Goldstone boson loops!
- OR assume they are included in effective parameters
- **Simply not tenable any longer !**
- Pion loops: δM_N » 300 MeV /// value for δM_Δ
- LNA term in n: $\mu_n = \alpha\, m_\pi$ » 0.6 μ_N is 1/3[rd] of physical μ_n!
- LNA term in $<r^2>_p$ is » 1 fm ^{2} at $m_\pi^{\ phys}$
- LNA terms *depend on hadron* and can ONLY come from Goldstone loops

Chiral Extrapolation Connects CQM to Physical Data

- Calculate CQM magnetic moments at M (strange) +/- 20 MeV (use exact SU(6) symmetry)
- Use Pade approximant to extrapolate to physical quark mass



Cloet et al., Phys. Rev. C65 (2002) 062201