QCD with Three Flavors of Improved Staggered Quarks

Robert Sugar

UCSB

Contributors

- The MILC Collaboration: C. Bernard, C. DeTar, S. Gottlieb, U.M. Heller, J. Hetrick, L. Levkova, F. Maresca, J. Osborn, D.B. Renner, R. Sugar, D. Toussaint
- The Fermilab Lattice, HPQCD, LHPC, and UKQCD Collaborations, and . . .

Outline

- MILC Three Flavor Gauge Configurations
- Initial Tests of the Configurations
- Properties of Light Pseudoscalar Mesons
- Quark and Hadron Masses
- Weak Decays of Heavy–Light Mesons
- High Temperature QCD
- Future Prospects

Challenges for Accurate Calculations

- Must include up, down and strange sea quarks, the only ones light enough to have a significant effect at the accuracy to which we aspire
- Must perform simulations with the masses of the up and down quarks small enough to enable extrapolations to their physical values
- Must perform simulations for a range of lattices spacings to enable extrapolations to the continuum limit
- Must control finite size effects which are expected to be of order $\mathbf{exp(-m_\pi L)}$, where $\mathbf{L}$ is the physical size of the box in which the simulations are performed

Sea Quarks in MILC Lattices

- Include three flavors of sea quarks: $\mathbf{u}$, $\mathbf{d}$, $\mathbf{s}$
- Attempt to fix $\mathbf{m_s}$ at its physical value
- Generate configurations with a range of light quark masses $\mathbf{m_l = (m_u + m_d)/2}$ to enable extrapolations to the physical value of $\mathbf{m_l}$.
- Take $\mathbf{m_u = m_d}$, which has a negligible ($< 1\%$) effect on isospin averaged quantities

Existing Gauge Configurations

- $\mathbf{a = 0.18}$ fm
  - $\mathbf{m_l / m_s = 0.1}$, 0.2, 0.4, 0.6, 1.0
  - lattice dimensions: $\mathbf{16^3 \times 48}$
  - $\mathbf{m_\pi \, L \geq 3.8}$
- $\mathbf{a = 0.125}$ fm
  - $\mathbf{m_l / m_s = 0.1}$, 0.14, 0.2, 0.4, 0.6, 0.8, 1.0
  - lattice dimensions: $\mathbf{20^3 \times 64}$ and $\mathbf{24^3 \times 64}$
  - $\mathbf{m_\pi \, L \geq 3.8}$
- $\mathbf{a = 0.09}$ fm
  - $\mathbf{m_l / m_s = 0.2}$, 0.4, 1.0
  - lattice dimensions: $\mathbf{28^3 \times 96}$
  - $\mathbf{m_\pi \, L \geq 4.2}$

Planned Gauge Configurations

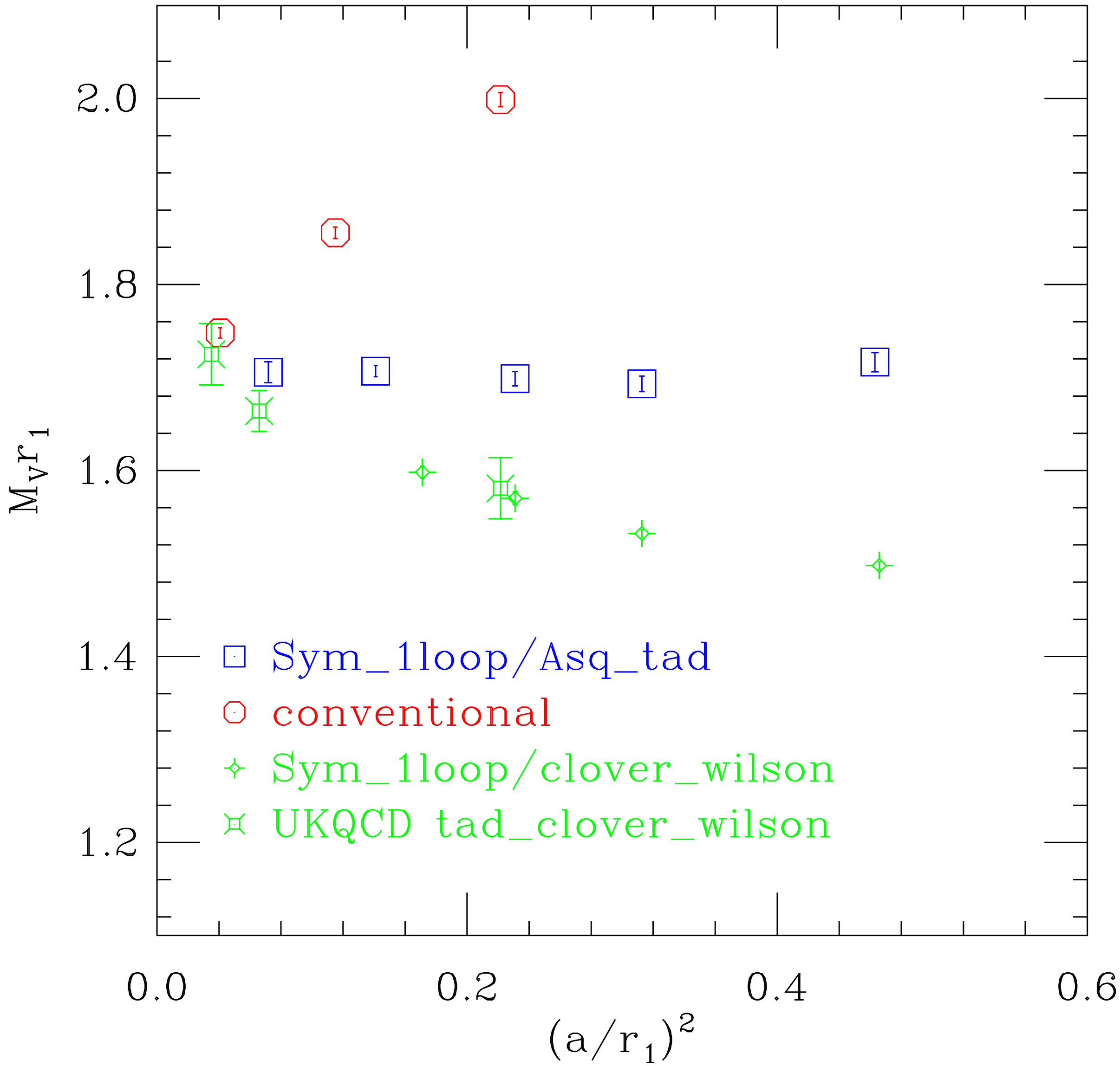
- $\mathbf{a = 0.09}$ fm
  - $\mathbf{m_l / m_s = 0.1}$
  - lattice dimensions: $\mathbf{40^3 \times 96}$
  - $\mathbf{m_\pi L = 4.2}$
- $\mathbf{a = 0.06}$ fm
  - $\mathbf{m_l / m_s = 0.2}$ and 0.4
  - lattice dimensions: $\mathbf{48^3 \times 144}$
  - $\mathbf{m_\pi L \geq 4.9}$

All configurations are publicly available

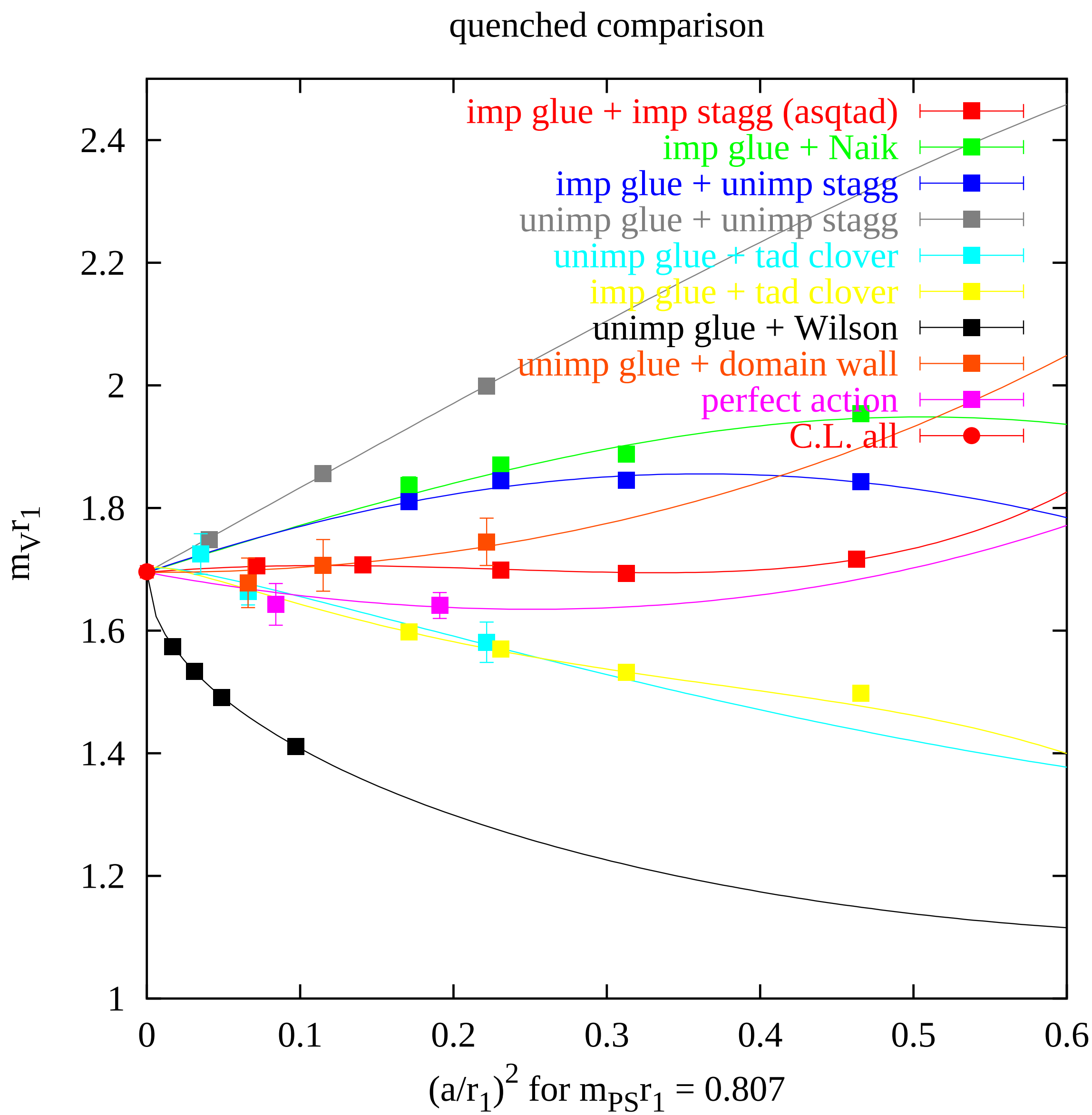
Gauge and Quark Actions

- One–loop Symanzik improved gauge action
- Improved staggered (Asqtad) quark action
- Tadpole improvement
- Leading discretization errors are of order $\mathbf{a^2/\log(a)}$

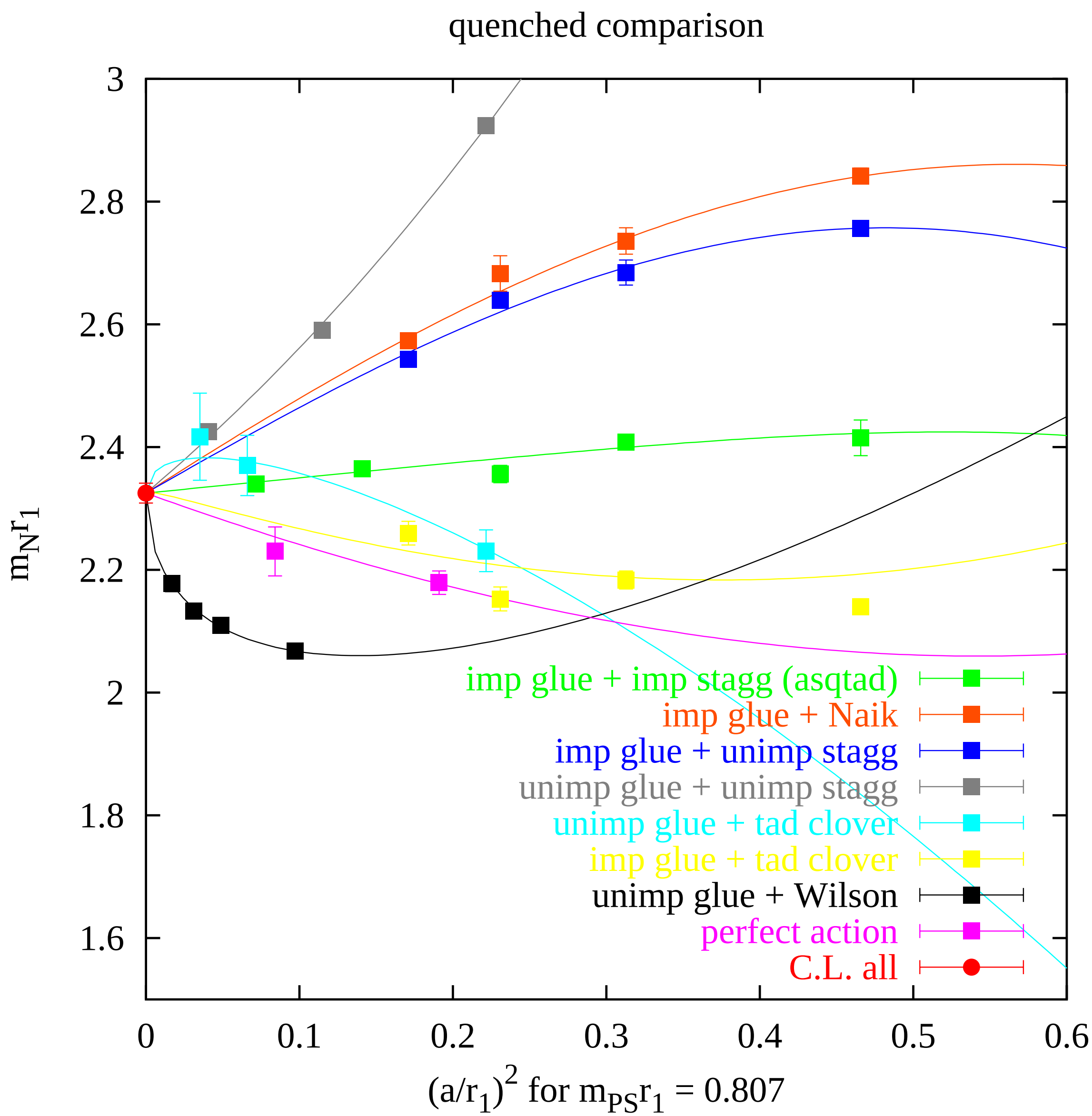
Quenched Rho Masses



Quenched Rho Masses



Quenched Nucleon Masses



Staggered Quarks

- Major advantage: Allow simulations with light sea quark masses using current computers
- Major disadvantage: Taste symmetry breaking
  - Each Kogut-Susskind field contains four quarks which become degenerate only in the continuum limit
  - The improved action significantly reduces taste symmetry breaking
  - Staggered chiral perturbation theory (Lee and Sharpe, Aubin and Bernard) incorporates the effects of taste symmetry breaking and enables controlled extrapolations to the chiral limit

Fourth Root of the Determinant

- One must integrate out the quark degrees of freedom before beginning a simulation, and incorporate the resulting fermion determinant into the weight function for the gluon fields
- A theory with $\mathbf{N_t}$ degerate quarks would have a fermion determinant of the form $[\mathbf{det\,M}]^{\mathbf{N_t}}$, where $\mathbf{M}$ is the Dirac operator for a single quark
- The staggered Dirac operator represents four quarks which become degenerate in the continuum limit
- In order to have one taste per flavor in the continuum limit, one uses $[\mathbf{det\,M_{stag}}]^{\mathbf{1/4}}$ in the simulations.

Fourth Root of the Determinant

- At finite lattice spacings, the fourth root corresponds to averaging over the contributions of the four tastes to each closed quark loops
- In simulations based on hybrid molecular dynamics, it corresponds to averaging over the contributions of the four tastes to the fermion force
- The question have been raised as to whether taking the fourth root of the determinant for finite lattice spacings introduces non-localities which persist in the continuum limit
- No evidence for such non-localities has been found to date

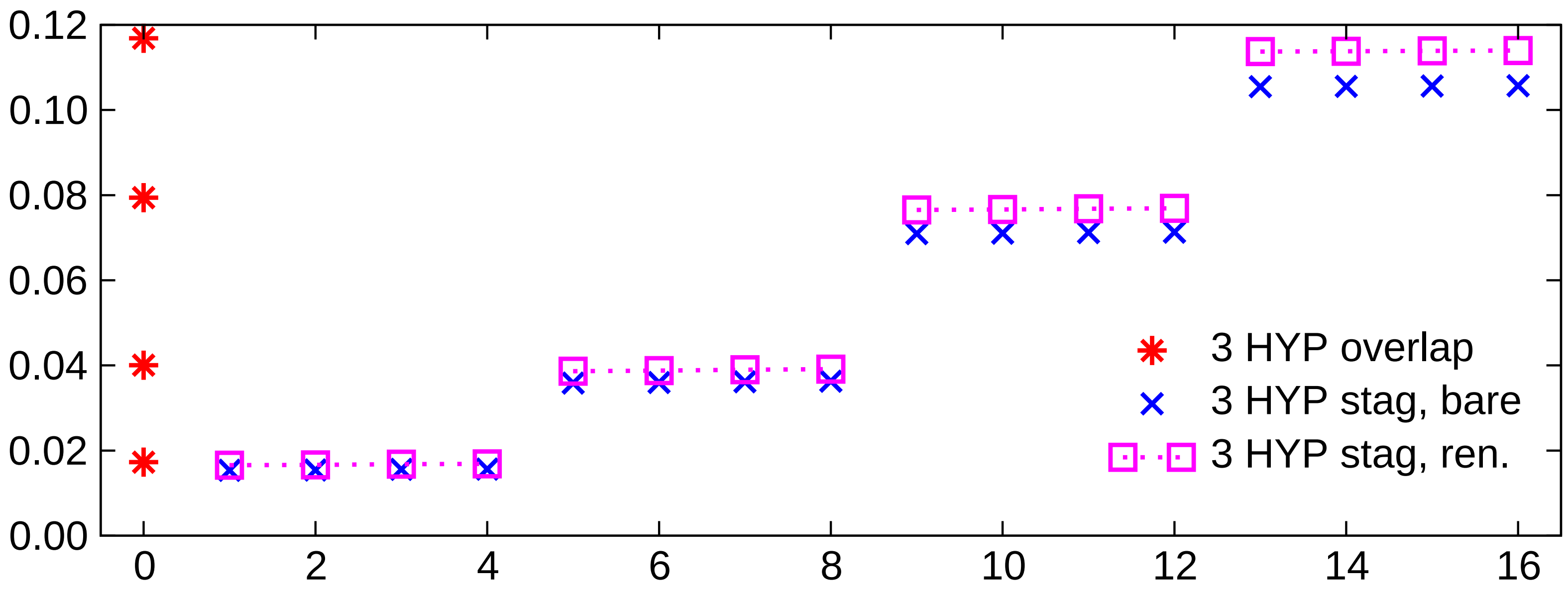
Fourth Root of the Determinant for Free Quarks

- For the free quark action local, single quark Dirac operators have been found with fermion determinants equal to the fourth root of the staggered fermion determinant
  - F. Maresca and M. Peardon
  - D. Adams
  - Y. Shamir

Eigenvalues of the Dirac Operator

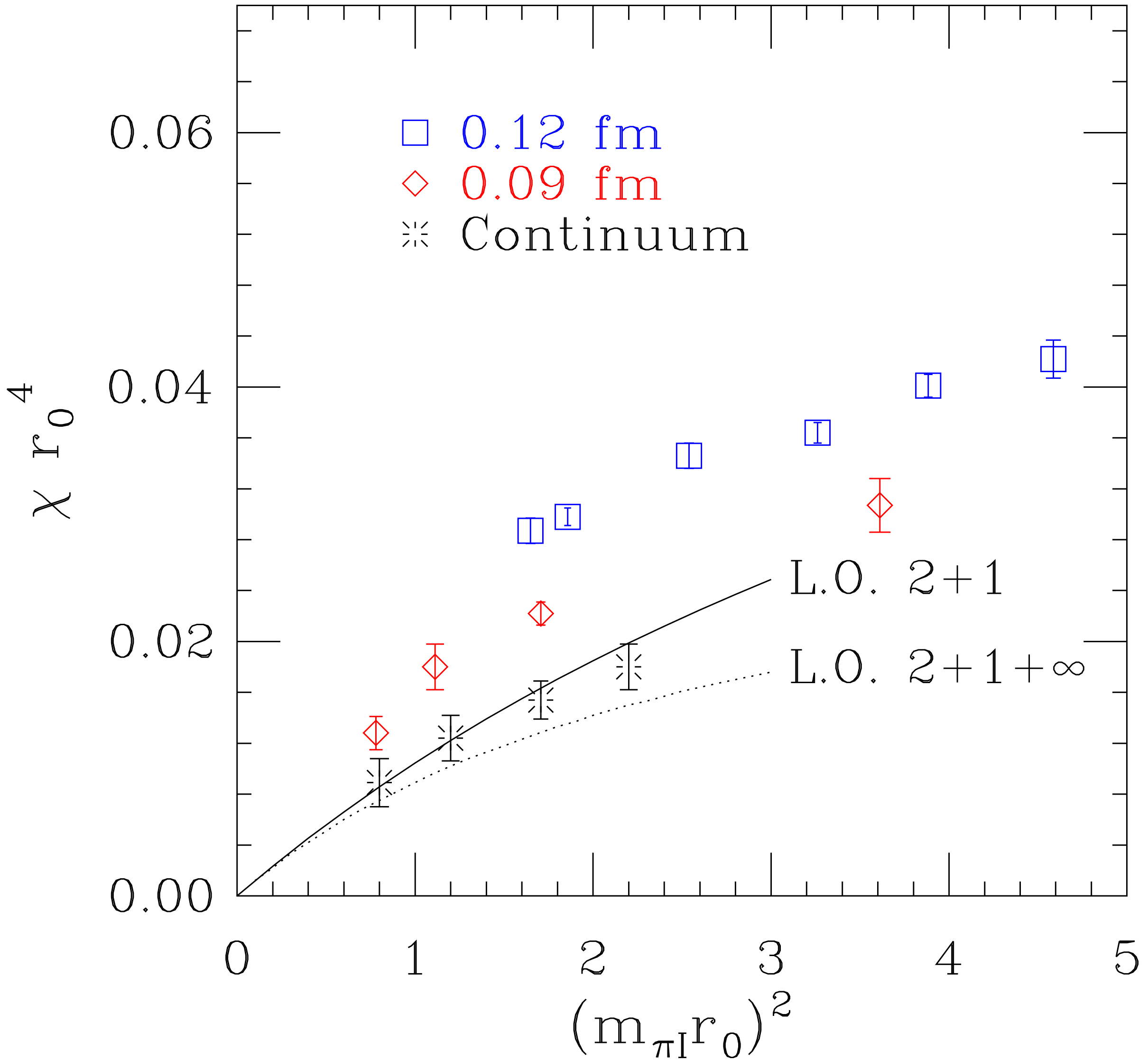
- For interacting quarks there is evidence that on smooth gauge configurations the eigenvalues of the staggered Dirac operator
  - Become four-fold degenerate
  - The near-zero and non-zero modes become well separated as the lattice spacing is decreased
  - The multiplicity of the near-zero modes is consistent with the index theorem
  - The density of the low lying non-zero eigenvalues is consistent with the predicted universal behavior
- See work by
  - S. Dürr and C. Hoelbling
  - E. Follana, A. Hart and C.T.H. Davies
  - E. Follana

Eigenvalues of the Dirac Operator



Eigenvalues of the staggered and overlap Dirac operator on a quenched configuration at $\mathbf{a \simeq 0.07}$ fm. S. Dürr and Ch. Hoelbling

Topological Susceptibiliy



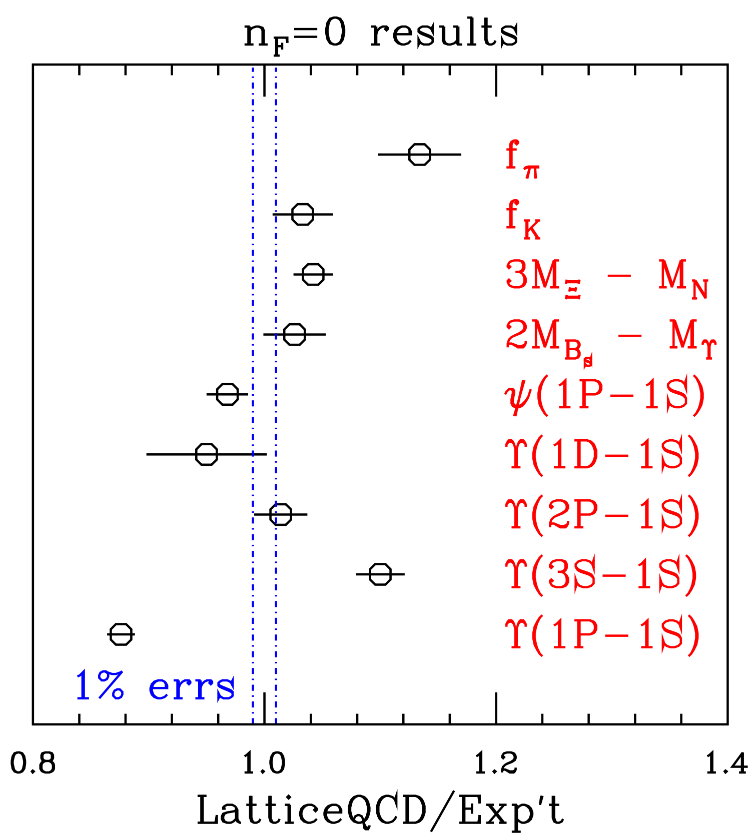
Parameters of the QCD Action

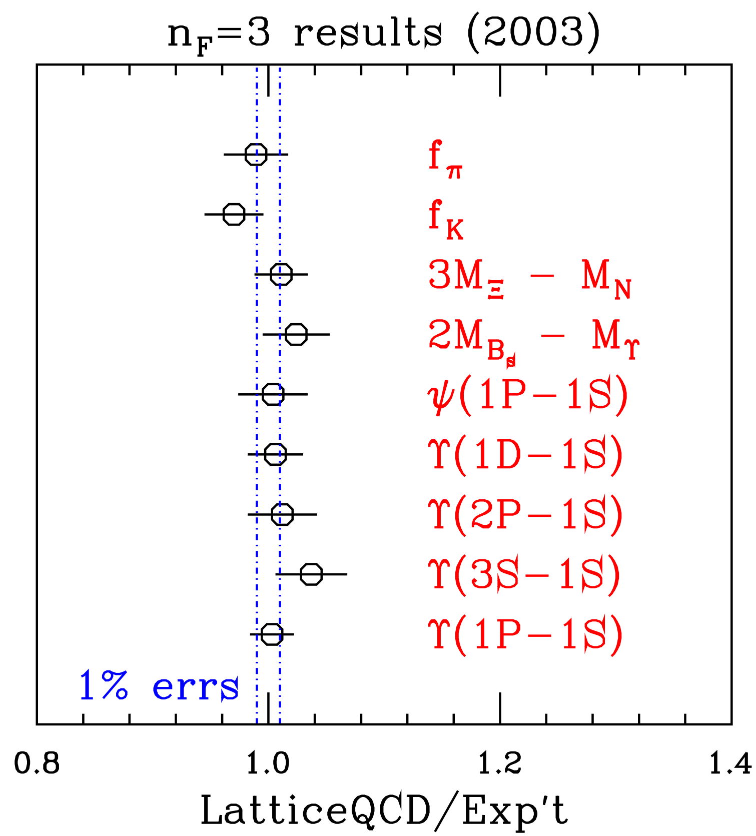
- One must fix five parameters in the QCD action from experiment and corresponding lattice calculations
  - Lattice spacing fixed from the $\mathbf{2S - 1S}$ or $\mathbf{1P - 1S}$ splitting in the upsilon spectrum
  - $\mathbf{m_l}$ fixed from the pion mass
  - $\mathbf{m_s}$ fixed from the kaon mass
  - $\mathbf{m_c}$ fixed from $\mathbf{m_{D_s}}$
  - $\mathbf{m_b}$ fixed from $\mathbf{m_\Upsilon}$
- Once these parameters are fixed, there is no remaining freedom in calculating all other physical quantities

Straightforward Calculations

- The most straightforward quantities to calculate
  - Involve stable hadrons or those with narrow widths
  - Have at most one hadron in the initial and final state
  - Allow good control over the chiral extrapolation
- Quantities that have these properties and are well determined experimentally provide stringent tests of our calculational approach
- There are many quantities with these properties that are important phenomenologically

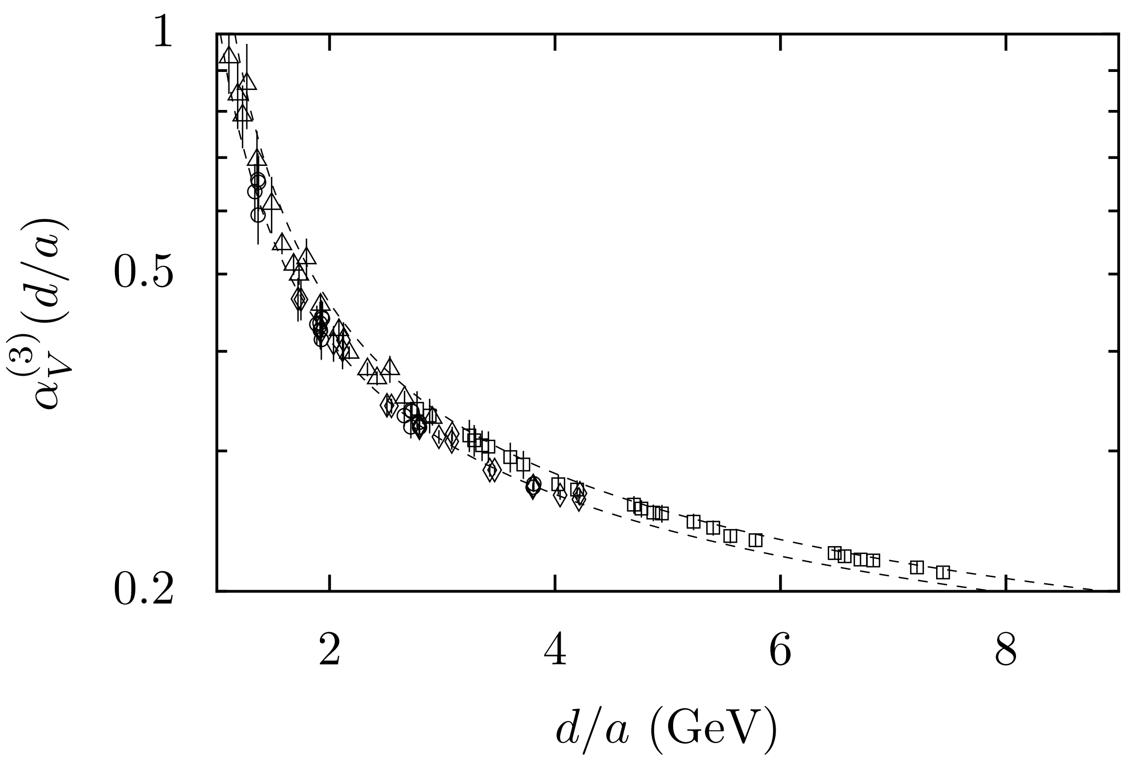
Initial Tests





Results of the Fermilab Lattice, HPQCD, MILC and UKQCD Collaborations

The Strong Coupling Constant



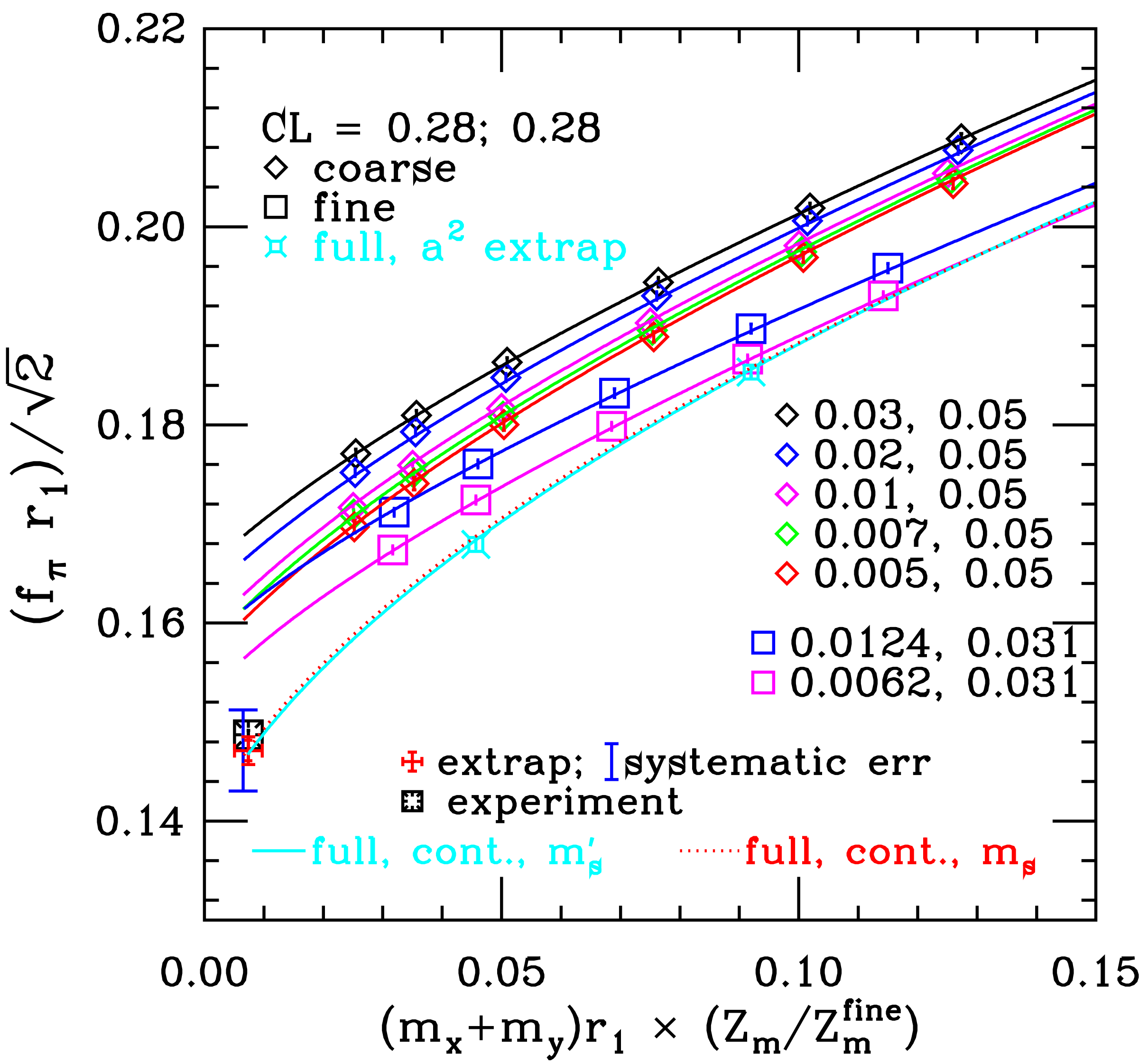
$$\alpha^{\overline{\mathsf{MS}}}(m_Z) \;=\; \mathbf{1177(13)} \quad \mathbf{HPQCD, UKQCD}$$

$$\alpha^{\overline{\mathsf{MS}}}(m_Z) \;=\; \mathbf{1187(20)} \quad \mathbf{Particle\ Data\ Group}$$

Light Pseudoscalar Mesons

- We can determine the masses and leptonic decay constants for fixed lattice spacing and quark masses to very high accuracy (0.1% to 0.7%)
- Staggered chiral perturbation theory allows us to fit the data accurately, including the effects of $\mathbf{O(a^2)}$ finite lattice spacing errors, and to make controlled extrapolations to the physical value of $\mathbf{m_l}$
- These calculations provide an opportunity to
  - Check our calculational methods to high accuracy
  - Obtain new results

Leptonic Decay Constant of the Pion



Results for Light Decay Constants

- Lattice Results

$$\begin{aligned} f_\pi &= 129.5 \pm 0.9 \pm 3.5 \text{ MeV} \\ f_K &= 156.6 \pm 1.0 \pm 3.6 \text{ MeV} \\ f_K/f_\pi &= 1.210 \pm 4 \pm 13 \end{aligned}$$

- Experimental Results

$$\begin{aligned} f_\pi &= 130.7 \pm 0.4 \text{ MeV} \\ f_K &= 159.8 \pm 1.5 \text{ MeV} \\ f_K/f_\pi &= 1.223 \pm 12 \end{aligned}$$

Determination of V_{us}

- Marciano has pointed out that an accurate lattice determination of f_K/f_π, coupled with experimental results for the decays $\pi \longrightarrow \mu\bar{\nu}(\gamma)$ and $K \longrightarrow \mu\bar{\nu}_\mu(\gamma)$, gives an accurate determination of the CKM matrix element $|V_{us}|$
- MILC results for f_K/f_π give $|V_{us}| = 0.2219 \pm 26$
- PDG value = 0.2196 ± 26
- Recent KTeV value = $0.2252 \pm 8 \pm 21$

Light Quark Masses

- The bare values of $\mathrm{m_l}$ and $\mathrm{m_s}$ are accurately determined from the masses of the pion and kaon
- Combining these results with one-loop perturbation theory gives at a renormalization scale of 2 GeV

$$
\begin{aligned}
m_s^{\overline{\text{MS}}} &= \mathbf{76(0)(3)(7)(0)\ MeV} \\
m_l^{\overline{\text{MS}}} &= \mathbf{2.8(0)(1)(3)(0)\ MeV} \\
m_s/m_l &= \mathbf{27.4(1)(4)(0)(1)}
\end{aligned}
$$

Masses of the Up and Down Quarks

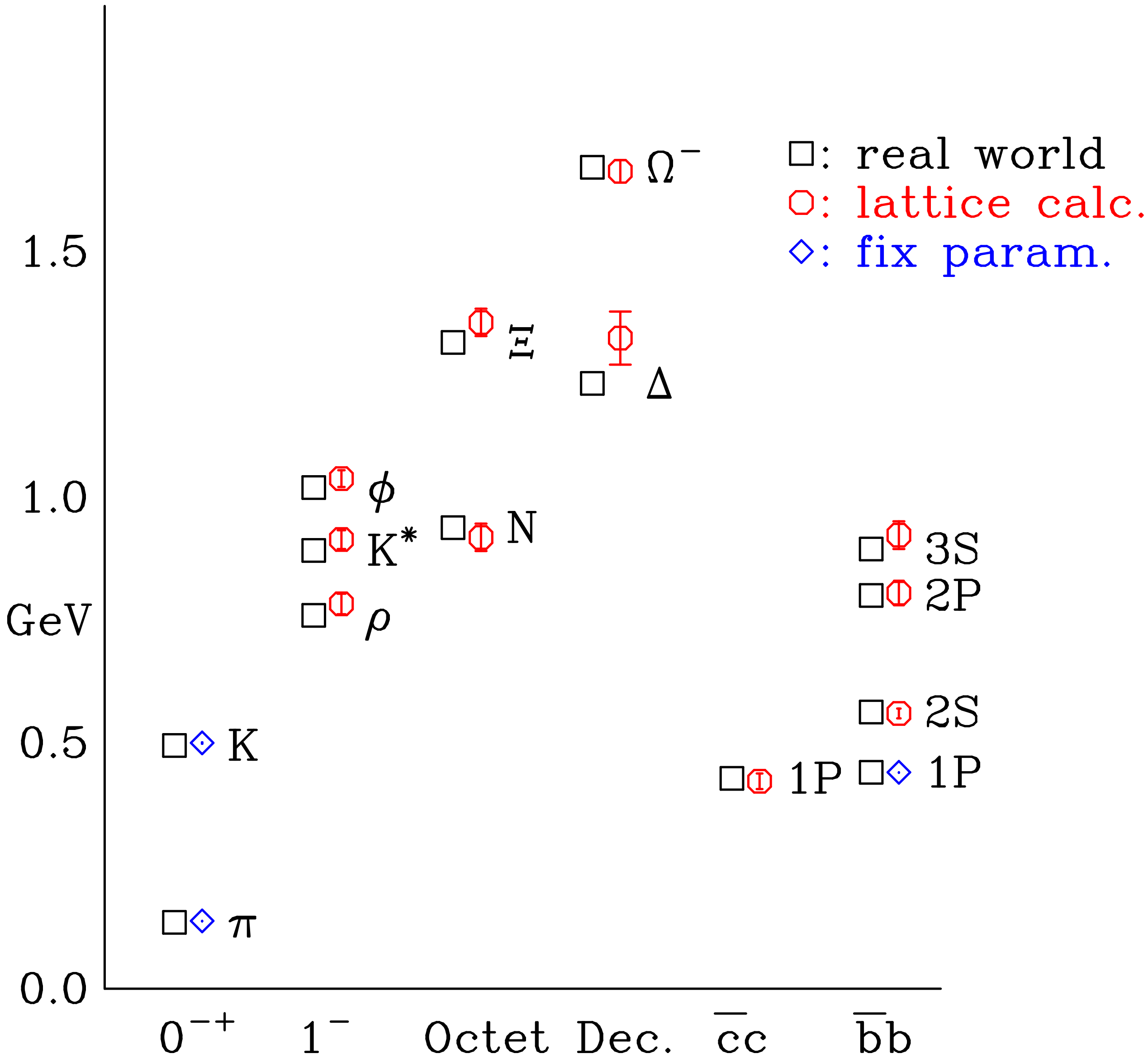
- One can estimate the $\mathbf{m_u - m_d}$ mass splitting from those of $\boldsymbol{\pi_+ - \pi_0}$ and $\mathbf{K_+ - K_0}$. We find

$$
\begin{aligned}
m_u/m_d &= \mathbf{0.43(0)(1)(8)} \\
m_u^{\overline{\text{MS}}} &= \mathbf{1.7(0)(1)(2)(2)\ MeV} \\
m_d^{\overline{\text{MS}}} &= \mathbf{3.9(0)(1)(4)(2)\ MeV}
\end{aligned}
$$

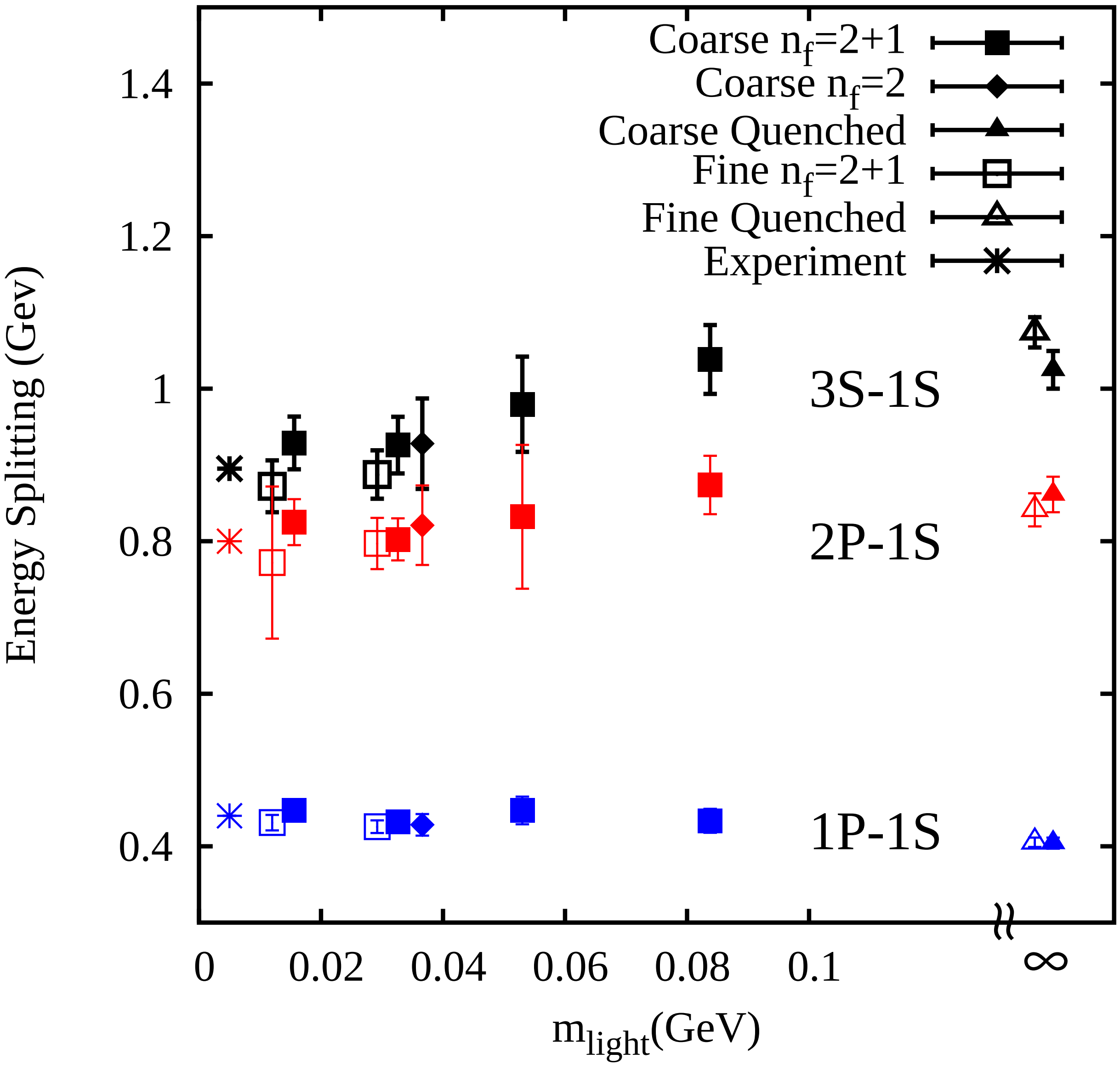
The Hadron Spectrum

- Calculation of the masses of light hadrons test our numerical methods
- Quark model assignments of many excited states are not well established, and lattice calculations can help to pin them down.
- Accurate determinations of the masses and decay properties of glueballs and particles with exotic quantum number could aid experimental searches for them

The Big Picture



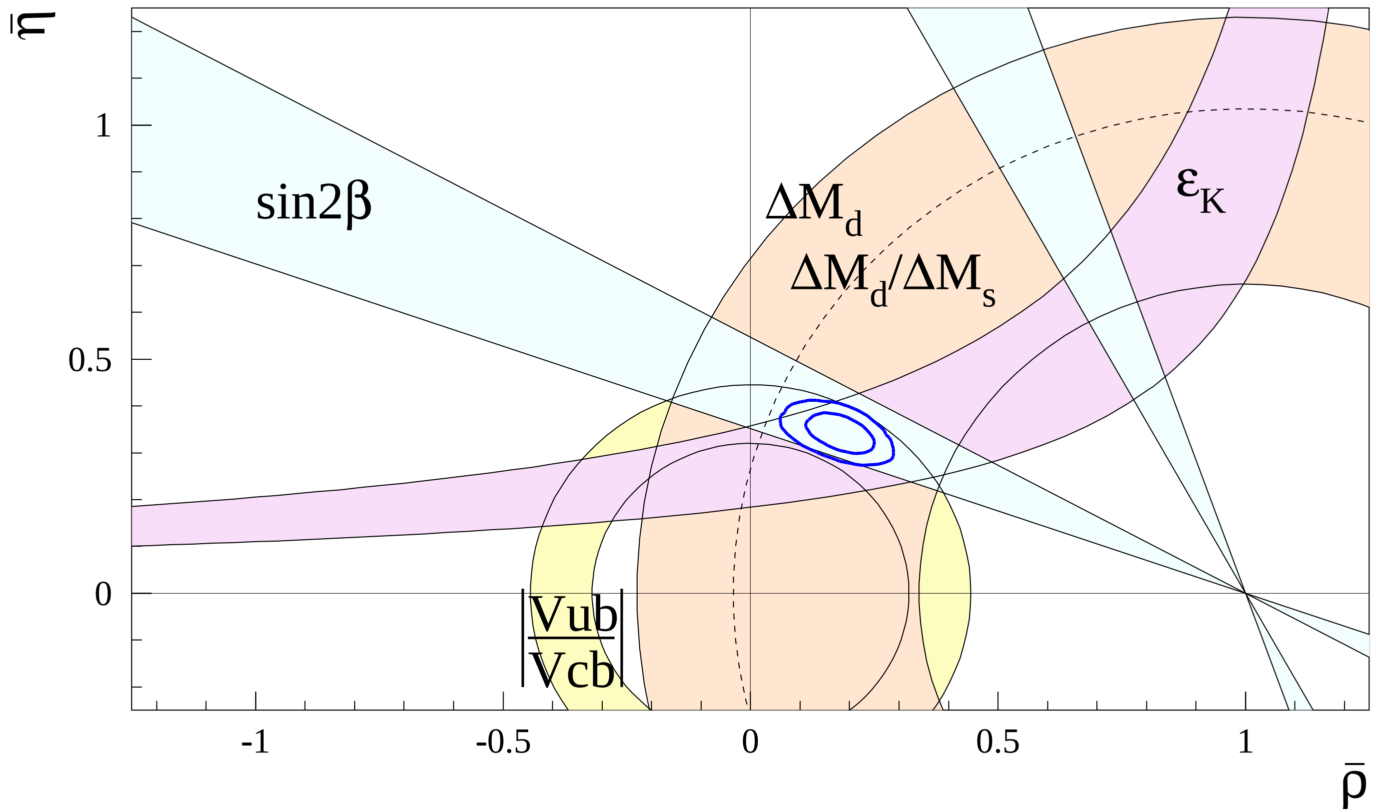
Upsilon Spectrum – HPQCD



Weak Interactions of Hadrons

- Lattice calculations can have an important impact on the study of the weak interactions of hadrons
- An important example concerns the major experimental efforts to study the decays and mixings of B and D mesons. The objectives are to:
  - Determine elements of the CKM Matrix
  - Make precise tests of the Standard Model
- In many cases a lattice calculation of a weak matrix element is needed to extract the CKM matrix element from the experimental result
- In most cases it is the uncertainty in the lattice calculation that limits the accuracy of the results

$\rho - \eta$ Plot



Allowed regions for $\bar{\rho}$ and $\bar{\eta}$, the least well known elements of the CKM matrix. Each of the measured quantities constrains $\bar{\rho}$ and $\bar{\eta}$ as shown: M. Ciuchini, *et al.* (2003)

Impact of Improved Lattice Calculations

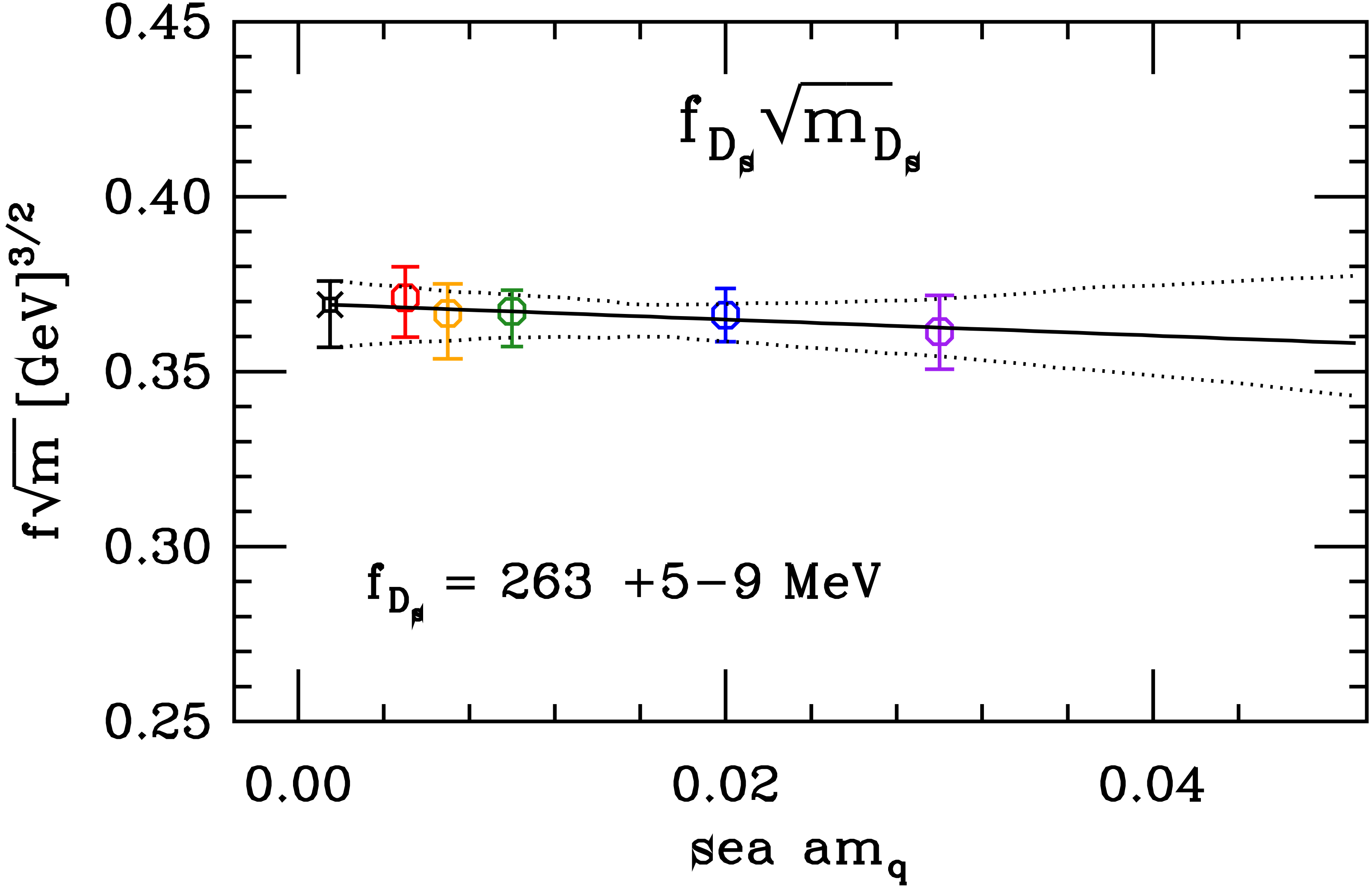
Measurement	CKM Matrix Element	Hadronic Matrix Element	Non-Lattice Errors	Current Lattice Errors	Lattice Errors MILC0	Lattice Errors MILC1
ε_K ($\bar{K}K$ mixing)	$\mathrm{Im}\, V_{td}^2$	$\hat{B}_K$	10%	20%	12%	5%
ΔM_d ($\bar{B}B$ mixing)	$\lvert V_{td}\rvert^2$	$f_{B_d}^2 B_{B_d}$	6%	30%	16%–26%	8%–10%
$\Delta M_d/\Delta M_s$	$\lvert V_{td}/V_{ts}\rvert^2$	ξ^2	—	12%	8%	6%
$B \to \binom{\rho}{\pi} l\nu$	$\lvert V_{ub}\rvert^2$	$\left\langle \begin{smallmatrix}\rho\\ \pi\end{smallmatrix} \right\rvert (V-A)_\mu \lvert B\rangle$	7%	15%	10%–13%	5.5%–6.5%

The error estimates are from the Lattice QCD Executive Committee Whitepaper (2004). MILC0 refers to the existing configurations and MILC1 to the ones planned for the coming year.

Decays of B and D Mesons

- The Fermilab Lattice and MILC Collaborations are calculating leptonic decay constants and semileptonic form factors of B and D mesons using improved staggered sea and light valence quarks, and Fermilab heavy quarks
- The HPQCD Collaboration is calculating B meson leptonic decays constants and bag parameters, and semileptonic form factors using improved staggered sea and light valence quarks, and NRQCD heavy quarks
- The CLEO-c studies of D decays will provide important tests for the lattice calculations
- These calculations will enable more accurate determinations of a number of CKM matrix elements such as $\mathbf{V_{ub}}$, $\mathbf{V_{cb}}$, $\mathbf{V_{cd}}$ and $\mathbf{V_{cs}}$

Preliminary Results for f_{D_s}



Leptonic Decay Constants of D Mesons

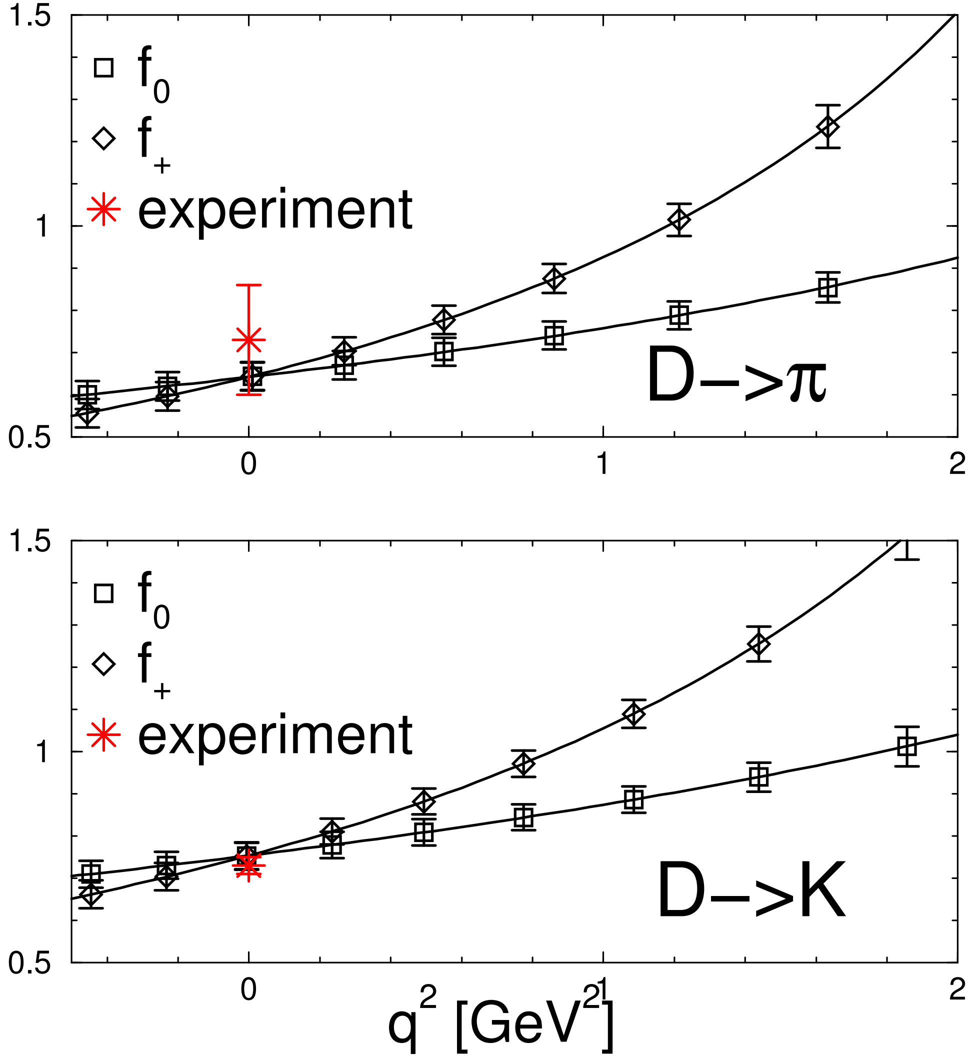
- Preliminary FNAL/MILC results

$$\begin{aligned} \frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} &= 1.20 \pm 0.06 \pm 0.06 \\ f_{D_s} &= 263^{+5}_{-9} \pm 24 \text{ MeV} \\ f_D &= 225^{+11}_{-13} \pm 21 \text{ MeV} \end{aligned}$$

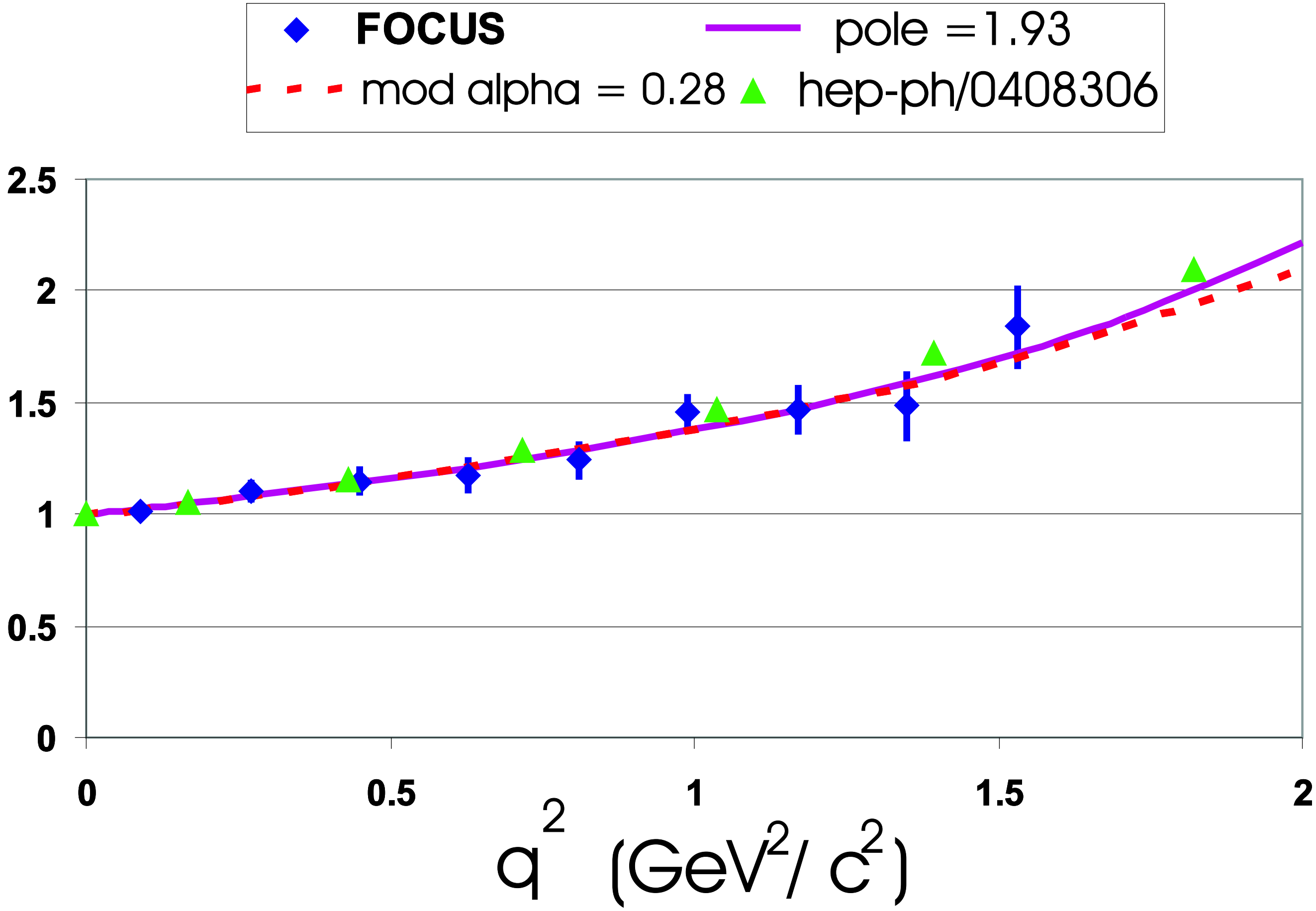
- Preliminary CLEO-c results

$$\begin{aligned} f_{D_s^+} &= 266 \pm 32 \text{ MeV} \\ f_{D^+} &= 202 \pm 41 \pm 17 \text{ MeV} \end{aligned}$$

Semileptonic Form Factors of D Mesons



Momentum Dependence of $D \to Kl\nu$



$f^+(q^2)/f^+(0)$ for $D \to Kl\nu$: The Focus Collaboration

$D \rightarrow K l \nu$ Form Factor

- FNAL/HPQCD/MILC Lattice Result

$$f_+^{D-K}(0) = 0.73 \pm 3 \pm 7$$

- BES Collaboration Experimental Result

$$f_+^{D-K}(0) = 0.78 \pm 8$$

$D \to \pi l \nu$ Form Factor

- FNAL/HPQCD/MILC Lattice Result

$$\begin{aligned} f_+^{D-\pi}(0) &= 0.64 \pm 3 \pm 6 \\ f_+^{D-\pi}(0)/f_+^{D-K}(0) &= 0.87 \pm 3 \pm 9 \end{aligned}$$

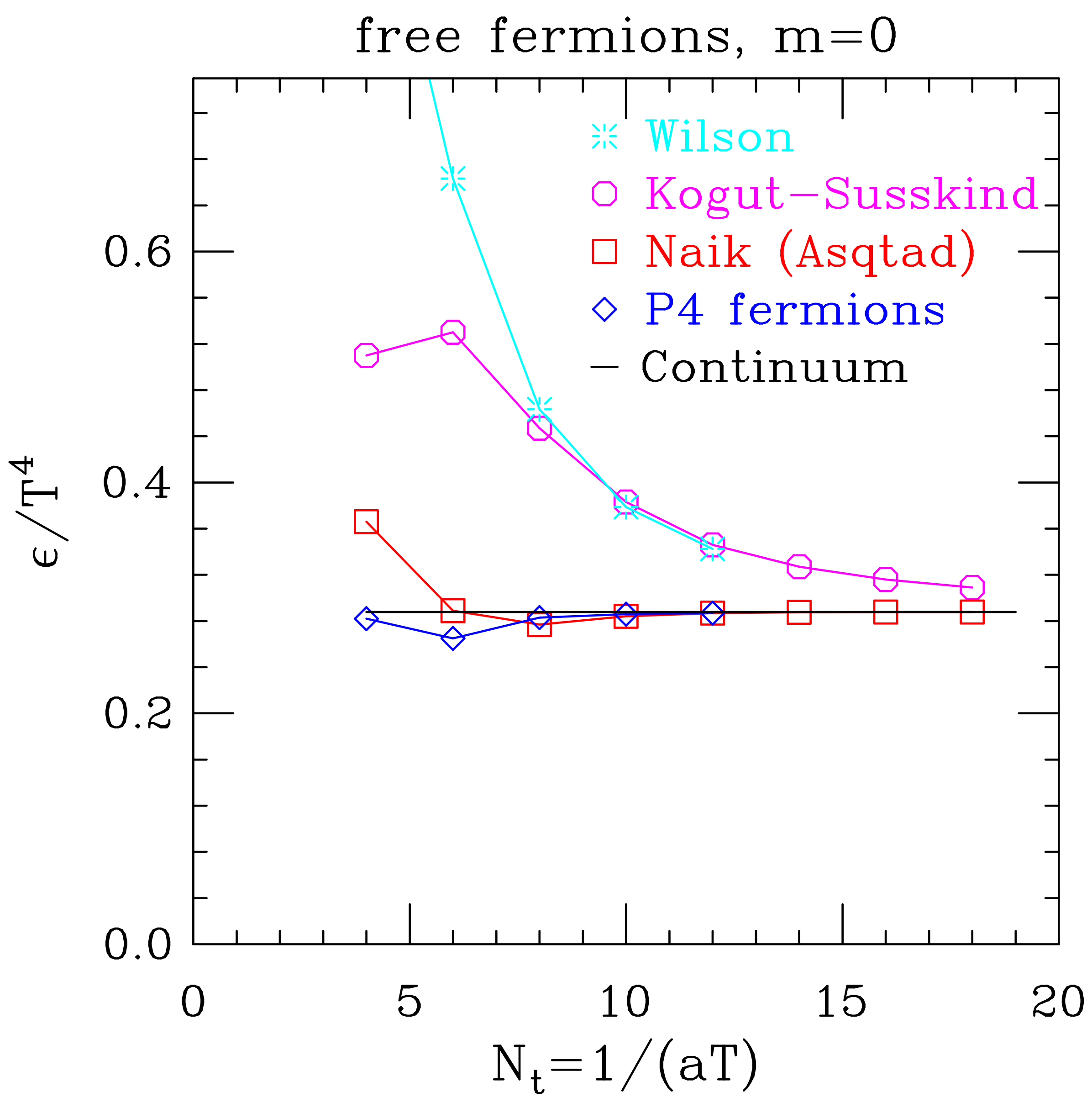
- CLEO Collaboration Experimental Result

$$f_+^{D-\pi}(0)/f_+^{D-K}(0) = 0.86 \pm 9$$

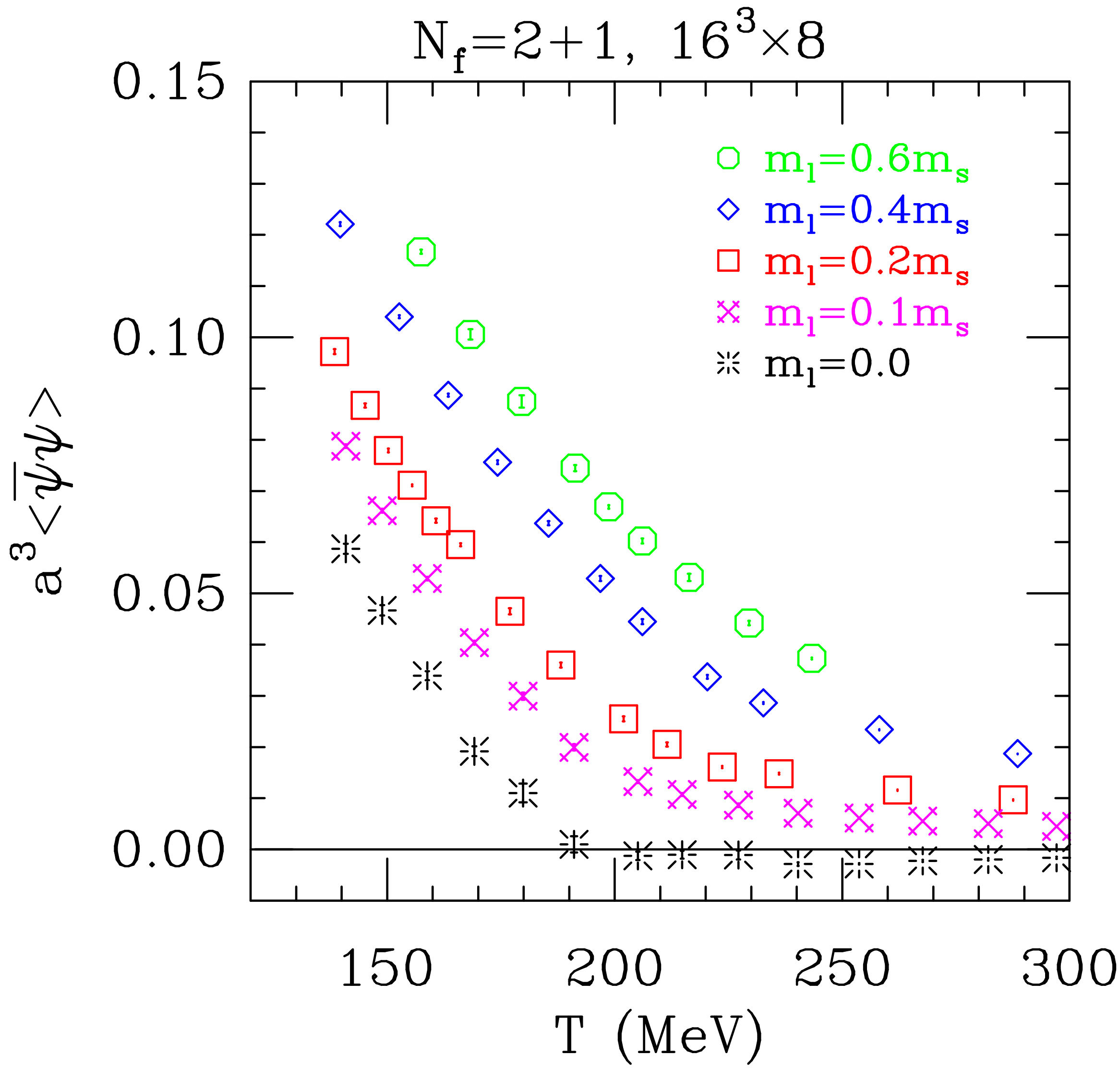
High Temperature QCD

- Improved actions play an important role
  - Improve the dispersion relations of quarks and physical particles
  - Reduce taste breaking effects in pion spectrum
  - Improve scaling in lattice spacing
- MILC Objectives
  - Determine phase diagram
  - Calculatate the equation of state
  - Calculate quark number susceptibilities

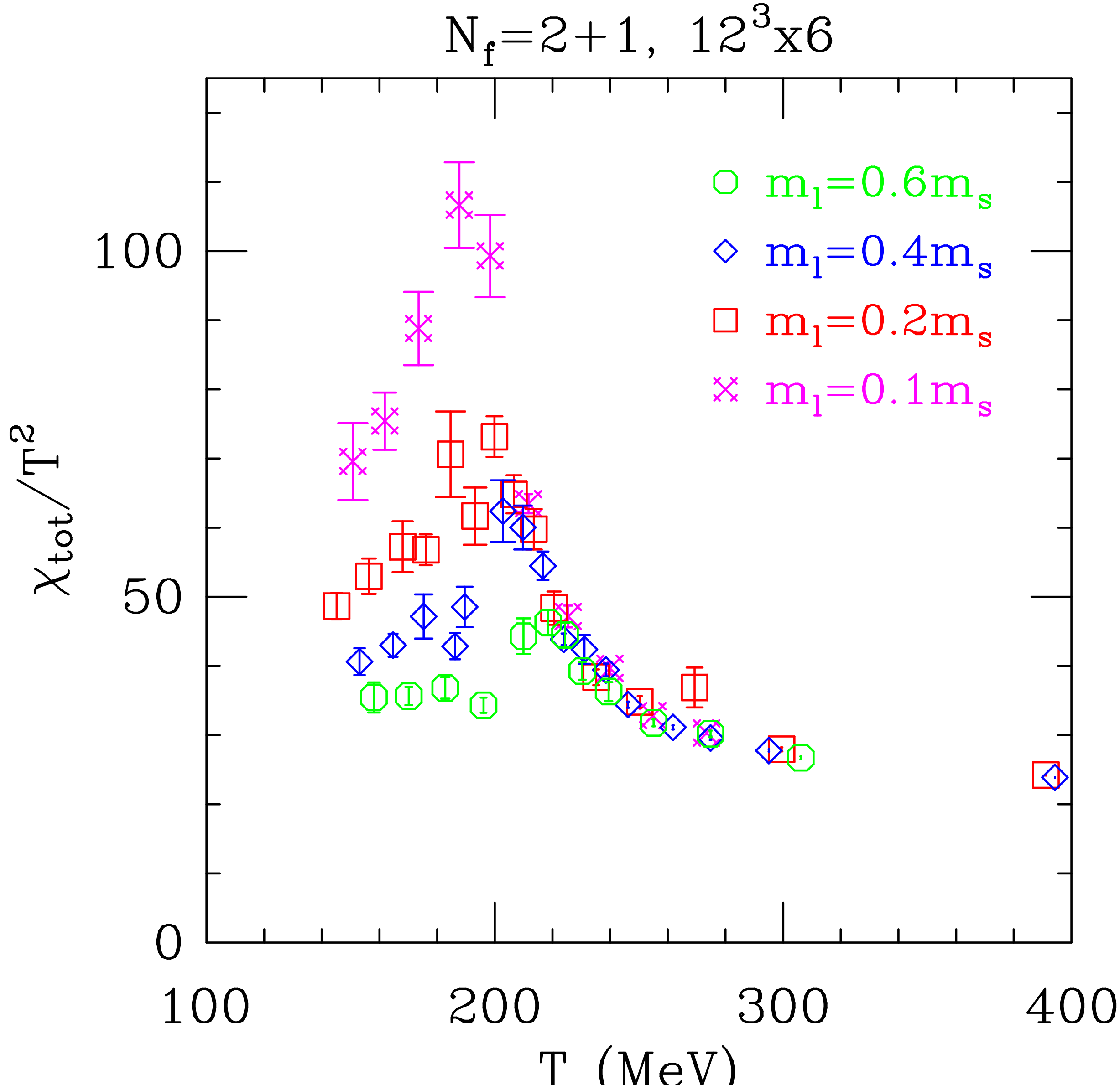
Free Energy of Free Massless Quarks



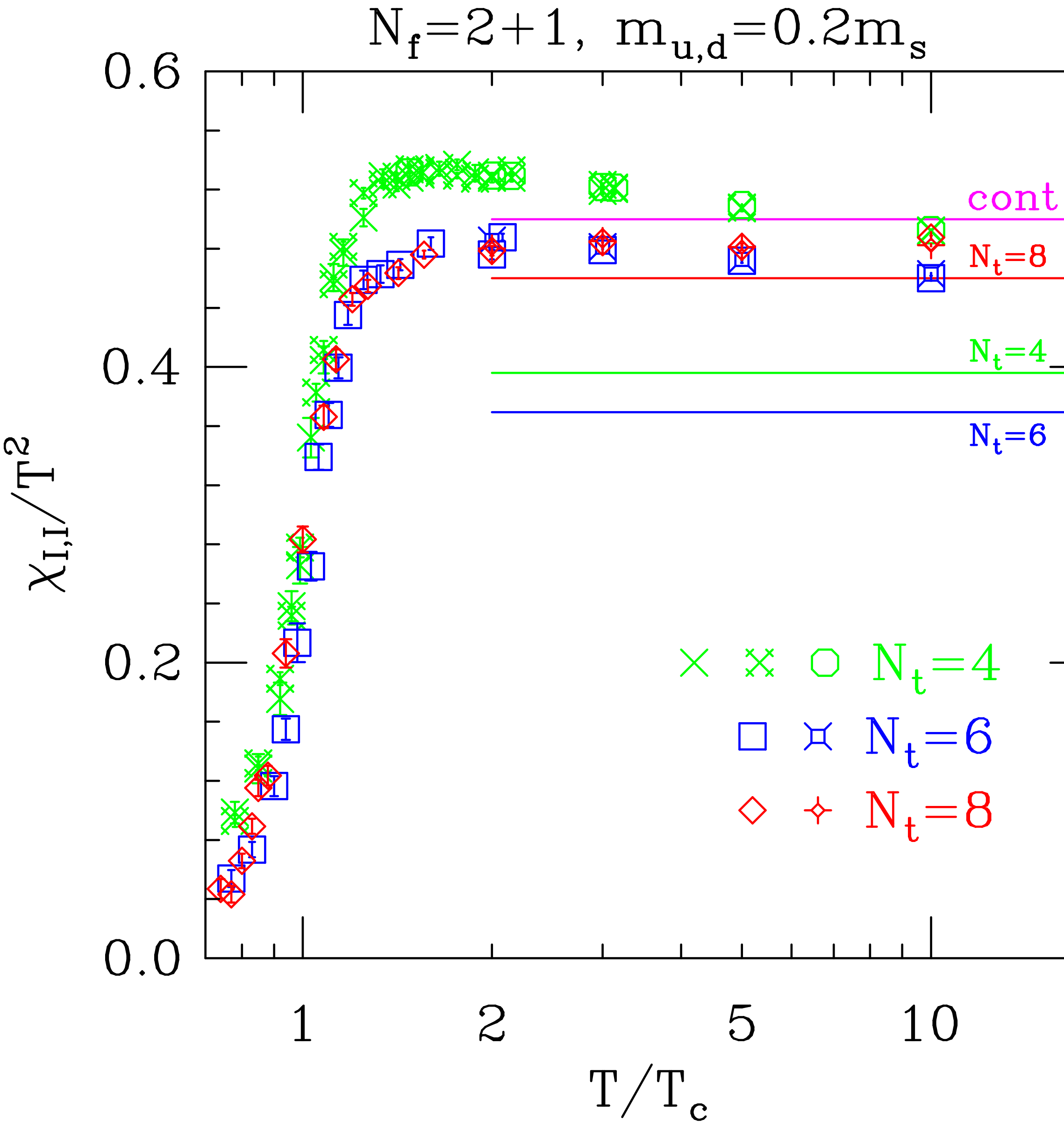
The Chiral Order Parameter



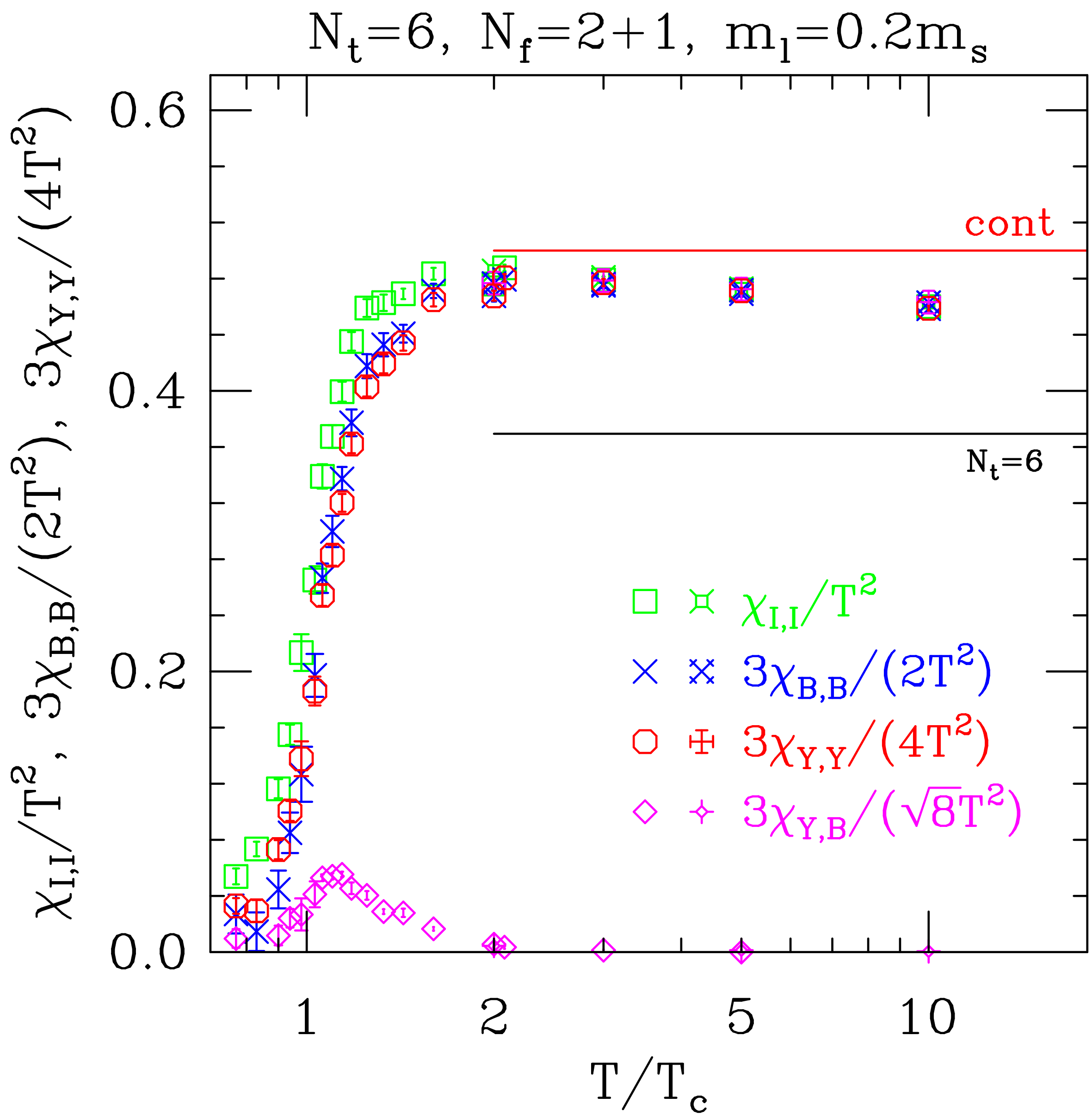
The $\bar{\psi}\psi$ Susceptibility



The Isospin Density Susceptibility



Quark Number Susceptibilities



Use of Configurations by Other Groups

- Spectroscopy of heavy-heavy and heavy-light mesons
- Decays of heavy-light mesons using Fermilab and NRQCD heavy quarks
- Calculations of $\mathrm{B_B}$ and $\mathrm{B_K}$
- Determination of α_S
- Calculation of light quark masses
- Parton distributions in nucleons
- Calculations of nucleon and meson form factors and spectral functions
- Hadronic contribution to the anomalous magnetic moment of the muon

Users of MILC Configurations

M. Alford (Washington U)
R.C. Brower (Boston U)
A. Dougall (Glasgow U)
R. Edwards (Jefferson Lab)
E. Follana (Glasgow U)
E. Gulez (Ohio State U)
G.P. Lepage (Cornell U)
M. Nobes (Simon Fraser U)
J.W. Negele (MIT)
M. Okamoto (Fermilab)
P. Petreczky (BNL)
D.B. Renner (MIT)
W. Schroers (MIT)
H. Trottier (Simon Fraser U)

T. Blum (Connecticut U)
C. Davies (Glasgow U)
P. Dreher (MIT)
G. Fleming (Jefferson Lab)
E. Gamiz (Glasgow U)
P. Hagler (MIT)
R. Lewis (Regina U)
A.S. Kronfeld (Fermilab)
P.B. Mackenzie (Fermilab)
M. Oktay (U Illinois)
K. Petrov (Columbia U)
D. Richards (William & Mary U)
J. Shigemitsu (Ohio State U)
M. Wingate (Washington U)

F.D.R. Bonnet (Regina U)
M. Di Pierro (DePaul U)
A. El-Khadra (U Illinois)
K. Foley (Cornell U)
A. Gray (Ohio State U)
J. Hein (Edinburgh U)
Q. Mason (Cornell U)
T. Lippert (Wuppertal U)
D. Menscher (Fermilab)
K. Orginos (MIT)
A.V. Pochinsky (MIT)
K. Schilling (Wuppertal U)
J. Simone (Fermilab)

Configuration Generation Plans

Plans for 2005–2006

a(fm)	m_l/m_s	$m_\pi L$	Lattice	Trajectories	TF-Yrs
0.09	0.1	4.2	$40^3 \times 96$	3,000	0.54
0.06	0.4	7.0	$48^3 \times 144$	3,500	0.52
0.06	0.2	4.9	$48^3 \times 144$	3,750	1.68

Possibilities for the Future?

a(fm)	m_l/m_s	$m_\pi L$	Lattice	Trajectories	TF-Yrs
0.06	0.1	4.2	$60^3 \times 144$	4,500	8.0
0.06	0.05	4.2	$84^3 \times 144$	6,300	93.2
0.045	0.4	6.1	$56^3 \times 192$	4,000	2.2
0.045	0.2	4.3	$56^3 \times 192$	5,000	7.5
0.045	0.1	4.2	$80^3 \times 192$	6,000	54.8

Prospects for the Future

- Existing configurations plus those planned for 2005–2006 will greatly improve the accuracy of the determination of:
  - Properties of light pseudoscalar mesons
  - Weak interaction matrix elements
  - Hadron spectrum
- The high statistics runs planned by UKQCD will enable accurate determinations of the masses of flavor singlet mesons, exotic mesons and glueballs
- Major progress will be made in the study of high temperature QCD in the next few years
- Calculations at finite baryon density will continue to pose a major challenge, but important progress is being made

Prospects for the Future

- The focus of large scale configuration generation will shift to actions with exact or nearly exact chiral symmetry
- Studies with chiral actions will test and confirm calculations with staggered actions