QCD and a Holographic Model of Hadrons

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Motivation and plan

- Large N_c:
  - planar diagrams dominate
  - resonances are infinitely narrow
- Effective theory in terms of resonances is weakly coupled?
- What is this effective theory?
- String theory explicit examples suggest it is a 5d effective theory.
- Bottom-up approach.

A holographic model: J. Erlich, E. Katz, D. Son and M.S., hep-ph/0501128.

(L. Da Rold, A. Pomarol, hep-ph/0501218)

This talk:

- ABC of AdS/CFT (holography)
- A simple model:
  - chiral symmetry breaking
  - quark-hadron duality, sum rules
  - OPE
  - etc.

AdS/CFT correspondence: formulation

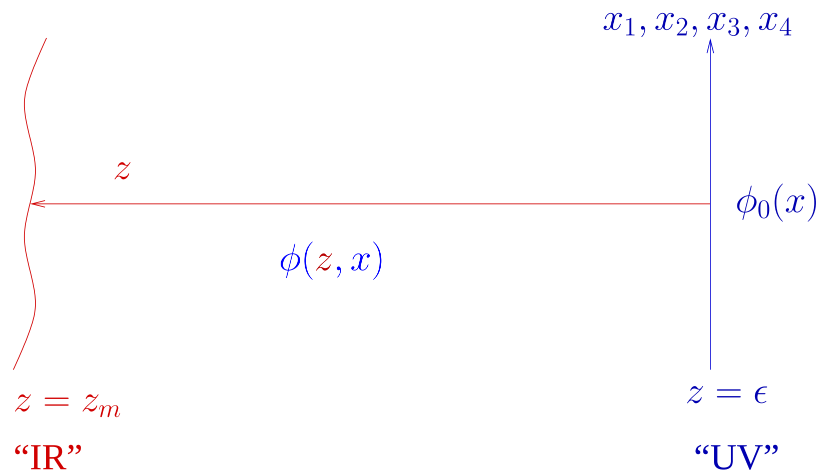
Begin with $S_4[G,q] = \int d^4x\, \mathcal{L}[G,q]\,.$
Generating functional for correlators of an operator $\mathcal{O}$ (examples of $\mathcal{O}$: $G^a_{\mu\nu}G^{\mu\nu a}$, $\bar{q}q$, $\bar{q}\gamma^\mu t^a q$, $\ldots$):

$$Z_4[\phi_0(x)] = \int \mathcal{D}[G,q]\ \exp\left\{ iS_4 + i\int_{x^4} \phi_0 \mathcal{O} \right\}.$$

5d bulk metric:
$ds^2 = z^{-2}\left(-dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$.
$\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$.

(Note: $x^m \to \lambda x^m$).

Polchinski-Strassler slice



$$Z_5[\phi_0(x)] = \int_{\phi(x,\epsilon)=\phi_0(x)} \mathcal{D}[\phi] e^{iS_5[\phi]}$$

$$Z_4 = Z_5$$

(Generating functional) [4d sources $\phi_0(x)$] = (Effective action) [fields $\phi_0(x)$].

Example: conserved current

Let $\mathcal{O}$ be a current: $J^{\mu a} = \bar{q}\gamma^\mu t^a q$.
ϕ_0: source for $J^{\mu a}$ is a vector potential $V_0^{\mu a}$. I.e.,

$$Z_4[V] = \int \mathcal{D}[G,q] \exp\left\{ iS_4 + i\int_{x^4} V_0 \cdot J \right\}.$$

We shall look at

$$\int d^4x e^{iqx} \langle J^{\mu a}(x) J^{\nu b}(y) \rangle = \delta^{ab}(q^\mu q^\nu - q^2 \eta^{\mu\nu}) \Pi(-q^2).$$

In QCD, scale invariance in the UV means $\Pi(Q^2) \sim \ln(Q^2)$.

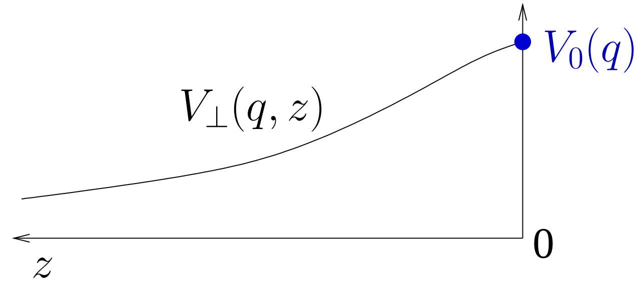
5d action for V_m^a? Let us take

$$S_5 = -\frac{1}{4g_5^2} \int d^5x \sqrt{g}\, V_{mn}^a V^{amn}.$$

At tree level, we need to minimize S_5 wrt V with the b.c. $V(x,\epsilon) = V_0(x)$.
Then take 2 variational derivatives wrt $V_0(x)$ and $V_0(y)$ to find $\langle J(x) J(y) \rangle$.

Example: current (contd)

$V_5 = 0$ gauge; linearize; Fourier $x^\mu \to q^\mu$:



$$\partial_z \left(\frac{1}{z} \partial_z V_\perp \right) + \frac{q^2}{z} V_\perp = 0.$$

Action to quadratic order in V, on eqs of motion (int. by parts):

$$S_5 = -\frac{1}{2g_5^2} \int d^4x \, \frac{1}{z} \, V_\mu^a \partial_z V^{a\mu} \Big|_{z=\epsilon}.$$

Let $V(q,z)$ be a solution with $V(q,z) = 1$, then we need $(V_\mu^a)_\perp = V_{\mu 0}^a(q) V(q,z)$.

$$\Pi(Q^2) = -\frac{1}{2g_5^2} \, \frac{1}{Q^2} \, \frac{\partial_z V(q,z)}{z} \Big|_{z=\epsilon}.$$

$$V(q,z) = (Qz) K_1(Qz) = 1 + \frac{Q^2 z^2}{2} \ln(Qz) + \mathcal{O}(z^2).$$

Thus

$$\Pi(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 + \text{contact terms}$$

5d coupling and N_c

In QCD

$$\Pi(Q) = -\frac{N_c}{24\pi^2} \ln Q^2 + \dots$$

In AdS$_5$:

$$\Pi(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 + \dots$$

Thus

$$g_5^2 = \frac{12\pi^2}{N_c}$$

Large N_c $\Leftrightarrow$ small coupling

Example: current (summary notes)

- $$\frac{\delta S_{5,\text{eff}}}{\delta (V_0)_{\|}} = 0 \qquad \text{corresponds to} \qquad \partial_\mu J^\mu = 0.$$
- $\log Q^2 \Leftarrow$ scale invariance of the 5d theory (metric).
- “Dictionary”:

4d	$\leftrightarrow$	5d
W_4	$\leftrightarrow$	$S_{5,\text{eff}}$
operator $\mathcal{O}(x)$ (ϕ_0 – source)	$\leftrightarrow$	field $\phi(x,z)$ (ϕ_0 – boundary value)
scale invariance ($\log Q$)	$\leftrightarrow$	scale invariance
$\partial_\mu J^\mu = 0$	$\leftrightarrow$	gauge invariance
large N_c	$\leftrightarrow$	small g_5
large Q	$\leftrightarrow$	small z
dimension of $\mathcal{O}$	$\leftrightarrow$	mass of ϕ

Dimension of operator and 5d mass

$$\mathcal{L}_5 = \frac{1}{2}\sqrt{g}\left(g^{mn}\partial_m\phi\,\partial_n\phi - m_5^2\,\phi^2\right)$$

$z \to 0$ ($qz \ll 1$) solution of LE equations for $\mathcal{L}_5$:

$$\phi \sim z^{\Delta_\phi} \qquad \text{with} \qquad (\Delta_\phi - 4)\Delta_\phi - m_5^2 = 0.$$

$$m_5^2 = 0 \quad : \quad \phi \to \text{const} = \phi_0 \qquad \text{OK;}$$

$$m_5^2 \neq 0 \quad : \quad \phi\, z^{-\Delta_\phi} \to \text{const} = \phi_0.$$

$$[\phi] = 0 \qquad \Rightarrow \qquad [\phi_0] = +\Delta_\phi \qquad ([x] = -1)$$

Thus $[\mathcal{O}] = 4 - \Delta_\phi \equiv \Delta_{\mathcal{O}}$ and

$$m_5^2 = (\Delta_\phi - 4)\Delta_\phi = \Delta_{\mathcal{O}}(\Delta_{\mathcal{O}} - 4)$$

For example,

$$T_\mu^\mu \quad : \quad m_5^2 = 0; \qquad \bar\psi\psi \quad : \quad m_5^2 = -3;$$

$$J^\mu \quad : \quad m_5^2 = (\Delta - 3)(\Delta - 1) = 0.$$

Spontaneous symmetry breaking

$$S_5 = \frac{1}{2} \int d^5x \sqrt{g}\, g^{mn} \partial_m \phi\, \partial_n \phi + \ldots$$

with b.c. at $z = 0$: $\phi\, z^{-\Delta_\phi} = \phi_0$. The extremum:

$$\phi_{\rm sol} = \phi_0\, z^{\Delta_\phi} + A\, z^{\Delta_\mathcal{O}} \qquad (\Delta_\phi + \Delta_\mathcal{O} = 4).$$

Vary the source: $\phi_0 \to \phi_0 + \delta\phi_0$:

$$\delta S_5 = \int d^4x\ z^{-3}\ \delta\phi\ \partial_z \phi \Big|_{z=0} + \ldots = (\Delta_\mathcal{O} - \Delta_\phi) \int d^4x\ \delta\phi_0\ A$$

Compare to W_4:

$$\delta W_4 = \int d^4x\ \delta\phi_0\ \langle \mathcal{O} \rangle$$

Therefore

$$A = \frac{1}{2\Delta_\mathcal{O} - 4} \langle \mathcal{O} \rangle$$

$A \nrightarrow 0$ as $\phi_0 \to 0$: spontaneous symmetry breaking

The model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	$\Delta_{\mathcal{O}}$	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

$$S = \int_0^{z_m} d^5x\,\sqrt{g}\,\mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

Symmetries: $X \to LXR^\dagger$, $F_L \to LF_LL^\dagger$, $F_R \to RF_RR^\dagger$.

Boundary conditions at $z = z_m$: $F_{z\mu} = 0$, $D_zX = 0$.

Chiral symmetry breaking:

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3.$$

Matching to QCD: $\Sigma^{\alpha\beta} = \langle\bar{q}^\alpha q^\beta\rangle$. We take $M = m_q\mathbf{1}$ and $\Sigma = \sigma\mathbf{1}$.

Four free parameters: m_q, σ, z_m and g_5.

Compared to three in QCD: m_q, Λ_{QCD} and N_c.

Hadrons and QCD sum rule

$$\partial_z \left(\frac{1}{z} \partial_z V_\perp \right) + \frac{q^2}{z} V_\perp = 0.$$

Normalizable modes: $\psi_\rho(\epsilon) = 0, \quad \partial_z \psi_\rho(z_m) = 0, \quad \int (dz/z)\, \psi_\rho(z)^2 = 1.$

$$\Pi_V(-q^2) = -\frac{1}{2g_5^2} \frac{1}{Q^2} \frac{\partial_z V(q,z)}{z} = -\frac{1}{g_5^2} \sum_\rho \frac{[\psi_\rho{}'(\epsilon)/\epsilon]^2}{(q^2 - m_\rho^2) m_\rho^2} \,.$$

$$F_\rho = \frac{1}{g_5} \frac{\psi'_\rho(\epsilon)}{\epsilon} \,.$$

QCD sum rule:

$$\Pi_V(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 + \ldots$$

Chiral symmetry breaking and GOR relation

$A = (A_L - A_R)/2, \quad X = X_0 \exp(i2\pi^a t^a), \quad A_\mu = A_{\mu\perp} + \partial_\mu \varphi, \quad v(z) = m_q z + \sigma z^3$

$$\delta A_\perp : \qquad \partial_z \left(z^{-1} \partial_z A_\perp \right) + z^{-1} q^2 A_\perp - z^{-3} g_5^2 v^2 A_\perp = 0;$$

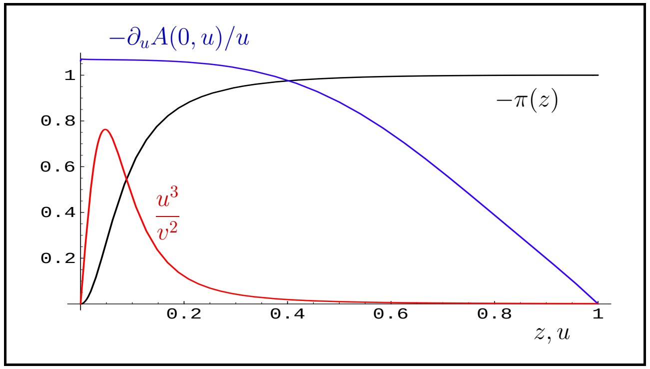
$$\delta A_\parallel : \qquad \partial_z \left(z^{-1} \partial_z \varphi \right) + z^{-3} g_5^2 v^2 (\pi - \varphi) = 0;$$

$$\delta A_z : \qquad -q^2 \partial_z \varphi + z^{-2} g_5^2 v^2 \partial_z \pi = 0.$$

$\Pi_\mathrm{A}(-q^2) \overset{q\to 0}{\longrightarrow} \dfrac{f_\pi^2}{-q^2}$. Analogous to V: $A(q,\epsilon) = 1$.

$$f_\pi^2 = -\frac{1}{g_5^2} \frac{\partial_z A(0,z)}{z}\bigg|_{z=\epsilon}.$$

For $m_q \to 0$: $\varphi(z) = A(0,z) - 1$, $\pi(z) = -1$.



$$\pi(z) = m_\pi^2 \int_0^z du \frac{u^3}{v(u)^2} \cdot \frac{1}{g_5^2 u} \partial_u A(0,u) = -m_\pi^2 f_\pi^2 \frac{1}{2 m_q \sigma} \quad \text{for} \quad z \gg \sqrt{m_q/\sigma}.$$

Thus $\boxed{m_\pi^2 f_\pi^2 = 2 m_q \sigma + \mathcal{O}(m_q^2)}$. (Still holds for $\Delta_\sigma \neq 3$, or deformed AdS.)

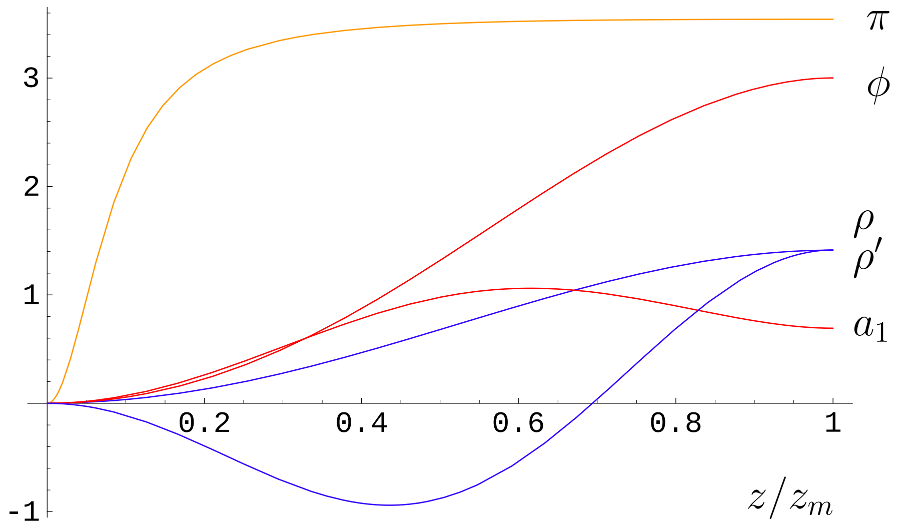
GOR (contd)

Arbitrary Δ_σ $\quad$ ($v(z) = m_q\, z^{\Delta_m} + \sigma\, z^{\Delta_\sigma}$):

$$1 = m_\pi^2 f_\pi^2 \int_0^\infty dz \frac{z^3}{(m_q\, z^{\Delta_m} + \sigma\, z^{\Delta_\sigma})^2} = m_\pi^2 f_\pi^2\, \frac{1}{(\Delta_\sigma - \Delta_m) m_q \sigma} = \frac{m_\pi^2 f_\pi^2}{2 m_q \langle \bar{q} q \rangle}$$

$$(\Delta_\sigma - \Delta_m)\sigma = 2\langle \bar{q} q \rangle$$

Meson wavefunctions and couplings



Couplings:

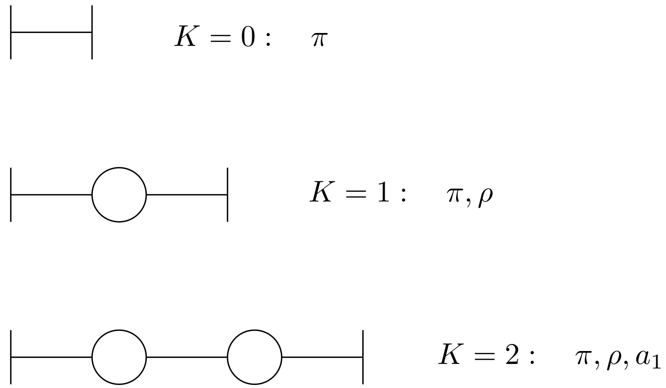
$$g_{\rho\pi\pi} = g_5 \int dz \, \frac{v^2(z)}{z^3} \, (\pi - \varphi)(z) \cdot \pi(z) \cdot \psi_\rho(z),$$

Comparison with experiment

Observable	Measured	Model	Units
m_π	139.6± 0.0004	139.6*	MeV
m_ρ	775.8± 0.5	775.8*	MeV
m_{a_1}	1230± 40	1363	MeV
f_π	92.4± 0.35	92.4*	MeV
$F_\rho^{1/2}$	345± 8	329	MeV
$F_{a_1}^{1/2}$	433± 13	452	MeV
$g_{\rho\pi\pi}$	6.03± 0.07	6.63	–
$m_{\rho'}$	1720± 20	1783	MeV

- $N_c \quad \Rightarrow \quad g_5 = \sqrt{12\pi^2/N_c} = 2\pi$.
- $m_\rho = 2.405/z_m \quad \Rightarrow \quad z_m = (323\ \mathrm{MeV})^{-1}$.
- f_π and $m_\pi \quad \Rightarrow \quad \sigma = (327\ \mathrm{MeV})^3$ and $m_q = 2.29\ \mathrm{MeV}$.

Holographic model vs open moose



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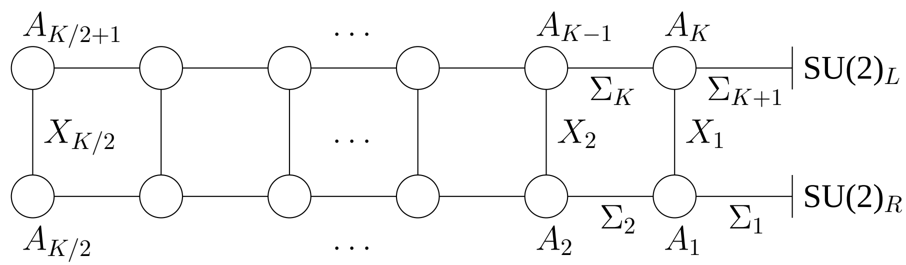

OPE and higher order terms

$$\Pi_V(Q^2) = -\frac{1}{2g_5^2}\ln Q^2; \qquad \Pi_A(Q^2) = \Pi_V(Q^2) + \#\frac{m_q\sigma}{Q^4} + \mathcal{O}\left(m_q^2, \frac{\sigma^2}{Q^6}\right);$$

(In open moose: $\Pi_A - \Pi_V \sim \exp(-\#Q)$.)
In QCD:

$$\Pi_V(Q^2) = \ldots - \frac{m_q\sigma}{Q^4}; \qquad \Pi_A(Q^2) = \ldots + \frac{m_q\sigma}{Q^4};$$

$$\Delta\mathcal{L}_5 = \sqrt{g}\,\mathrm{Tr}\left[\gamma_{XFXF}(X^\dagger F_L X F_R) + \gamma_{X^2F^2}(XX^\dagger F_L^2 + X^\dagger X F_R^2)\right]$$

$$\left.\begin{aligned} \Pi_A - \Pi_V &= \frac{4}{3}m_q\sigma\,(4\gamma_{XFXF}+1)\frac{1}{Q^4} \\ &= 2m_q\langle\bar{q}q\rangle\frac{1}{Q^4} \end{aligned}\right\} \quad\Rightarrow\quad \gamma_{XFXF} = \frac{1}{8}$$

Similarly: $\Pi_V = \ldots + \#\frac{\langle\alpha_s G^2\rangle}{Q^4}$.

Need source h_0 for operator $\alpha_s G^2 \Rightarrow$ massless scalar field h in 5d.

Classical solution: $h = h_0 + A_h z^4$. $A_h = (1/4)\langle\alpha_s G^2\rangle$.

Coupling $\Delta\mathcal{L}_5 = \gamma h(F_L^2 + F_R^2)$ gives $\Pi_V = \ldots\#\gamma A_h/Q^4$.

Outlook

- $h(z,x) \longrightarrow$ glueball spectrum (Polchinski-Strassler, Boschi-Filho-Braga)
- strange mesons
- chiral anomaly (WZW)
- Baryons (Teramond-Brodsky)
  - finite density?
- running of α_s (log violations of scaling)