Finite Density Algorithms and Mesoniums

- Finite Density Algorithms and Progress
- Scalar Mesons and Tetraquark Mesoniums

χQCD Collaboration:
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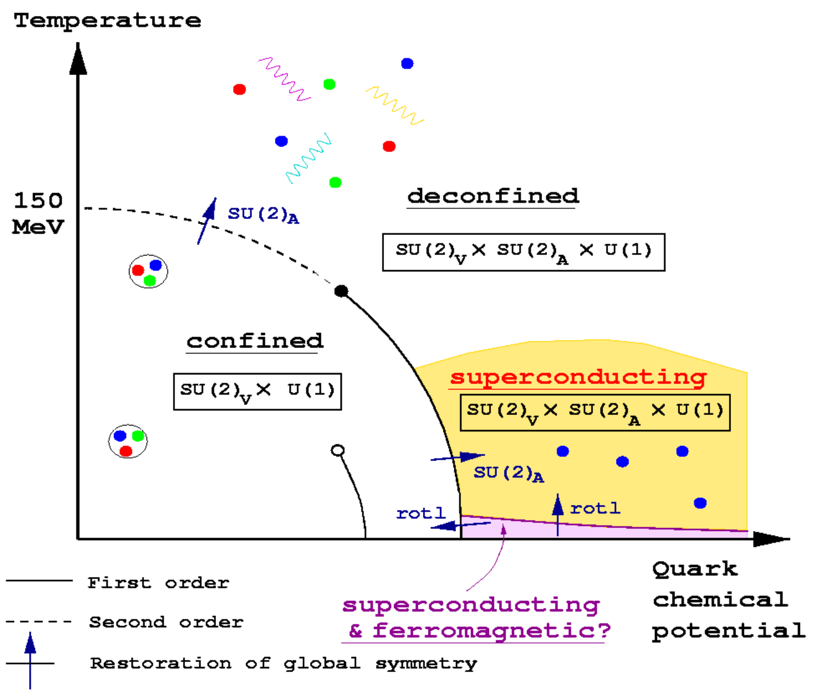
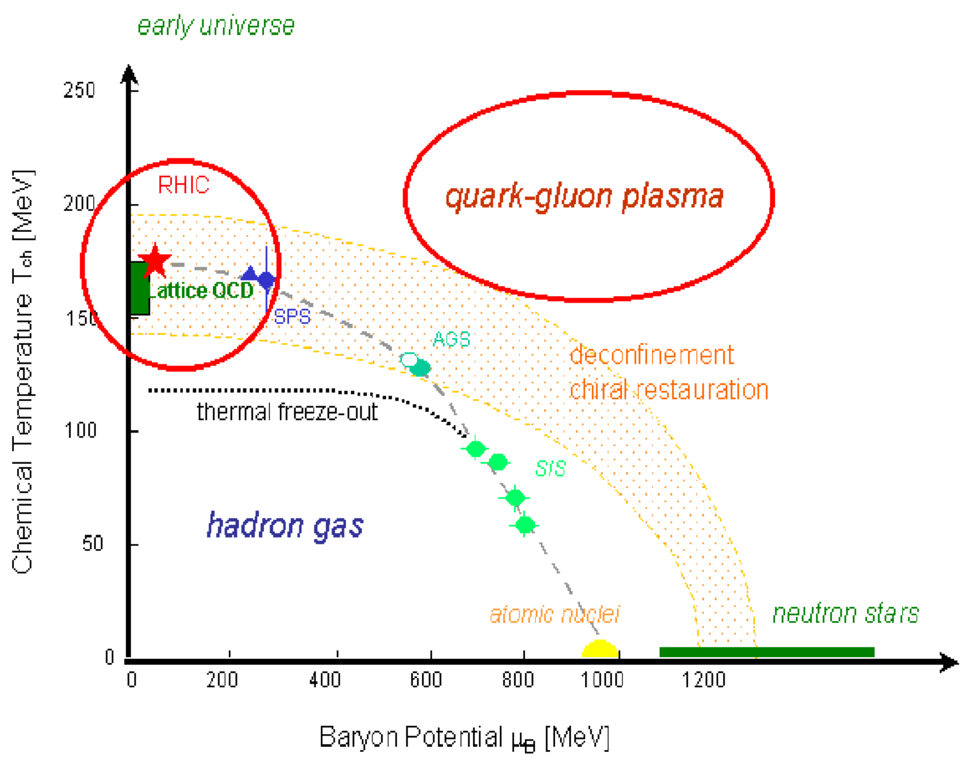
F.X. Lee, K.F. Liu, N. Mathur, S. Tamhankar, H. Thacker, J.B. Zhang

Modern Challenges for LFT, Mar. 28, 2005

Early Universe, Quark Gluon Plasma
Neutron Star, Strange Quark Star
Color Superconductor
High Temperature Superconductor – Hubbard Model





Charles Gale, McGill University

Finite Density Algorithms

- Grand canonical ensemble approach with chemical potential

$$Z_{GC} = e^{-\beta H - \mu N}, \quad \langle \hat{\mathrm{N}} \rangle = N$$

$$S_F = \overline{\psi} M \psi = \sum_x \{ \overline{\psi}(x)\psi(x) + \kappa[\overline{\psi}(x+a_4)(1+\gamma_4)U_4^+(x)e^{\mu a}\psi(x)$$

$$+\overline{\psi}(x)(1-\gamma_4)U_4(x)e^{-\mu a}\psi(x+a_4)] + \ldots \}$$

- No γ_5 hermiticity → $\gamma_5 M \gamma_5 \neq M^+$ → Det M is complex
- The signal $\langle O \rangle = \frac{\langle O e^{i\phi} \rangle_P}{\langle e^{i\phi} \rangle_P}; \ P = | \det M | e^{-s_g}$ can be exponentially damped.
- This leads to a sign problem!

- Improved Approaches:
  - Fugacity Expansion (Glasgow Algorithm)

$$Z_{GC}(\mu/T, T, V) = \sum_{B=-V}^{V} z^B Z_B(T, V); \quad z = e^{\mu/T}$$

$$\mathrm{Z}_{\mathrm{GC}} = \langle \frac{\det M(\mu)}{\det M(0)} \rangle_{\mu=0}$$

  - Multi-parameter Reweighting (Z. Fordor and S.D. Katz)
  - Taylor Expansion (C.R. Allton et al.)

➢ Alleviates the sign problem somewhat, but restricted to small μ and high T.

- Imaginary Chemical Potential
  - E. Dagotto, A. Moreo, R.L. Sugar, and D. Toussaint; M. Alford, A. Kapustin, and F. Wilczek

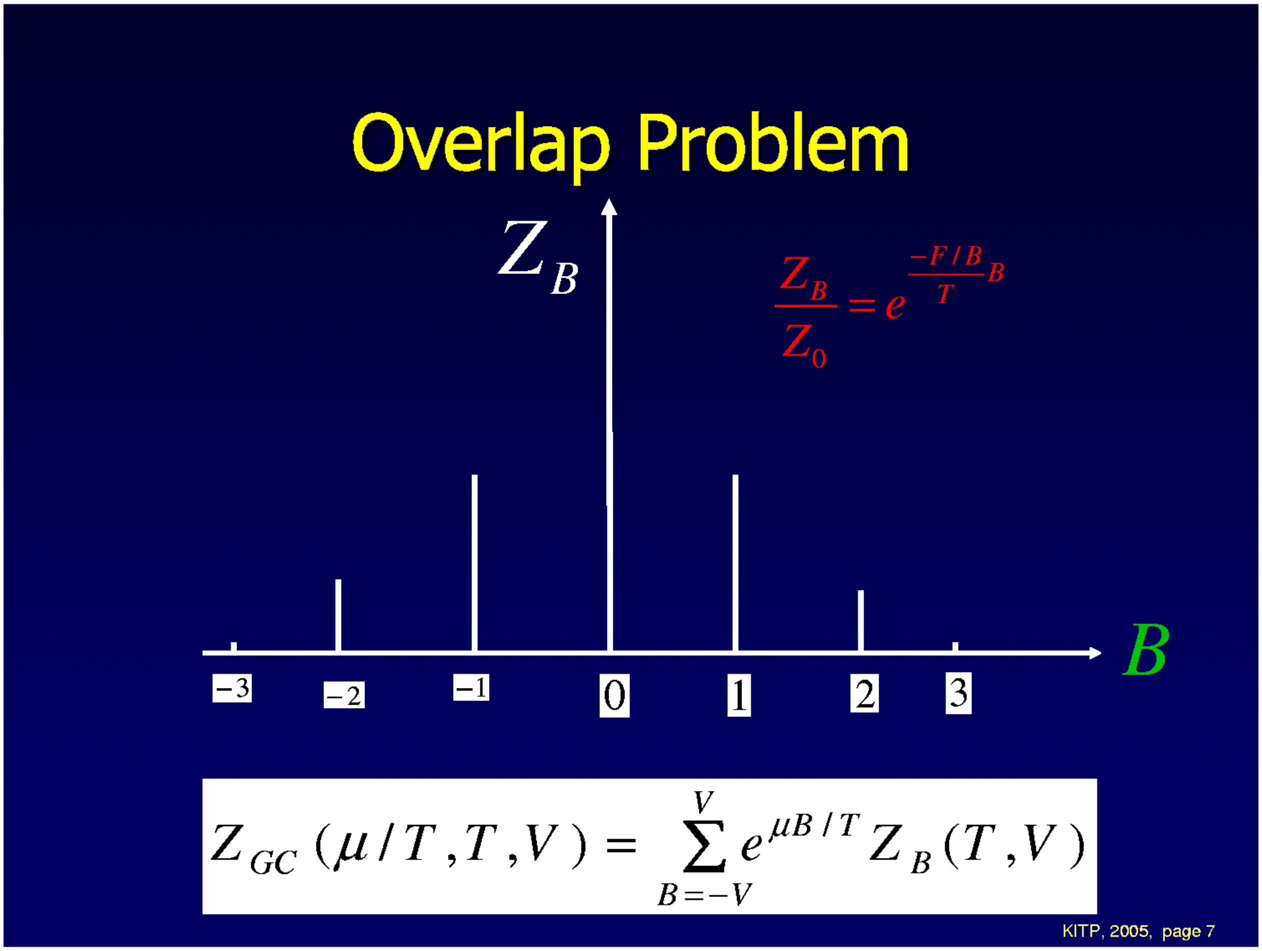
$$Z_B(T, V) = \frac{\beta}{2\pi} \int_0^{2\pi/\beta} d\mu \, Z_{GC}(i\mu) e^{-i\beta\mu B}$$

$$\frac{\mathrm{Z}_{\mathrm{GC}}(i\mu)}{\mathrm{Z}_{\mathrm{GC}}(i\mu_{update})} = \langle \frac{\det M(i\mu)}{\det M(i\mu_{update})} \rangle$$

  - Extrapolate to real μ (P. de Forcrand and O. Philipsen)

▪ Det M (iμ) is real (no sign problem)

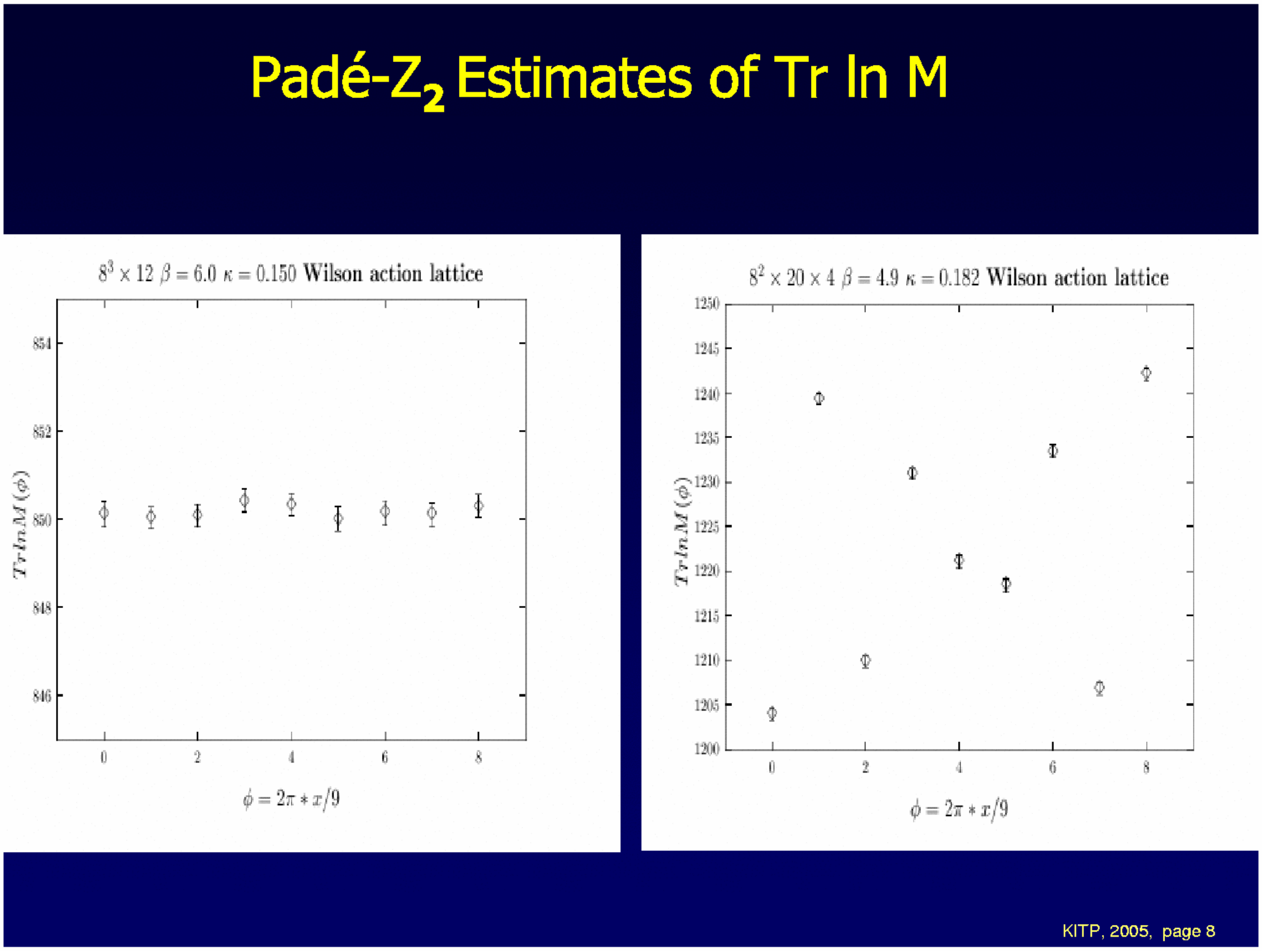
➢ But, there is overlap problem!



Overlap Problem

$$\frac{Z_B}{Z_0} = e^{\frac{-F/B}{T}B}$$

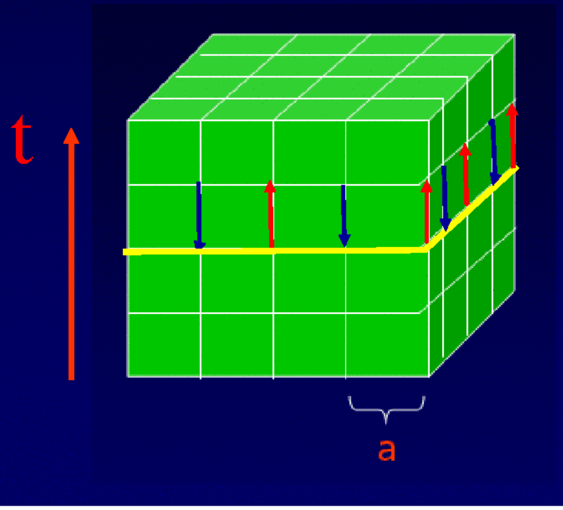
$$Z_{GC}(\mu/T, T, V) = \sum_{B=-V}^{V} e^{\mu B/T} Z_B(T, V)$$



Padé-Z_2 Estimates of Tr ln M

- Canonical Ensemble Approach:
  - Equivalent to imaginary chemical potential

$$Z_B(T,V) = \frac{\beta}{2\pi}\int_0^{2\pi/\beta} d\mu\ Z_{GC}(i\mu) e^{-i\beta\mu B}$$
$$= \int DU\ e^{-S_g} \int_0^{2\pi} d\theta / 2\pi\ e^{-i3B\theta} \det\ M(\theta);$$
$$M(\theta)_{m,n} = \delta_{m,n} - \kappa[(1+\gamma_4)U_4^+(n)e^{i\theta}\delta_{m,n+4} + (1-\gamma_4)U_4 e^{-i\theta}(m)\delta_{m+4,n} + ...]$$



$$\det M = e^{Tr \log M(\theta)}$$

is real

- Avoid the Overlap Problem
  - KFL (Int. Jou. Mod. Phys. B16, 2017 (2002))

- The earlier procedure $\frac{Z_{GC}(i\mu)}{Z_{GC}(i\mu_{update})} = \langle \frac{\det\ M(i\mu)}{\det\ M(i\mu_{update})} \rangle$

  is like projection after variation (Peierls and Yoccoz)

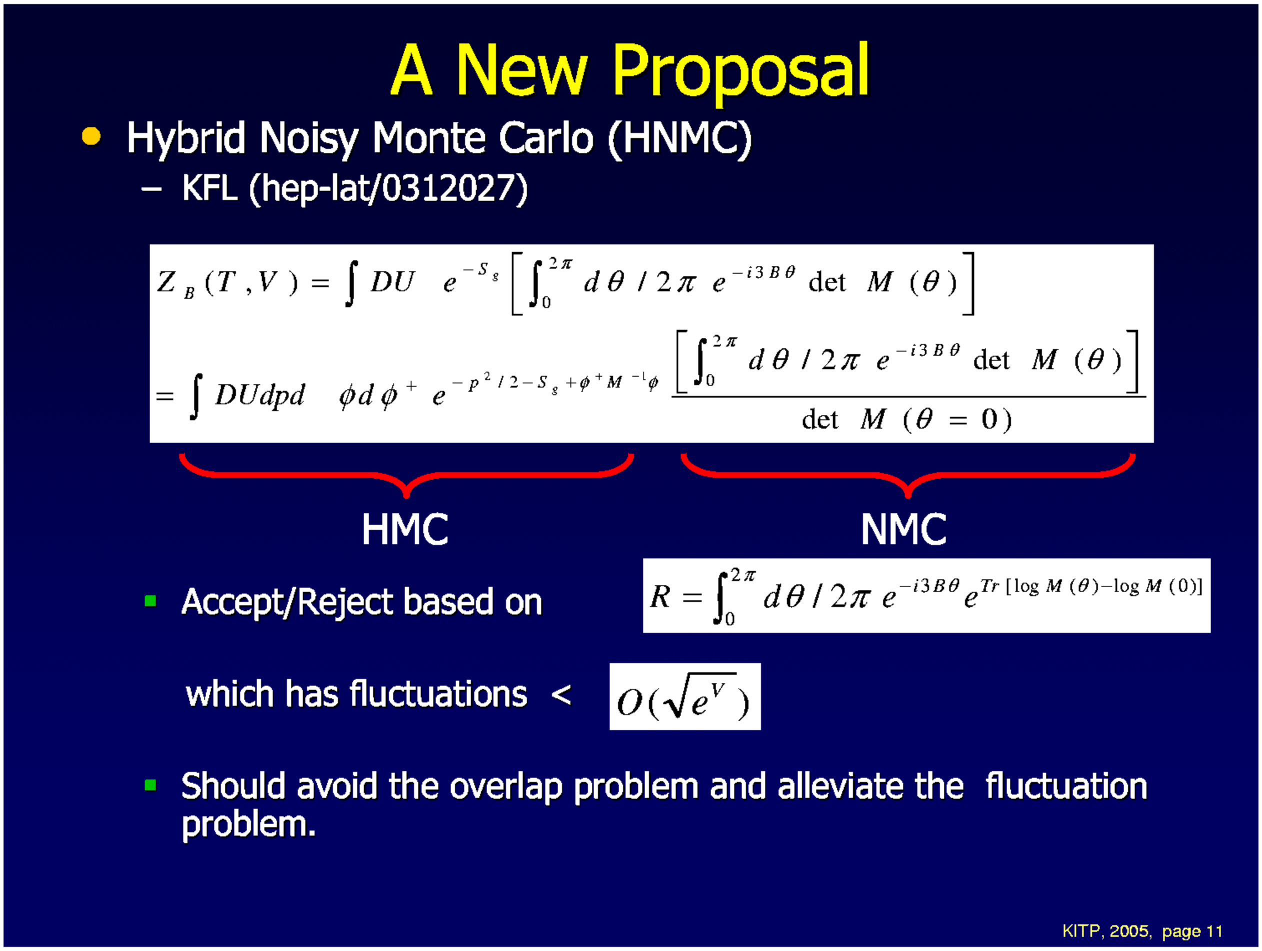
- Need variation after projection (Zeh-Rouhaninejad-Yoccoz)

$$Z_B(T,V) = \int DU\ e^{-S_g}\left[\int_0^{2\pi} d\theta / 2\pi\ e^{-i3B\theta} \det\ M(\theta)\right]$$

- Accept/reject based on $\det_B$.

➢ Unfortunately, this introduces fluctuation problem!

Because $\det M = e^{Tr \log M} \sim O(e^V)$



A New Proposal

- Hybrid Noisy Monte Carlo (HNMC)
  - KFL (hep-lat/0312027)

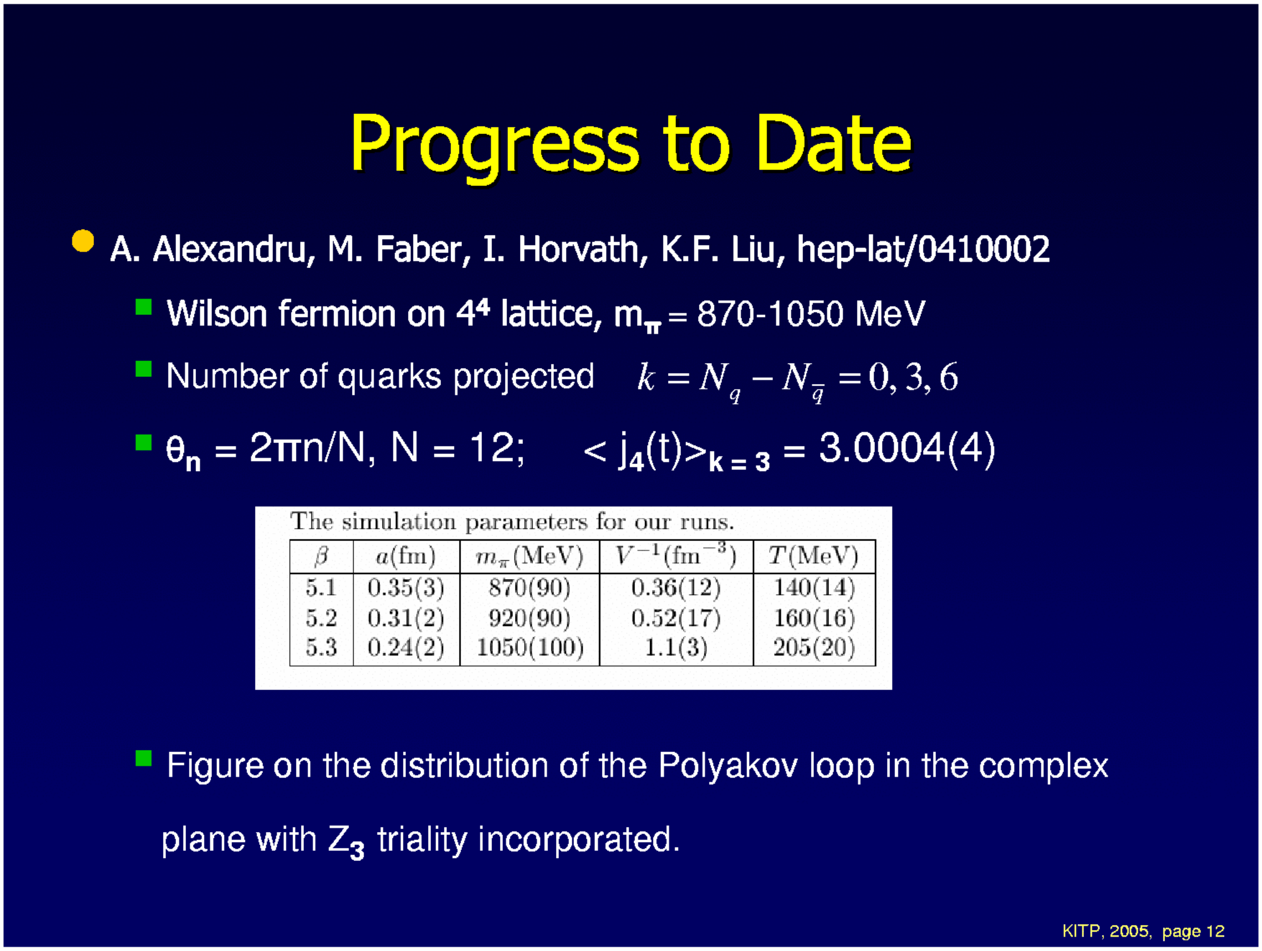
$$Z_B(T,V) = \int DU \;\; e^{-S_g}\left[\int_0^{2\pi} d\theta/2\pi \; e^{-i3B\theta} \det \; M(\theta)\right]$$

$$= \underbrace{\int DUdpd \;\; \phi d\phi^+ \; e^{-p^2/2-S_g+\phi^+M^{-1}\phi}}_{\text{HMC}} \; \underbrace{\frac{\left[\int_0^{2\pi} d\theta/2\pi \; e^{-i3B\theta} \det \; M(\theta)\right]}{\det \; M(\theta=0)}}_{\text{NMC}}$$

- Accept/Reject based on $R = \int_0^{2\pi} d\theta/2\pi \; e^{-i3B\theta} e^{Tr\,[\log M(\theta) - \log M(0)]}$

  which has fluctuations < $O(\sqrt{e^V})$

- Should avoid the overlap problem and alleviate the fluctuation problem.



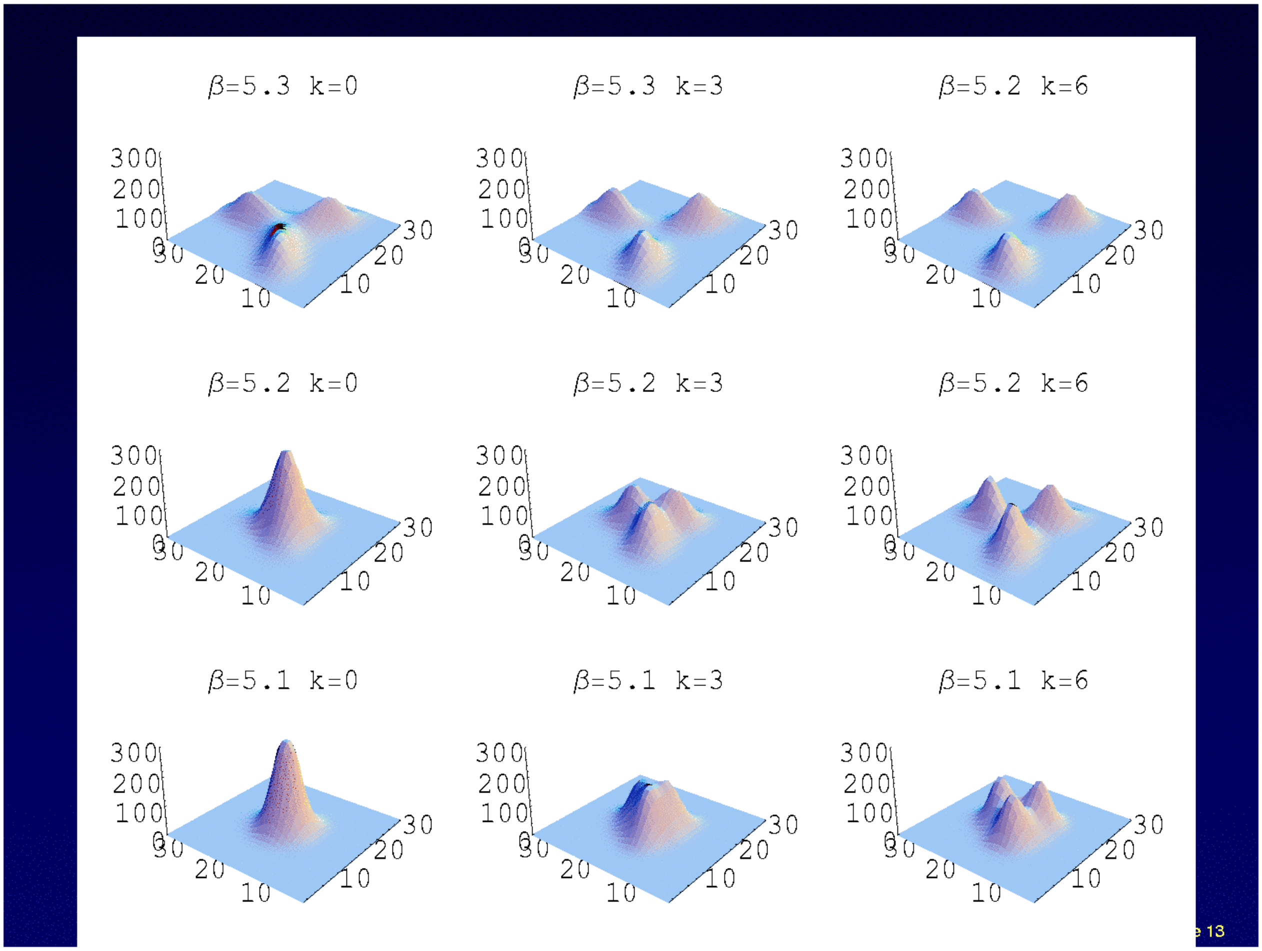
Progress to Date

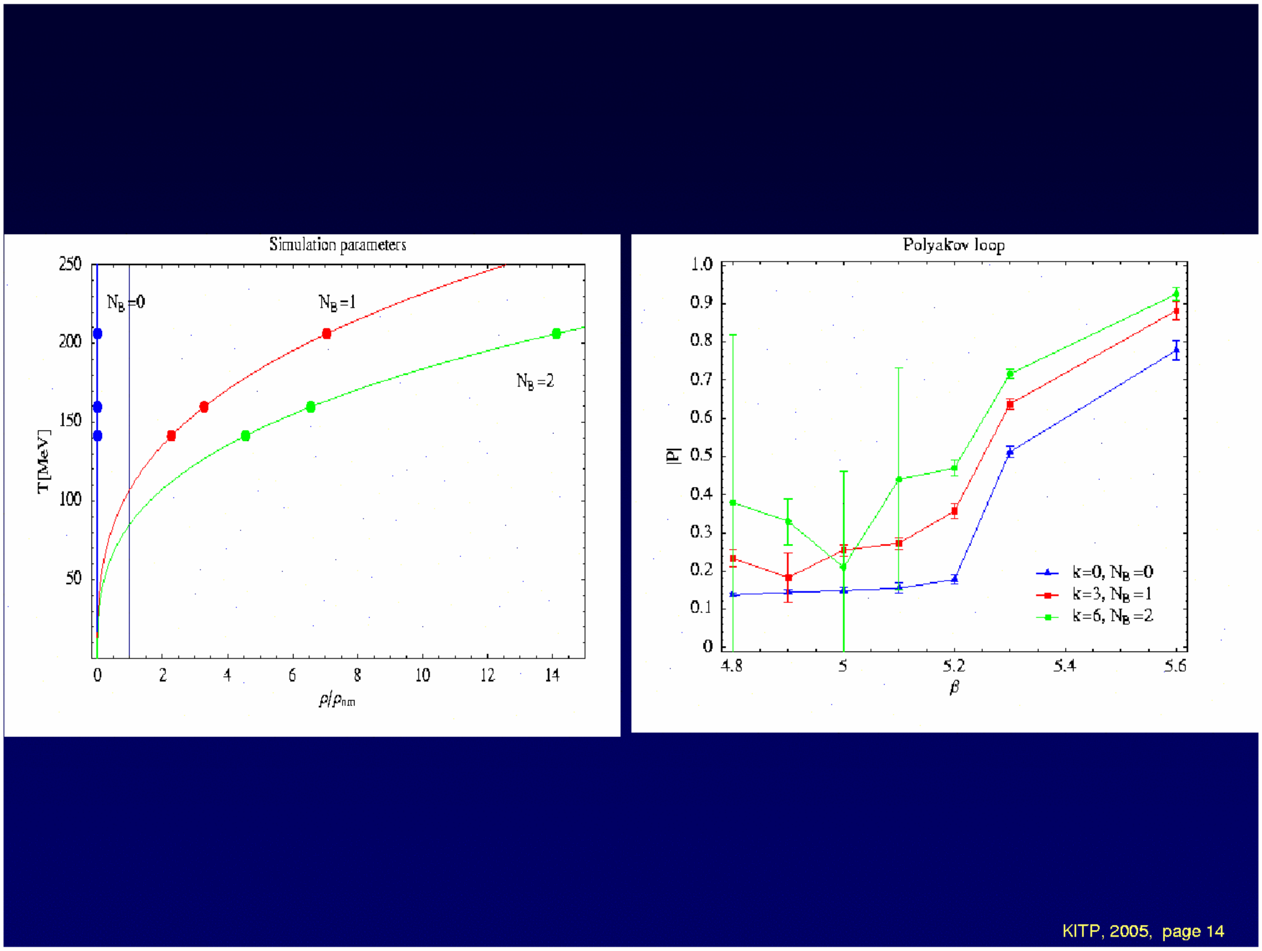
- A. Alexandru, M. Faber, I. Horvath, K.F. Liu, hep-lat/0410002
  - Wilson fermion on 4^4 lattice, m_π = 870-1050 MeV
  - Number of quarks projected $k = N_q - N_{\bar{q}} = 0, 3, 6$
  - $\theta_n = 2\pi n/N$, N = 12; $< j_4(t) >_{k=3} = 3.0004(4)$

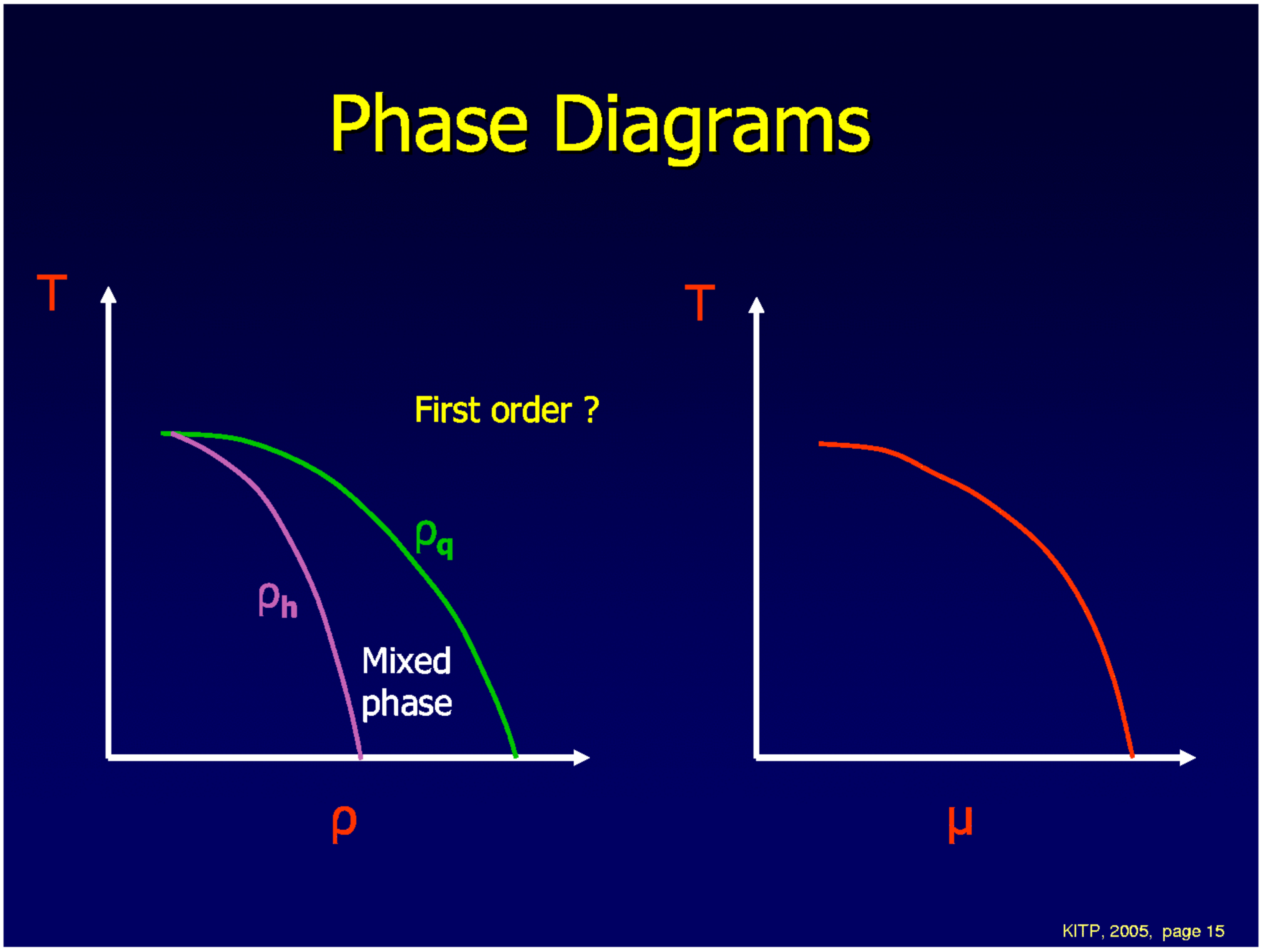
The simulation parameters for our runs.

β	a(fm)	m_π(MeV)	V^{-1}(fm^{-3})	T(MeV)
5.1	0.35(3)	870(90)	0.36(12)	140(14)
5.2	0.31(2)	920(90)	0.52(17)	160(16)
5.3	0.24(2)	1050(100)	1.1(3)	205(20)

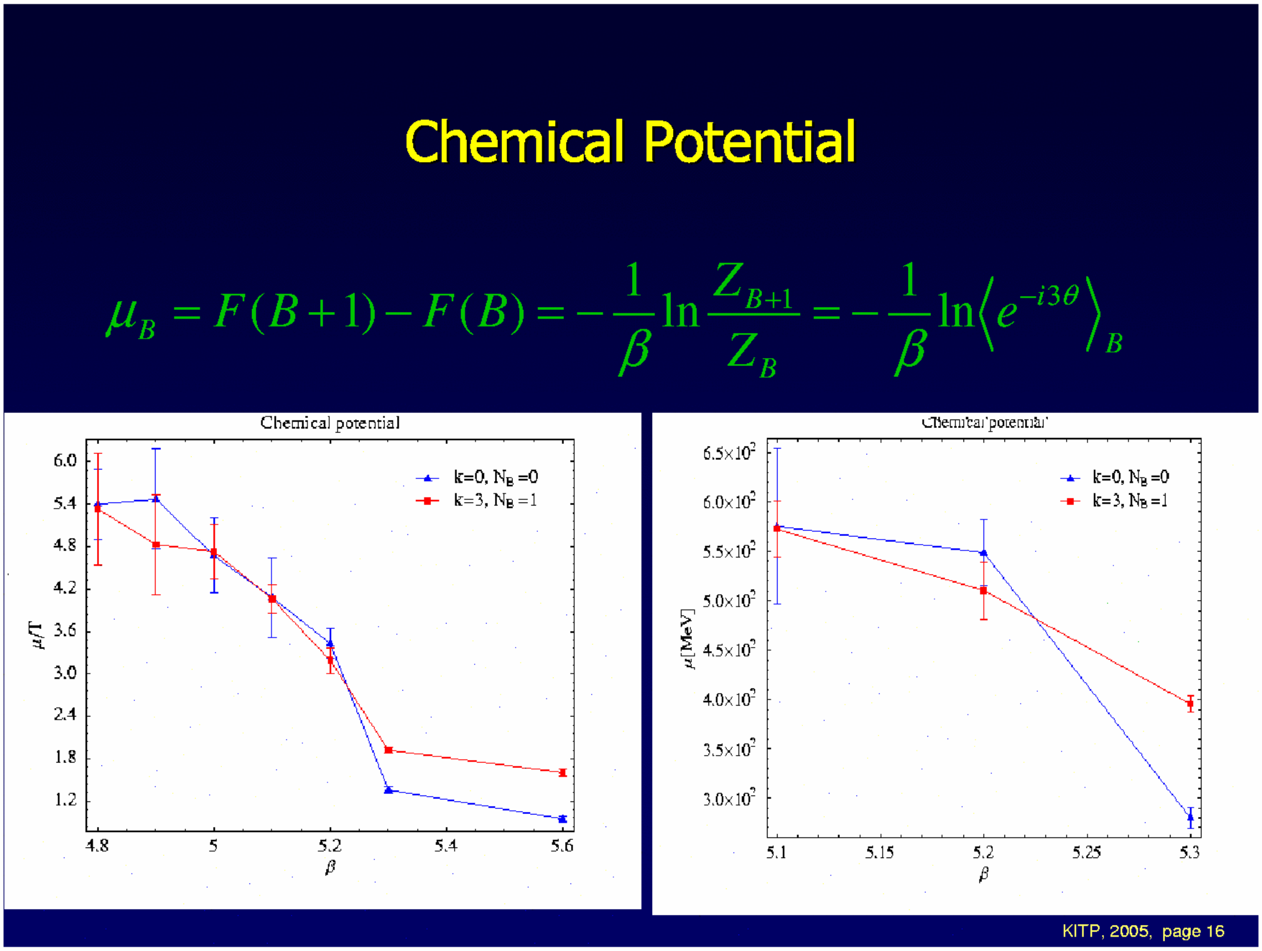
  - Figure on the distribution of the Polyakov loop in the complex plane with Z_3 triality incorporated.


β=5.3 k=0
β=5.3 k=3
β=5.2 k=6
β=5.2 k=0
β=5.2 k=3
β=5.2 k=6
β=5.1 k=0
β=5.1 k=3
β=5.1 k=6
300
200
100
0
30
20
10
e 13


Simulation parameters
T[MeV]
250
200
150
100
50
$N_B=0$
$N_B=1$
$N_B=2$
0
2
4
6
8
10
12
14
ρ/ρ_{nm}
Polyakov loop
|P|
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
4.8
5
5.2
5.4
5.6
β
k=0, $N_B=0$
k=3, $N_B=1$
k=6, $N_B=2$
KITP, 2005, page 14

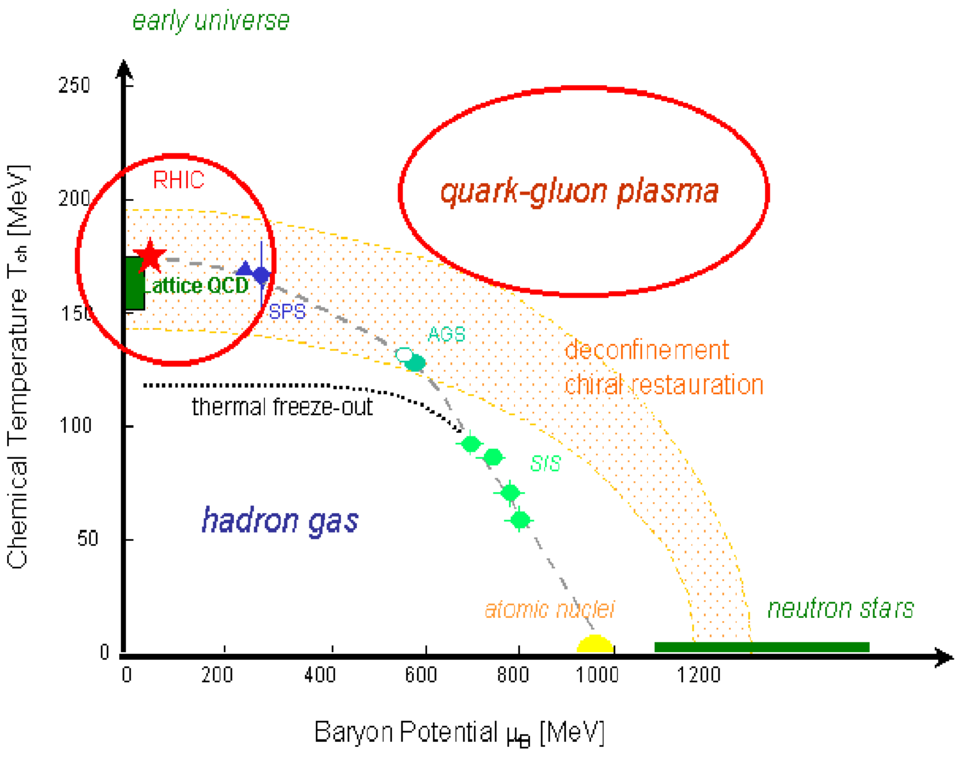


Phase Diagrams



Chemical Potential

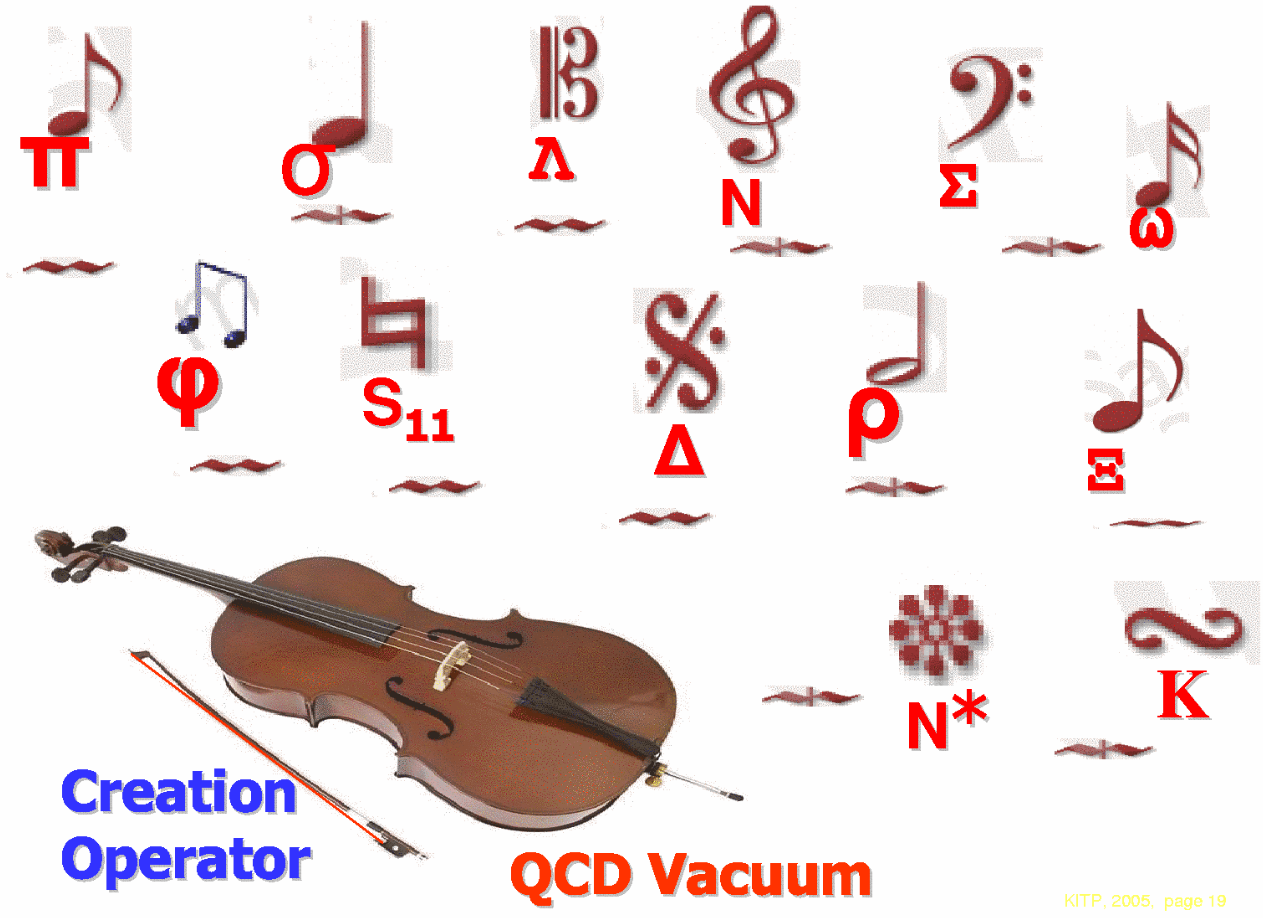
$$\mu_B = F(B+1) - F(B) = -\frac{1}{\beta}\ln\frac{Z_{B+1}}{Z_B} = -\frac{1}{\beta}\ln\left\langle e^{-i3\theta}\right\rangle_B$$



Charles Gale, McGill University

Scalar Mesons and Mesoniums

QCD Vacuum



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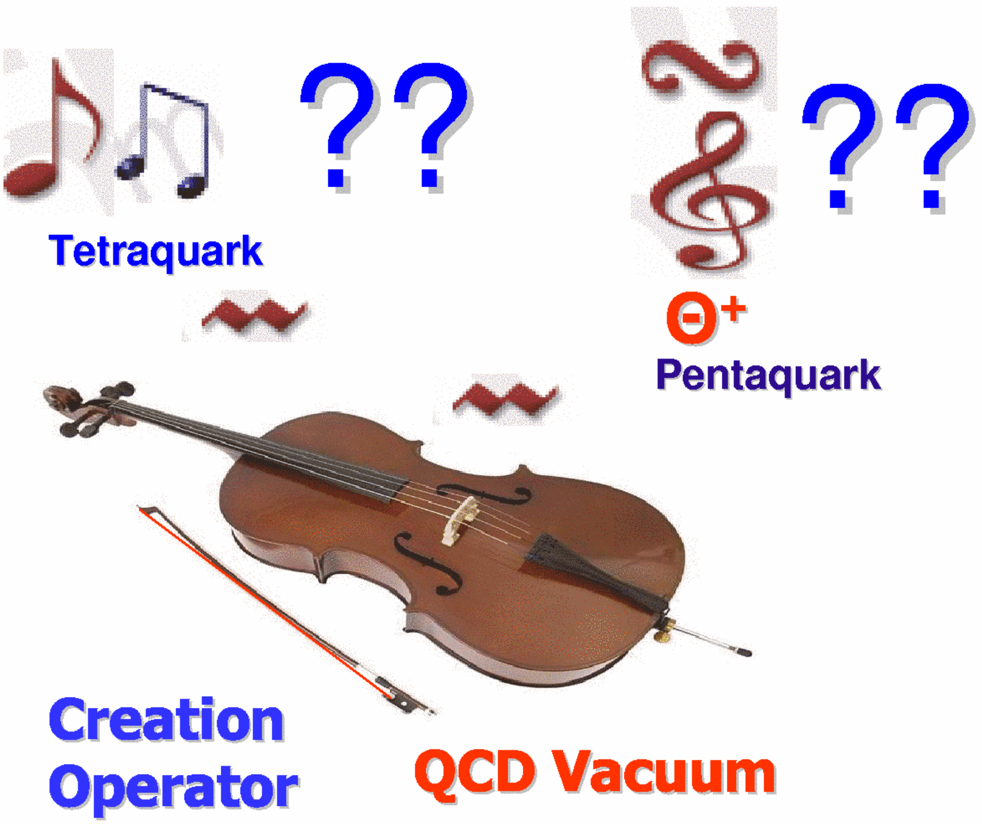

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$D_{sJ}(2317)\ 0(0^+)$

Found in $D_s\pi^0$ **Channel** ... **PRL 90, 242001(2003)** BABAR
PRL 92, 012002(2004) BELLE,
PRD 68, 032003(2004)CLEO, hep-ex/0406044 FOCUS

Mass : 2317.4 ± 0.9 MeV
Width : < 4.6 MeV (90% CL)

M(DK) – M(D_{sJ}(2317)) ~ 45 MeV

Below the DK threshold → **Isospin violation decay**

Masses much lower than potential model P-level predictions

Quark models could not accommodate this state

$C\bar{S}$ **P-level state?**
DK molecule? $D_s\pi$atom?

$D_{sJ}(2460)\ 0(1^+)$

Found in $D^*_s\pi^0$ **Channel** ... **PRD 68, 032003(2004) CLEO**
PRL 92, 012002(2004) BELLE
PRL91, 262002 (2003) BELLE
hep-ex/0405081 BABAR

Mass : 2459.3 ± 1.3 MeV
Width : < 5.5 MeV (90% CL)

M(D*K) – M(D_{sJ}(2460)) ~ 45 MeV

Below the DK threshold → **Isospin violation decay**

Masses much lower than potential model P-level predictions

Quark models could not accommodate this state

$C\bar{S}$ **P-level state?**
D*K molecule? $D^*_s\pi$ atom?

$D_{sJ}(2632)$

Found in $D_s\eta$ **and D_0K^+ Channels ...hep-ex/0406045**

Mass : 2632 MeV
Width : < 17 MeV

$$\frac{\Gamma(D_0K^+)}{\Gamma(D_s\eta)} = 0.16 \pm 0.06$$

Not $c\bar{s}$ state

Quark models could not accommodate this state

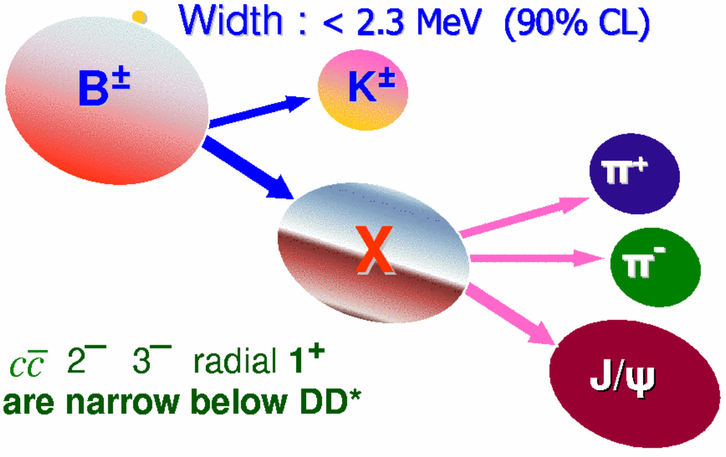
$D_{sJ}(2632) - M(D_s\eta) \approx 116$ MeV
$D_{sJ}(2632) - M(D_0K^+) \approx 274$ MeV

Not molecular state

What is it??

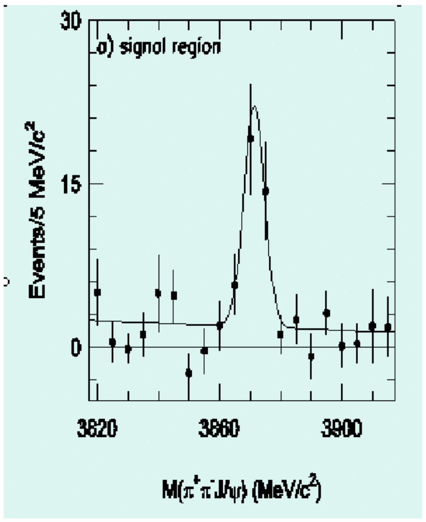
X(3872)

- Mass : 3871.9 ± 0.5 MeV ≈ M(D^0) + M($\underline{D}^{0*}$)
- Width : < 2.3 MeV (90% CL)

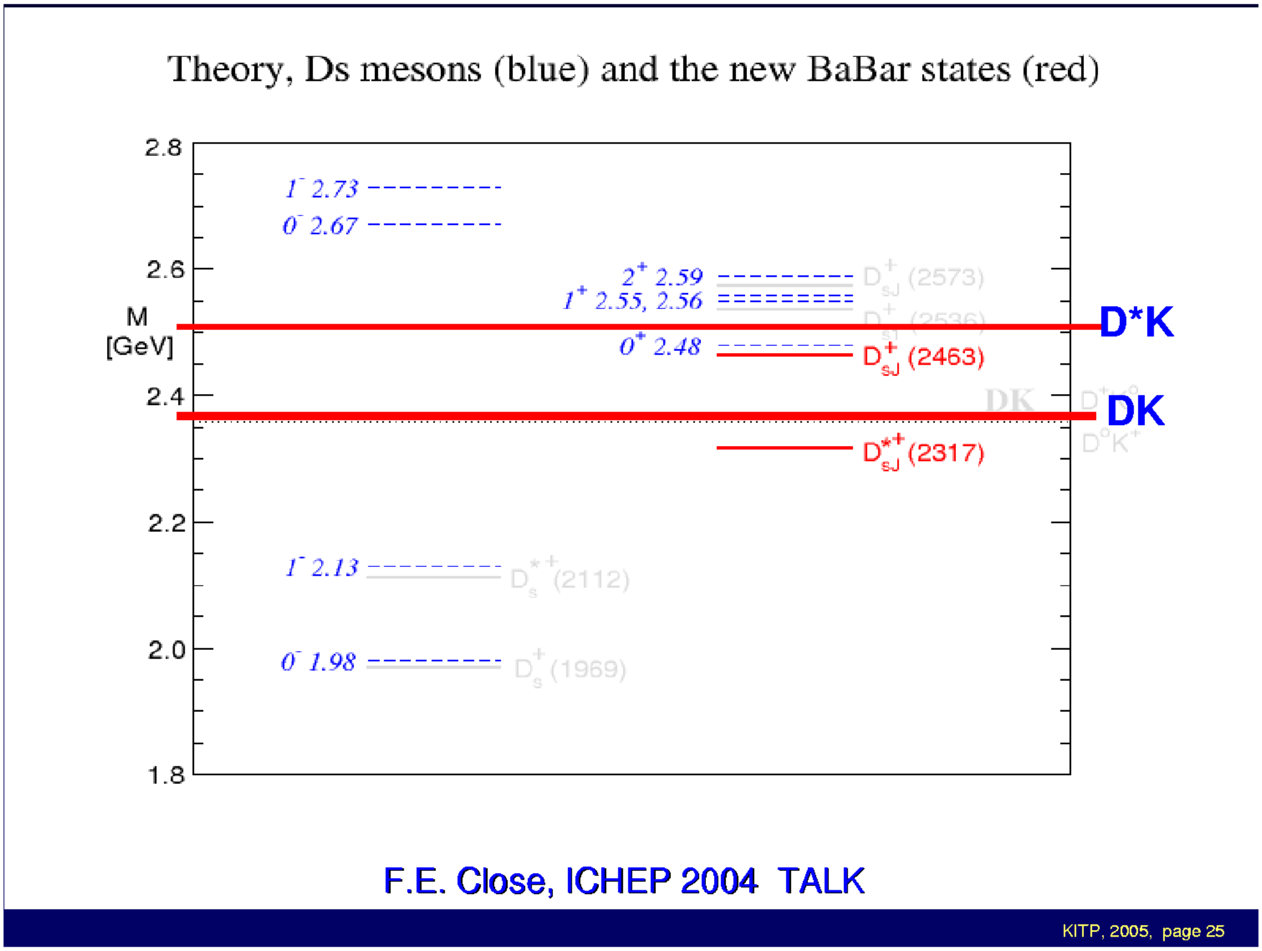


$c\bar{c}$ 2^- 3^- radial 1^+
are narrow below DD*

Mass, width, angular dist *etc.*
are all inconsistent with cc*



DD* Molecule? J/ψ ρ? Vector Glueball?



Theory, Ds mesons (blue) and the new BaBar states (red)

F.E. Close, ICHEP 2004 TALK

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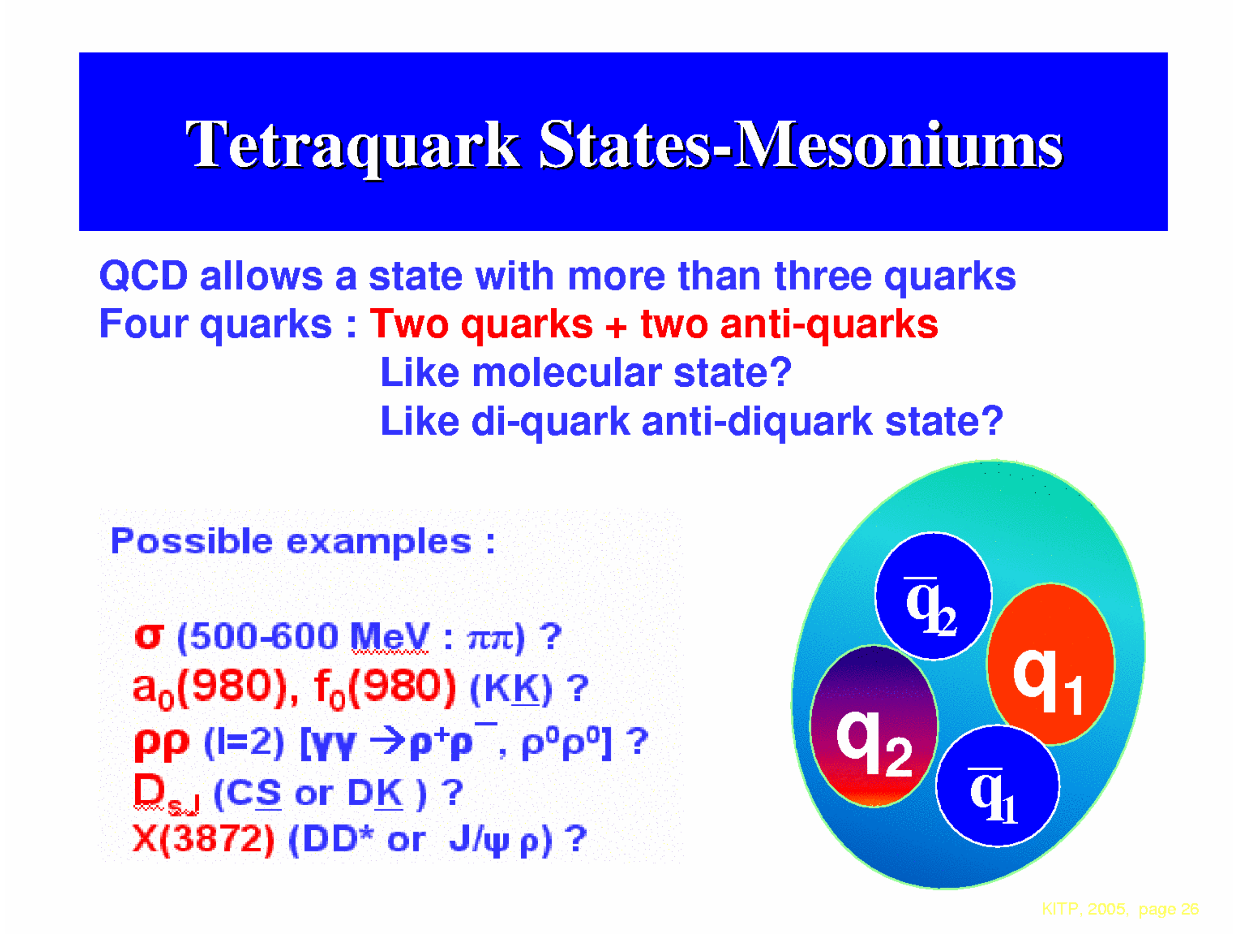

Tetraquark States-Mesoniums

QCD allows a state with more than three quarks
Four quarks : Two quarks + two anti-quarks
Like molecular state?
Like di-quark anti-diquark state?

Possible examples :

σ (500-600 MeV : $\pi\pi$) ?
$a_0(980)$, $f_0(980)$ ($K\underline{K}$) ?
$\rho\rho$ (I=2) [$\gamma\gamma \rightarrow \rho^+\rho^-$, $\rho^0\rho^0$] ?
D_{sJ} (C$\underline{S}$ or D$\underline{K}$) ?
X(3872) (DD* or J/ψ ρ) ?

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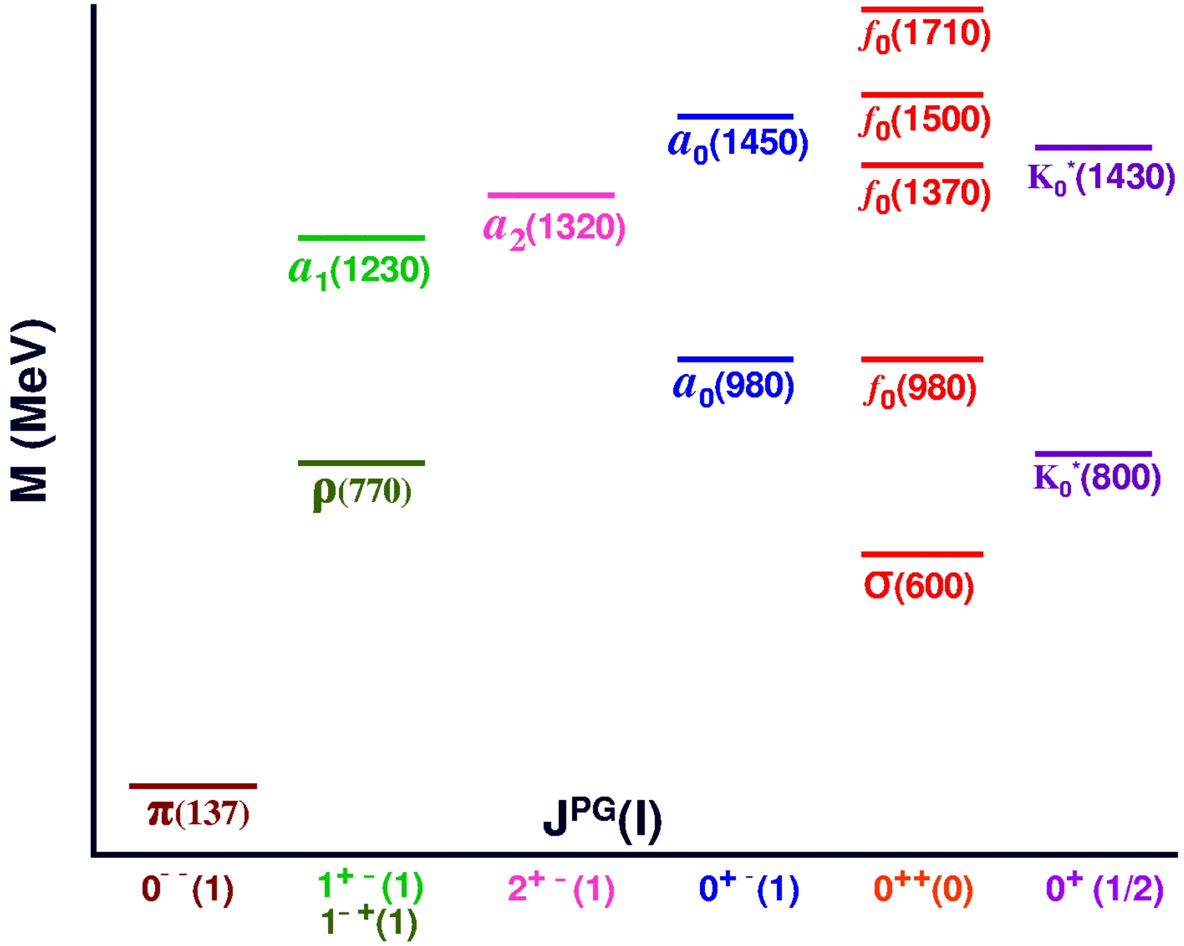

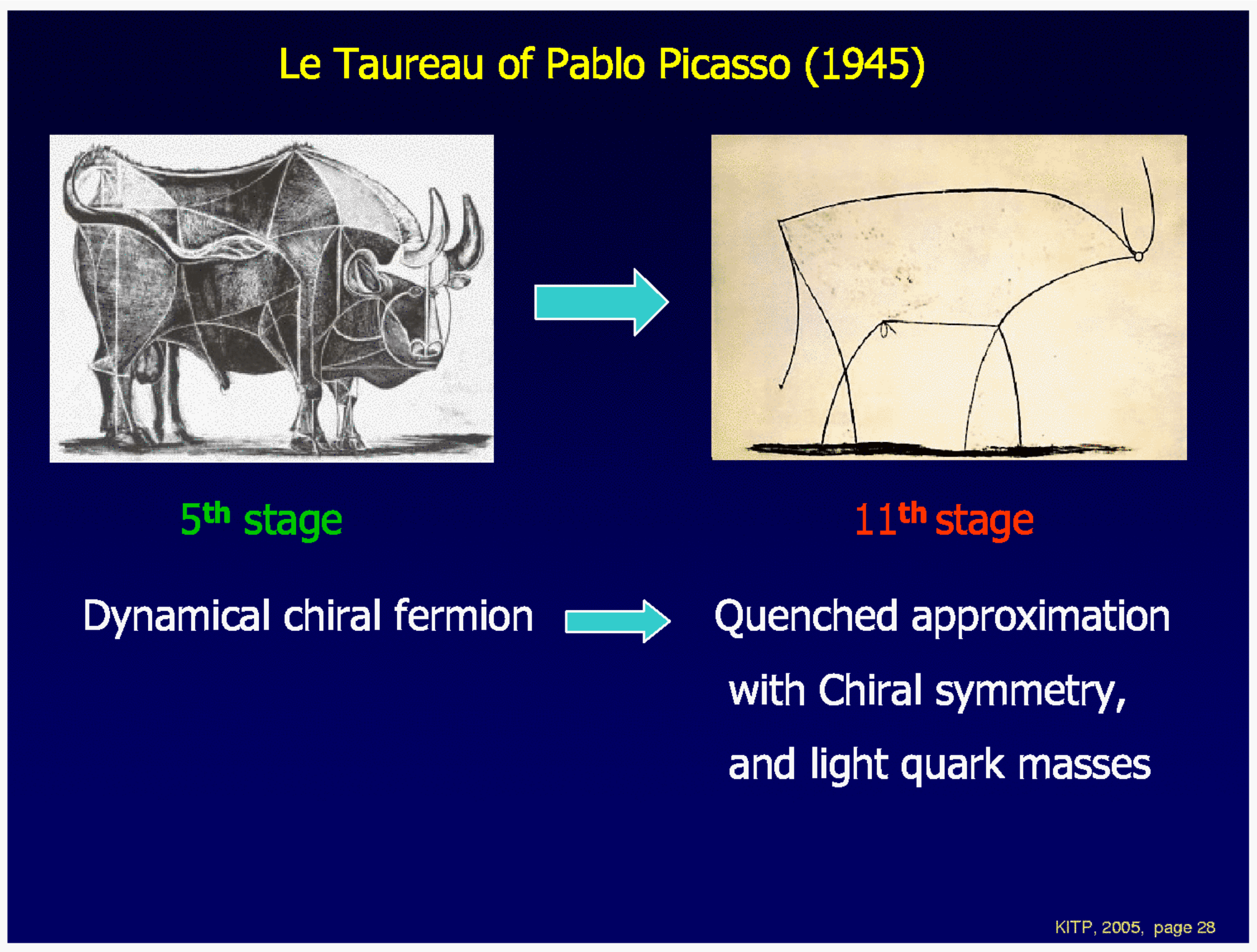
Le Taureau of Pablo Picasso (1945)

5th stage

11th stage

Dynamical chiral fermion → Quenched approximation with Chiral symmetry, and light quark masses

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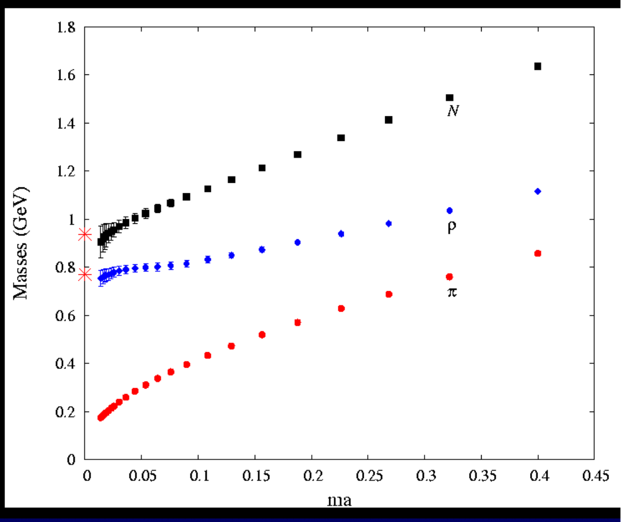
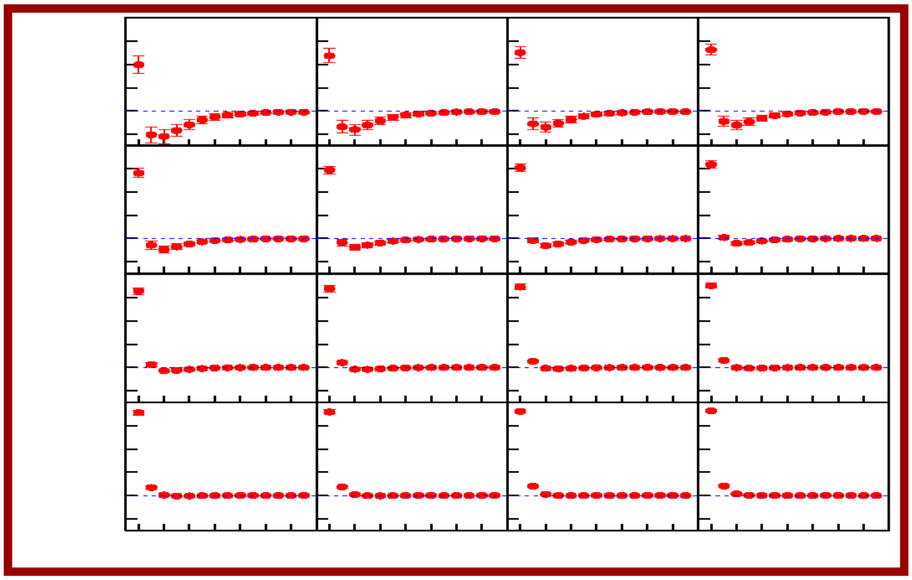
Masses of N, ρ, and π

- 16^3 x 28 quenched lattice, Iwasaki action with a = 0.200(3) fm
- Overlap fermion
- Critical slowing down is gentle
- Smallest $m_\pi \sim 180$ MeV
- $m_\pi L > 3$



Is a_0 (1450) (0^{++}) a two quark state?

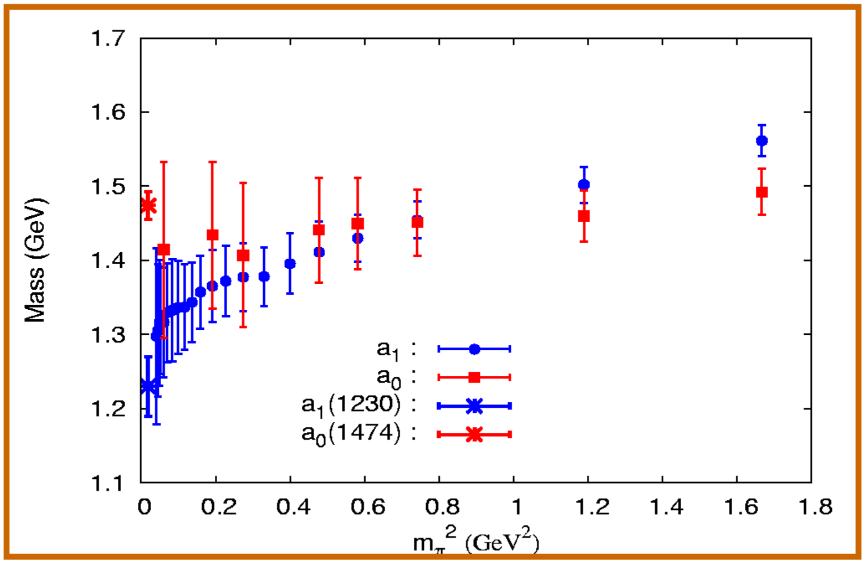


Correlation function for Scalar channel

Ground state : π η ghost state.

First excited state : a_0

$\overline{\psi}\psi$ $I^G(J^{PC}) \equiv 1^-(0^{++}),\ 1^-(1^{++})$

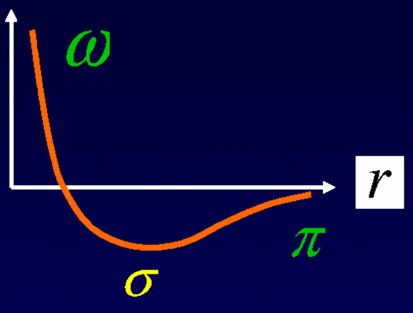


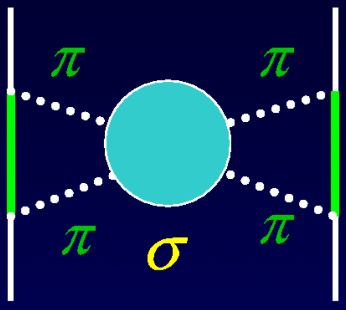
Our results shows scalar mass around 1400-1500 MeV, suggesting

$a_0(1450)$ is a two quark state.

Further study is ongoing with large statistics.

What is the nature of σ (600)?



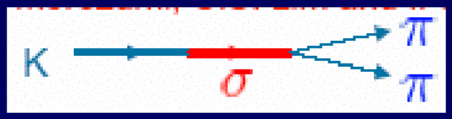


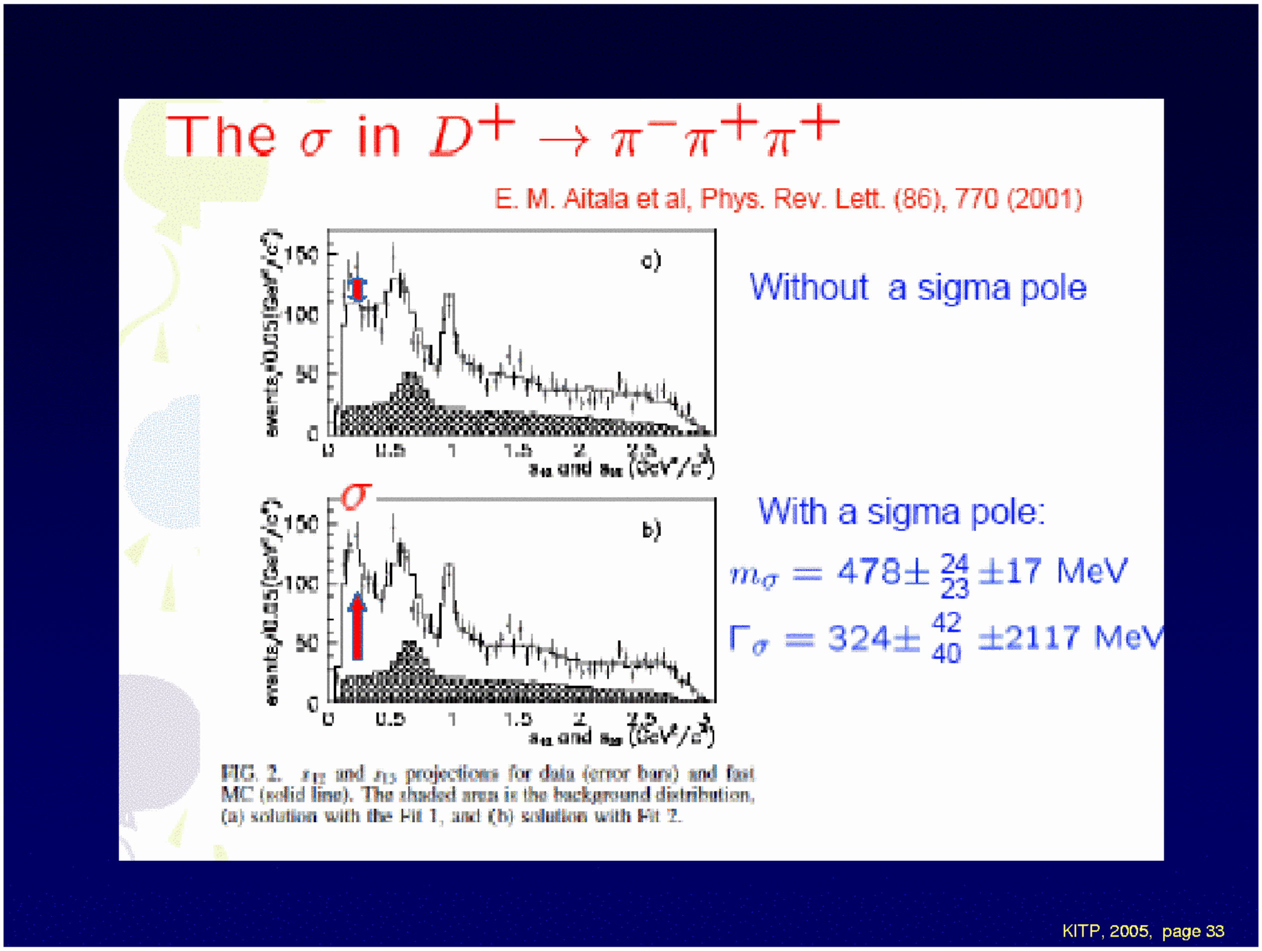
σ (400): Edward Teller

Two-pion exchange potential:

Chembto, Durso, Riska; Stony Brook, Paris, …

σ enhancement of Δ I = ½ rule





The σ in $D^+ \rightarrow \pi^- \pi^+ \pi^+$

E. M. Aitala et al, Phys. Rev. Lett. (86), 770 (2001)

FIG. 2. s_{12} and s_{13} projections for data (error bars) and fast MC (solid line). The shaded area is the background distribution, (a) solution with the Fit 1, and (b) solution with Fit 2.

Without a sigma pole

With a sigma pole:

$m_\sigma = 478 \pm {}^{24}_{23} \pm 17$ MeV

$\Gamma_\sigma = 324 \pm {}^{42}_{40} \pm 2117$ MeV

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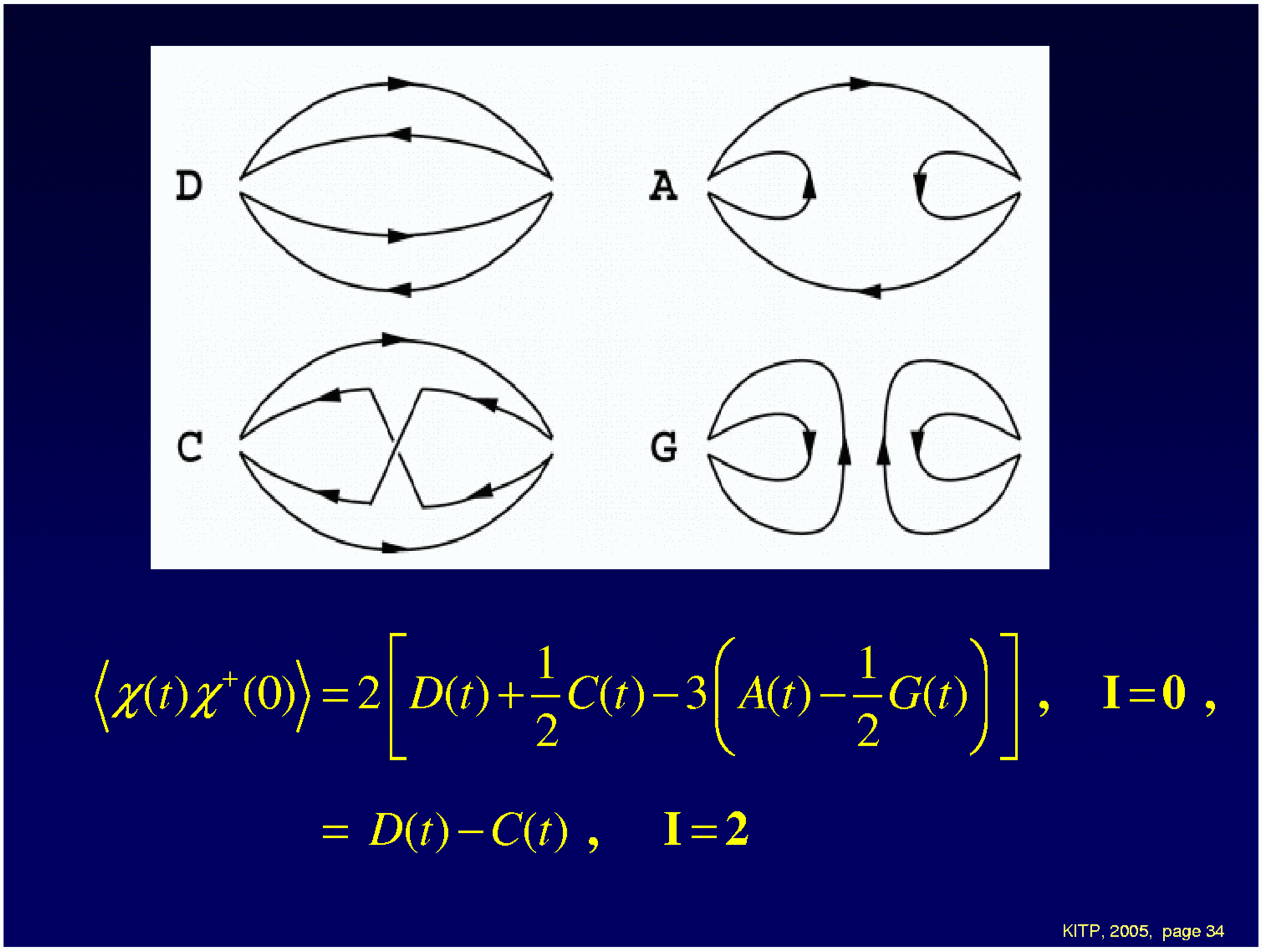

$$\langle \chi(t)\chi^{+}(0) \rangle = 2\left[D(t) + \frac{1}{2}C(t) - 3\left(A(t) - \frac{1}{2}G(t) \right) \right], \quad \mathbf{I=0},$$

$$= D(t) - C(t), \quad \mathbf{I=2}$$

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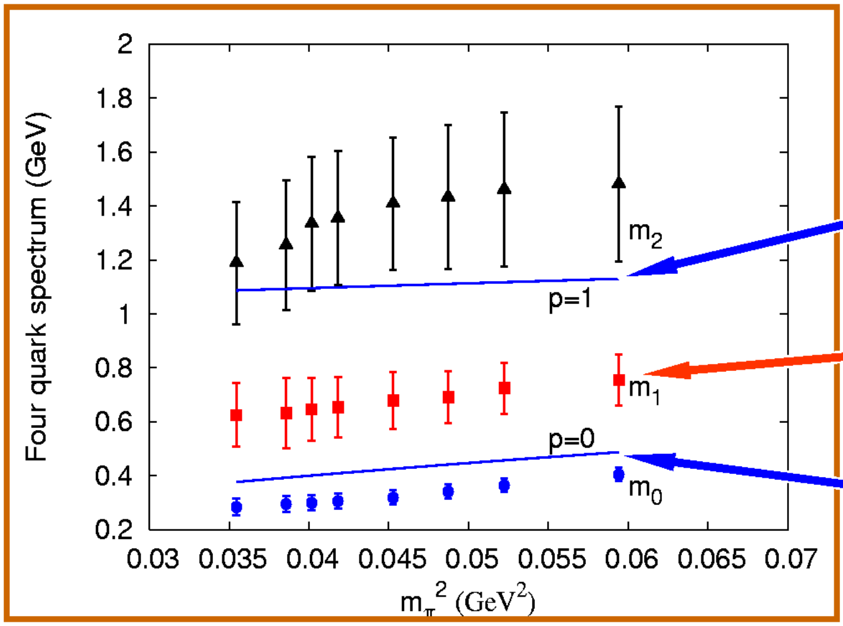
$\overline{\psi}\gamma_5\psi\,\overline{\psi}\gamma_5\psi\,[\pi\pi, \mathrm{I}^{\mathrm{G}}(\mathrm{J}^{\mathrm{PC}}) \equiv 0^+(0^{++})]$



Scattering states

$E_\pi(p=1)+E_\pi(p=1)$

Possible BOUND state
σ(600)?

$E_\pi(p=0)+E_\pi(p=0)$

Scattering states
(Negative scattering length)

Further Study is ongoing to check the volume dependence of the observed states

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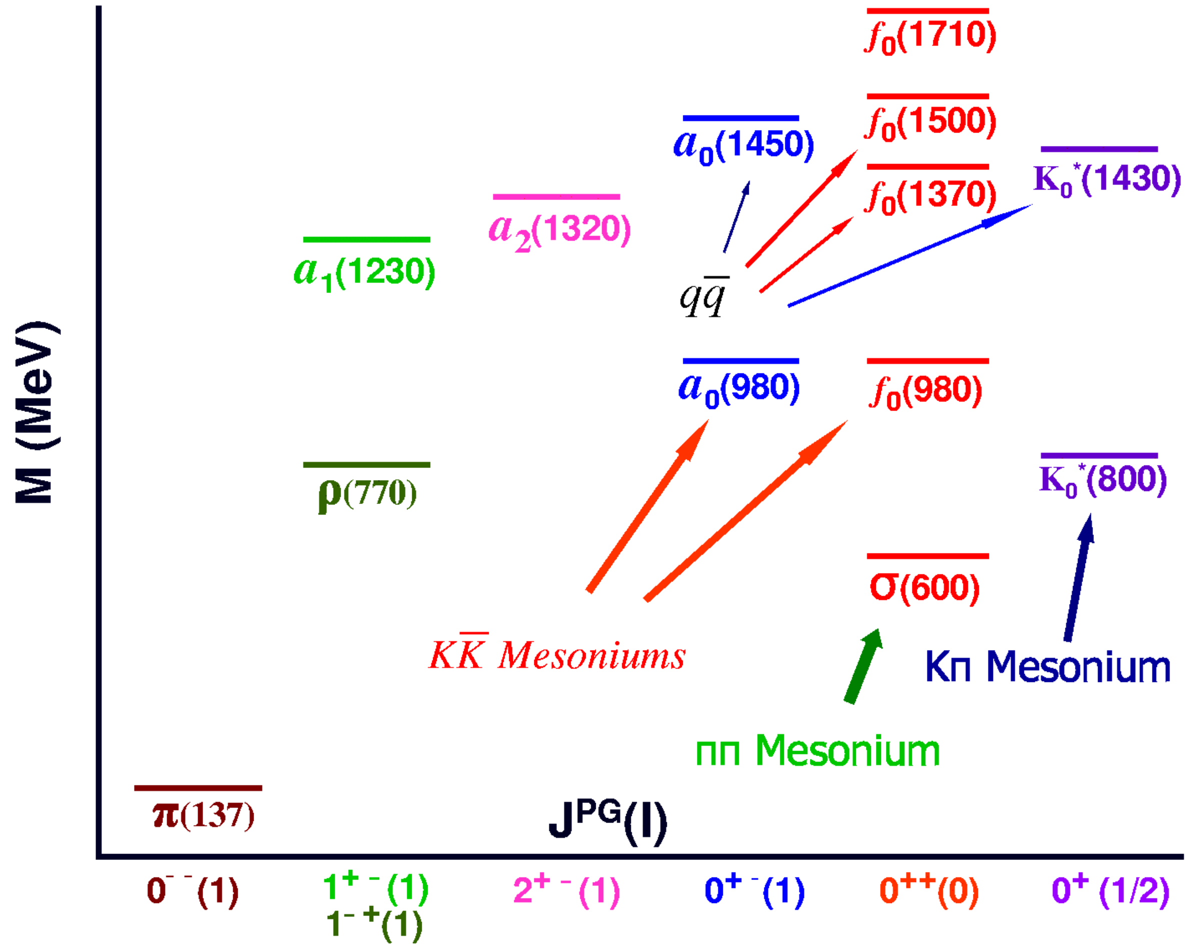

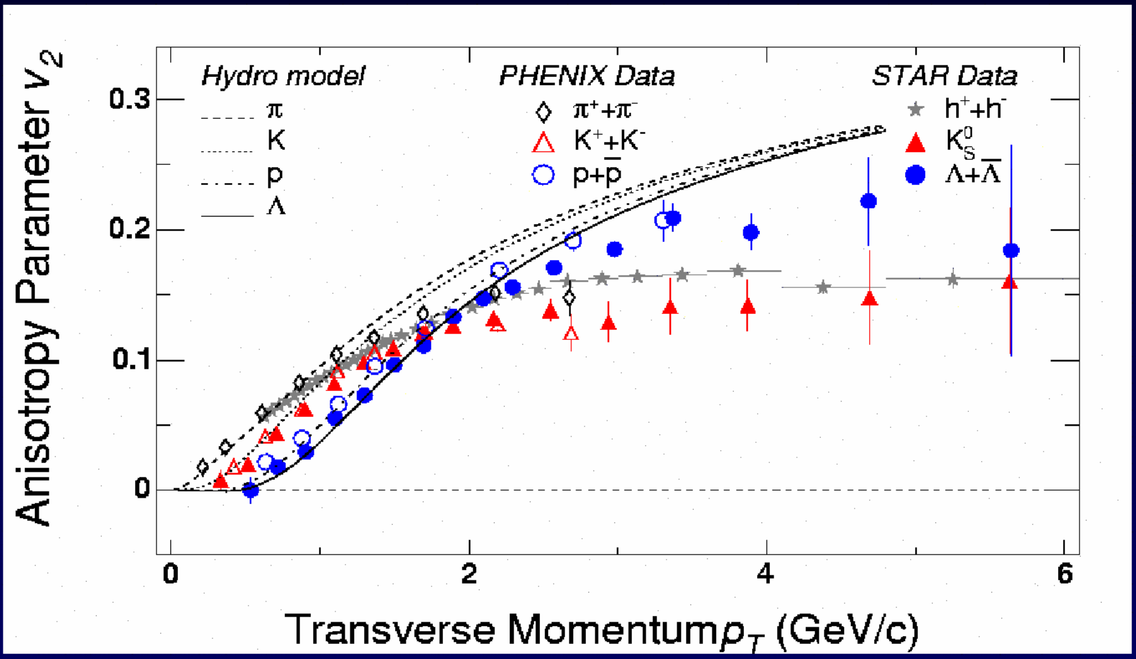

Azimuthal anisotropy in Au + Au collisions with $\sqrt{S_{NN}}$ = 200 GeV (STAR collaboration)

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