“Frontiers in Intense Laser Physics”, KITP, Santa Barbara, CA, Aug 4, 2014

Pair creation in external electric and magnetic fields

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Support: National Science Foundation, NSFC

www.phy.ilstu.edu/ILP

Acknowledgement: Illinois State University

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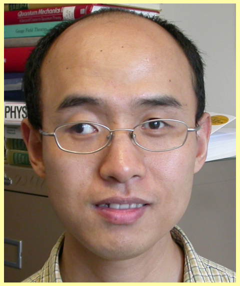
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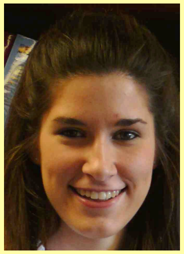
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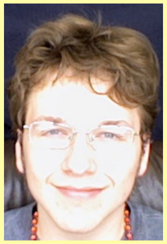
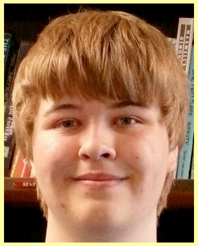
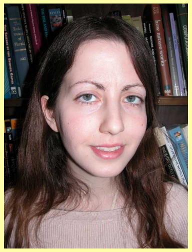

Rogers


Ware


Shields


Lamb

Acknowledgement: China

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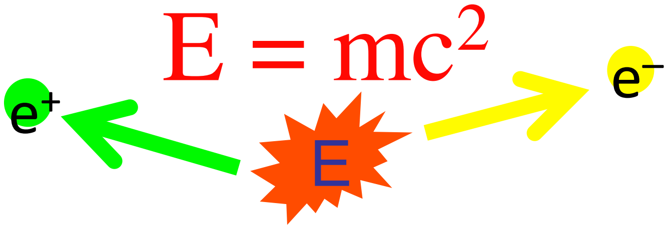
ultra-intense laser development → Vacuum physics

Vacuum breakdown


$E = mc^2$

Pair creation

Extreme Light Infrastructure

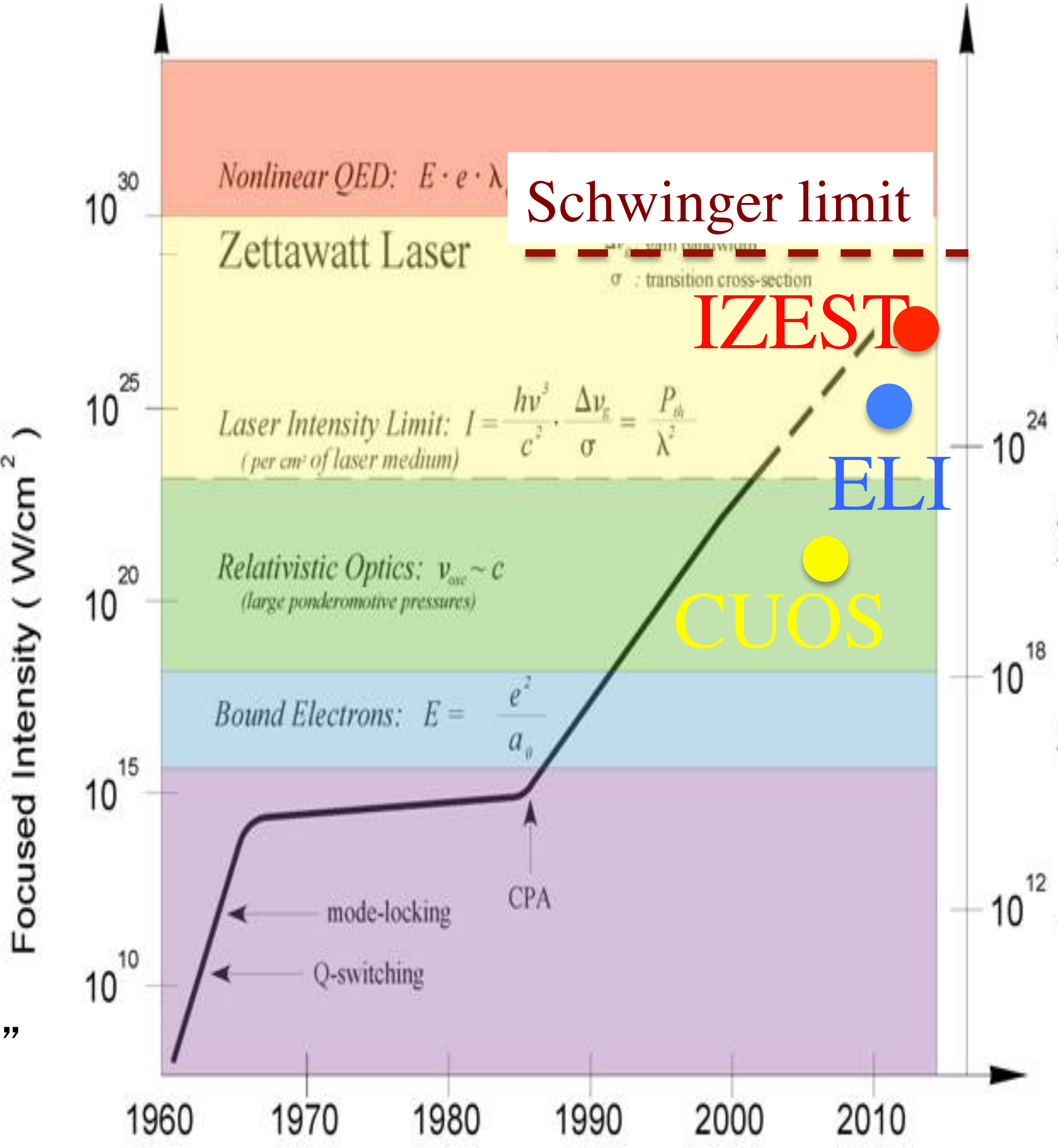
I ∼ 10^{25} W/cm^2

Int. Zetta-Exawatt Sci. & Tech.

I > 10^{27} W/cm^2

"Zeptotechnology is just around the corner"

The Economist, page 77 Feb 28 2004



Relevant experiments

heavy ion 1980s, Argonne, GSI
pairs observed
nuclear triggered

laser-electron 1997, Stanford, SLAC
electron-laser collision
indirect

pure laser ELI, IZEST, …
pure light —> matter

eli
extreme light infrastructure

Theoretical exploration

- goals
  - new concepts
  - threshold estimation
  - possible experiments ?

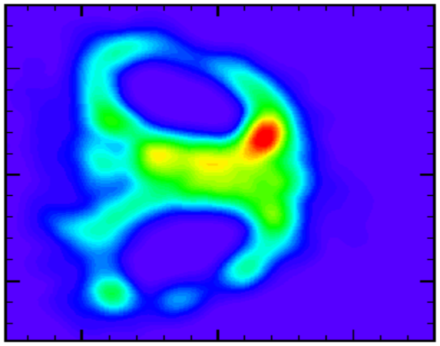
- theory
  - Europe (Sweden, ELI, …)
  - US (CA, CT, …)

- simulation
  - US (ILP)
  - Europe (Heidelberg)
  - Asia (CAS)

Progress obtained with related numerical models

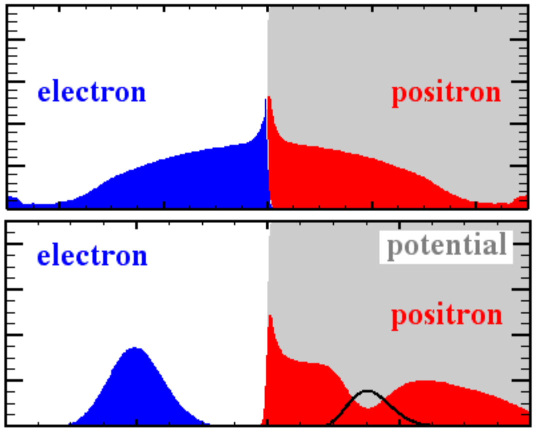
Relativistic quantum mechanics (1996–2003)

numerical solution to Dirac equation
motion in electric and magnetic fields
superluminal in barrier tunneling
harmonic generation
retardation effect
resonances in cycloatoms



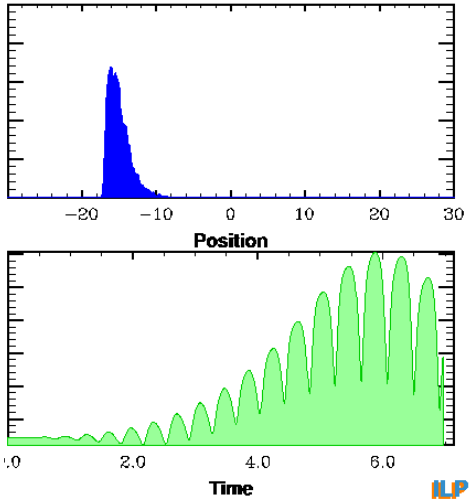
Computational quantum field theory (2003–)

resolution of Klein paradox
electron-electron correlation
transition to negative Dirac sea
Zitterbewegung
entanglement
supercritical bound states
interference in charge density
pair creation in varying forces



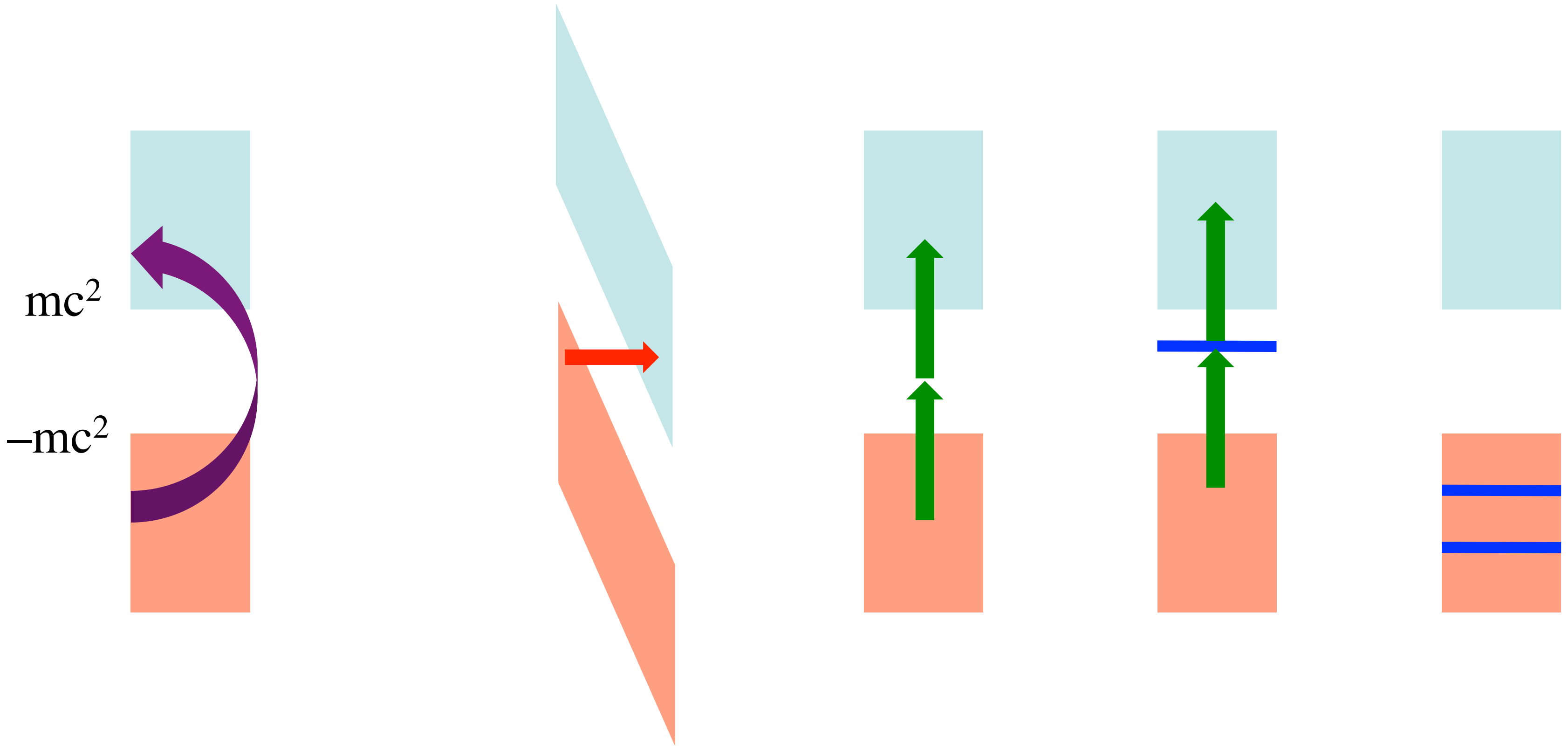
Quantum fermion-boson interaction (2008–)

numerical solution to Yukawa model
virtual boson formation
transfer of fermion source correlation to bosons
pair creation with Klein-Gordon field



Processes leading to pair creation

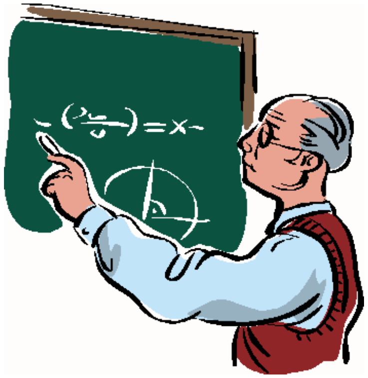
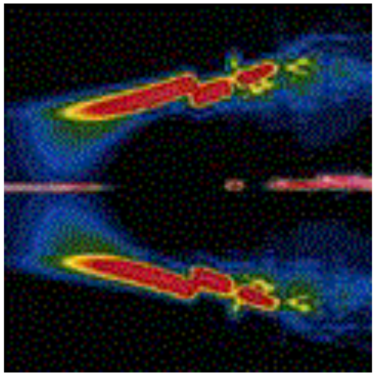
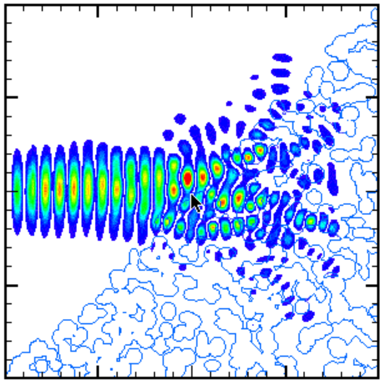
connection with ionization?



pair creation = Schwinger tunneling or photon transition or resonance enhancement or dived bound state

Outline of my talk

- Introduction to the subject ✔
- Introduction to numerical approach
- Application to the Klein paradox
- Time dependent field
- Field-induced bound states
- Magnetic field influence
- Boson versus fermion
- Dived bound states
- Outlook

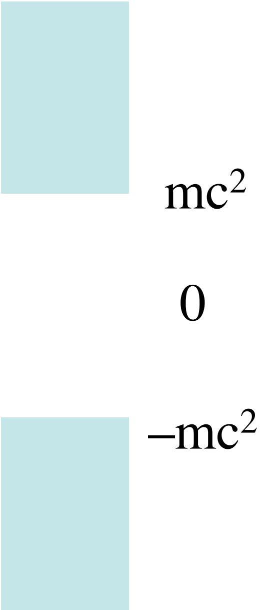




QM Dirac equation

$$i\hbar \frac{\partial}{\partial t}\phi = h\phi$$

$$h = c\alpha(p - qA/c) + mc^2\beta + qV$$

when A=V=0 $E=\pm\sqrt{m^2c^4 + p^2c^2}$



but $<\phi(t)|\phi(t)>=1$ → How to describe creation ?

Quantum field description

$$\hat{\Psi} = \sum_{\lambda} \hat{b}_{\lambda} |\lambda\rangle$$

QF operator

QM state

$$i\hbar \frac{\partial}{\partial t} \hat{\Psi} = [\hat{\Psi}, \hat{H}] \quad \xleftrightarrow{\hat{H} = \hat{\Psi}^{\dagger} h \, \hat{\Psi}} \quad i\hbar \frac{\partial}{\partial t} \hat{\Psi} = h \hat{\Psi}$$

➡ $$\hat{\Psi}(t) = \sum_{\lambda} \hat{b}_{\lambda}(t) \, |\lambda\rangle = \sum_{\lambda} \hat{b}_{\lambda} \, |\lambda(t)\rangle$$

From wave functions to operator solutions

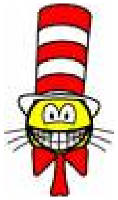
$$\hat{b}_\lambda(t) = \sum_\alpha \underbrace{\hat{b}_\alpha}_{\text{operator}} \underbrace{\langle\lambda|\alpha(t)\rangle}_{\text{time involution of } |\alpha>}$$

$$i\hbar\frac{\partial}{\partial t}|\alpha\rangle = h|\alpha\rangle$$

$$\hat{\Psi}_{e-}(t) = \sum_\lambda \hat{b}_\lambda(t)|\lambda\rangle$$

J. Braun, Q. Su, R. Grobe, PRA 59, 604 (1999)

Now we know the electron’s quantum field



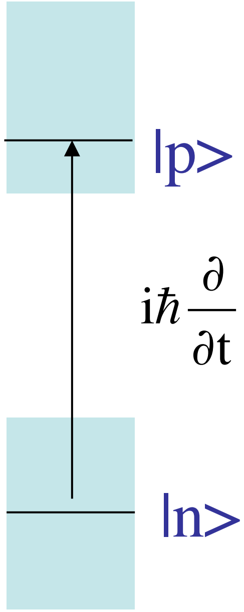
Apply $\hat{\Psi}$ to pair creation

$$\hat{\Psi} = \hat{\Psi}_{e-} + C\hat{\Psi}_{e+}$$

Number of created pairs

$$N(t) = \langle\langle vac \| \quad \hat{\Psi}^{\dagger}_{e-}(t) \quad \hat{\Psi}_{e-}(t) \, \| vac \rangle\rangle$$

$$= \sum_{n}\sum_{p} \left| < p \, | \, u(t) \, | \, n > \right|^{2}$$



$$i\hbar \frac{\partial}{\partial t}\left|n(t)\right\rangle = h\left|n(t)\right\rangle$$

$$\rightarrow \left|n(t)\right\rangle = u(t)\left|n\right\rangle$$

How to get pairs

step1: WF start with: |n>

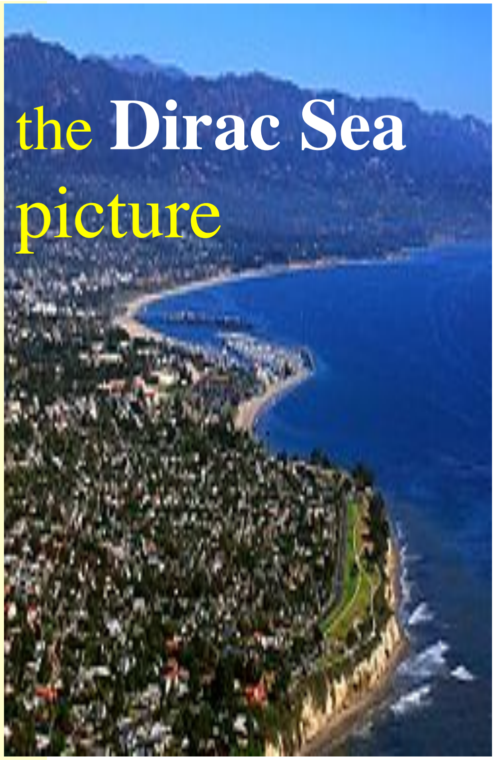
step2: solve $i\hbar \frac{\partial}{\partial t}|n(t)\rangle = h|n(t)\rangle$ to get |n(t)>

step3: project |n(t)> on |p>, call it <p|n(t)>

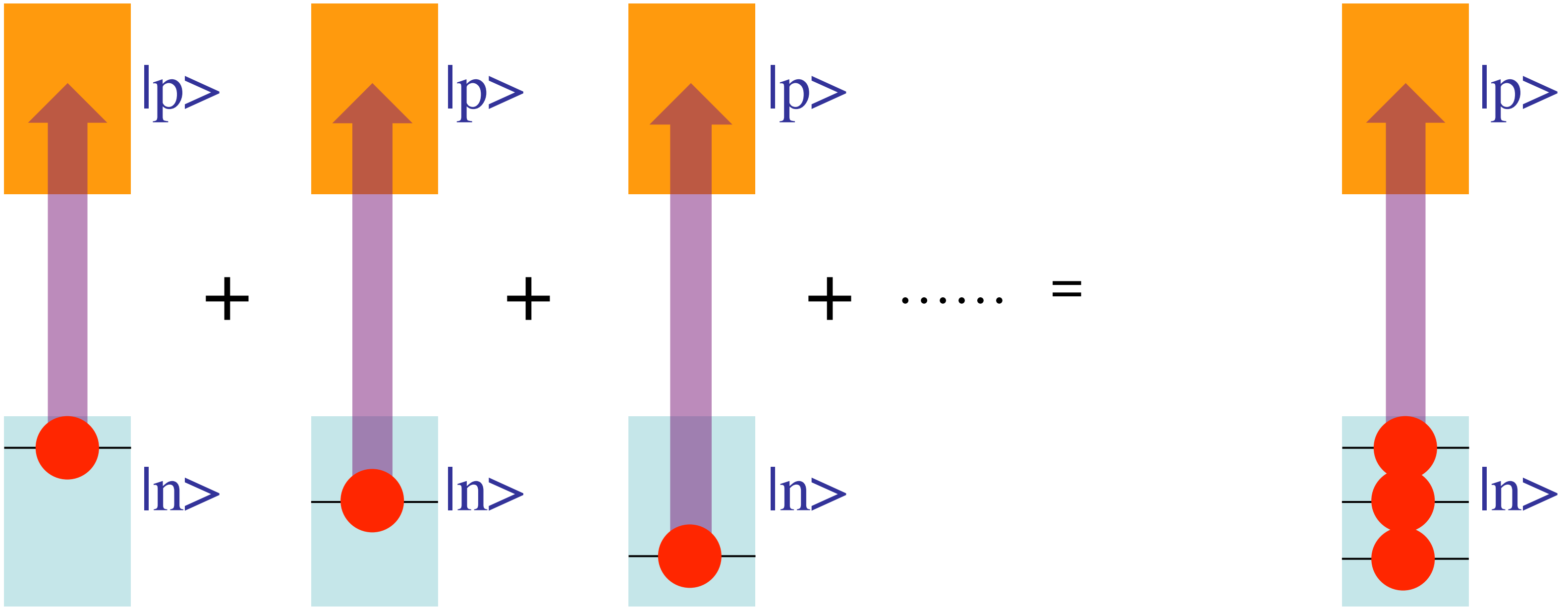
step4: repeat steps 1 to 3, …

Number of the created pairs

step5: add up results $N(t) = \sum_{p,n} |< p \mid n(t) >|^2$

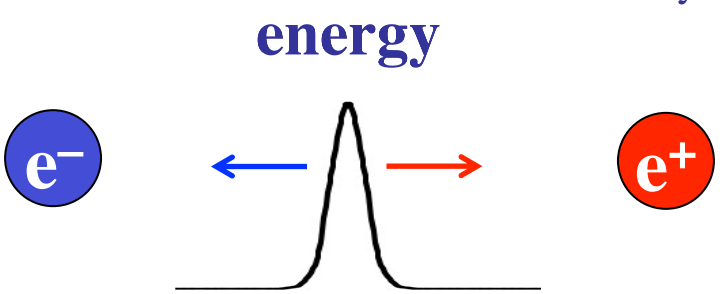


the **Dirac Sea** picture



The space-time resolved pair creation

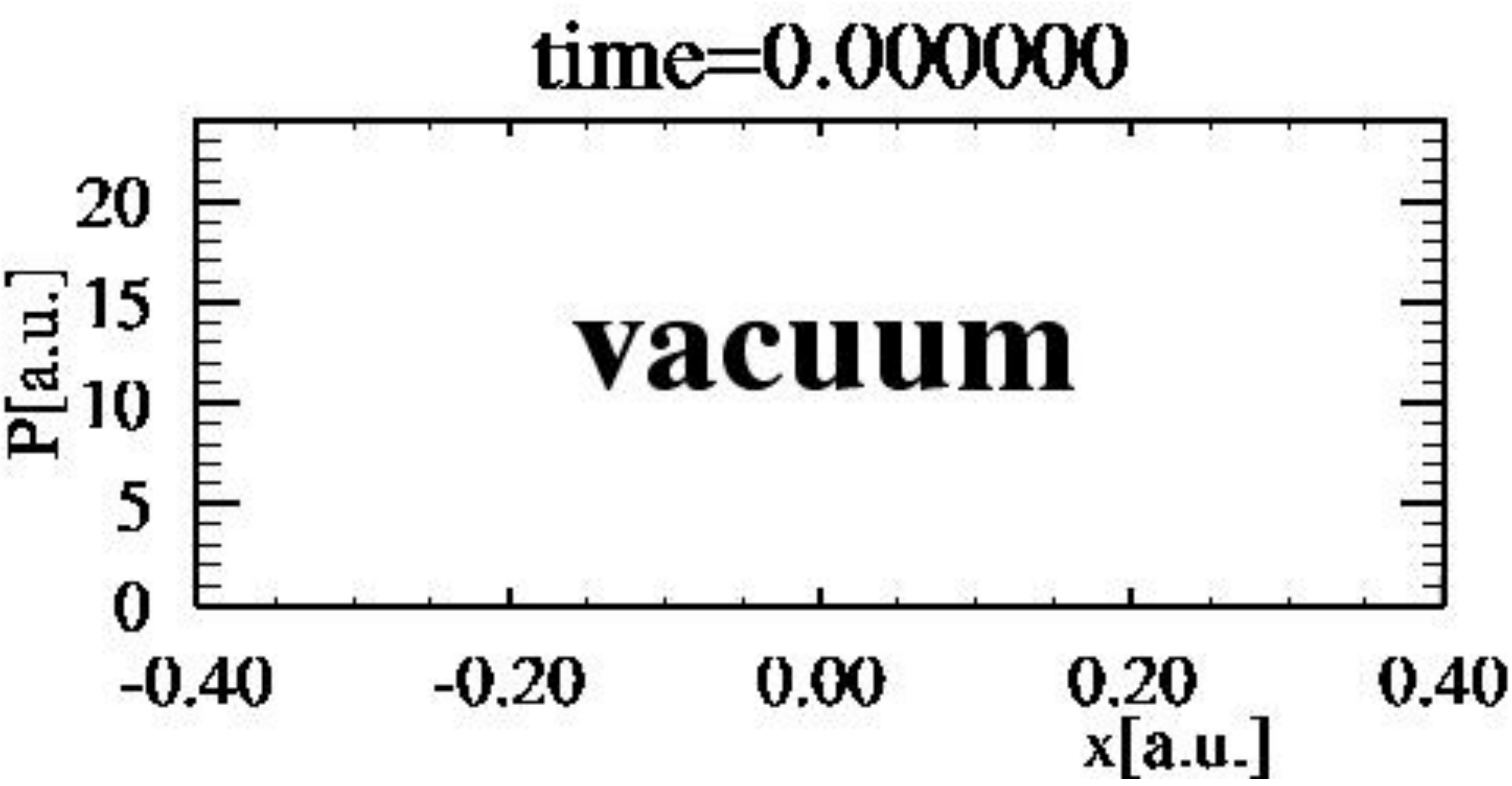
Phys. Rev. Lett. 92, 040406 (2004)



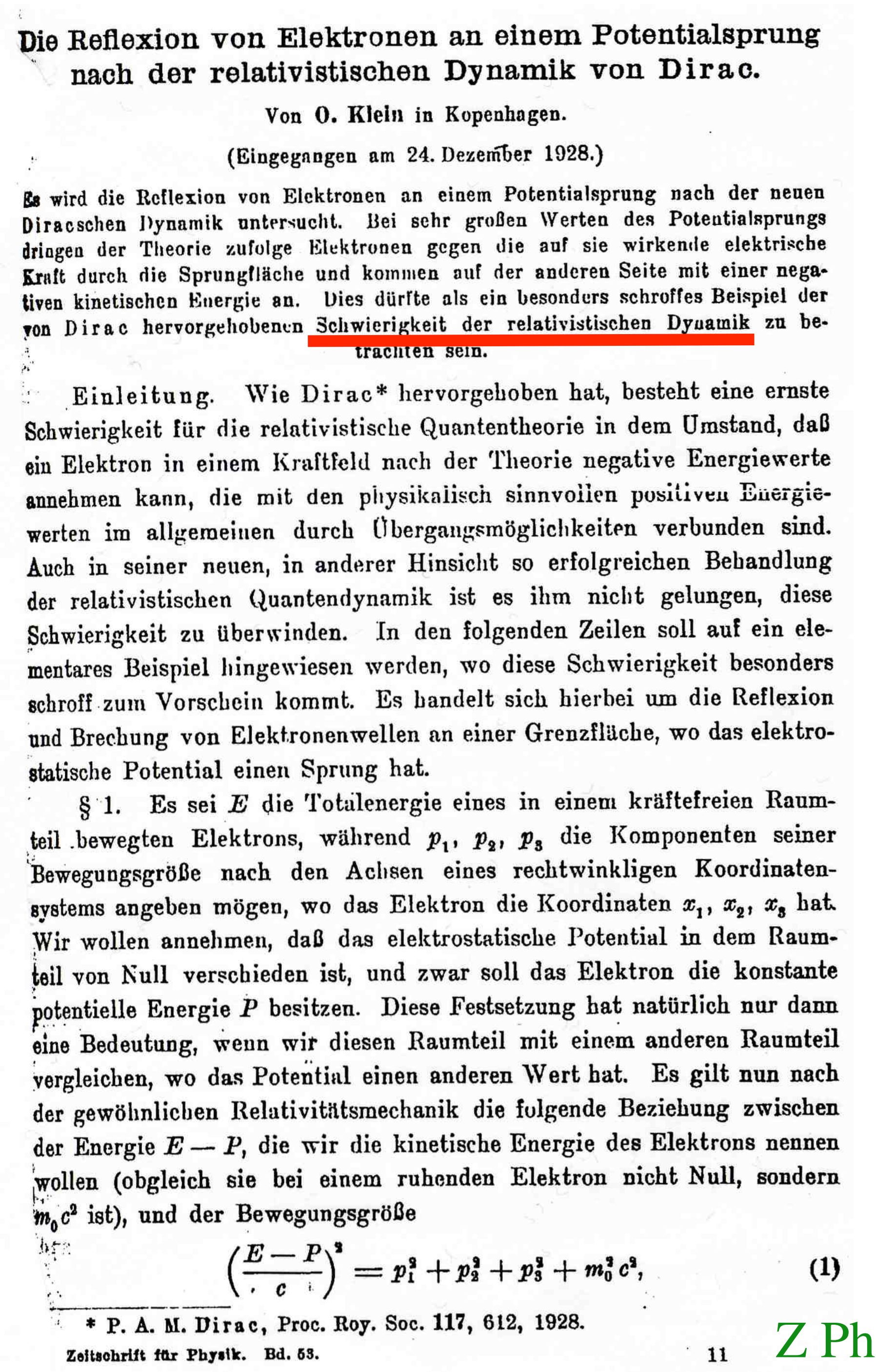
energy

e^-

e^+



Klein paradox



Die Reflexion von Elektronen an einem Potentialsprung nach der relativistischen Dynamik von Dirac.

Von O. Klein in Kopenhagen.

(Eingegangen am 24. Dezember 1928.)

Es wird die Reflexion von Elektronen an einem Potentialsprung nach der neuen Diracschen Dynamik untersucht. Bei sehr großen Werten des Potentialsprungs dringen der Theorie zufolge Elektronen gegen die auf sie wirkende elektrische Kraft durch die Sprungfläche und kommen auf der anderen Seite mit einer negativen kinetischen Energie an. Dies dürfte als ein besonders schroffes Beispiel der von Dirac hervorgehobenen Schwierigkeit der relativistischen Dynamik zu betrachten sein.

Einleitung. Wie Dirac* hervorgehoben hat, besteht eine ernste Schwierigkeit für die relativistische Quantentheorie in dem Umstand, daß ein Elektron in einem Kraftfeld nach der Theorie negative Energiewerte annehmen kann, die mit den physikalisch sinnvollen positiven Energiewerten im allgemeinen durch Übergangsmöglichkeiten verbunden sind. Auch in seiner neuen, in anderer Hinsicht so erfolgreichen Behandlung der relativistischen Quantendynamik ist es ihm nicht gelungen, diese Schwierigkeit zu überwinden. In den folgenden Zeilen soll auf ein elementares Beispiel hingewiesen werden, wo diese Schwierigkeit besonders schroff zum Vorschein kommt. Es handelt sich hierbei um die Reflexion und Brechung von Elektronenwellen an einer Grenzfläche, wo das elektrostatische Potential einen Sprung hat.

§ 1. Es sei E die Totalenergie eines in einem kräftefreien Raumteil bewegten Elektrons, während p_1, p_2, p_3 die Komponenten seiner Bewegungsgröße nach den Achsen eines rechtwinkligen Koordinatensystems angeben mögen, wo das Elektron die Koordinaten x_1, x_2, x_3 hat. Wir wollen annehmen, daß das elektrostatische Potential in dem Raumteil von Null verschieden ist, und zwar soll das Elektron die konstante potentielle Energie P besitzen. Diese Festsetzung hat natürlich nur dann eine Bedeutung, wenn wir diesen Raumteil mit einem anderen Raumteil vergleichen, wo das Potential einen anderen Wert hat. Es gilt nun nach der gewöhnlichen Relativitätsmechanik die folgende Beziehung zwischen der Energie $E - P$, die wir die kinetische Energie des Elektrons nennen wollen (obgleich sie bei einem ruhenden Elektron nicht Null, sondern $m_0 c^2$ ist), und der Bewegungsgröße

$$\left(\frac{E-P}{c}\right)^2 = p_1^2 + p_2^2 + p_3^2 + m_0^2 c^2, \qquad (1)$$

---

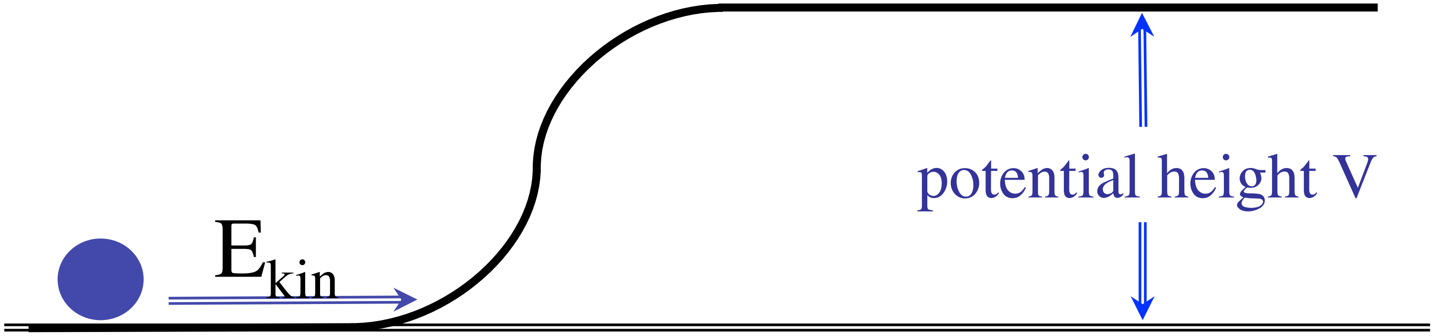
* P. A. M. Dirac, Proc. Roy. Soc. 117, 612, 1928.

Zeitschrift für Physik. Bd. 53. 11

Z Physik **53**, 157 (1929)

Oskar Klein
(1894-1977)

“Normal” potential height $(V << 2mc^2)$



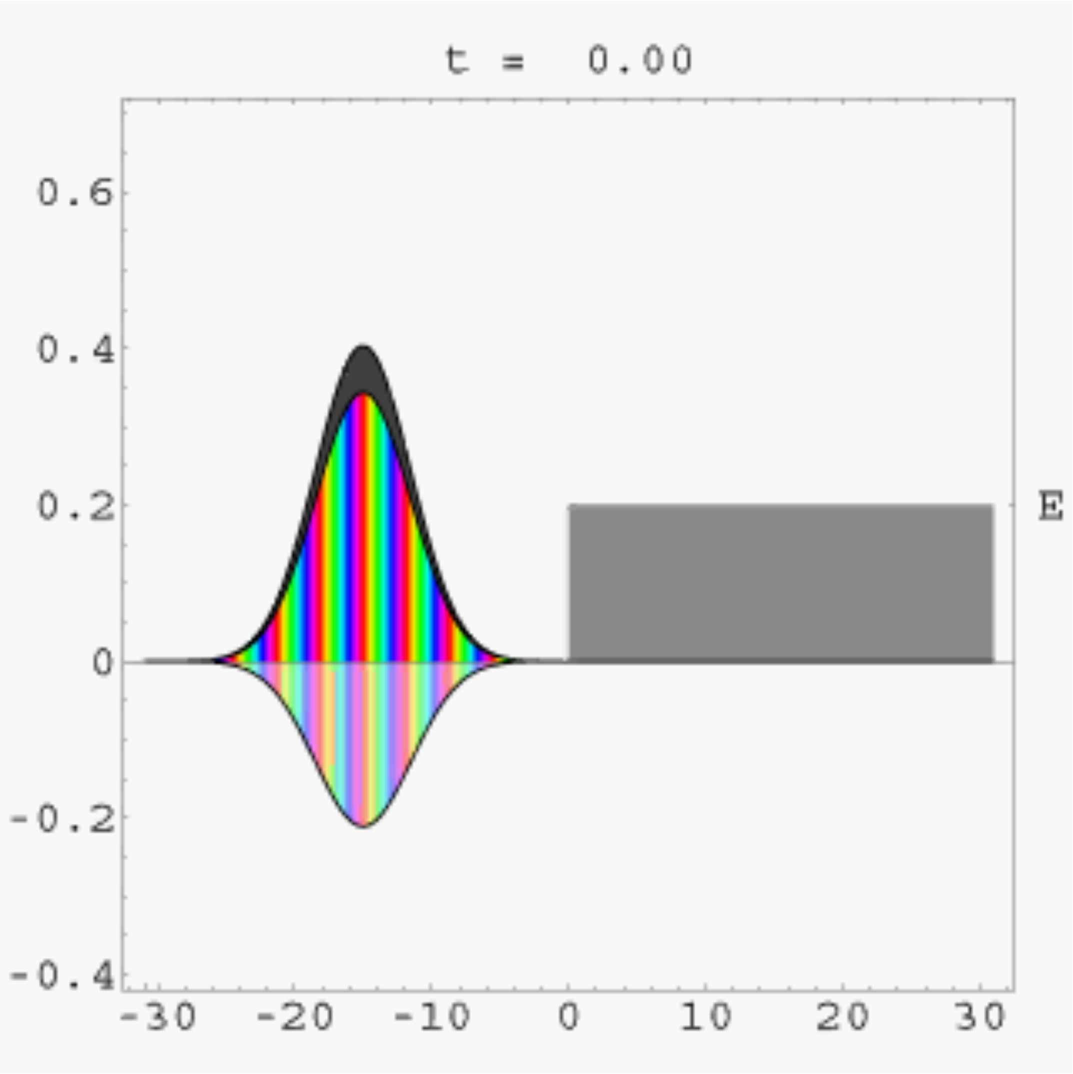
Classical mechanics predicts:

if $E_{kin} < V$ ⇒ ball cannot roll up

Traditional quantum mechanics

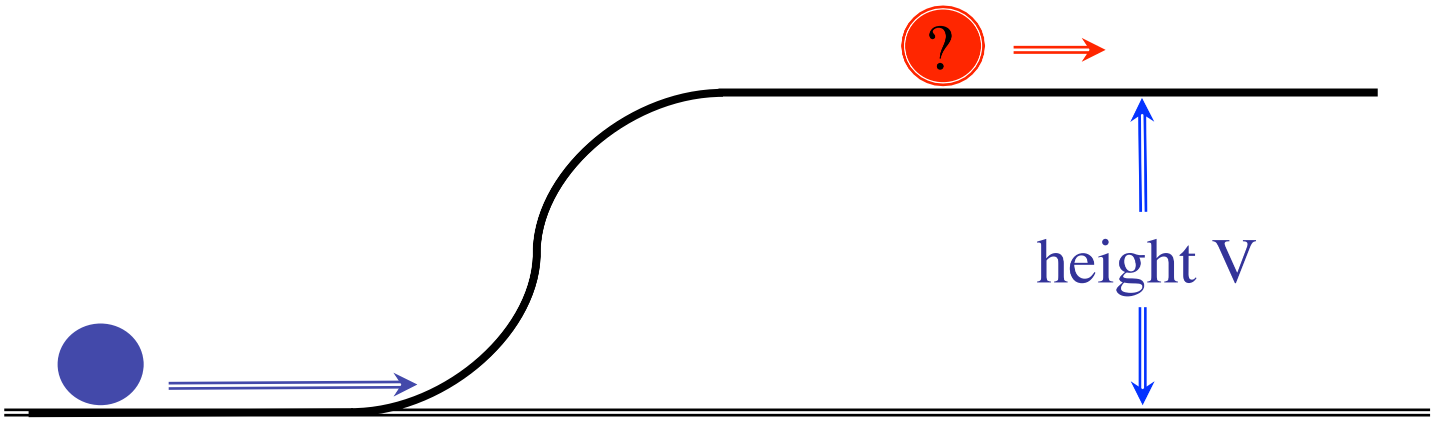
if $E_{kin} < V$

$\Rightarrow$ bounces back



Movie: Courtesy of Bernd Thaller
http://www.kfunigraz.ac.at/imawww/thaller/

“Abnormal” potential height ($V > 2mc^2$)



Single-particle quantum mechanics predicts:

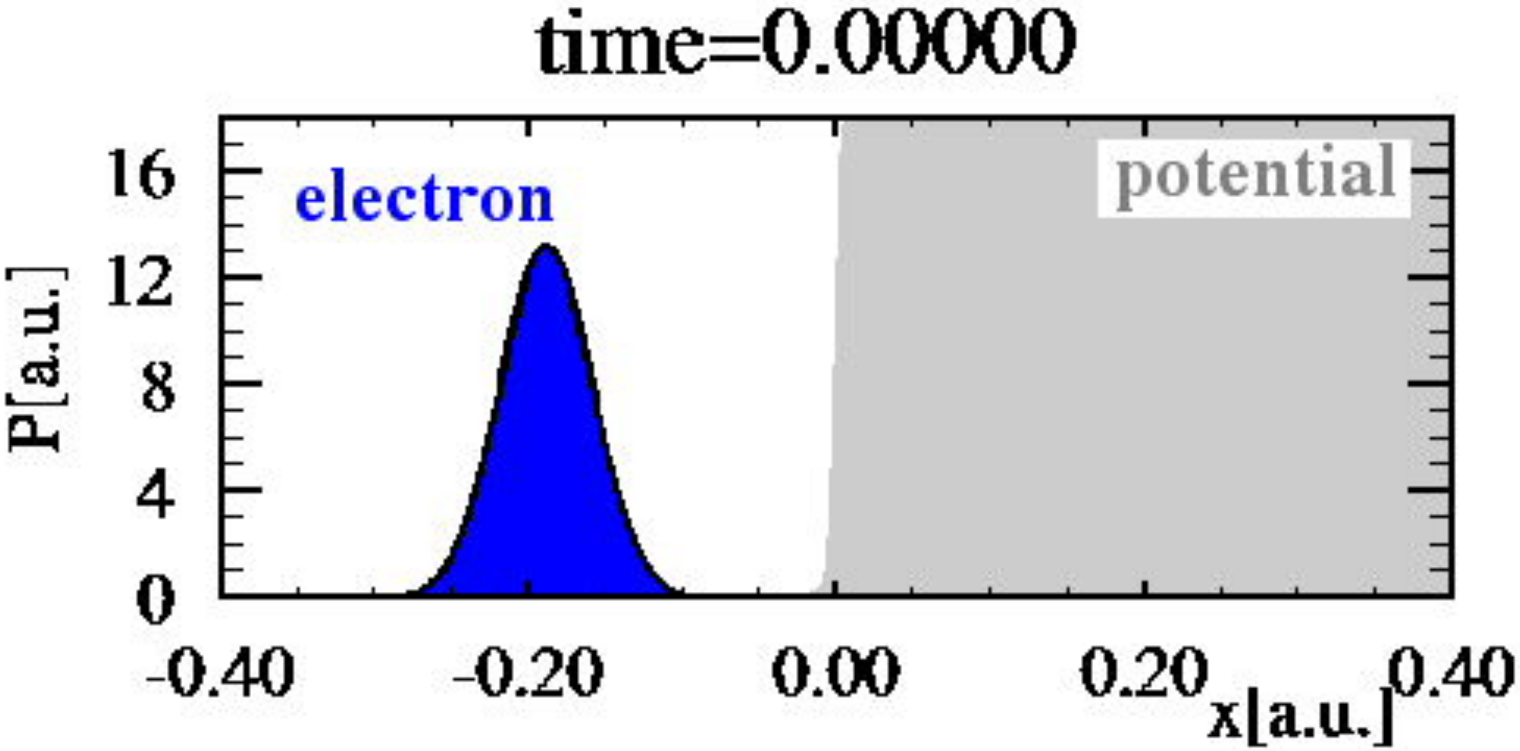
even if $E_{kin} < V \Rightarrow$ some of the “**particle**” rolls up

??

Wave packet evolution under single particle Dirac equation

$V > 2\,mc^2$

$E_{kin} < V$

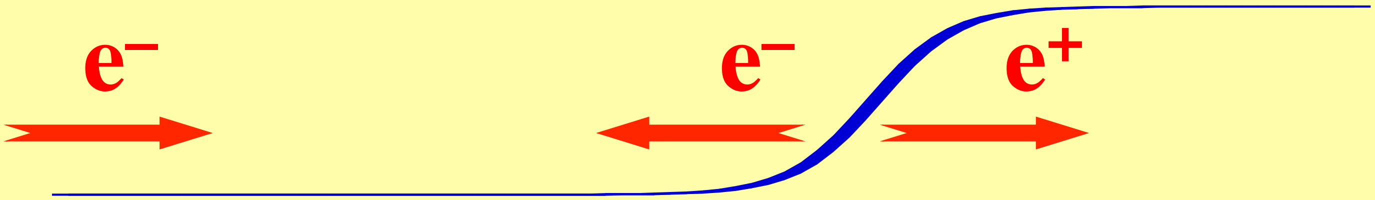


Interpretation of mysterious transmitted portion unclear

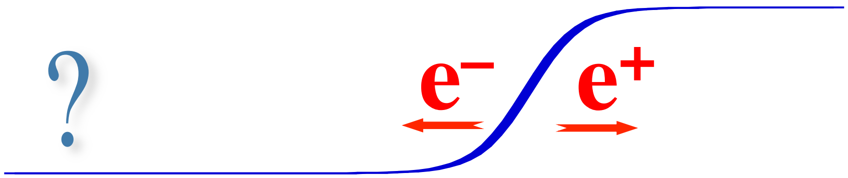
Braun, Su and Grobe, PRA 59, 604 (1999)

The Klein paradox

electron affects pair-creation ????

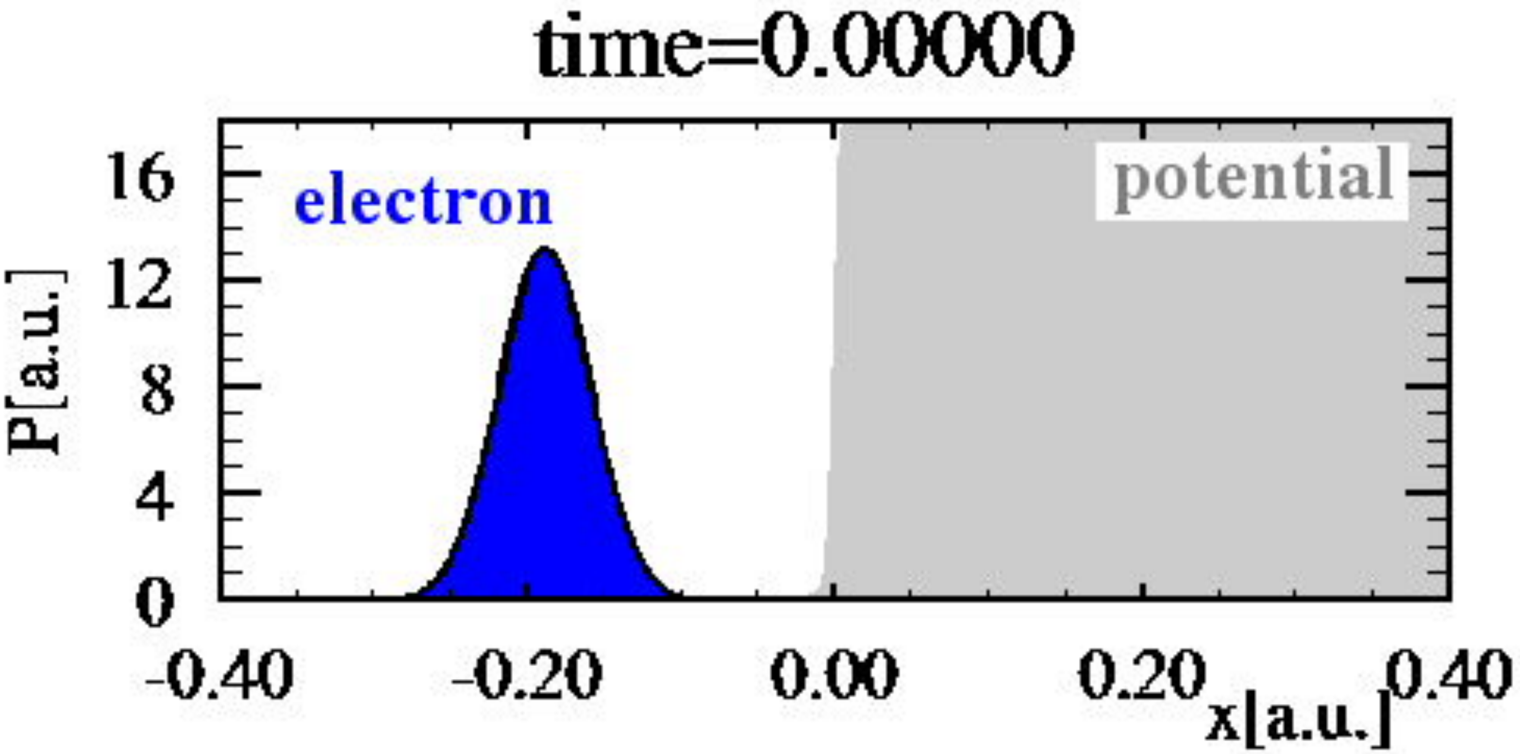


before 2004:

simplification:		approach:
no e^-- e^+		Quantum mechanics
no e^-		Quantum field theory

2004: Resolution of the Klein-paradox

$E_{kin} < V$ $V > 2\ mc^2$



- Electron cannot move into the barrier
- Klein's state is the amount of **suppressed** positrons

P. Krekora, Q. Su and R. Grobe, Phys. Rev. Lett. 92, 040406 (2004).

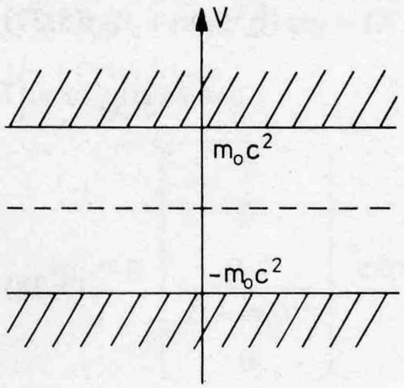


Fig. 5.3

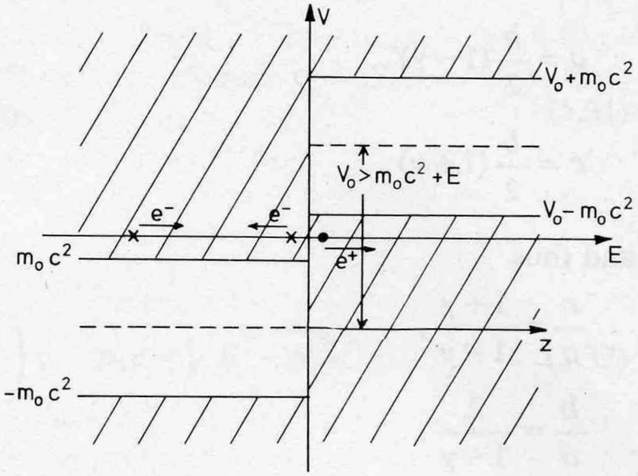


Fig. 5.4

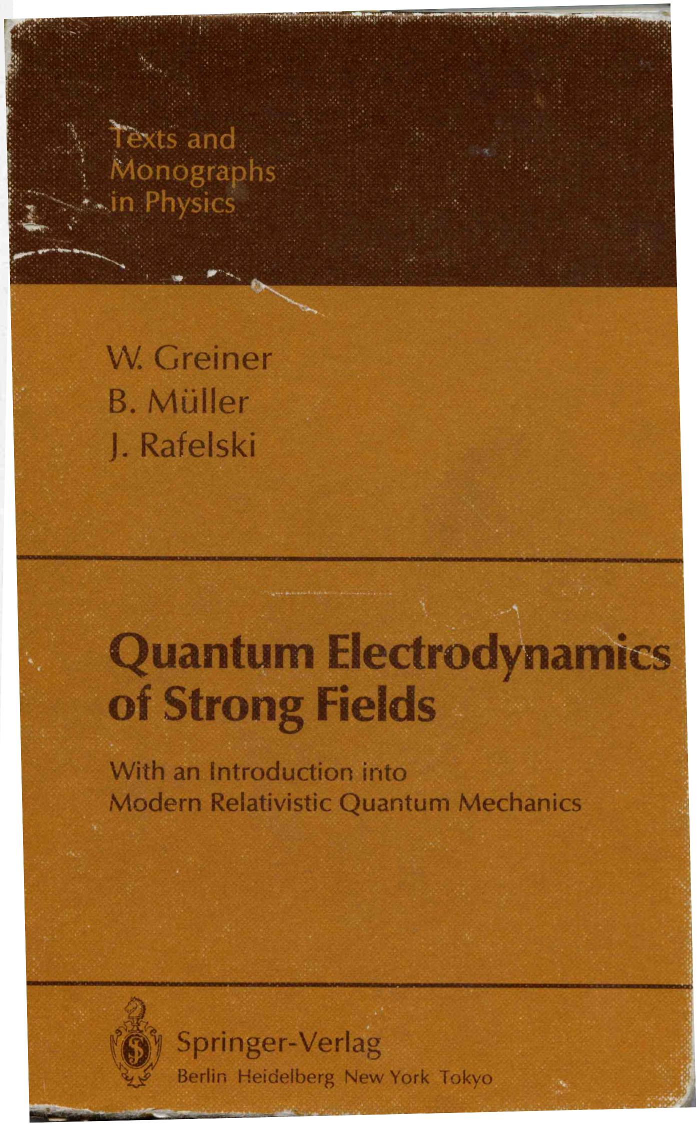
Fig. 5.3. Positive and negative energy continua of the free Dirac equation

Fig. 5.4. Upper and lower continua in the regions with and without potential. For $V_0 > m_0c^2 + E$, the electron impacting from the left is confronted with electrons from the lower, occupied continuum at the right

region I. However, according to our assumptions there are no electrons in region II. Hence the results must be reinterpreted. This is achieved by interpreting the Dirac field as a many-body problem, i.e. by means of the hole theory. In hole theory the formally obtained solutions to negative energy are taken seriously and thus two electron continua exist (Fig. 5.3).

As outlined in Chap. 4, the negative energy states must be occupied by electrons to stabilize the vacuum. This hypothesis now allows the following explanation for the Klein paradox. If the potential $V_0 > m_0c^2 + E$, where E is the energy of the electron in region I, then the energies of the level spectrum in region II are lifted by V_0. As seen in Fig. 5.4, then a part of the positive energy spectrum of region I overlaps with a fraction of the lower energy continuum of region II. Therefore, electrons impacting on the potential barrier from the left can knock out electrons from the occupied lower continuum states at the right. This explains that the reflected electron current is larger than the incoming electron current, (5.43). In the domain of the potential (region II) a positron current (i.e. hole current) is produced.

We can now understand within this picture, why according to (5.33) plane waves may exist in region II. These are *positron waves*. Furthermore, one can also understand the sign of the current j_{II} of (5.41). It is a positron current in $+\boldsymbol{e}_z$



Texts and Monographs in Physics

W. Greiner
B. Müller
J. Rafelski

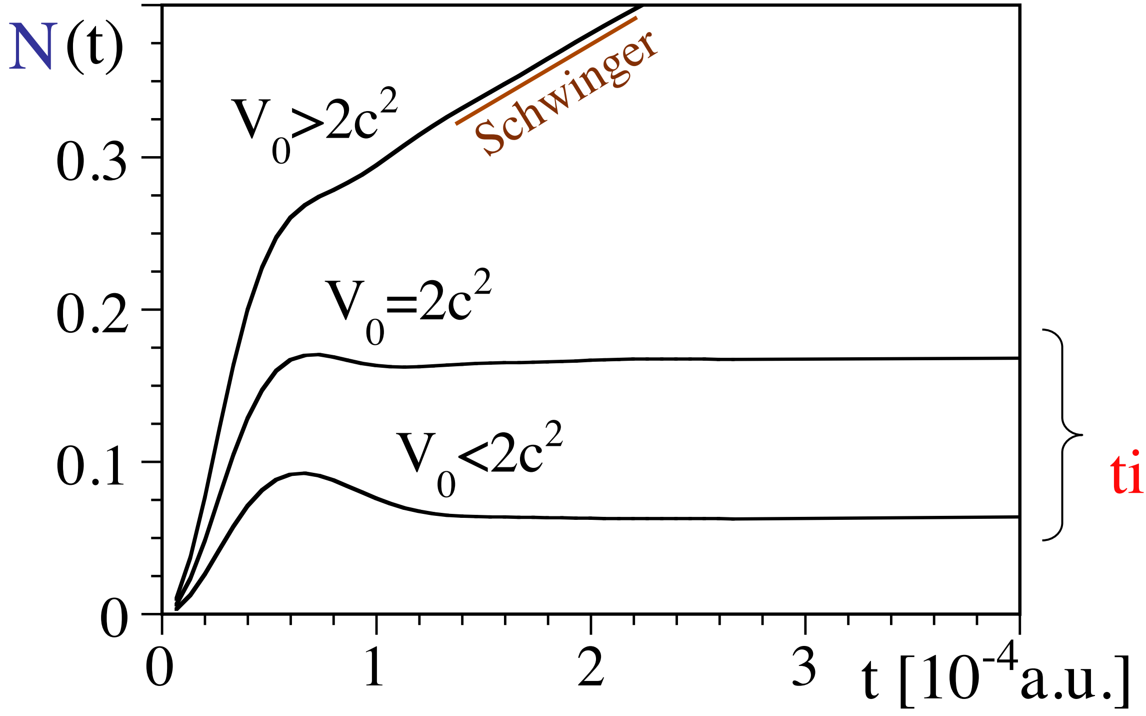
Quantum Electrodynamics of Strong Fields

With an Introduction into Modern Relativistic Quantum Mechanics

Springer-Verlag
Berlin Heidelberg New York Tokyo

ways to create pairs

super-critical, large force

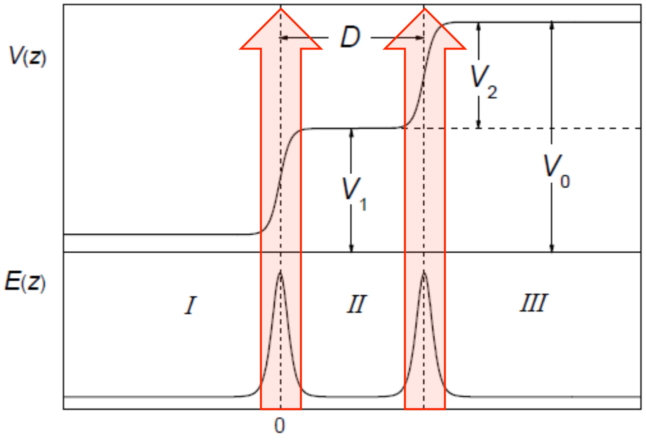


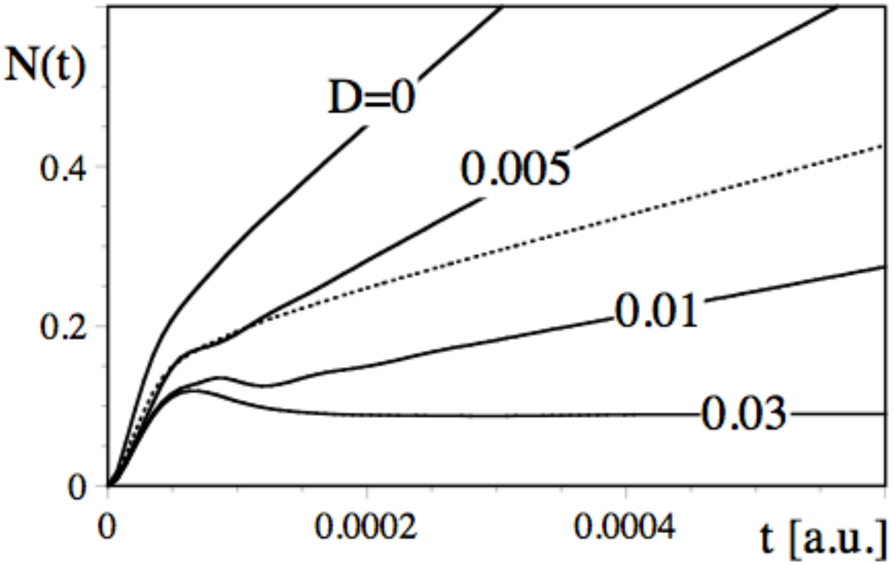
under-critical,
time dependent force

EuroPhys. Lett. 86, 13001 (2009)

multi-field combination

Lowering the field threshold with 2 subcritical constant fields ?





2 subcritical fields
displaced by space D

$V_1 = V_2 = 1.5c^2$, dash is for $V=2.5c^2$

M. Jiang, et al, Phys. Rev. A 83, 053402 (2011)

Lowering the field threshold with alternating field + constant field ?

Different mechanisms for pair creation: $\eta = E / c\omega$

1. **Multi-photon transition for an alternating field** $\eta << 1$

   Key parameter:

   field frequency

   Threshold: $\boldsymbol{\omega_{cr} > 2c^2}$

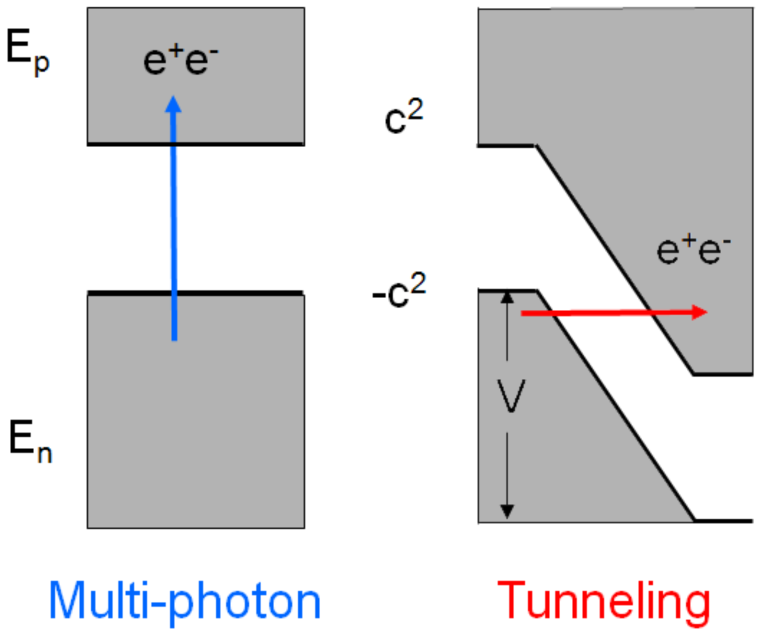
2. **Schwinger tunneling for a constant field** $\eta >> 1$

   Key parameter:

   field strength

   Threshold: $\boldsymbol{V_{cr} > 2c^2}$

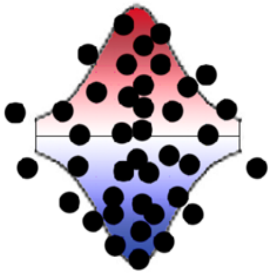
[with $W \sim 1/c$ while $E_0 = V_0/(2W)$]



M. Jiang, et al, Phys. Rev. A 85, 033408 (2012)

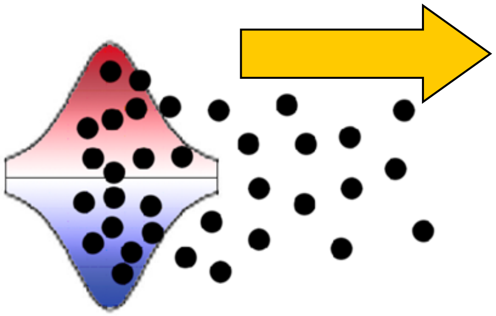
Pair creation enhancement due to combined external fields

Motivation:



Only alternating field:

· suppression due to **Pauli blocking**



Possible solution:

· pull out the created pairs

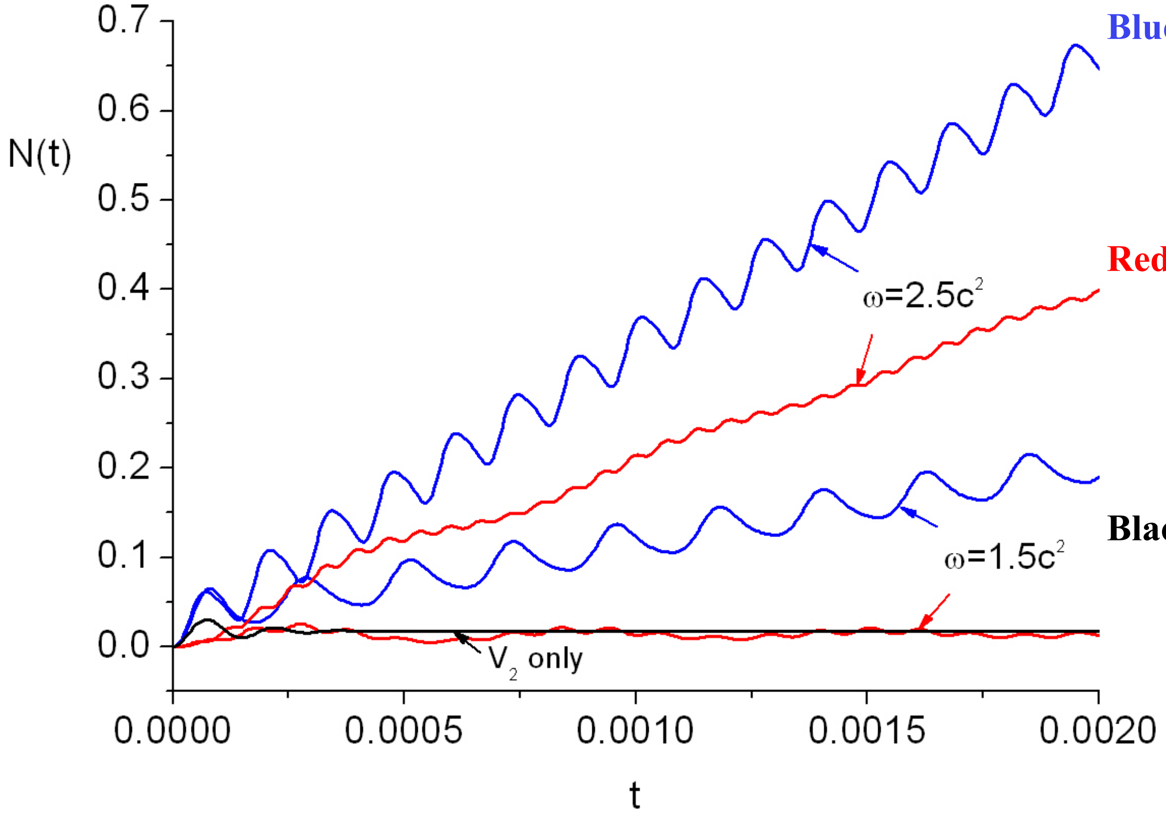
Model:

$F_1 = V_1 S(z)\ \sin(\omega t)$ with $V_1 = 1.5c^2$ and

$F_2 = V_2 S(z)$ with $V_2 = 2.5c^2$.

PRA 85, 033408 (2012)

Pair creation enhanced in combined fields !

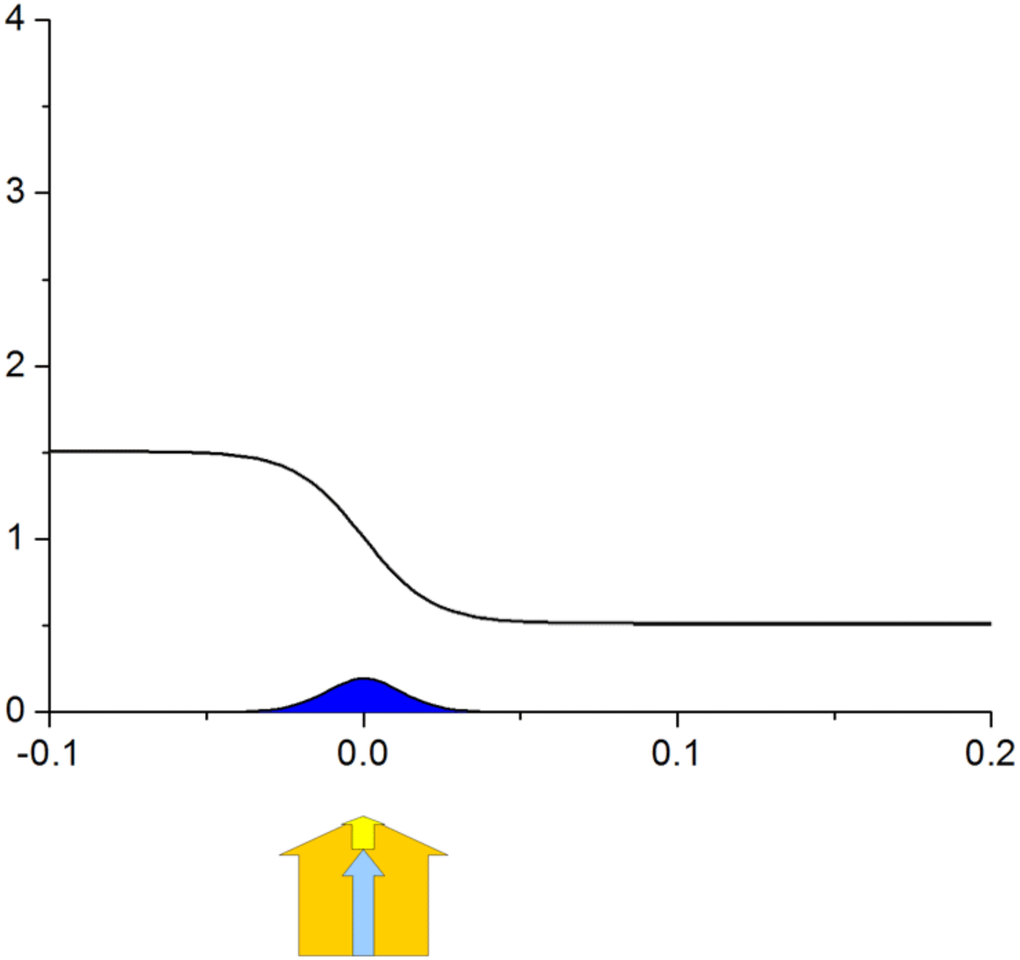


Blue lines: *both added*

Red lines: *only alternating*

Black line: *only static*

Time evolution of the spatial probability



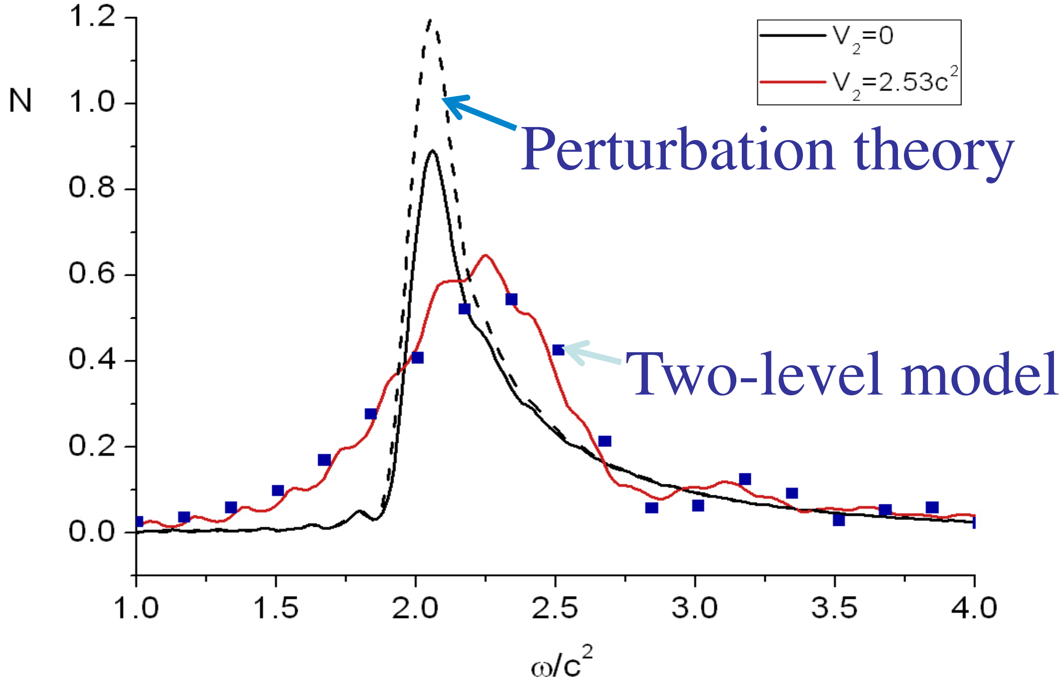
F_1: creates particles

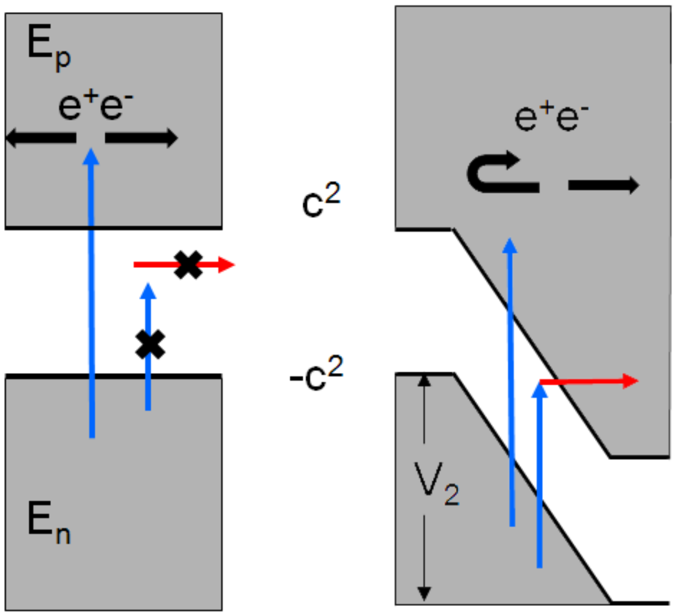
F_2: drags the created particles out.

For k>0: accelerated (green balls)

For k<0: decelerated then accelerated (red balls)

Multiphoton or Schwinger tunneling ??



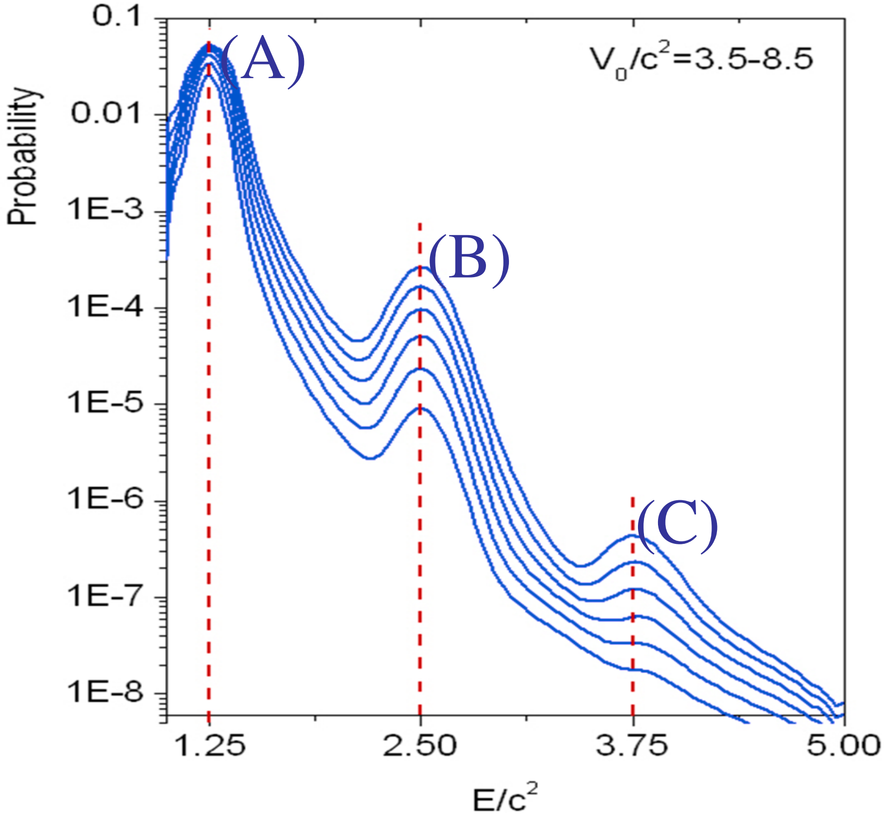


N(t) for F_1 only and F_1+F_2 [W = 5/c]

- For F_1 **only** (black line), sudden rise at $\omega = 2c^2$, suggests the start of single photon transition
- By **adding** F_2 (red line), the region $\omega < 2c^2$ is no longer forbidden, due to **single photon transition** and **Schwinger tunneling**

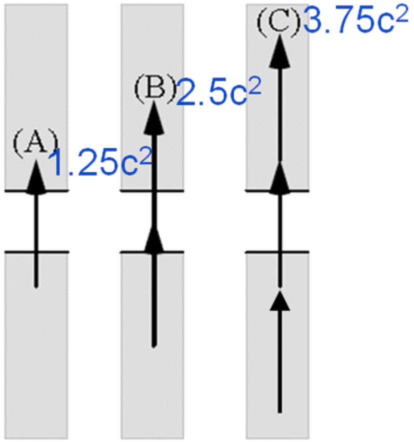
Above Threshold Pair-creation

M. Jiang, et al, Phys. Rev. A 85, 033408 (2012)



Pair production vs energy,
for F_1 only and with six field strengths.
$[W = 3/c, \omega = 2.5c^2, T = 0.002]$

$$E^2 = m^2c^4 + c^2p^2$$



ω=2.5c²

Multi-photon process

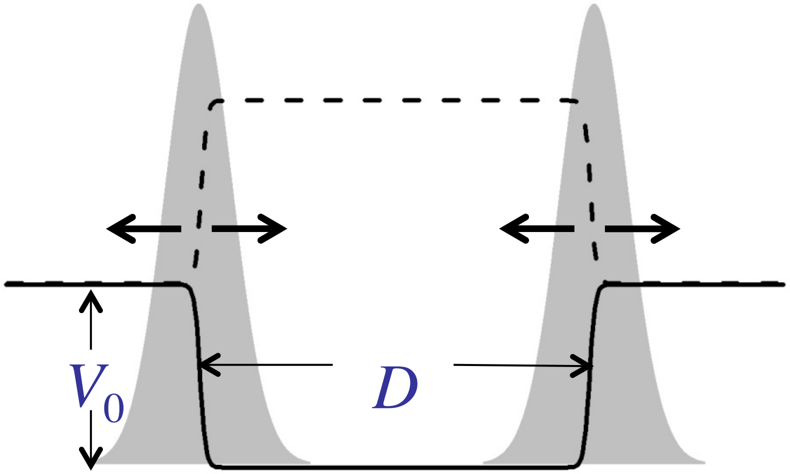
- \> **location of the peaks: independent of V_1.**
- ➢peak **(A): one-photon transition** $E_{initial}(E_A) + \omega = E_{final}(E_A)$;
- ➢peak **(B)** and **(C): two** and **three-photo transitions.**

Field-induced bound states enhance pair creation ?

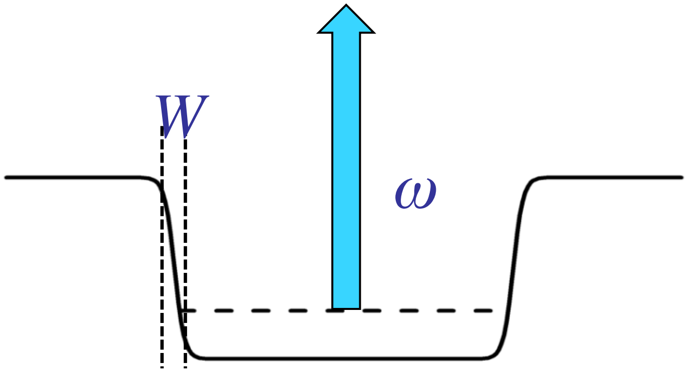
Model: a time dependent potential well

$$V(z,t) = V_0\,[\,S(z-D/2)-S(z+D/2)\,]\,\sin(\omega t)$$

Quantum coherent effect



Transition from the bound states



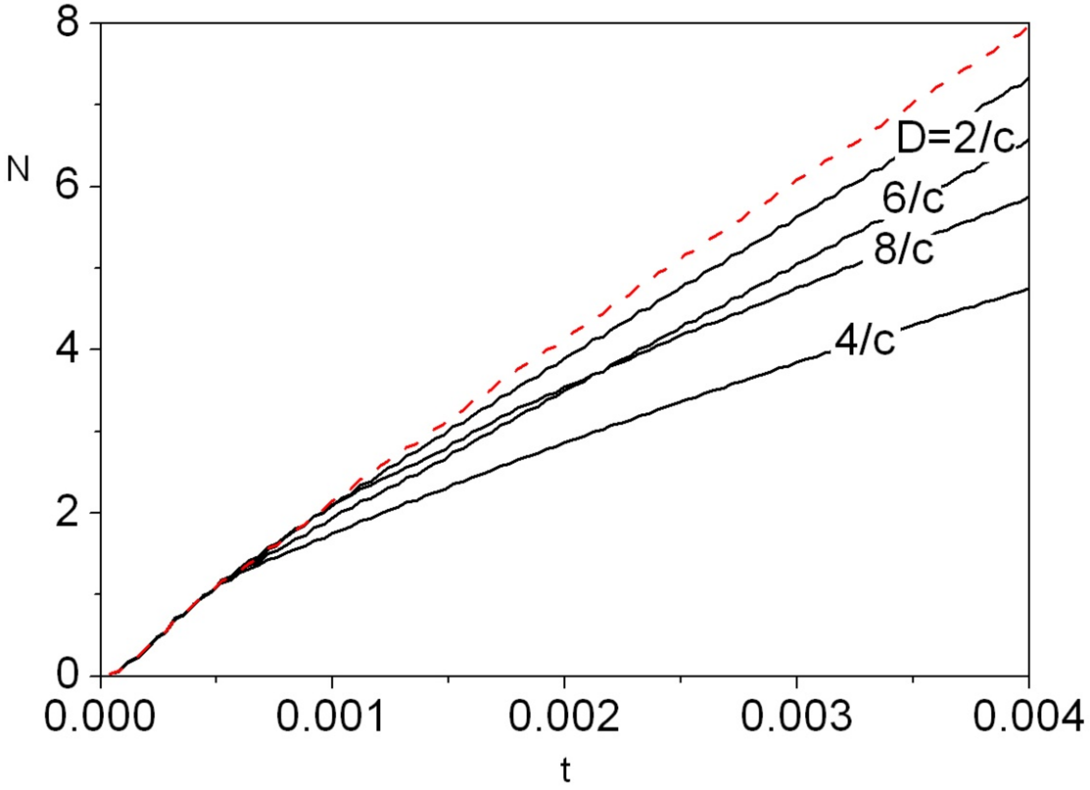
Key parameters:

V_0-depth D-width

W-field width ω-frequency

PRA 87, 042503 (2013)

Pair creation non-monotonic with D



D = 2/c, 4/c, 6/c, 8/c and ∞ (red dashed line).

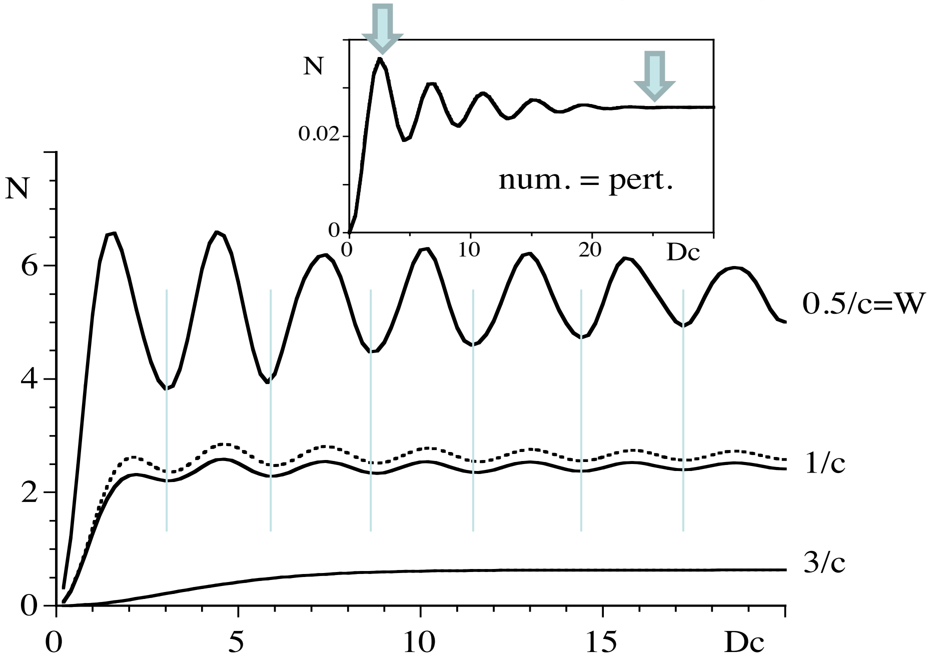
[$V_0 = 1.5c^2$, W = 0.5/c and $\omega = 2.5c^2$]

> Supercritical frequency, particles are produced continuously
> The slope varies with the well width D, and has an oscillatory behavior

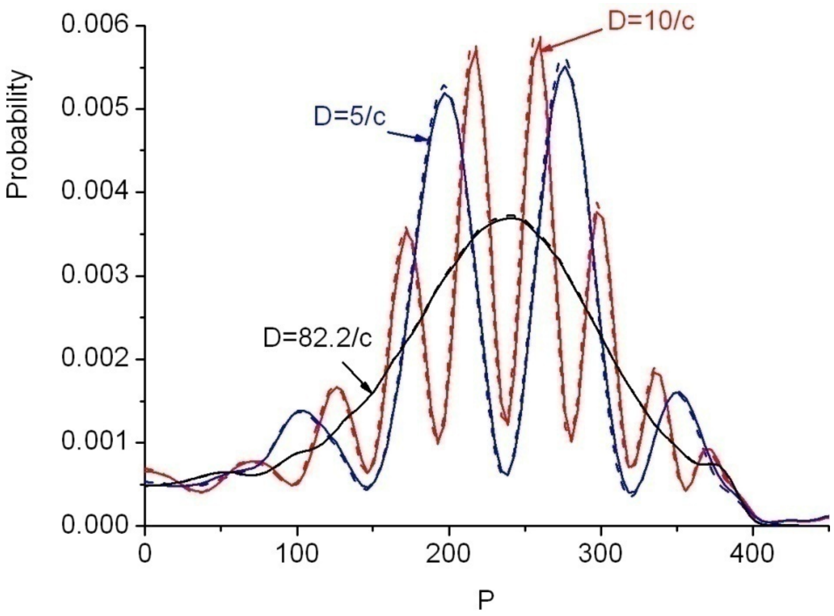
Perturbation theory works

$$N^{(1)}(t)=\frac{\pi^4 W^2 V_0^2}{L^2}\sum_{p,n}\frac{\sin^2\left[(\omega_{pn}-\omega)t/2\right]}{\left[(\omega_{pn}-\omega)/2\right]^2}A_{pn}^2\,\mathrm{csch}^2\left[\frac{\pi W(p+n)}{2}\right]\sin^2\left[(p+n)D/2\right]$$

The coherent term



The N-D plot, at time t=0.004. [$V_0=1.5c^2$, $\omega=3c^2$].
The inset [$V_0=0.1c^2$, $W=0.5/c$, $\omega=2.5c^2$, $t=0.002$]

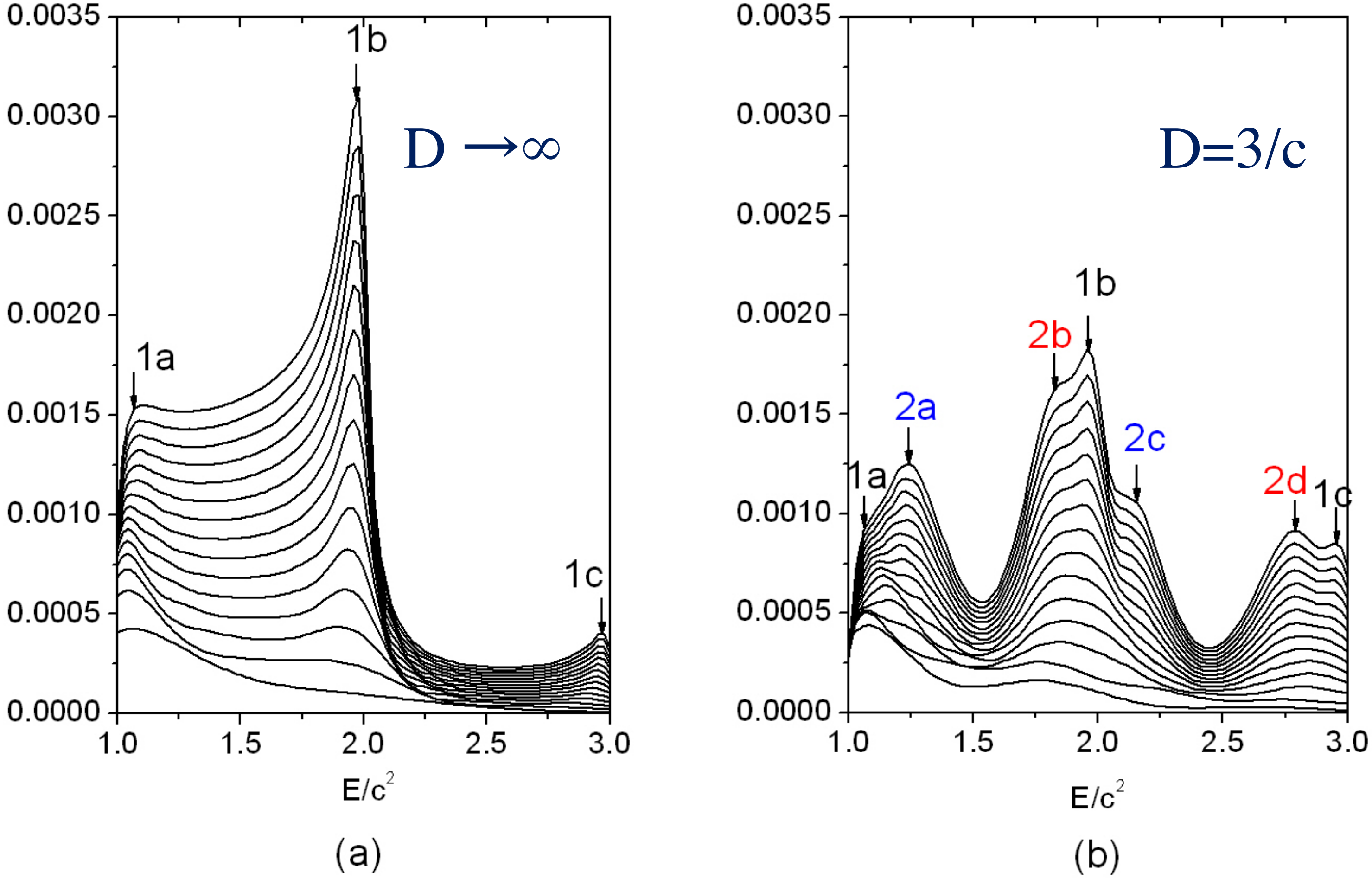


Momentum distribution at t=0.002
[$V_0=1.5c^2$, $W=1/c$, $\omega=4c^2$, and for
D=5/c, D=10/c, and D=82.2/c (∞)]

$T_D=\pi/p$ while $p\sim n$, $E=\omega/2=c\sqrt{c^2+p^2}$

$T_p=\pi/D$

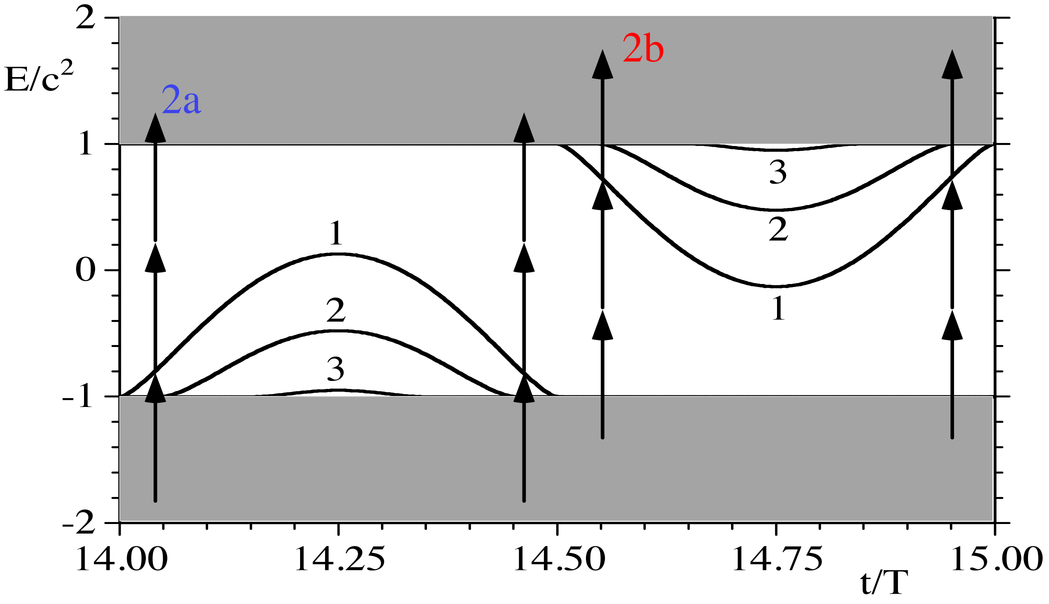
New peaks in the electron energy spectrum (D/c ~ 2π/ω)



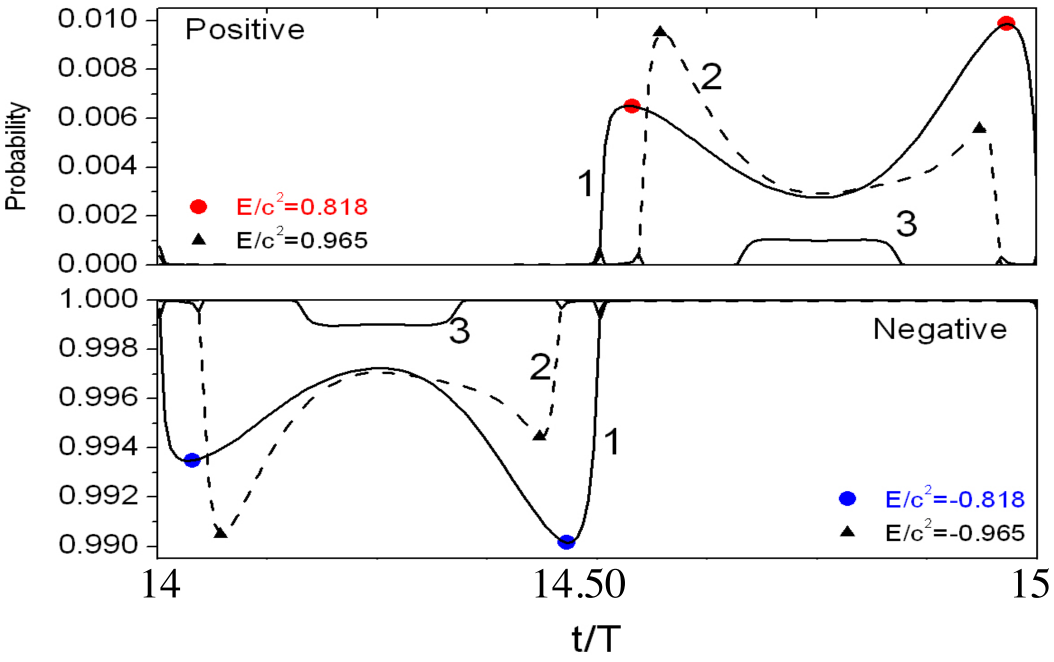
The time evolution of the energy distribution from t=0 to 0.005 (a) D →∞, and (b) D=3/c. [$\omega=c^2$, $V_0=1.5c^2$, $W=0.5/c$]

$E_{2a}\sim1.2c^2$, $E_{2b}\sim1.8c^2$, $E_{2c}=E_{2a}+\omega$ and $E_{2d}=E_{2b}+\omega$

New pathways due to field induced bound states



Instantaneous eigenvalues of the potential well in one temporal period [D=3/c, V_0=1.5c^2, W=0.5/c]

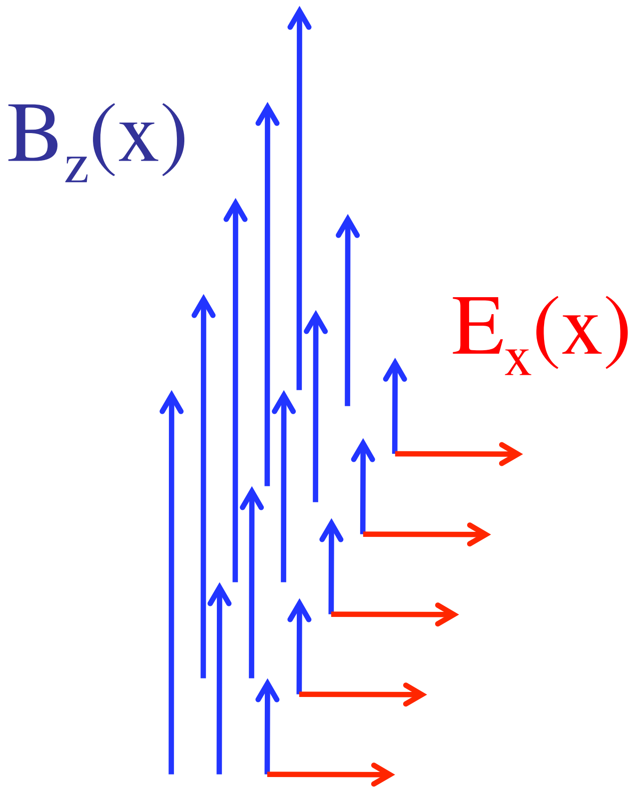


The excitations of the three electron (positron) bound states

$E + 2\omega = -0.8c^2 + 2c^2 = 1.2c^2$

$E + \omega = 0.8c^2 + c^2 = 1.8c^2$

Magnetic field influence



- J. Schwinger (1951）

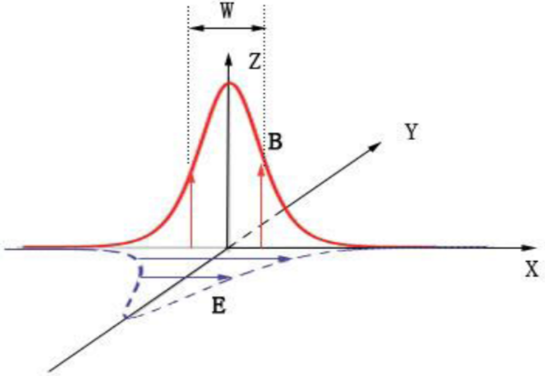
  considered pure E-field

  creation threshold:

$$\Rightarrow E_c \sim 1.32\times10^{16}\,V/cm$$

- magnetic field can not be neglected
- model from laser plasma physics
- static fields, both change along x
- uniform along y, p_y conserved

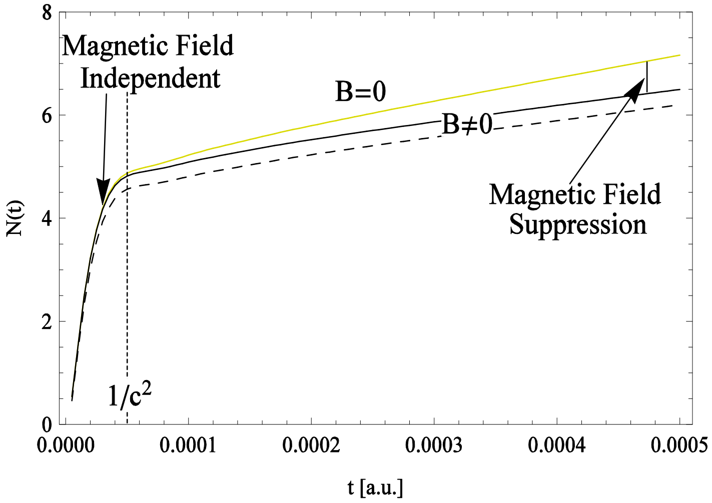
Suppression of pair creation due to a steady magnetic field



Model:

$$\mathbf{E}(x) = (E_x(x),0,0) = (V_0 / 2W_E \,\mathrm{sech}^2(x/W_E),0,0)$$

$$\mathbf{B}(x) = (0,0,B_z(x)) = (0,0,M_0 / 2W_B \,\mathrm{sech}^2(x/W_B))$$



[$W_E = W_B = 0.1/c$]

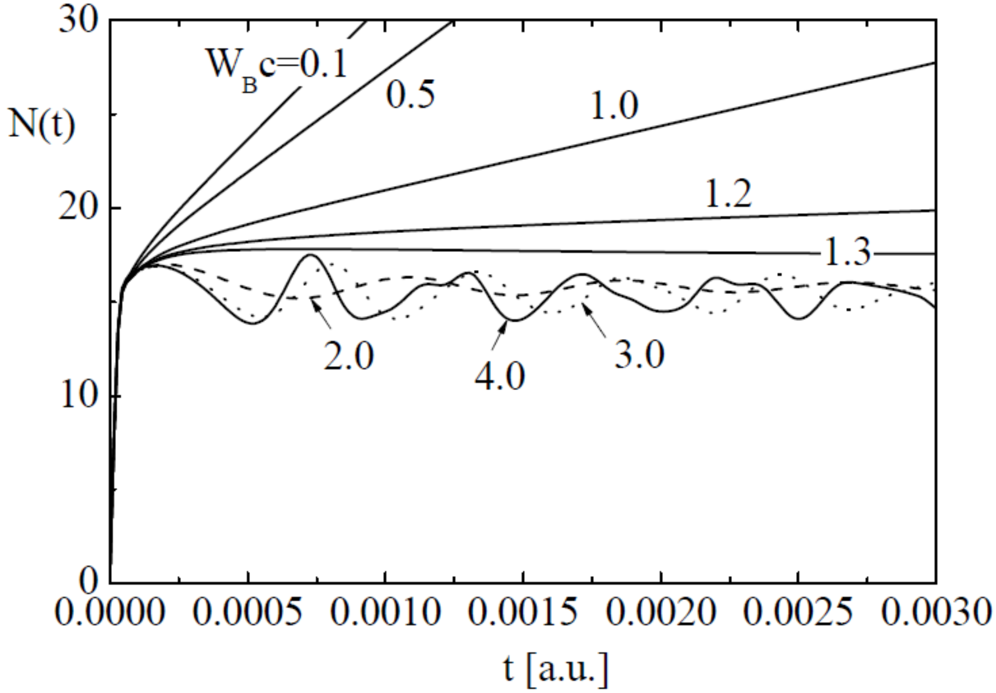
Yellow line:
$V_0 = 2.5c^2$, $M_0 = 0$

Black solid line:
$V_1 = 2.5c^2$, $M_1 = 0.6c^2$

Black dashed line:
$V_2 = \sqrt{(V_1^2 - M_1^2)}$, $M_2 = 0$

PRA 86, 013422 (2012)

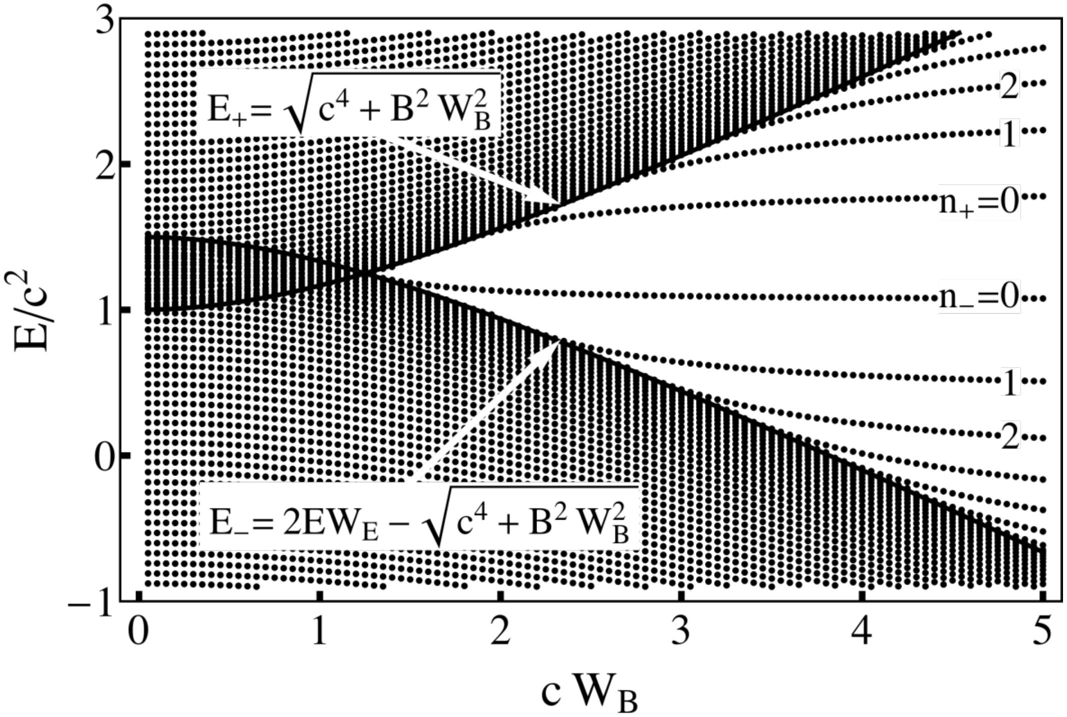
Pair creation in unequal width, while **W_B>W_E**



Time evolution of N for different W_B

Phys. Rev. Lett. 109, 253202 (2012)

Energy spectrum of the total h vs magnetic field width W_B

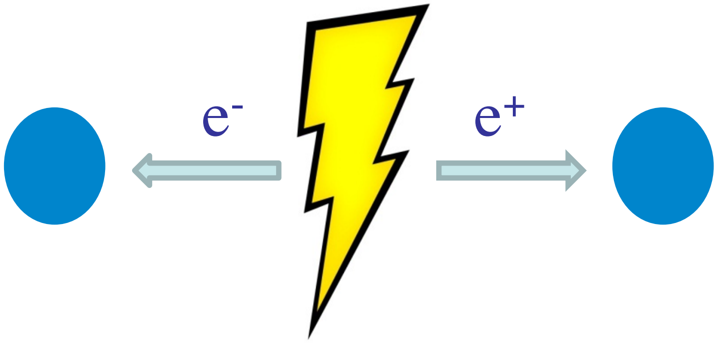


When $W_B > 1.25/c$, continua begin to separate, the system become subcritical. In the gap area new discrete energy levels emerge.

Oscillations in N(t): transition between field dressed Landau levels

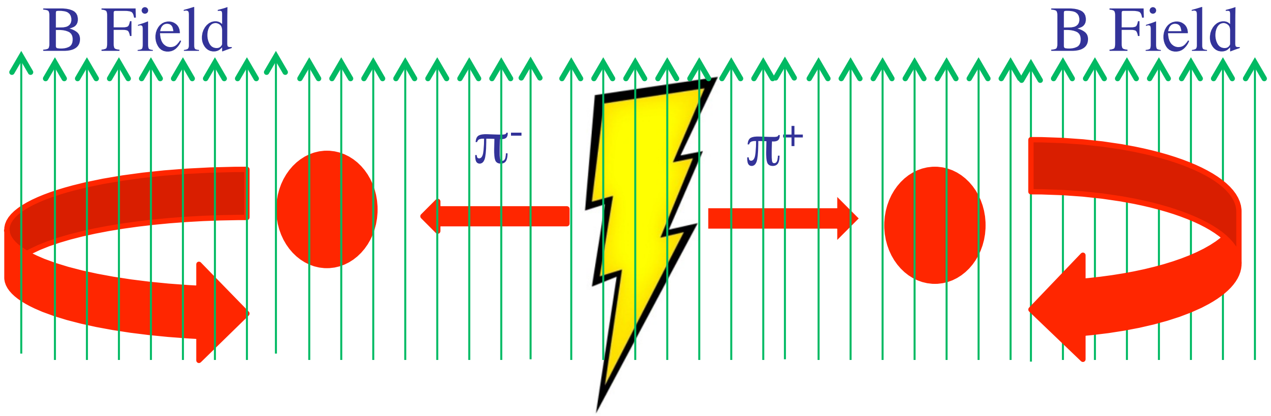
Why might boson creation work better ?

- Traditional:



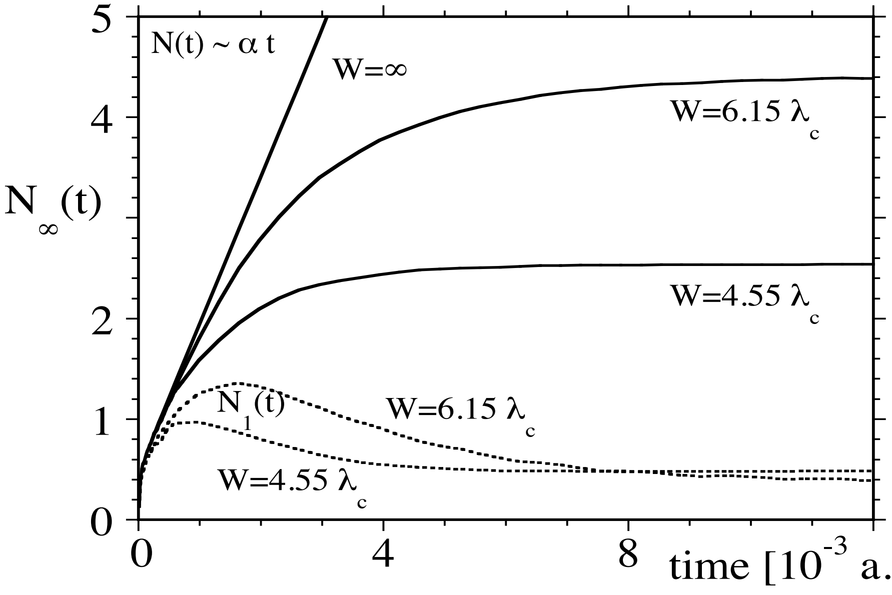
Wolfgang Pauli
(1900-1958)

- New idea: force π^-, π^+ to return to creation zone

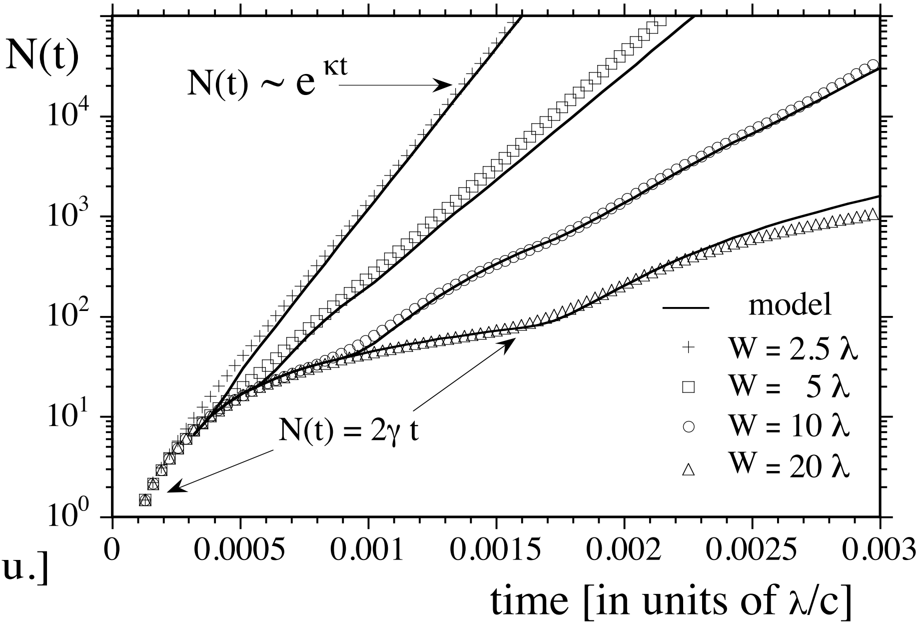


Dream: stimulated emission amplifies π^-, π^+ creation

fermions (time)



bosons (time)



Pauli exclusion principle

⇓

Pair production **stops**

Stimulated emission

⇓

Pair production **increases**

P. Krekora, K. Cooley, Q. Su and R. Grobe, Phys. Rev. Lett. 95, 070403 (2005)
R.E. Wagner, M.R. Ware, Q. Su and R. Grobe, Phys. Rev. A 81, 052104 (2010)

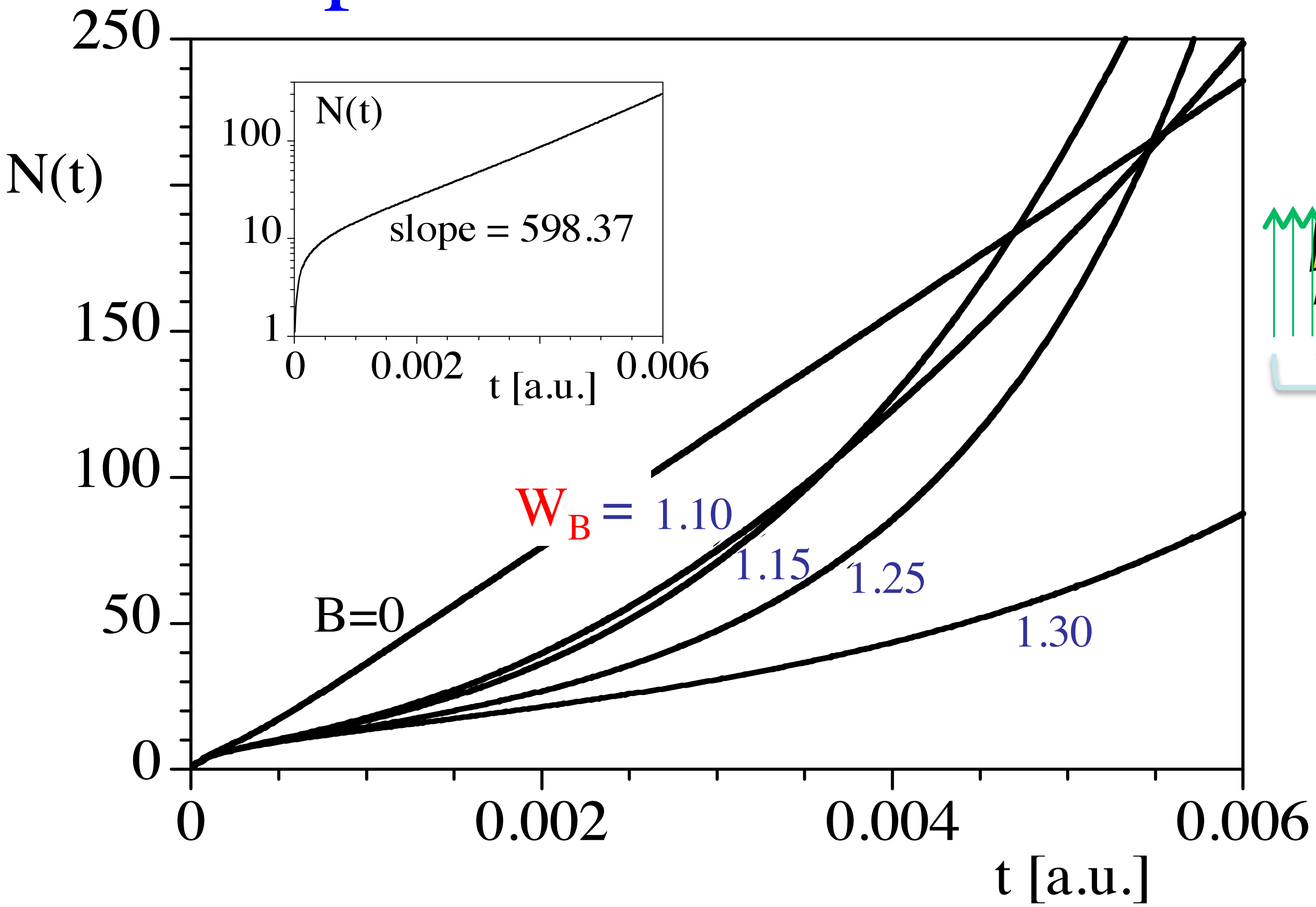
Speculation

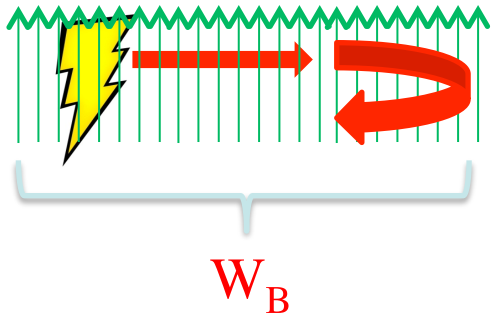
Number of pairs in a magnetic field

e^-, e^+ attenuation

π^-, π^+ amplification ?

Exponential creation confirmed

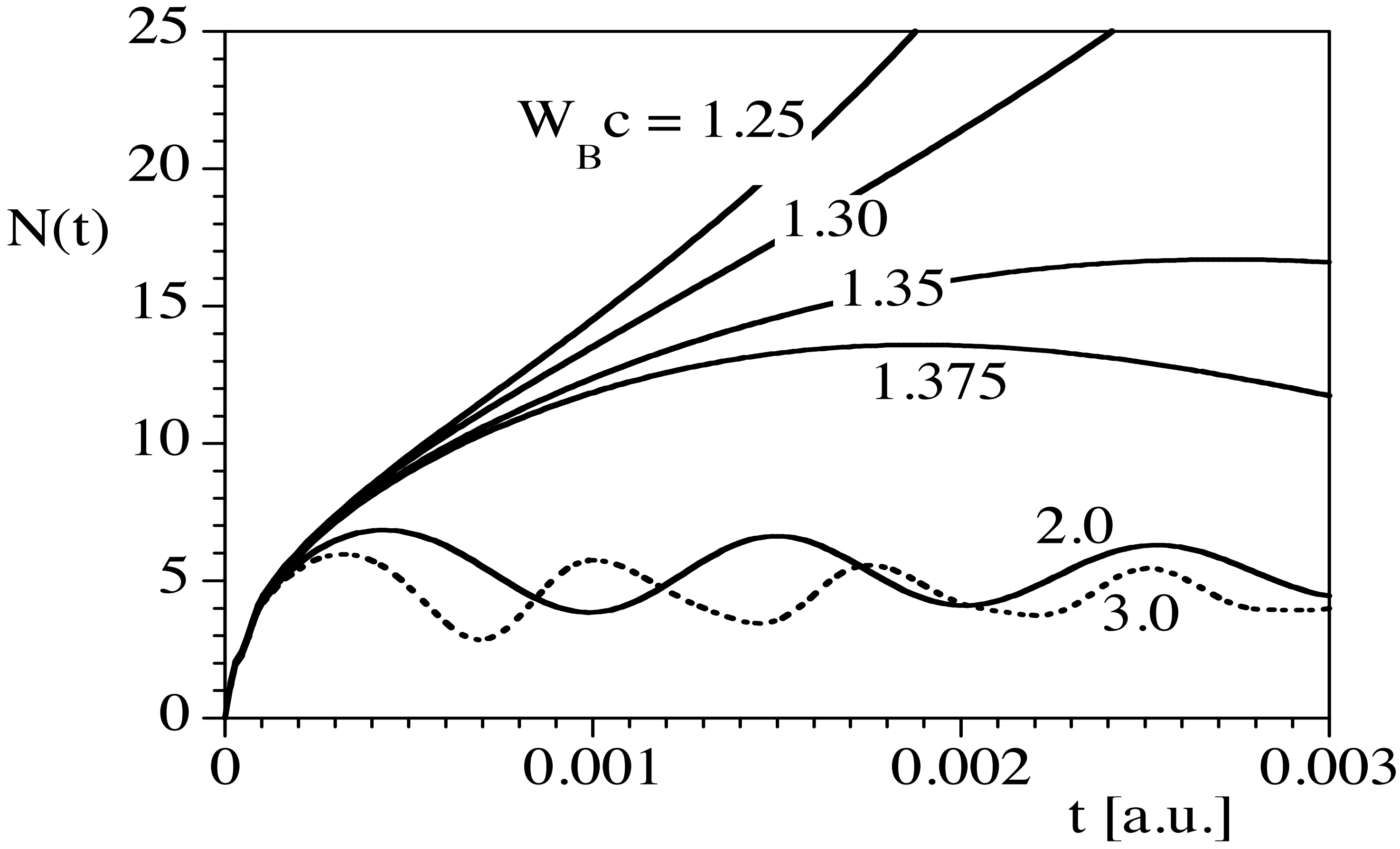




- Four different widths W_B of the magnetic field
- Inset: log scale shows exponential growth for W_B=1.25

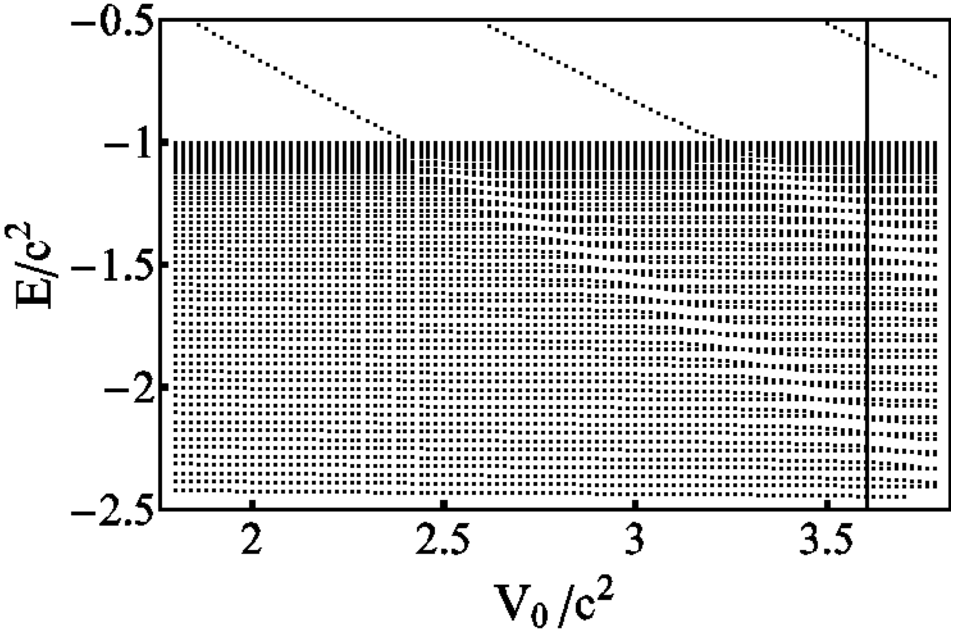
$V_0=2.5c^2$, $W_E=0.3=0.0022$ a.u., $B=0.6c^3$ and $W_B=1.25=0.0091$ a.u.

suppression for very large widths



$V_0=2.5c^2$, $W_E=0.3$, $B=0.6c^3$

Multi-channel non-competing mechanism



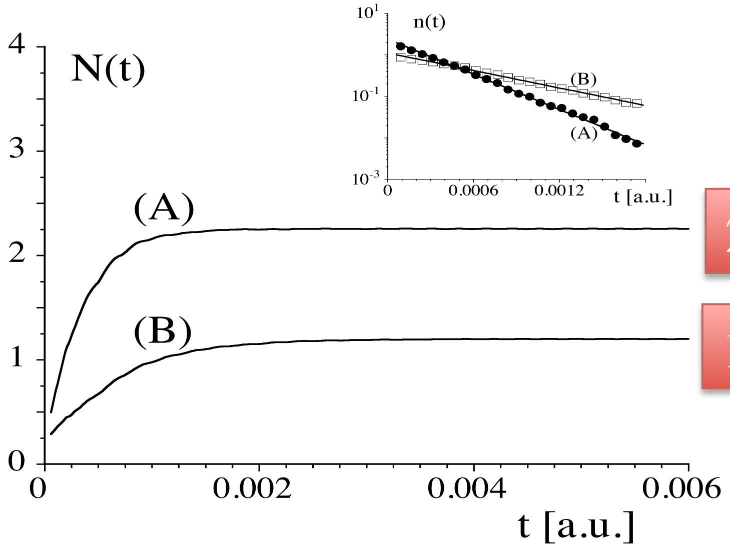
strength of interaction

ways lead to pair creation:

- Dirac +/– Sea overlap
- bound states dive in the Dirac Sea

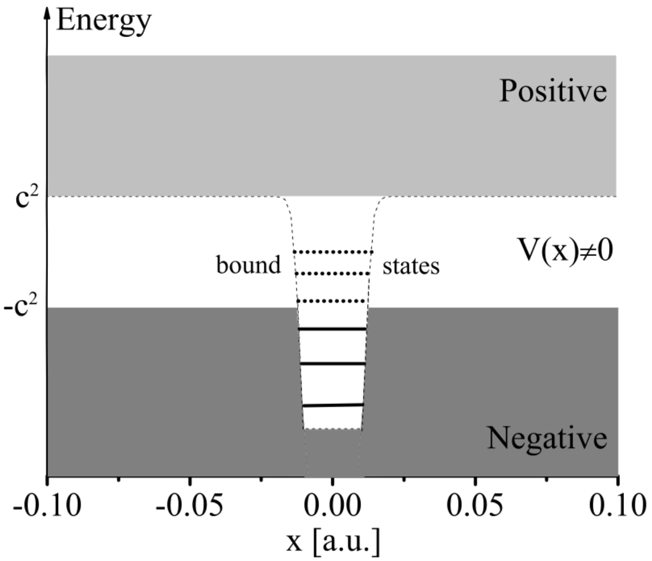
general belief：

- when multiple states dive in, dominant channel matters



2 states dive in

1 state dive in

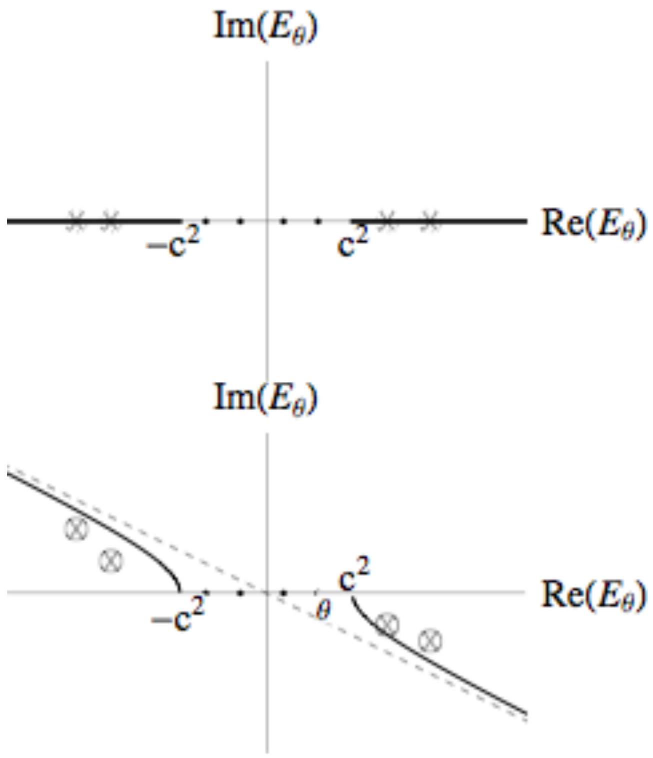


$$E = E_r - i\, E_i$$

$$N(t) = N\,[\,1 - \exp(-\Gamma t)\,]$$

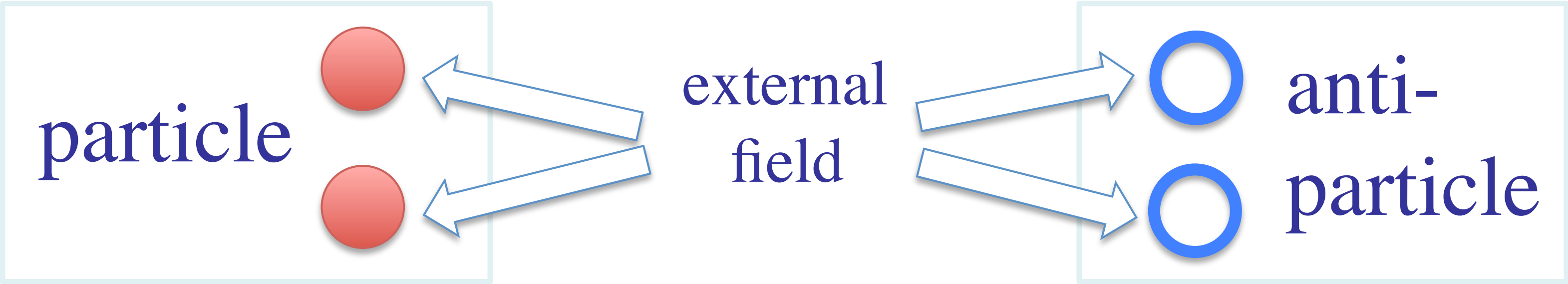
$$\Gamma = E_{i1} + E_{i2} + \ldots$$

PRL, 111, 183204 (2013)
PRA 90, 013405 (2014)



Next step: extend our model

Dirac field model


particle

external field

anti-particle

neglected：（1）inter-particle “forces”
（2）particle reaction on external field

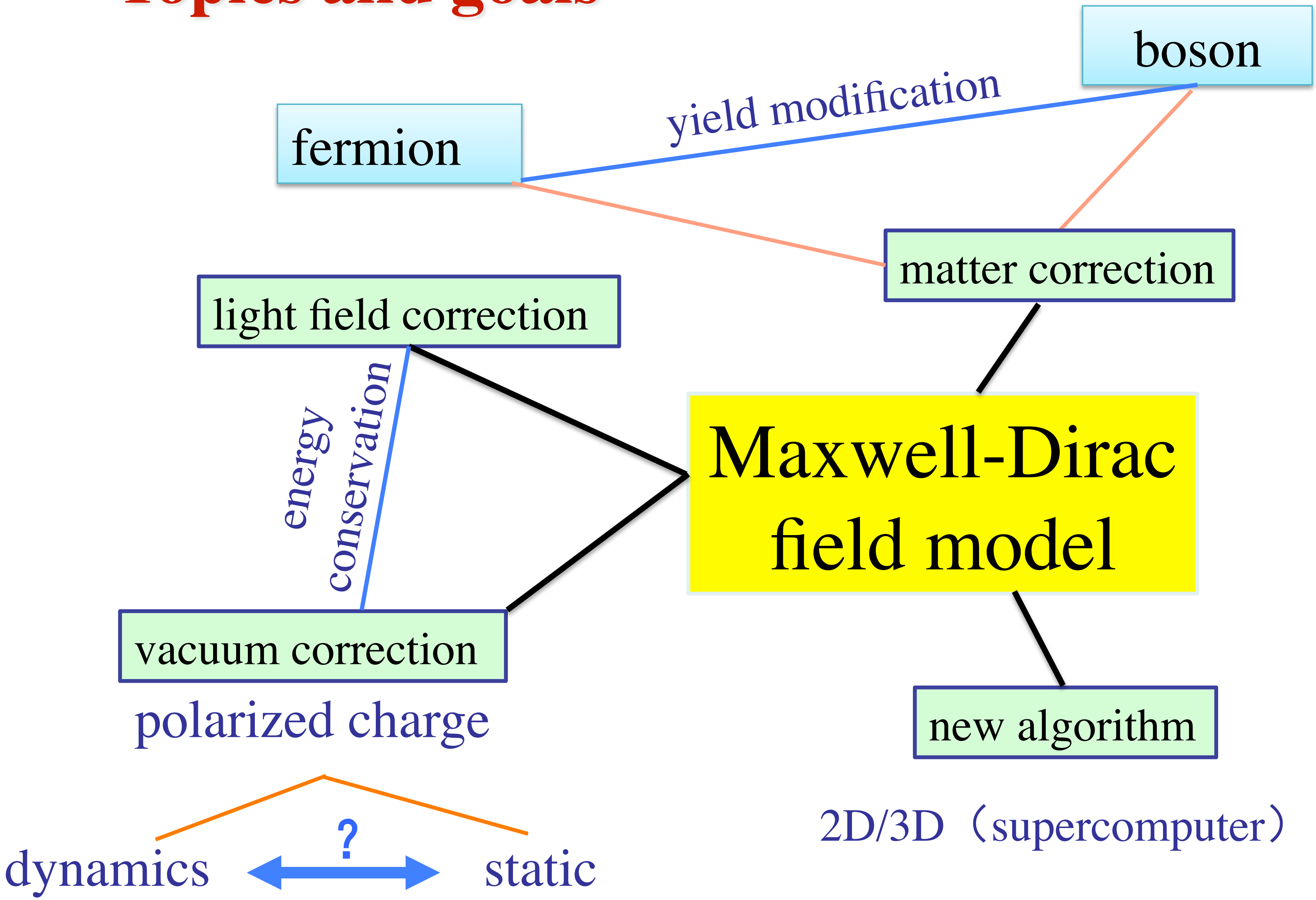
Maxwell-Dirac field model


particle

Maxwell field

anti-particle

Topics and goals



boson

fermion

yield modification

matter correction

light field correction

energy conservation

Maxwell-Dirac field model

vacuum correction

polarized charge

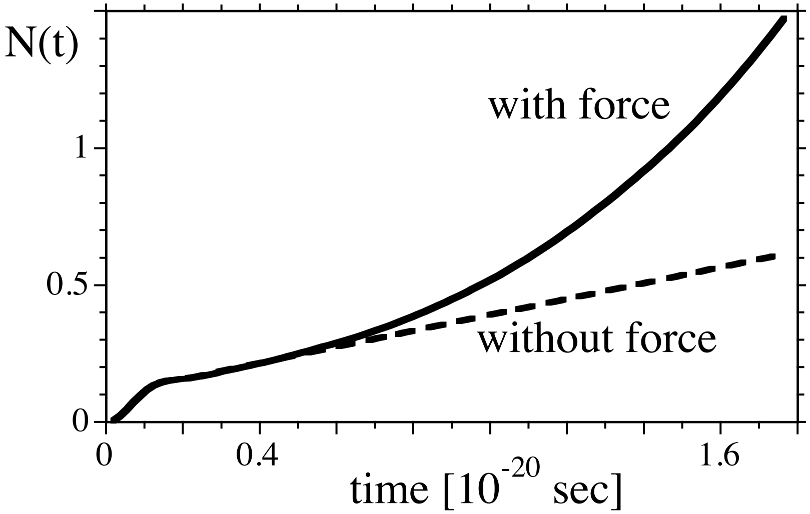
dynamics ? static

new algorithm

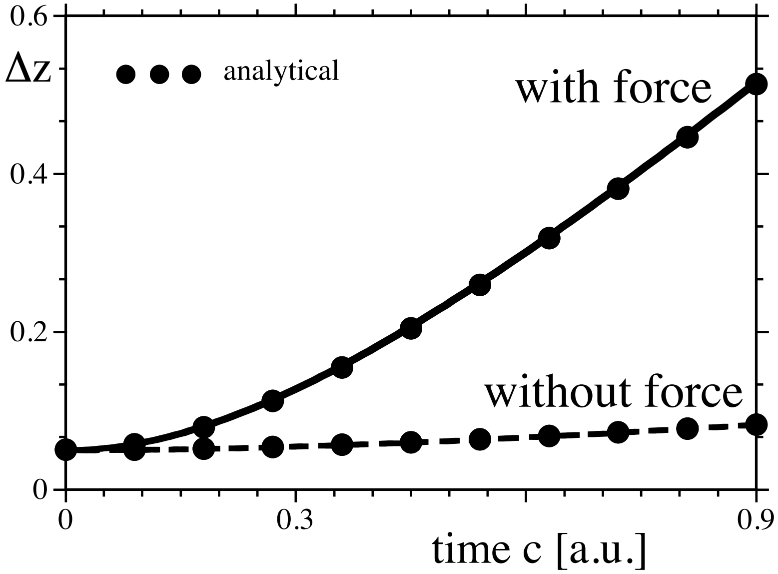
2D/3D（supercomputer）

Initial attempts

correction on pairs



correction on width



before (no force):

external field =》 Dirac field

now (yes force):

Maxwell field 《=》 Dirac field

new formulas:

Maxwell field =》 Dirac field

$$i\,\partial\psi(r,t)/\partial t = h(V,A)\,\psi(r,t)$$

Dirac field =》 Maxwell field

$$(c^{-2}\partial^2/\partial t^2 - \nabla^2)\, V(r,t) = 4\,\pi\,\varrho(\psi)$$

$$(c^{-2}\partial^2/\partial t^2 - \nabla^2)\, A(r,t) = (4\pi/c)\, j(\psi)$$

Thank you for your attention!

Thank you to the organizers!