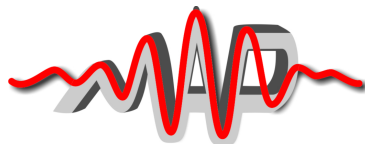


Precision Photo Electron Spectra

Discretization, Boundaries, Gauge

Armin Scrinzi
Ludwig Maximilians University, Munich

Frontiers of Intense Laser Physics
KITP Santa Barbara, August 19, 2014



Munich Advanced Photonics
Excellence Cluster



Vienna Computational Materials Science
FWF Special Research Program



Marie Curie ITN



**Vinay
Majety**
2-electron
& molecules



**Mattia
Lupetti**
Solids and
surfaces



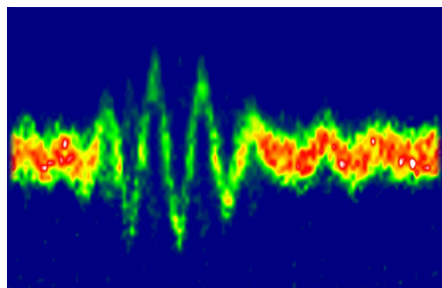
**Alejandro
“the convergator”
Zielinski**
1-e elliptic
2-electron



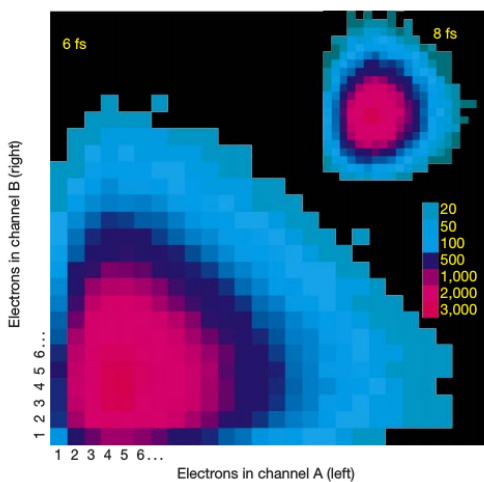
**Jakob
Liss**
Solids and
surfaces

A wealth of measured photo-electron spectra...

Few-cycle IR pulse

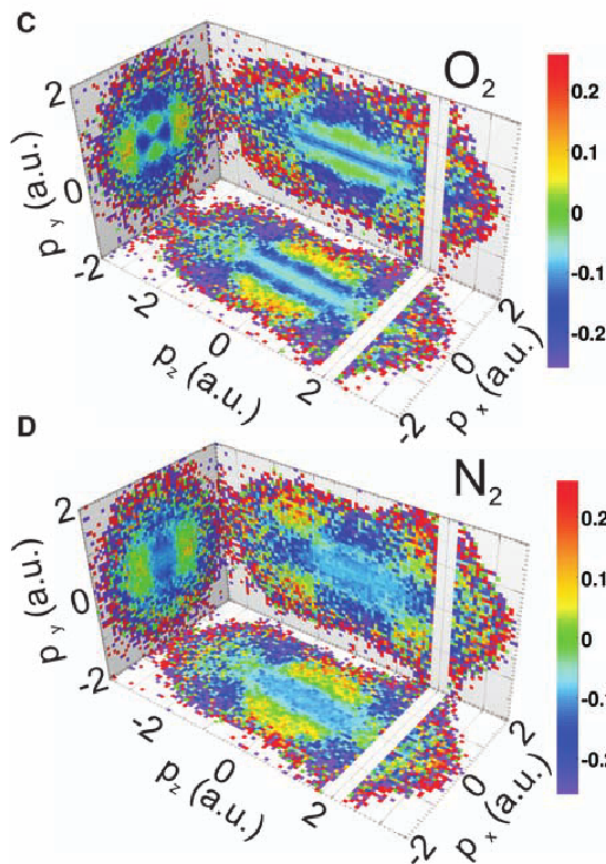


The first stereo-ATI result

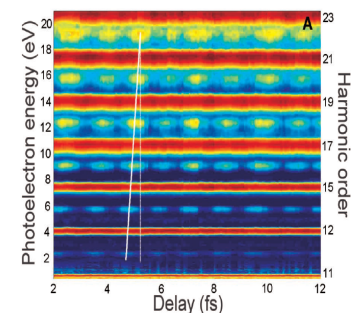


Laser-Induced Electron Tunneling and Diffraction

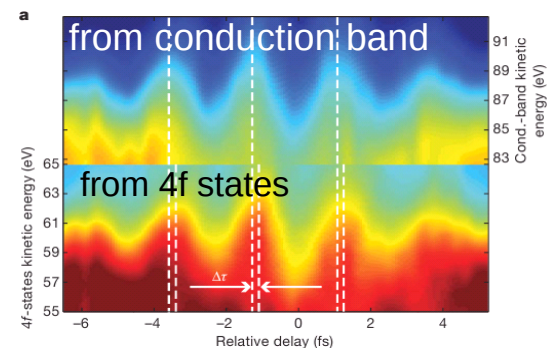
M. Meckel,^{1,2} D. Comtois,³ D. Zeidler,^{1,4} A. Staudte,^{1,2} D. Pavičić,¹ H. C. Bandulet,³ H. Pépin,³ J. C. Kieffer,³ R. Dörner,² D. M. Villeneuve,¹ P. B. Corkum^{1*}



RABITT spectrogram



Emission from surface



...but modelling and calculation are hard

Two main difficulties:

- (1) solution covers large (phase) space
- (2) complexity of few-electron calculations

Large box sizes due to ionization:

irECS perfect absorption method

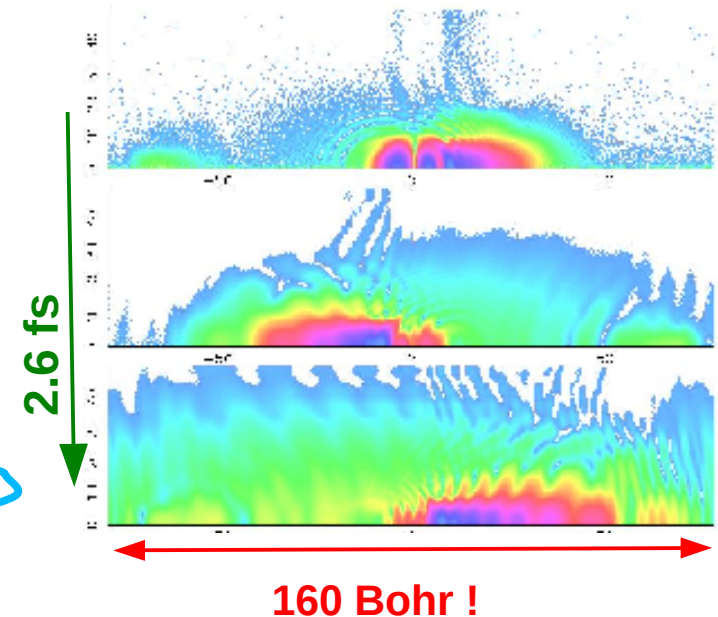
Spectra from a truncated calculation:

sTurff time dependent surface flux

Complexity of quantum chemistry wave function:

→ Integrate quantum chemistry with strong field dynamics

**Single electron density
during 2.6 fs**



Messages: Methods

- (1) Advertisement for the flexibility of **finite element** discretizations
- (2) **irECS** - The problem of **absorbing boundary** conditions for the TDSE is solved for all practical purposes
- (3) **tSURFF** – computing **photo-electron spectra** without spectral projections
- (4) **Mixing of length and velocity gauges** in the same calculations
Unites numerical efficiency of length gauge
with intuitive modeling of length gauge
It is mandatory for numerical modeling using quantum chemical structure

Messages: Results & Code

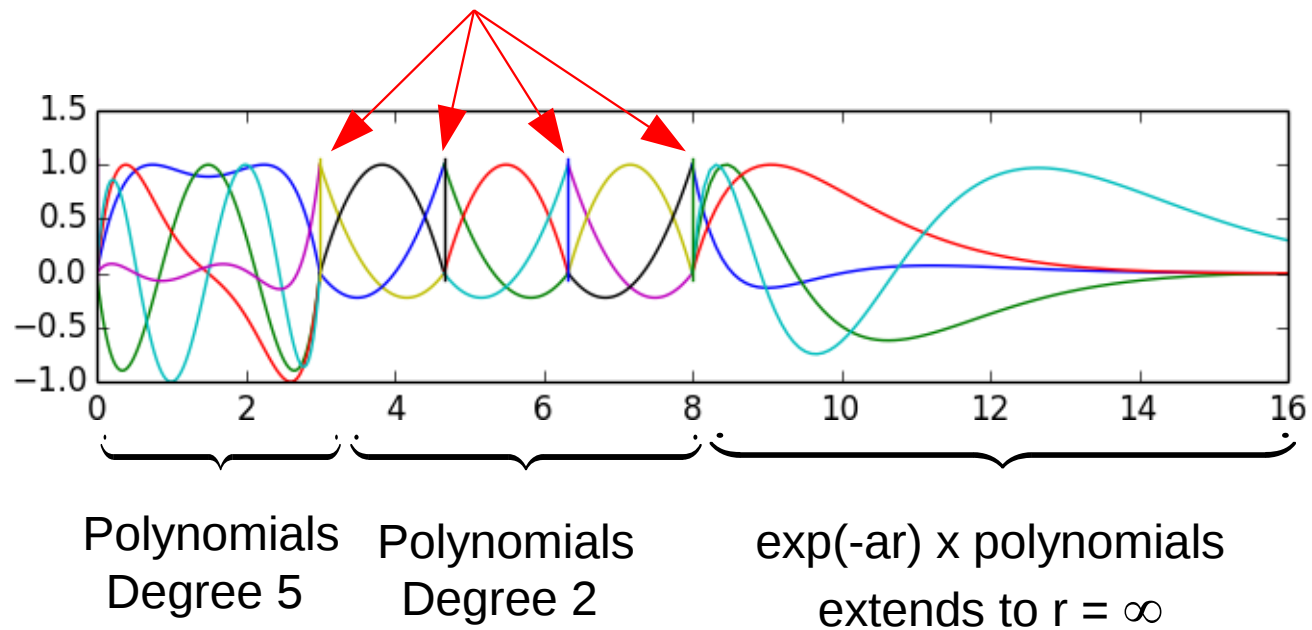
- (5) Single photo-electron spectra of He, H₂, N₂
converged @ 400 nm wave-length
- (6) Fano line-shapes, IR control: numerics + analytic theory
- (7) Double photo-electron spectra of He
converged at XUV wave length
- (8) Correlation in double emission and its measure
- (9) tRecX – the code that does it all is available for use!

on the Discretization (Finite Elements)

Finite elements

Approximation by piece-wise analytic functions (mostly polynomials)

Element boundaries
Functions continuous
Derivatives discontinuous



Flexible, can be adjusted to local properties of the solution

Discretization: why finite elements?

Required basis sizes

d degrees of freedom
phase space volume V

$$N \gtrsim V / \hbar^d$$

There are **no smart tricks** to beat this number unless we have **additional information**

Additional information

E.g. **perturbative ionization**, i.e. initial state or free motion
or: **SFA**: initial state or Volkov wave packet
or: we “know” only **bound states** play a role or ...

Basis sets

Pseudo-spectral (e.g. field-free eigenstates, momentum-space)

Build energy or momentum information into ansatz

Local basis sets (B-splines, finite-element, FEM-DVR)

Exploit **locality** of operators (differentiation, multiplication)
Numerically robust

High order finite elements

Locally adjustable (\rightarrow **irECS**)
Well-defined points of non-analyticity (element boundaries)
Rapid convergence due to high order (e.g. 10-20)

Boundary conditions (aka Absorption)

Exterior complex scaling (ECS)

General principle for perfect absorbers (PML, ECS)

[A.S., H-P. Stimming, N. Mauser, J. Comp. Phys. 269, 98 (2014)]

Outside some inner region $[0, R_0]$

analytically continue a **unitary transformation** U_λ (e.g. coordinate scaling)
to **contractive (non-unitary)** U_θ

$$i \frac{d}{dt} \Psi = H(t) \Psi \rightarrow i \frac{d}{dt} \Psi_\theta = U_\theta H(t) U_\theta^{-1} \Psi_\theta$$

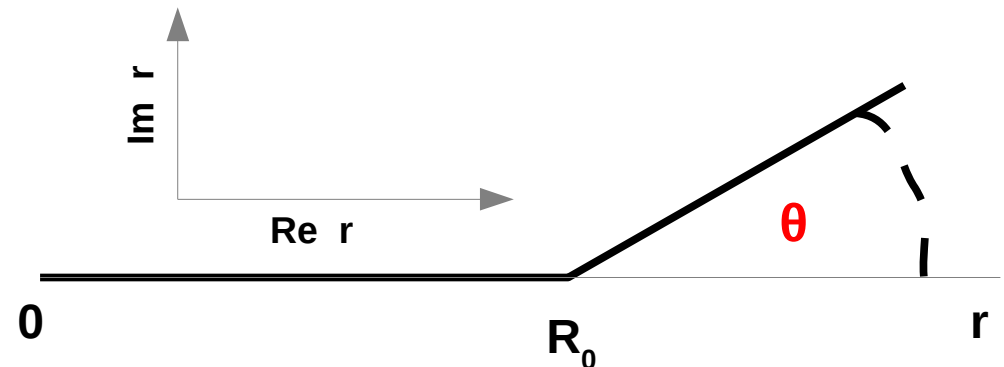
Unitarity + analyticity guarantee unchanged solution Ψ_θ on $[0, R_0]$

!!! Caution: Domain issues for $U_\theta H U_\theta^{-1}$!!!

Note: **analytic continuation** is $\lambda \rightarrow \theta$ (not in coordinate r)

Translates into:
Complex coordinates
beyond a finite distance R_0

$$\begin{aligned} r &\rightarrow r && \text{for } r < R_0 \\ r &\rightarrow R_0 + e^{i\theta} (r - R_0) && \text{for } r > R_0 \end{aligned}$$



Implementation of exterior complex scaling

Important technical complications

Bra and ket functions are not from the same set!!!

Exterior scaled Laplacian $\Delta_{R_0, \theta}$ is defined on **discontinuous** functions

$$\Psi(R_0 - 0) = e^{3i\theta/2} \Psi(R_0 + 0)$$

Discontinuity as we need **unitarity** for real transformation

Discontinuity is reversed for the left hand functions

$$\Psi^*(R_0 - 0) = (e^{-3i\theta/2} \Psi)^*(R_0 + 0)$$

Matrix elements of $\Delta_{R_0, \theta}$

are computed by piece-wise integration $[0, R_0] + [R_0, \infty)$

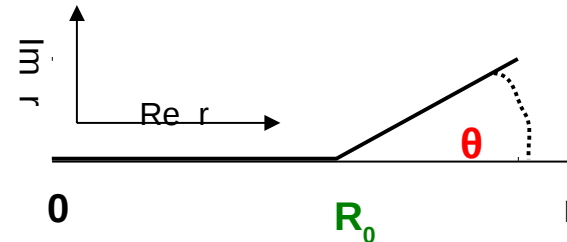
Conditions easy to implement with a local basis set

irECS – a perfect absorber

[A.S., Phys. Rev. A81, 53845 (2010)]

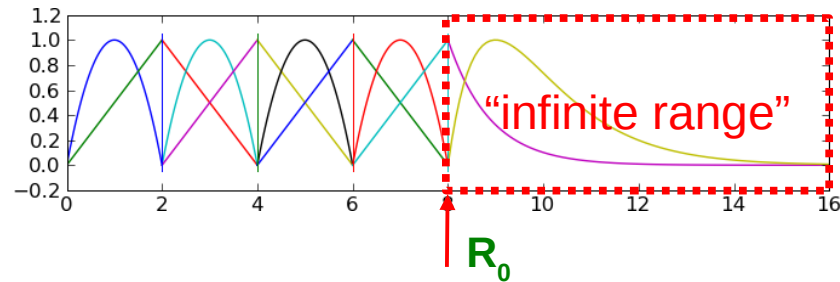
Infinite range Exterior Complex Scaling

$$\text{ECS: } \begin{aligned} r &\rightarrow r && \text{for } r < R_0 \\ r &\rightarrow R_0 + e^{i\theta} (r - R_0) && \text{for } r > R_0 \end{aligned}$$



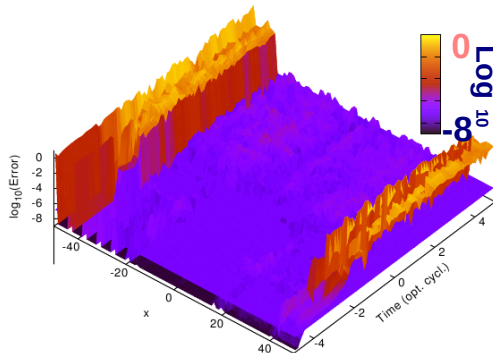
Discretization

High (~8th) order finite elements
infinite size last element $[R_0, \infty)$



Accuracy

$$|\Psi_{\theta R_0}(x, t) - \Psi(x, t)| / |\Psi(x, t)|$$



Accuracy inside $R_0 \sim 10^{-7}$

Efficiency

Method	Number of points				Accuracy
	M_A	A	θ or σ	q	$\mathcal{E}[-R_0, R_0]$
irECS	21	∞	0.6	—	2×10^{-15}
ECS	20	10	0.6	—	2×10^{-4}
ECS	40	20	0.5	—	1×10^{-7}
CAP	20	10	10^{-4}	4	3×10^{-3}
CAP	20	10	2×10^{-6}	6	4×10^{-3}
CAP	40	20	4×10^{-6}	4	3×10^{-4}
CAP	60	30	6×10^{-7}	4	1×10^{-5}

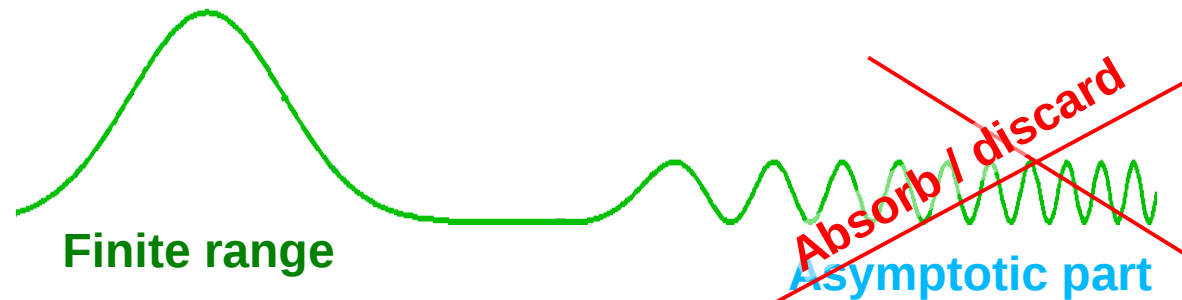
CAP = complex absorbing potential

Spectra

(tSURFF)

tSURFF – how to obtain spectra from a finite range wave function

Scattering spectra = asymptotic information by definition



If we solve only on a finite range, exactly the asymptotic information is **missing**

Solution:

Continue beyond the box using some **known solution - Volkov**

[Caillat et al., Rev. A 71 , 012712 (2005)]
[L. Tao and A.S., New. J. Phys. 14, 013021 (2012)]

How we usually calculate spectra from TDSE

Get $\Psi(r,t)$ at the end of the pulse $t=T$: $\Psi(r,T)$

← **problem 1**

Needs
very large box

Scattering solution $\psi_{\vec{k}}$

$$H(T)|\psi_{\vec{k}}\rangle = \frac{\vec{k}^2}{2}|\psi_{\vec{k}}\rangle$$

← **problem 2**

Time-independent
scattering

With asymptotics

$$\psi_{\vec{k}}(\vec{r}) \sim (2\pi)^{-3/2} \exp(i\vec{k} \cdot \vec{r})$$

Spectrally analyze $\Psi(x,t)$

$$b(\vec{k}) = \langle \psi_{\vec{k}} | \Psi(T) \rangle$$

Spectral density

$$\sigma(\vec{k}) \propto |b^2(\vec{k})|$$

Solve by using additional information

(1) TDSE is a 2nd order PDE

Value and derivative at a surface $r = R_c$ suffice to continue the solution beyond the surface

(2) Beyond distances $R_c \sim 50$ a.u. motion is ~ free

Use Volkov solution for free motion in the field instead of numerically solving

Compare R-matrix theory!

How things are done...

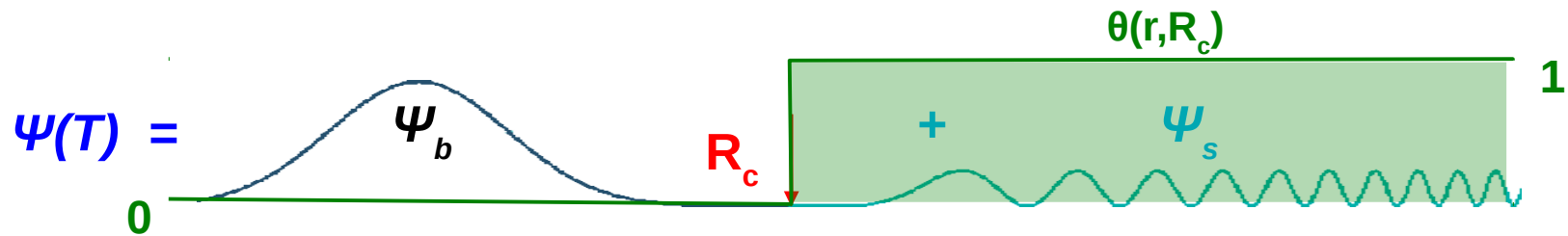
- for a given pulse, solve with irECS absorption (box size ~ 50 a.u., laser-dependent)
- save values and derivatives at surface(s) as function of time
- properly time-integrate surface values for asymptotic momenta p of your choice (one integration for each p , ordinary integrals, very cheap!)
- can zoom in onto areas of interest (important for 2-electron problems)
- Effort grows only linearly with pulse duration T (cf. $T^2 \sim T^4$ if time and box-size grow)

tSURFF – time-dependent surface flux method

[L. Tao and A.S., New. J. Phys. 14, 013021 (2012)]

Propagate until large T where bound Ψ_b and scattering Ψ_s parts separate

Beyond distance R_c scattering solutions χ_k are known



Spectral amplitude $\sigma(k)$:

with Volkov solutions χ_k $\sigma(\vec{k}) \propto |\langle \chi_{\vec{k}} | \theta(R_c) | \Psi(T) \rangle|^2$

Volume integral \rightarrow Time-integral & surface integral

$$\langle \chi_k(T) | \theta(R_c) | \Psi_s(T) \rangle = i \int_0^T dt \chi_k(t) \left[-\frac{1}{2} \Delta + i \vec{A}(t) \cdot \vec{\nabla}, \theta(R_c) \right] | \Psi_s(t) \rangle$$

Commutator depends only on $\Psi(R_c, t)$ and $\partial \Psi(R_c, t)$

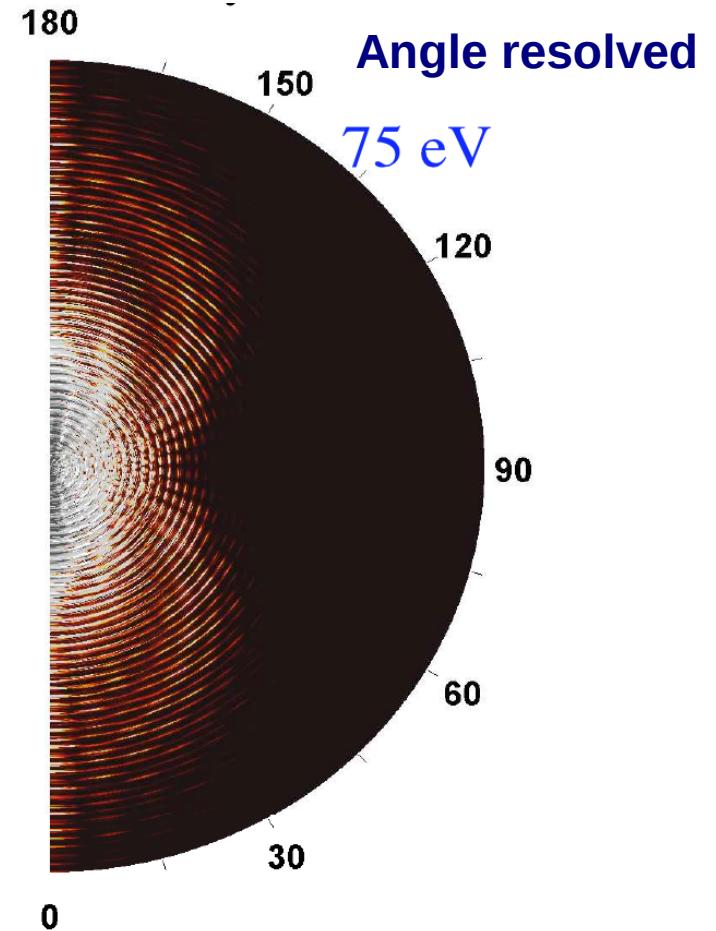
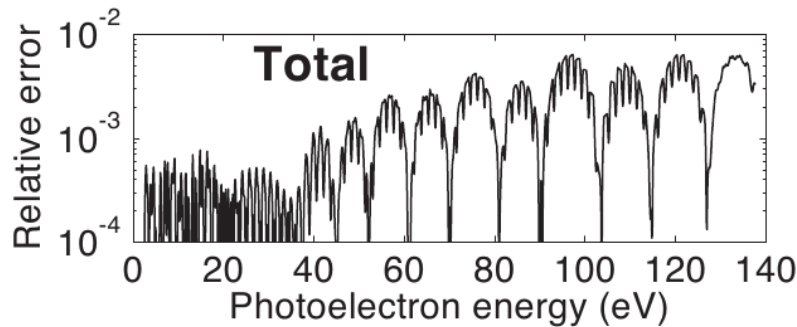
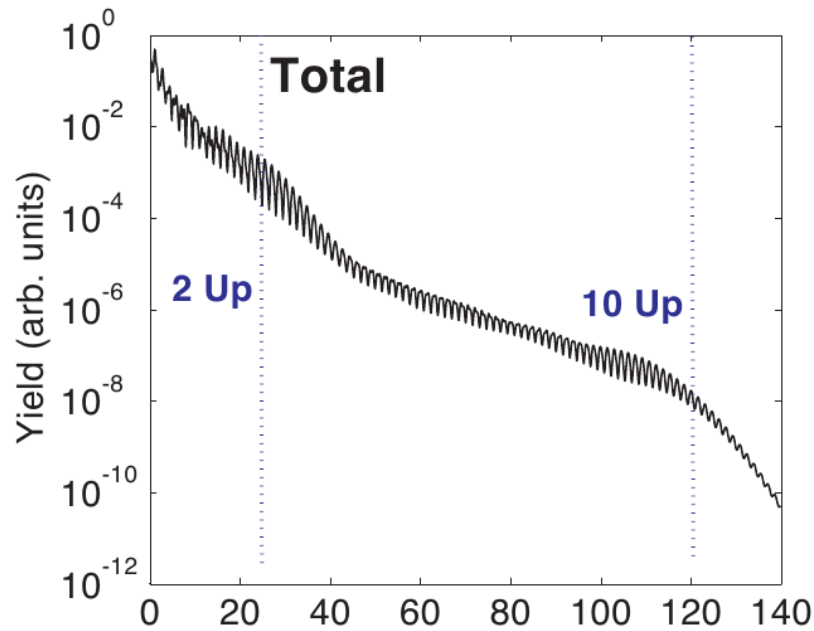
Note: need time-dependent bra-solutions \approx Volkov (or better, if available)

Single photo-electron spectra

Photo-electron spectra – single electron, 3d

[L. Tao and A.S., New. J. Phys. 14, 013021 (2012)]

Hydrogen atom, Laser: 2×10^{14} W/cm² @ 800 nm, 20 opt.cyc. FWHM
Linear polarization



90 radial discretization points, 30 angular momenta

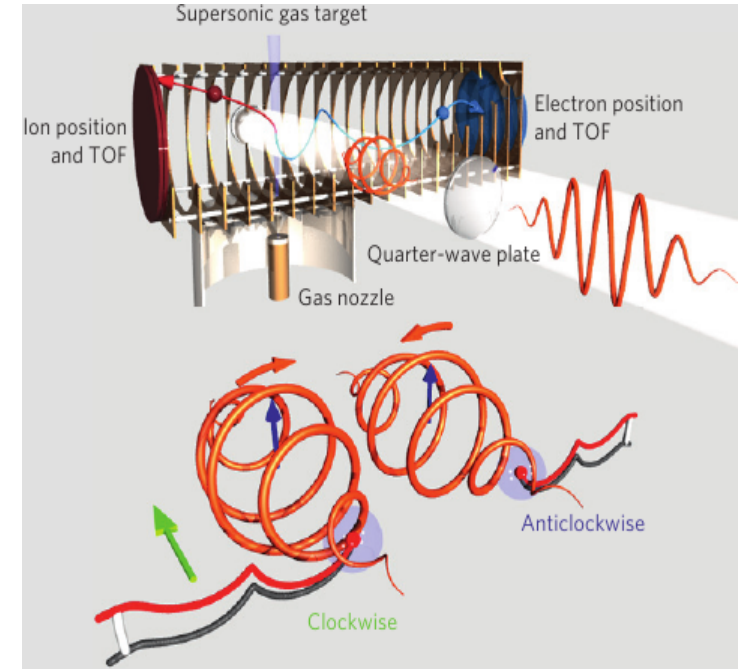
Attoclock – ionization by elliptically polarized IR

[Pfeiffer et al, Nat. Phys. 8, 76 (2012)]

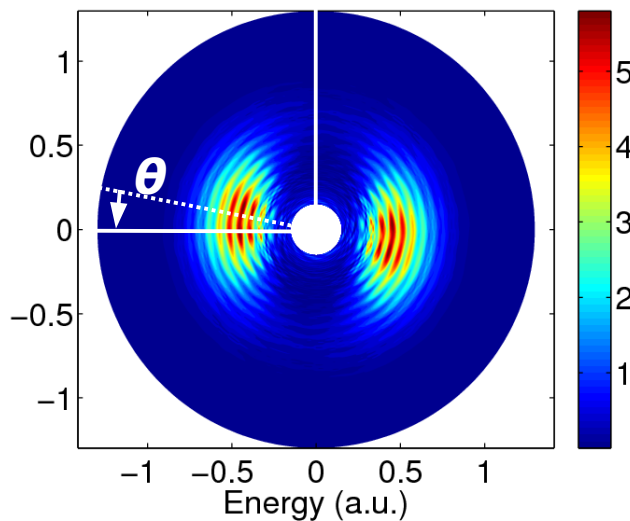
Angle-resolved photo-electron spectra

Peak emission direction
deviates from
Peak field direction

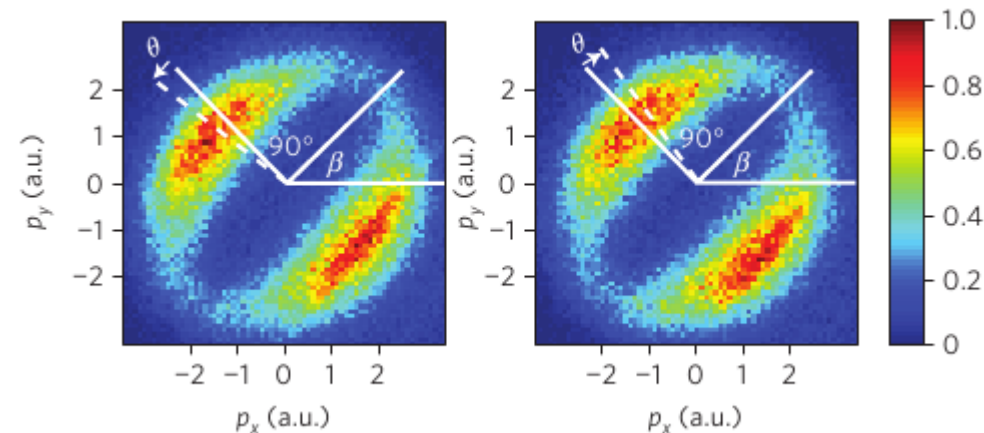
=> deduce delay in release of electron



Solution of the TDSE



Use oppositely handed polarizations
to calibrate peak field direction



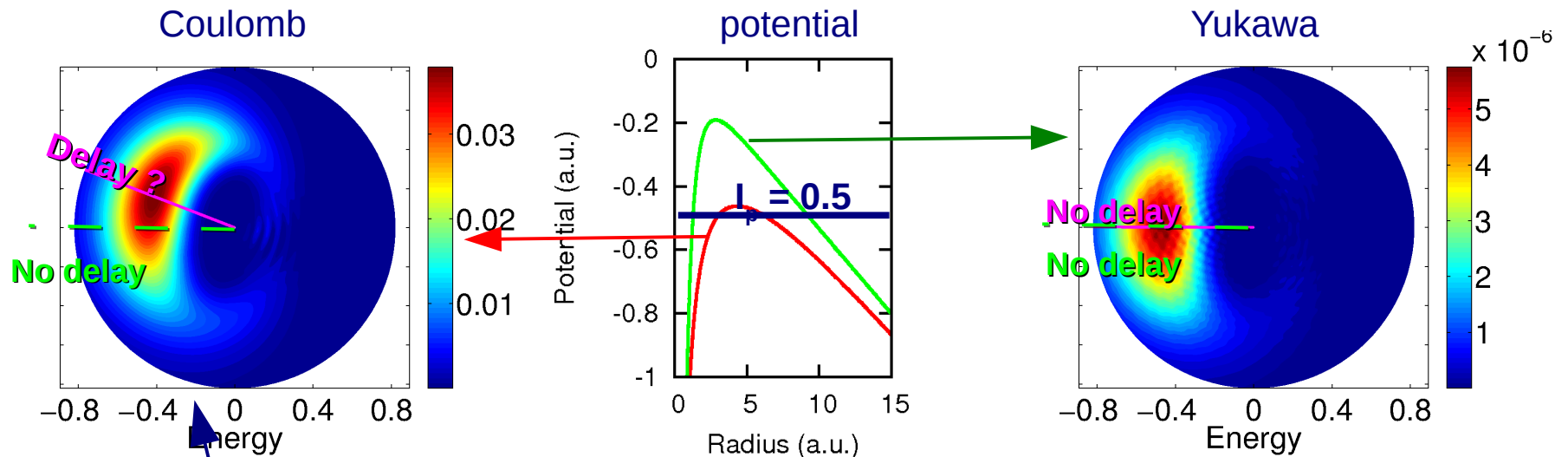
Tunneling times (?) in IR ionization

Is the offset angle θ related to a “tunneling time”?

If the time-delay is related to tunneling, expect wider barrier longer tunneling delay?

Numerical result

Laser: 800nm, single cycle, $10^{14}\text{W}/\text{cm}^2$, ellipticity as in Pfeiffer et al.
Ionization potential: 0.5 a.u. (Hydrogen)



No evidence for “tunneling time” in this setting

(Reasons for the observed delay in Coulomb – long range correction)

Comparison theory and experiment

Angles of peak photo-emission

Helium, elliptically polarized pulse

Measurements:

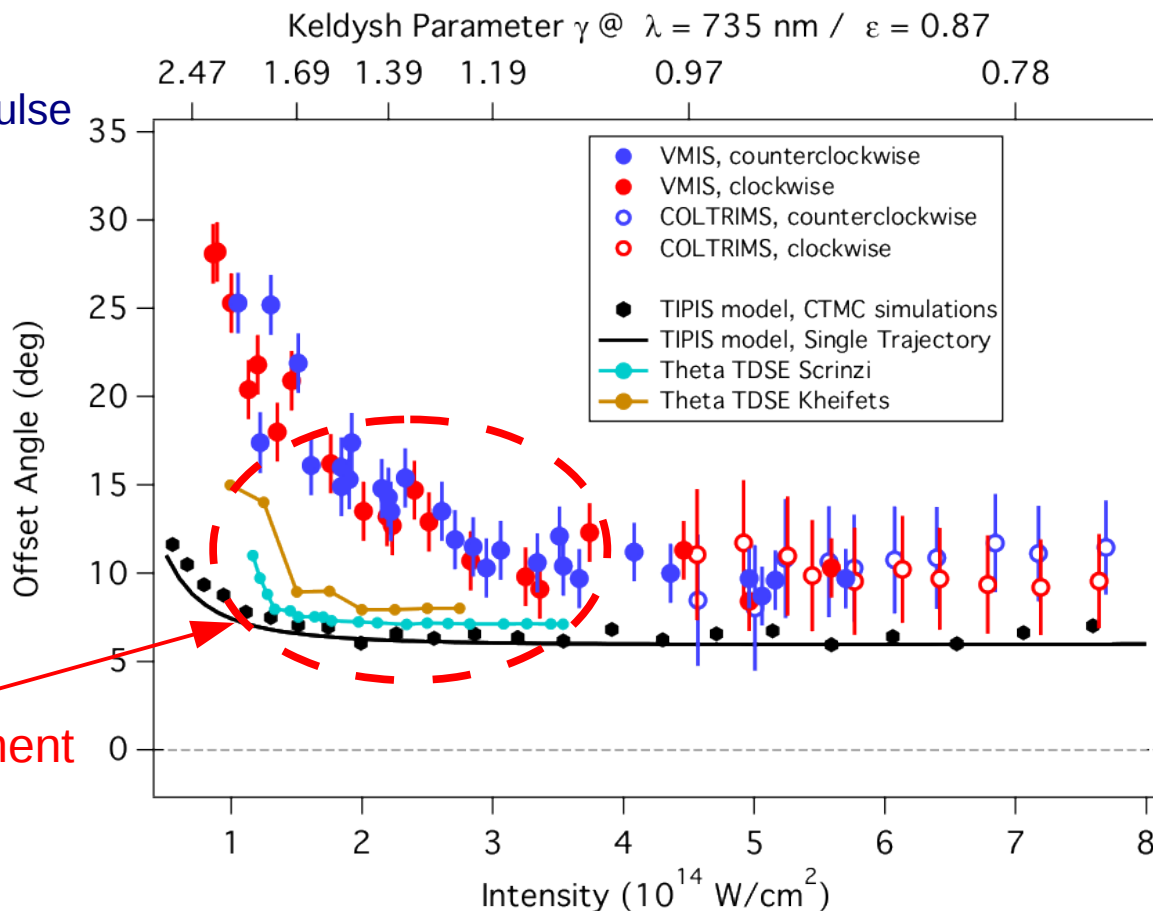
C. Cirelli et al. ETH

Calculations:

L. Madsen (TIPIS)

Kheifets/Ivanov

A. Zielinski/A.S.



Very disquieting disagreement

Note: calculations are all single-electron...

Multi-electron effects?

Two-electron systems

- ✓ Extension of tSURFF to multi-channel emission
- ✓ Extension to double-emission
- ✓ Technical remarks
- ✓ Fano-resonances, correlation in double emission

tSURFF for 2-electron systems

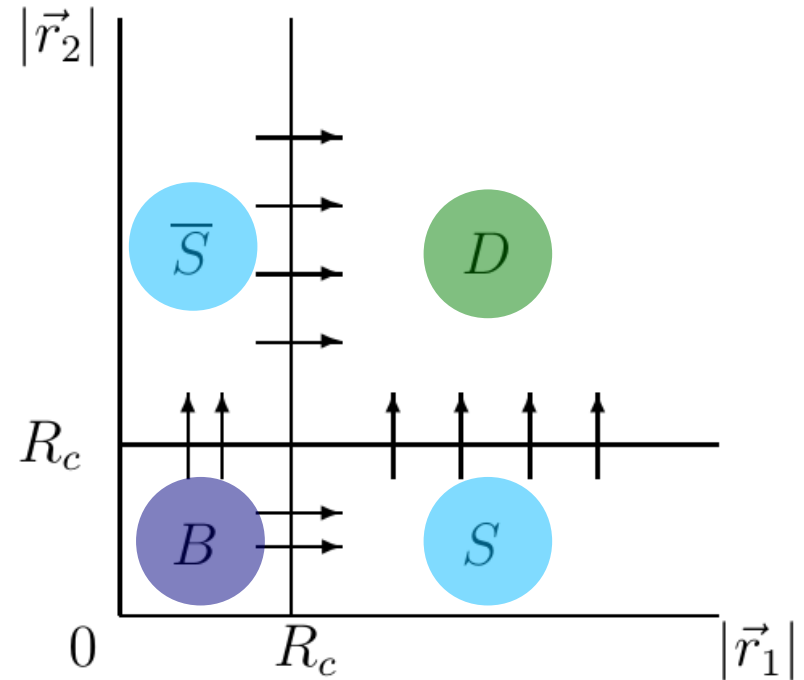
[A. S., New. J. Phys., 14, 085008 (2012)]

Split two-electron coordinate space

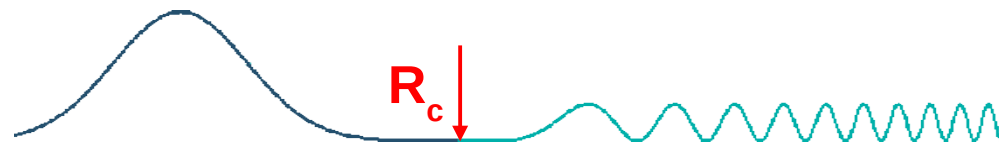
B... $|\mathbf{r}_1|, |\mathbf{r}_2| < R_c$ “bound” region
Numerical solutions on \mathbf{r}_1 and \mathbf{r}_2

S... $|\mathbf{r}_2| < R_c, |\mathbf{r}_1| > R_c$ “singly asymptotic” region
Numerical ionic solution on \mathbf{r}_2 : $\Phi_c(\mathbf{r}_2, t)$
Volkov solution on \mathbf{r}_1

D... $|\mathbf{r}_1|, |\mathbf{r}_2| > R_c$ “doubly asymptotic” region
Volkov solutions on \mathbf{r}_1 and \mathbf{r}_2



R_c as before:

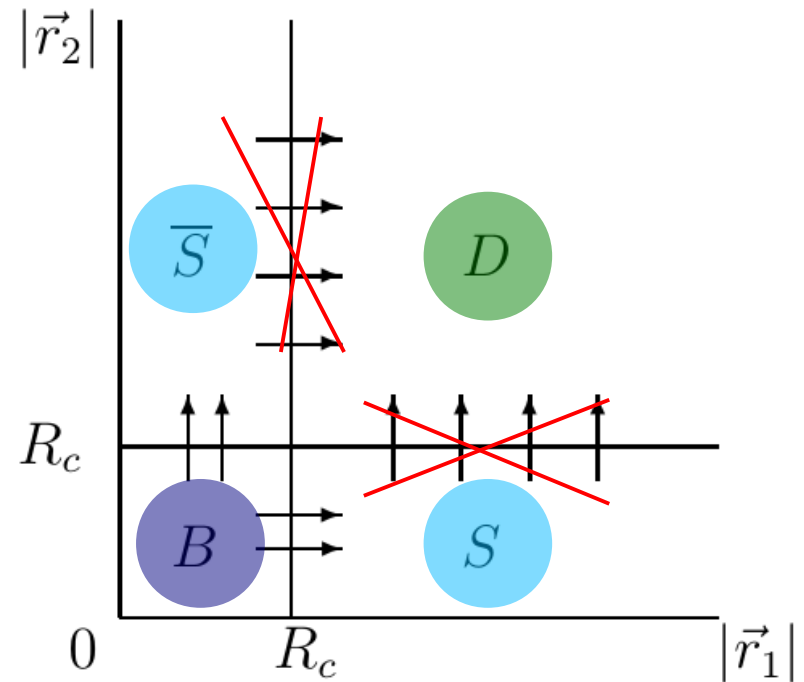


Multi-channel single emission

If one can neglect double ionization

Computational tasks for ionic channels reduces to:

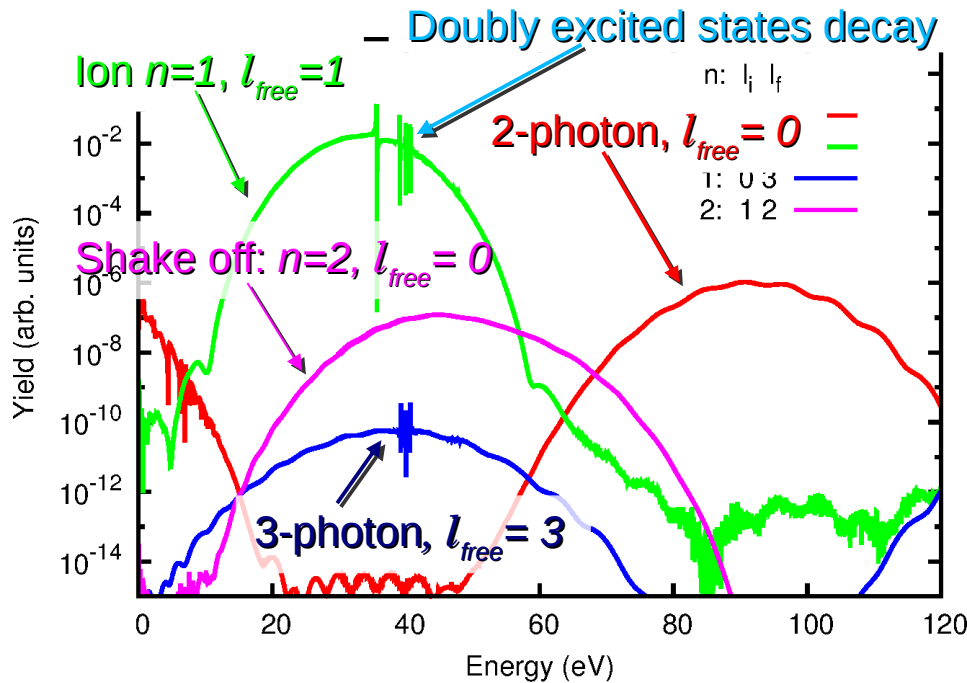
- solve full 2-electron problem on B
- for each single ionization channel, solve a single ionic problem in $[0, R_c]$



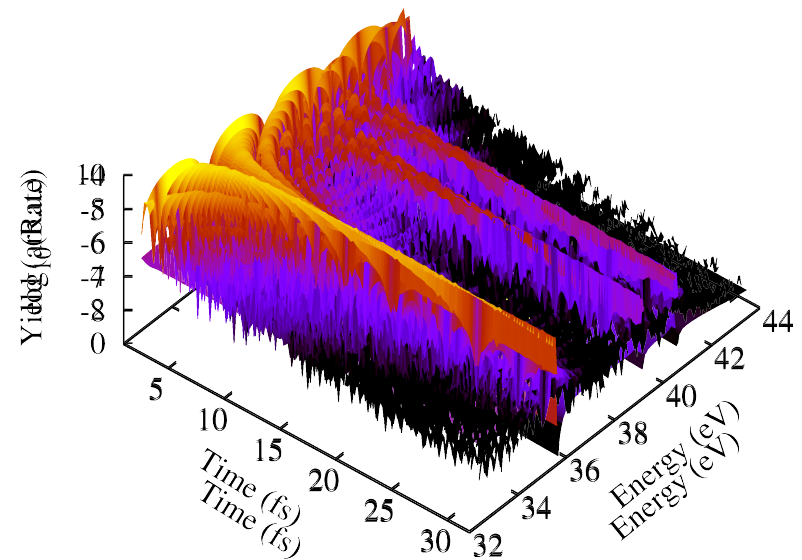
3d He: shake up photo-electron spectra @ XUV

Ionic channels & partial waves

(Laser: 2 opt.cyc. FWHM @ $h\nu=54\text{eV}$, perturbative intensity regime)



Buildup rate and decay times



Note:

Doubly excited content after the end of the pulse can also be obtained by projection / window operator

Control of a Helium Fano line shape by IR

[A. Zielinski et al., arXiv:1405.4279]

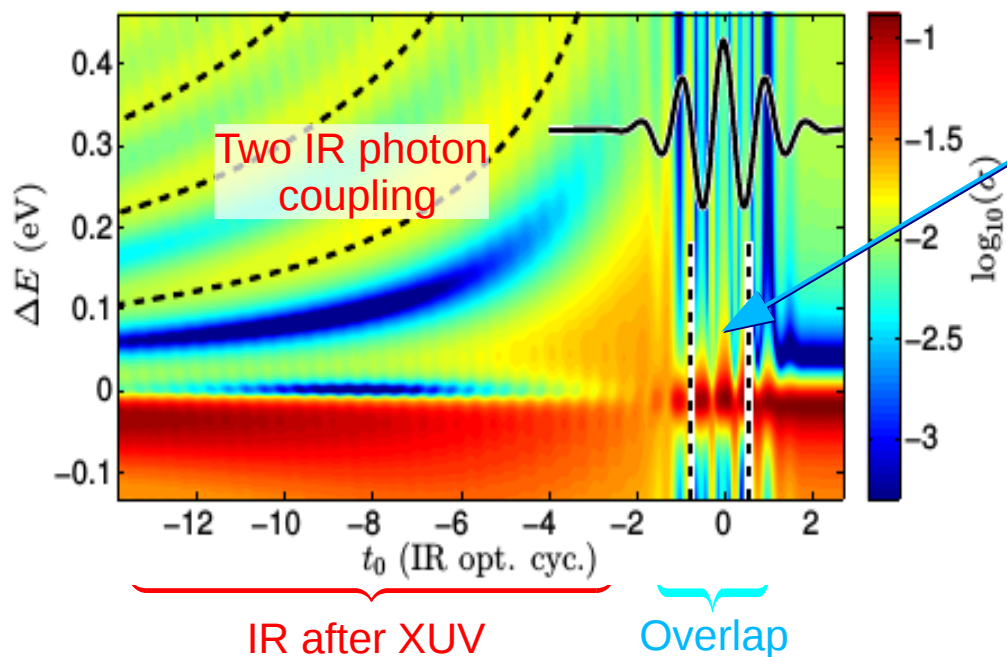
Pulse parameters

XUV @ 21 nm, 150 as duration

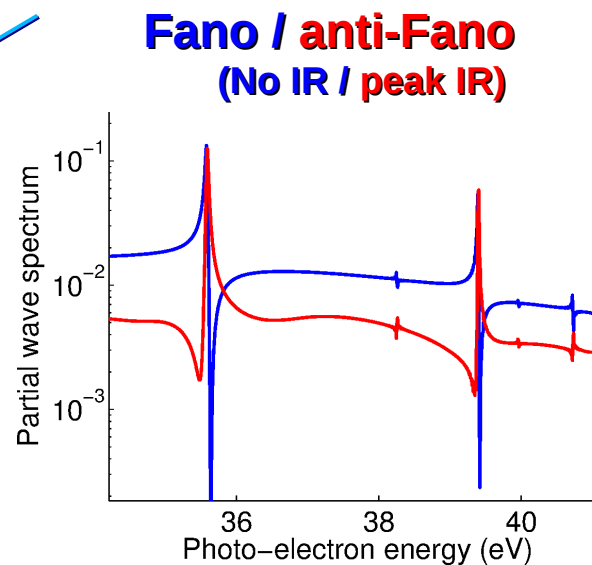
Time-delayed 800 nm few-cycle IR probe

Intensity 2×10^{12} W/cm²

Photo-electron spectrum



Line-shape modulations



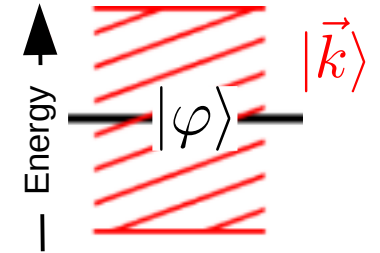
Compare experiment by Ott et al. Science 340, 716 (2013)

Modified Fano theory

[A. Zielinski et al., arXiv:1405.4279]

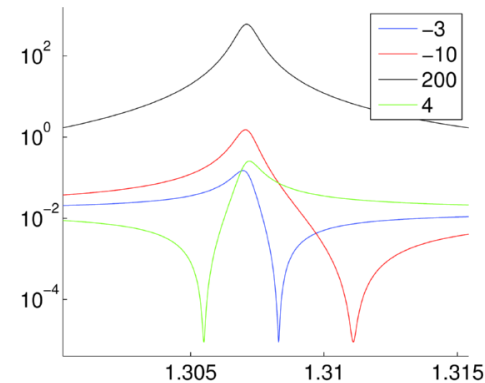
Standard Fano Hamiltonian – resonant state $|\varphi\rangle$ embedded in **continuum** $|\vec{k}\rangle$

$$H = \underbrace{|\varphi\rangle E_\varphi \langle\varphi| + \int d^3k |\vec{k}\rangle \frac{\vec{k}^2}{2} \langle\vec{k}|}_{H_0: \text{interaction-free eigenstates}} + \underbrace{\int d^3k \left(|\vec{k}\rangle V(\vec{k}) \langle\varphi| + |\varphi\rangle V(\vec{k})^* \langle\vec{k}| \right)}_{V: \text{coupling}}$$



Lineshape:

$$\sigma := \frac{|\mathcal{E} + q|^2}{\mathcal{E}^2 + 1}$$



Modification during IR

Neglect coupling: $V = 0$

Effects of IR:

$$H_0 \rightarrow H_{IR}(t) = |\varphi\rangle \underbrace{E_\varphi(t)}_{\text{Stark shift}} \langle\varphi| + \underbrace{\int d^3k |\vec{k} - \vec{A}_{IR}(t)\rangle \frac{(\vec{k} - \vec{A}_{IR}(t))^2}{2} \langle\vec{k} - \vec{A}_{IR}(t)|}_{\text{free motion in IR / Volkov}}$$

Modified Fano parameter $q \rightarrow q_{IR}$

Complex Fano q-parameter

[A. Zielinski et al., arXiv:1405.4279]

Modification of Fano q-parameter

$$q_{\text{IR}} = q + \underbrace{\alpha}_{\text{coupling}} \left[e^{-i \int_{t_0}^{t_1} dt \left(E_{\varphi}(t) - E_{\varphi}^{(0)} \right) + \frac{i}{2} \int A_{\text{IR}}^2} \underbrace{\left(\mathcal{J}(A_{\text{IR}}, k) - 1 \right)}_{\text{IR streaking}} \right]$$

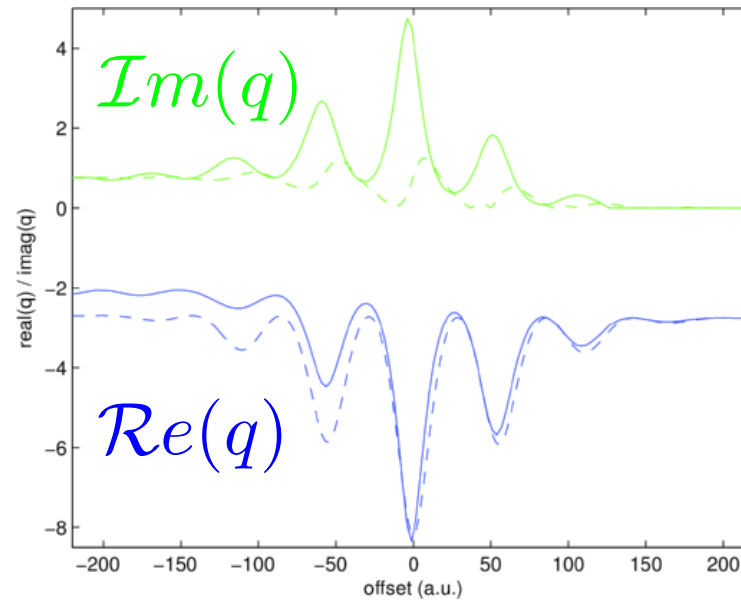
[Click for details...](#)

Model vs. TDSE

Solid: fit to full 2-electron TDSE
Dashed: q_{IR}

Need **complex** q_{IR} !

(no time-reversal symmetry)



Double-ionization

t-SURFF for two-electron systems: double ionization

[A. S., New. J. Phys., 14, 085008 (2012)]

Spectrum in D – integrate flux $S \rightarrow D$

S-D-surface values/derivatives

Need solution in region S

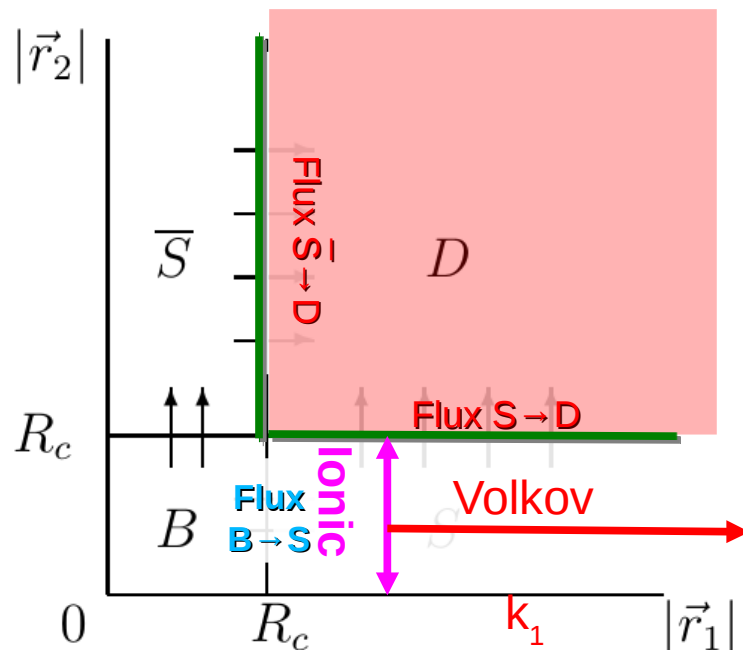
Dynamics is entangled:

Independently for each Volkov k_1 ,

solve one ionic problem in $[0, R_c]$

(perfectly parallelizable)

Similar for flux $\bar{S} \rightarrow D$



Equations on S

$b(k_1, n, t)$... coefficients for ionic basis $|\xi_n\rangle$ in $[0, R_c]$

$$i \frac{d}{dt} b(\vec{k}_1, n, t) = \sum_m \langle \xi_n | H_{ion}(t) | \xi_m \rangle b(\vec{k}_1, m, t) \dots \text{ionic time-evolution}$$

$$- \langle \vec{k}_1, t | [H_v(t), \theta_1] \langle \xi_n | \Psi(t) \rangle \rangle \dots \text{flux } B \rightarrow S$$

Double-ionization of Helium at XUV wave length

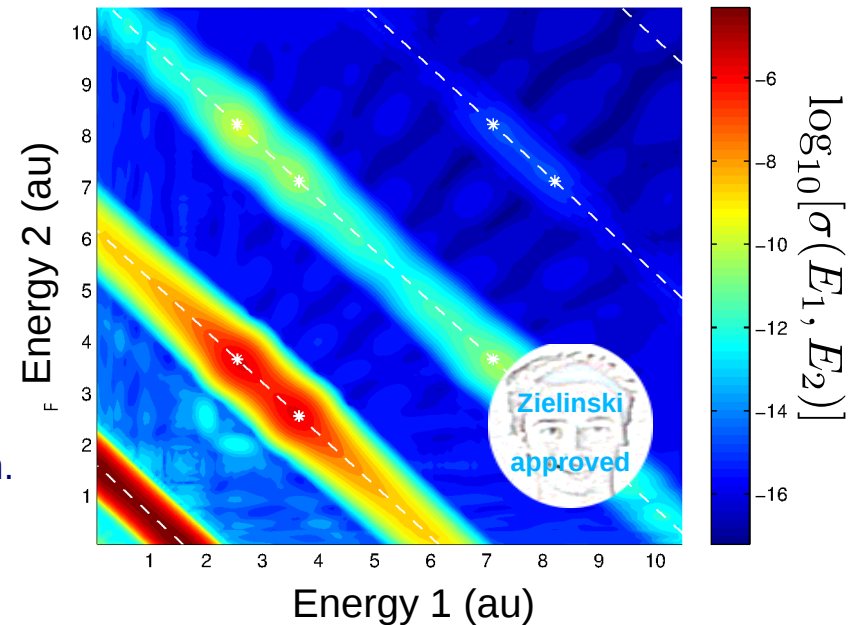
Helium atom

XUV pulse 10nm
Duration 8 cycles
Intensity $10^{15}\text{W}/\text{cm}^2$

Discretization

Simulation volume: 15 au
Angular momenta: 16
Radial basis functions: 56 per electron & ang.mom.
Propagation time: 400 optical cycles

Angle-integrated spectrum



Matches in detail with calculations by S. Nagele et al., box size ~ 1000 au

Run time:

4 hours on 16 CPUs (2-electron part)

3* hours on 128 CPUs (spectra from surfaces)

(*)Spectral calculation is fully parallel, effort $\propto \sqrt{\text{number of spectral points}}$

Effort for two-electron calculations

Inner region (B)

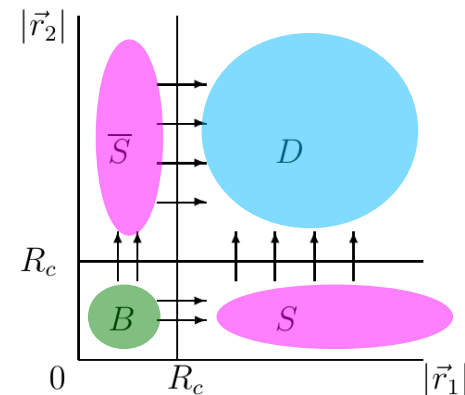
Box sizes $R_c \times R_c \sim 20 \times 20$ a.u.

Radial discretization points $N_1 \times N_2 \sim 40 \times 40$

Angular momenta: wave-length dependent

XUV: $M \times L_1 \times L_2 \sim 2 \times 4 \times 4$

NIR: $M \times L_1 \times L_2 \sim 4 \times 40 \times 40$



Single ionization spectra (S)

Solve one (hydrogen-like) ionic TDSE for each channel c

$$i \frac{d}{dt} \Phi_c(\vec{r}_1) = H_{ion}(\vec{r}_1) \Phi_c(\vec{r}_1)$$

One time-integral for each momentum p in the channel

$$\int dt f[\vec{p}, \Phi_c(|\vec{r}_1| = R_c)]$$

Double ionization spectra (D)

One ionic TDSE with source term for each momentum p_2 ,

$$i \frac{d}{dt} \Phi_{\vec{p}_2}(\vec{r}_1) = H_{ion}(\vec{r}_1) \Phi_{\vec{p}_2}(\vec{r}_1) + S_{\vec{p}_2}(\vec{r}_1)$$

One time-integral for each momentum pair (p_1, p_2)

$$\int dt g[\vec{p}_1, \Phi_{\vec{p}_2}(|\vec{r}_1| = R_c, t)]$$

Complex atoms and small molecules

- ✓ Integration with quantum chemistry (COLUMBUS)
- ✓ Technical remarks
- ✓ Emission from Ar and N₂

Ionic core dynamics (quantum chemical)

for molecular photo-emission...

Combine complex scaled basis χ_i
with ionic CI functions Φ_c (COLUMBUS)

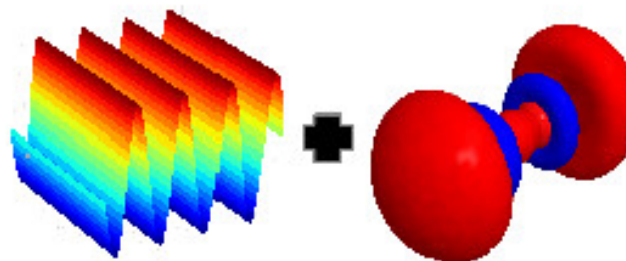
Goals:

Reliable strong-field ionization rates
Accurate photo-electron spectra

Anti-symmetrize

$$\psi_{i,c}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n) = \mathcal{A} [\chi_i(\vec{r}_1) \Phi_c(\vec{r}_2, \dots, \vec{r}_n)]$$

$$\Psi = \sum_{c=1}^{N_{core}} \sum_i C_{i,c}(t) \psi_{i,c}$$



Difficulties:

Get CI wave function (solved)
Basis size (endless story)
Messy matrix elements (solved)
Over-completeness issues (solved)
Gauge (solved)

Many thanks for access to
COLUMBUS wave functions

H. Lischka
Th. Müller
J. Pittner

**Core-dynamics
Anti-Symmetrization
Gauge**

Gauge dependence of the approach

Idea of the quantum chemistry basis

Core electron dynamics largely within field free bound states Ψ_i

Length gauge:

x and p have their standard meaning

Functions Ψ_i correspond to field free states also in presence of IR

Velocity gauge:

Corresponds to a time-dependent boost $p \rightarrow p + eA(t)/c$

Functions $e^{-i\vec{r}\cdot\vec{A}(t)}\Psi_i(\vec{r})$ correspond to field free state

At strong IR fields $\exp[-i\vec{r}\cdot\vec{A}(t)]$ can strongly differ from 1 across the Ψ_i

(Compare the debate about the “correct” gauge in SFA)

Computations more efficient in velocity gauge
=> local gauge transform on the bound state range
(tricky business)

Physics implications of gauge

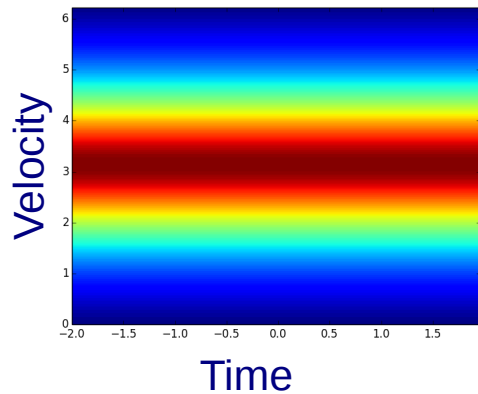
Gauge is nothing but math – multiply everything by $U(t) = \exp[-i\vec{A}(t) \cdot \vec{r}]$

Velocity gauge is an **accelerated** frame of reference

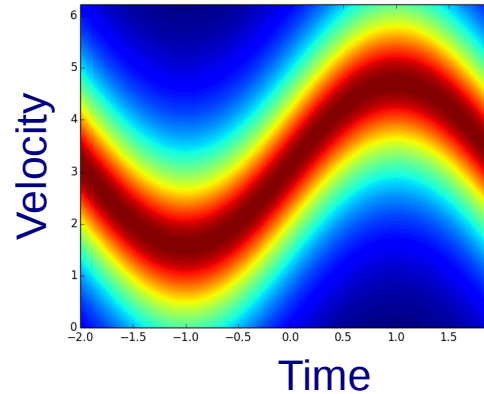
Change of momentum coordinate $\vec{p} \rightarrow \tilde{\vec{p}} = \vec{p} + \vec{A}(t)$

Velocity distribution of $\Phi_0(\vec{r}) = \exp(-r)$

Length gauge



Velocity gauge



So, which distribution is your choice?

Length gauge is the natural* choice IF we believe the system remains close to Φ_0

(* for SFA, Coupled-Channels, R-matrix,...

Numerical aspects of gauge

While length gauge may be more “natural”...

Velocity gauge is “better” for numerical calculations

Smaller momenta

When the laser-imposed boosts become comparable to the initial state momentum spread

the canonical momentum-range covered is smaller

- Fewer phase-wiggles in your wave-function
- Fewer angular momenta $L = \vec{r} \times \vec{p}$ - important at large $|r|$

Efficient absorption

irECS – the by far most efficient (and the only exact) absorption method for TDSE
requires velocity gauge

Break the spell – mix the gauges!

Issues: rapid, time-dependent change or discontinuity of solution
in the transition region length \leftrightarrow velocity

Core-discretization is gauge sensitive

Non-interacting system $H(x, y) = h(x) + h(y)$

Ansatz + exchange (triplet) $\Psi(x, y, t) = \phi(x, t)\chi(y, t) - \chi(x, t)\phi(y, t)$

Matrix element

$$\langle \Psi | H | \Psi \rangle = \langle \phi | \phi \rangle \langle \chi | h | \chi \rangle + \langle \phi | h | \phi \rangle \langle \chi | \chi \rangle - \langle \chi | \phi \rangle \langle \phi | h | \chi \rangle - \langle \chi | h | \phi \rangle \langle \phi | \chi \rangle$$

$\langle \chi(t) | \phi(t) \rangle = 0$ for exact time-evolution

$\langle \chi(t) | \phi(t) \rangle \neq 0$ for approximate evolution of $|\phi(t)\rangle$

Importance of the effect

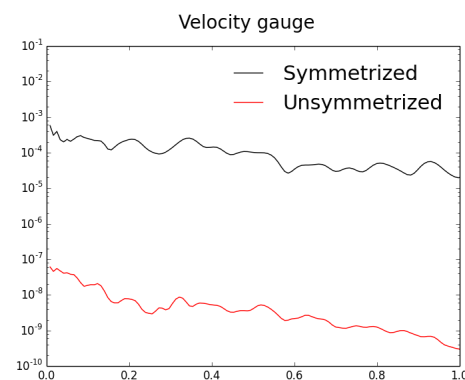
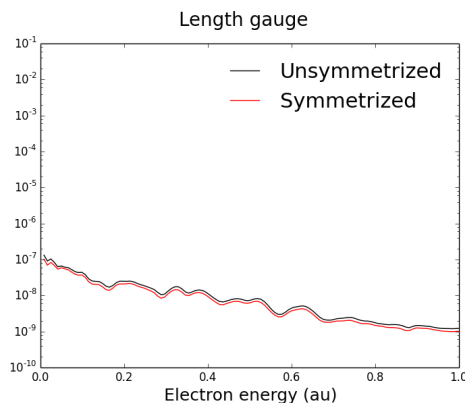
System: non-interacting He

Pulse: $1 \times 10^{15} \text{ W/cm}^2$

400 nm

Discretization:

4 coupled channels



**** Minimize core-polarization (→ length gauge)**

**** Describe remaining polarization correctly:**

e.g. xSCF, small phase-space grid for one more electron

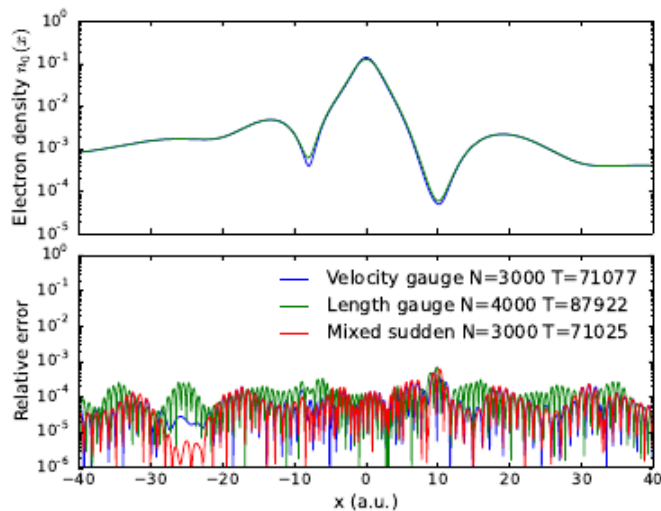
**** Possibly: separate spin from exchange symmetry (?)**

Numerical efficiency of gauges – single electron

1d model electron density

Single-cycle pulse,
800 nm, 25% ionization
No absorption (large box)

Discretization adjusted
for 10^{-4} relative error

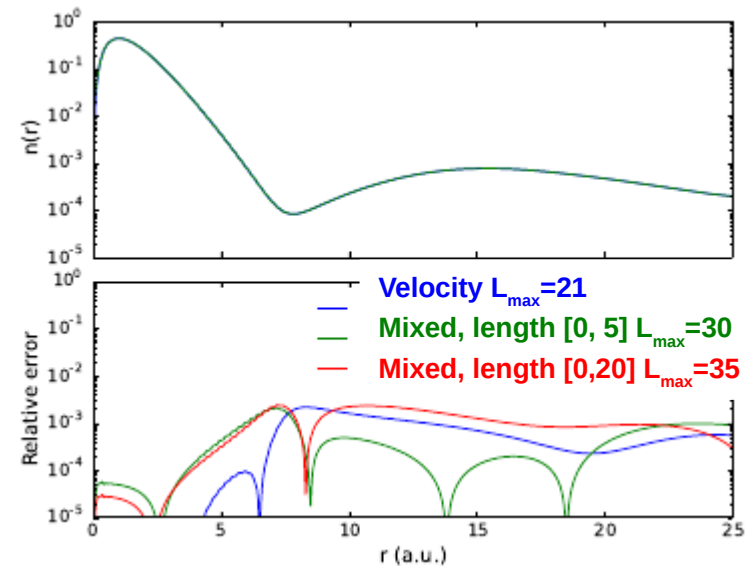


**Velocity and mixed comparable
Length less efficient**

3d H-atom electron density

3-cycle pulse,
800 nm, 16% ionization
irECS absorption (box 30 au)

Discretization adjusted
For $\sim 10^{-3}$ relative error



Penalty for size of length-gauge region

Numerical efficiency of gauges – He and H₂

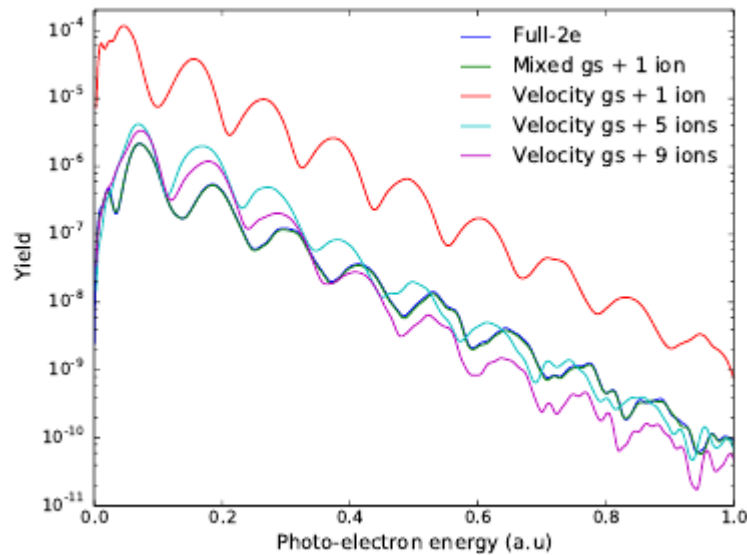
Total photoelectron spectra

3-cycle pulse, 400 nm, 10¹⁴ W/cm²

Compare

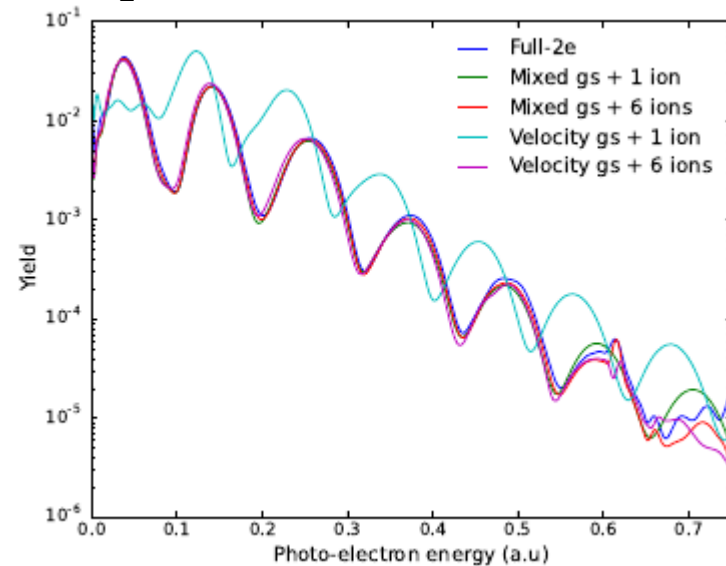
Converged 2-electron calculation
with ionic core dynamics calculations
in velocity and mixed gauges

Helium



Mixed gauge converges with 1 ion state
Velocity gauge not converged with 9 ion states

H₂ molecule



Mixed gauge converges with 1 ion state
Velocity gauge barely converged with 6 ion states

A flavor of the complexity of matrix elements

Two-particle (electron-electron) interaction: $H_2 = \sum_{i,j,i < j} h_{ij}$

Electron + ion basis function:

$$\psi_{i,I} = \mathcal{A} [\chi_i(\vec{r}_1) \Phi_I(\vec{r}_2 \dots \vec{r}_n)]$$

Matrix element:

$$\begin{aligned} \langle \psi_{i,I} | H_2 | \psi_{j,J} \rangle &= (n-1) \sum_{kl} \langle \phi_k \chi_i | \phi_l \chi_j \rangle \rho_{kl;IJ}^{(1)} \\ &+ \frac{(n-1)(n-2)}{2} \langle \chi_i | \chi_j \rangle \sum_{klmn} \langle \phi_k \phi_l | \phi_m \phi_n \rangle \rho_{klmn;IJ}^{(2)} \\ &- (n-1) \sum_{kl} \langle \phi_k \chi_i | \chi_j \phi_l \rangle \rho_{kl;IJ}^{(1)} \\ &- (n-1)(n-2) \sum_{klmn} \langle \phi_k \chi_i | \phi_m \phi_n \rangle \rho_{klmn;IJ}^{(2)} \langle \phi_l | \chi_j \rangle \\ &- (n-1)(n-2) \sum_{klmn} \langle \phi_k \phi_l | \chi_j \phi_n \rangle \rho_{klmn;IJ}^{(2)} \langle \chi_i | \phi_m \rangle \\ &- \frac{(n-1)(n-2)(n-3)}{2} \sum_{abcdef} \langle \phi_a \phi_b | \phi_d \phi_e \rangle \rho_{abcdef;IJ}^{(3)} \langle \chi_i | \phi_f \rangle \langle \phi_c | \chi_j \rangle \end{aligned}$$

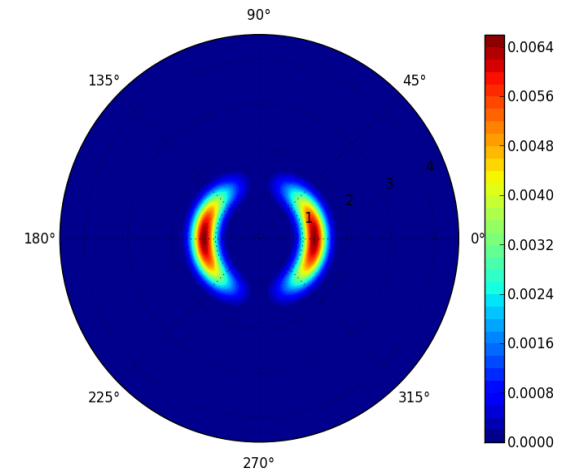
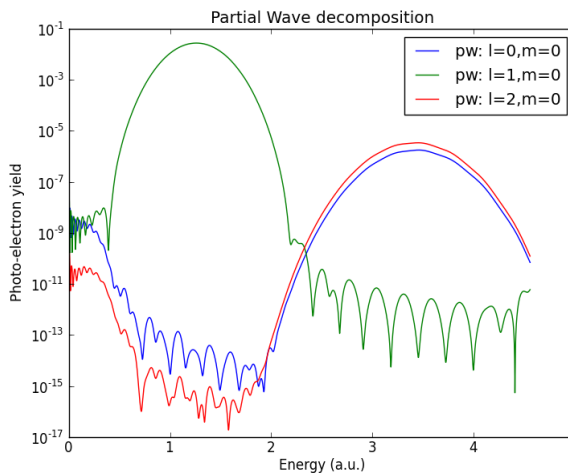
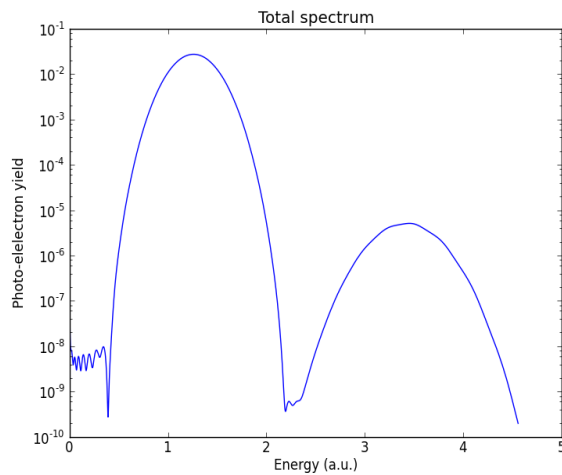
Non-standard 3-particle reduced density matrix for ionic states Φ_I, Φ_J

Photo-electron spectra of He, H₂, N₂

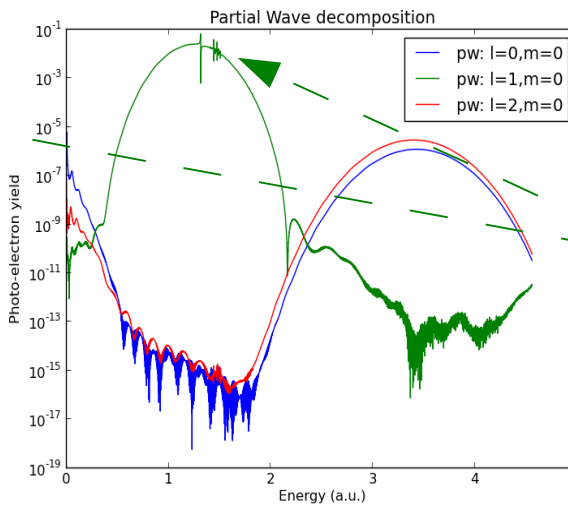
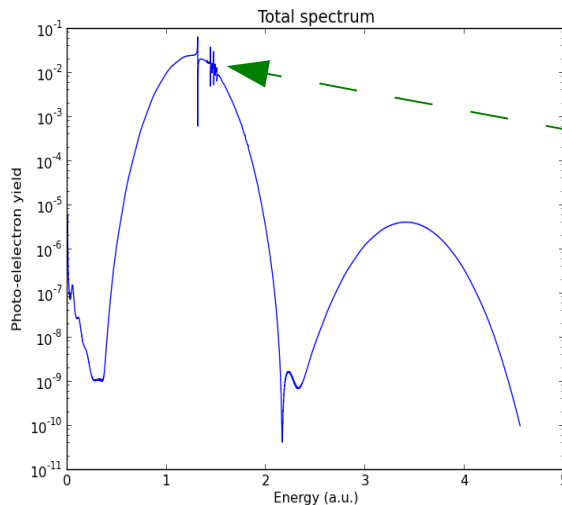
XUV photo-ionization of Helium

Pulse parameters: $\lambda = 21\text{nm}$, 3-cycle, \cos^8 envelope, linear polarization

1 ionic state



6 ionic states



Doubly excited states / Fano resonances

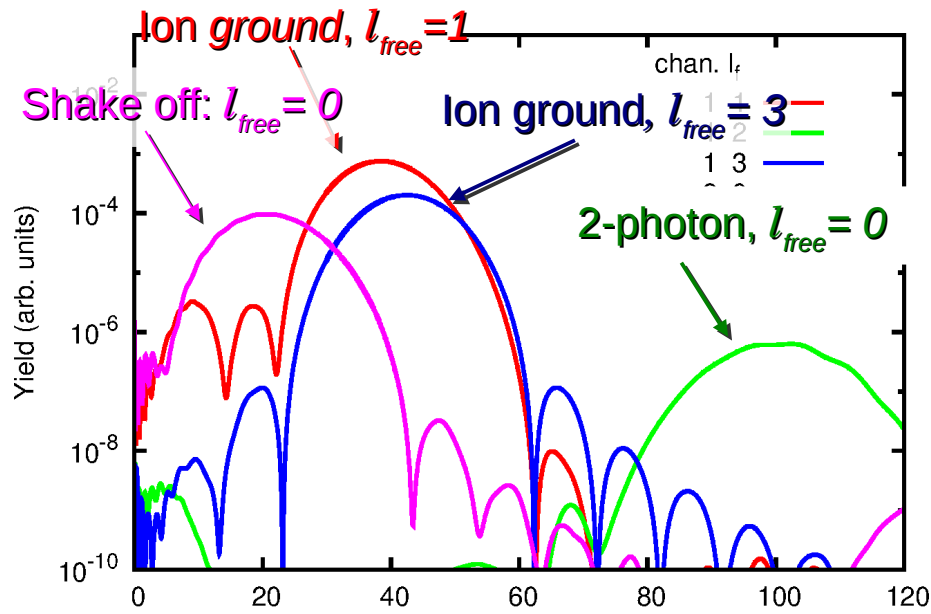
State	Literature[a]	Calculation
2s 2p	1.307	1.313
2s 3p	1.436	1.441
2s 4p	1.466	1.474

[a] J Chem. Phys. 139, 104314 (2013)

Photo-ionization of H₂ and N₂

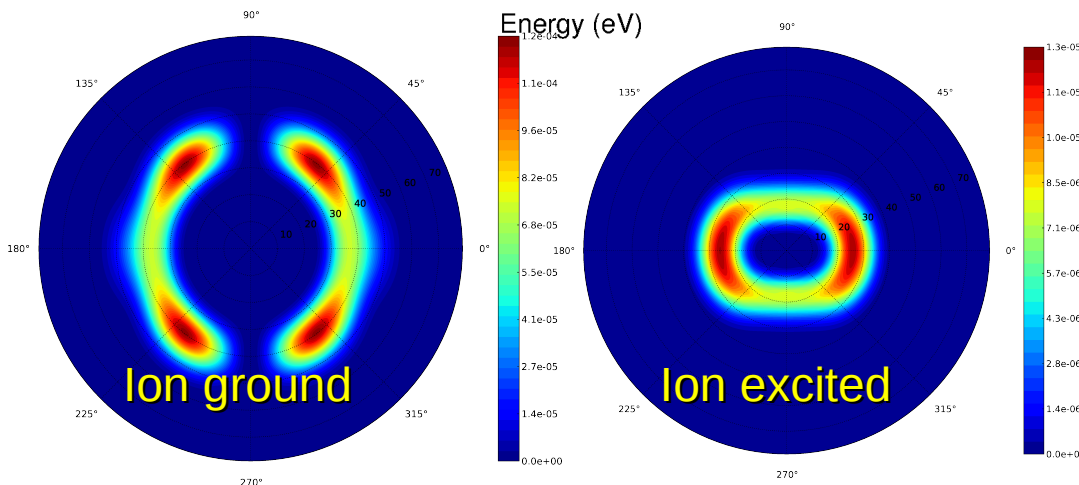
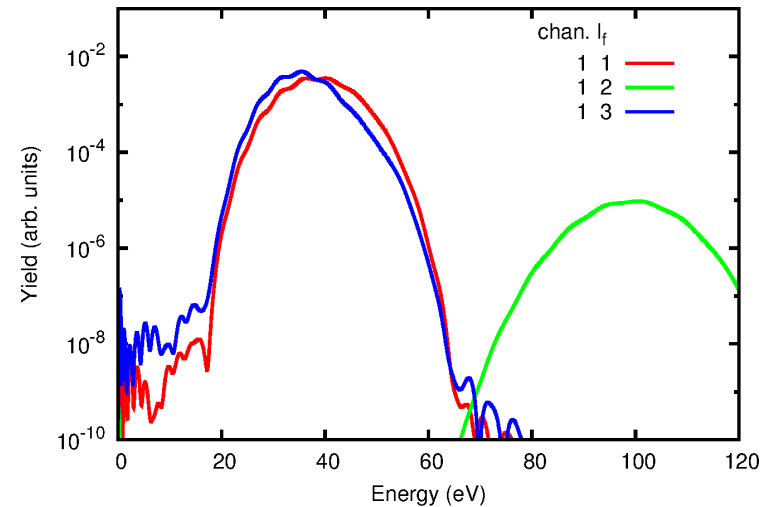
H₂ XUV photo-ionization

Pulse: 21 nm, 3 cycle FWHM, 10¹⁵W/cm²



N₂ XUV photo-ionization

More of the same...



Angle-resolved spectra

H₂ ionic channels
(disc diameter=80 eV)

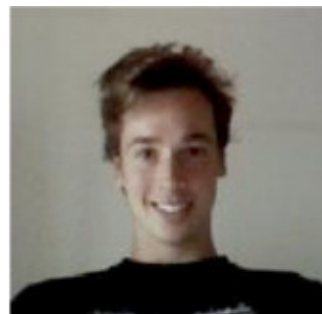
Team



**Vinay
Majety**
2-electron
& molecules



**Mattia
Lupetti**
Solids and
surfaces



**Alejandro
“the convergator”
Zielinski**
1-e elliptic
2-electron



**Jakob
Liss**
Solids and
surfaces

Publications

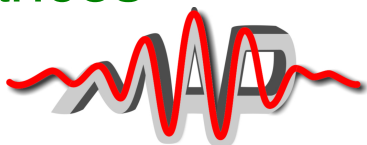
[A.S., Phys. Rev. A81, 53845 (2010)]

[L. Tao and A.S., New. J. Phys. 14, 013021 (2012)]

[A. S., New. J. Phys., 14, 085008 (2012)]

[A.S., HP. Stimming, N. Mauser, J.Comp.Phys 269, 68 (2014)]

Finances



Munich Advanced Photonics
Excellence Cluster



Vienna Computational Materials Science
FWF Special Research Program



Marie Curie ITN

Code: tRecX = tSURFF + irECS (working title)

All codes united in a single, C++ code based on recursive structures

General dimensions and coordinate systems

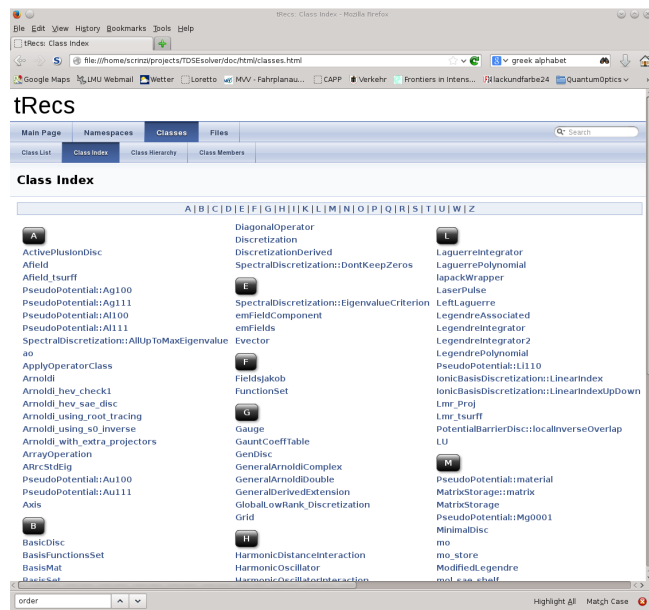
PDEs of the form

$$d/dt\Psi = D\Psi$$

(includes e.g. Maxwell's equations)

Preparing for public access
Can be made available for collaborations immediately

Doxygen documentation



tRecX demo input

```
--- coordinates and discretization ---  
Axis: coordinate,nCoefficients,lower end, upper end,functions,order  
M,1  
Eta,3  
R,120,0.,60.,legendre,20  
  
--- Hamiltonian for the Hydrogen atom ---  
Operator: Hamiltonian  
<1><1><1/2.dJd-Jr>+<1><1/2.dJ1q2d><nJr>  
[]  
--- laser pulse definition ---  
Laser: shape, I(W/cm2), FWHM, lambda(nm),phiCEO  
gauss, 0.5e14, 110., 100000.,0  
  
--- what to plot along which coordinates ---  
Plot: coordinate,points,lowerBound,upperBound  
R,51,0.,12.  
Eta,21,-1.,1.
```