Spin effects in relativistic strong-field processes

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Outline

- **Introduction**

  What has already been done?

- **Spin effects in relativistic ionization**

  Electron motion in strong Coulomb and laser fields

- **Spin dynamics in the Kapitza-Dirac effect**

  Electron scattering on a standing light wave of high intensity

- **Spin effects in strong-field e^+e^- pair creation**

  Multiphoton Breit-Wheeler & Bethe-Heitler pair creation

Spin effects in strong laser fields

Free and bound electron dynamics
- Walser, Urbach, Hatsagortsyan, Hu & Keitel, Phys. Rev. A 65, 043410 (2002)
- Hu & Keitel, Phys. Rev. Lett. 83, 4709 (1999)
- Bauke, Ahrens, Keitel & Grobe, NJP 16, 043012 (2014)

Strong-field photoionization
- Faisal & Bhattacharyya, Phys. Rev. Lett. 93, 053002 (2004)

Laser-assisted Mott scattering
- Szymanowski, Taieb & Maquet, Laser Phys. 8, 102 (1998)
- Panek, Kaminski & Ehlotzky, Phys. Rev. A 65, 033408 (2002)

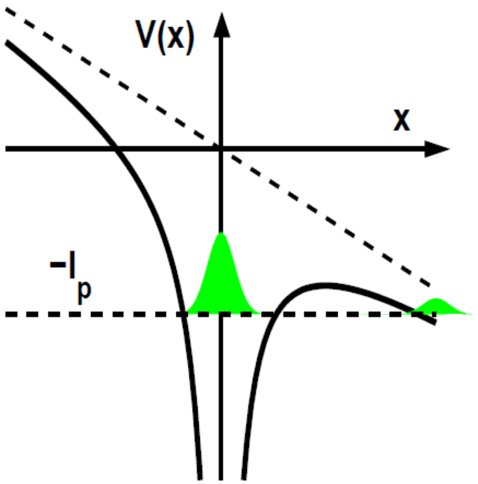
Multiphoton Compton scattering
- Panek, Kaminski & Ehlotzky, Phys. Rev. A 65, 022712 (2002)
- Ivanov, Kotkin & Serbo, Eur. Phys. J. C 36, 127 (2004)
- Boca, Dinu & Florescu, Nucl. Instrum. Meth. Phys. Res. B 279, 12 (2012)

Pair creation by energetic non-laser photon + laser field
- Tsai, Phys. Rev. D 48, 96 (1993)
- Ivanov, Kotkin & Serbo, Eur. Phys. J. C 40, 27 (2005)

Pair creation in an oscillating electric field & Klein paradox (fermions vs. bosons)
- Popov, Sov. Phys. JETP 34, 709 (1972)
- Krekora, Su & Grobe, Phys. Rev. Lett. 92, 040406 (2004)
- Wagner, Ware, Su & Grobe, Phys. Rev. A 81, 024101 (2010)

Spin effects in relativistic ionization in strong laser fields

Theoretical approach: Strong-field approximation



Dirac Hamiltonian with laser field:

$$\hat{H} = c\alpha \cdot (\hat{\mathbf{p}} + \mathbf{A}) + \beta c^2 + V^{(c)} - \Phi = \hat{H}_0 + \hat{H}_{\mathrm{int}}$$

Transition amplitude:

$$M_{\mathrm{fi}} = -i \int dt\, d^3\mathbf{r}\, \psi_{\mathrm{f}}^{\dagger}(\mathbf{r}, t) H_{\mathrm{int}} \phi_{\mathrm{i}}^{(c)}(\mathbf{r}, t)$$

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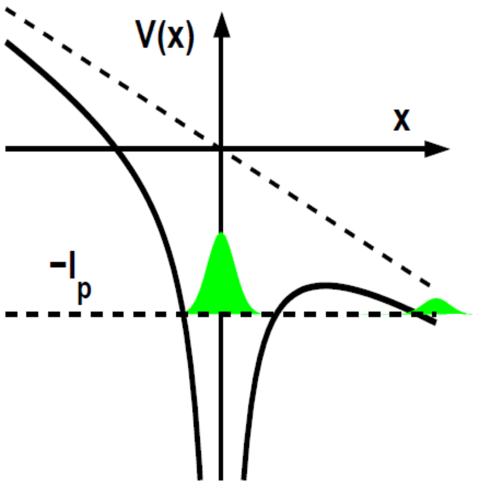

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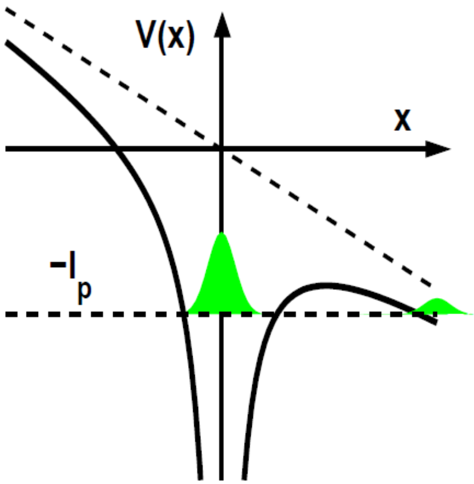
Standard form of strong-field approximation relies on:

$$\hat{H}_0^{(S)} = c\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta c^2 + V^{(c)}, \quad \hat{H}_{\text{int}}^{(S)} = c\boldsymbol{\alpha} \cdot \mathbf{A} - \Phi$$

Theoretical approach: Strong-field approximation



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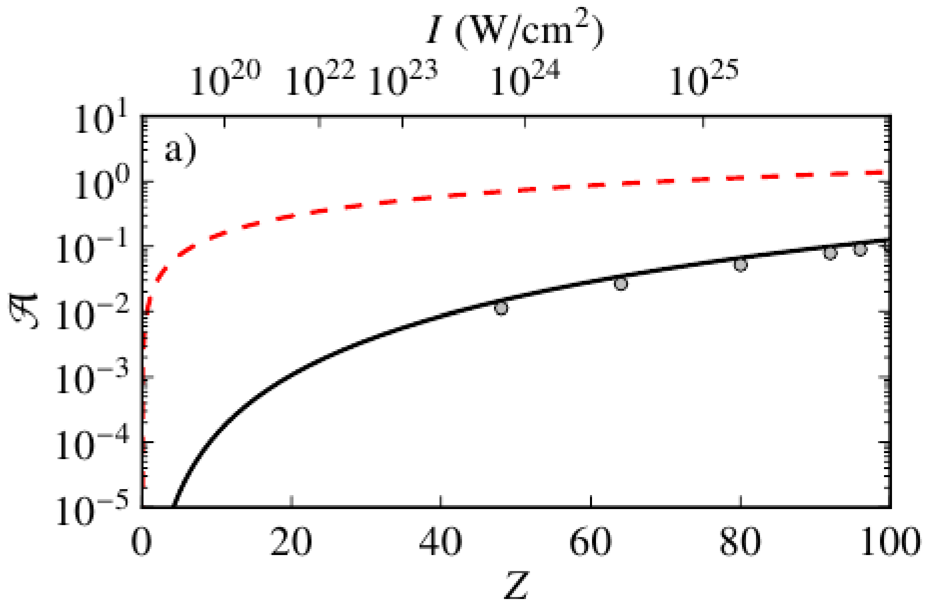
$$\hat{H}_0^{(S)} = c\alpha\cdot\hat{\mathbf{p}}+\beta c^2+V^{(c)},\qquad \hat{H}_{\mathrm{int}}^{(S)} = c\alpha\cdot\mathbf{A}-\Phi$$

Dressed SFA:

$$\hat{H}_0^{(D)} = c\alpha\cdot[\hat{\mathbf{p}}-\hat{\mathbf{k}}(\mathbf{r}\cdot\mathbf{E})/c]+\beta c^2+V^{(c)},\qquad \hat{H}_{\mathrm{int}}^{(D)} = \mathbf{r}\cdot\mathbf{E}$$

Spin asymmetry

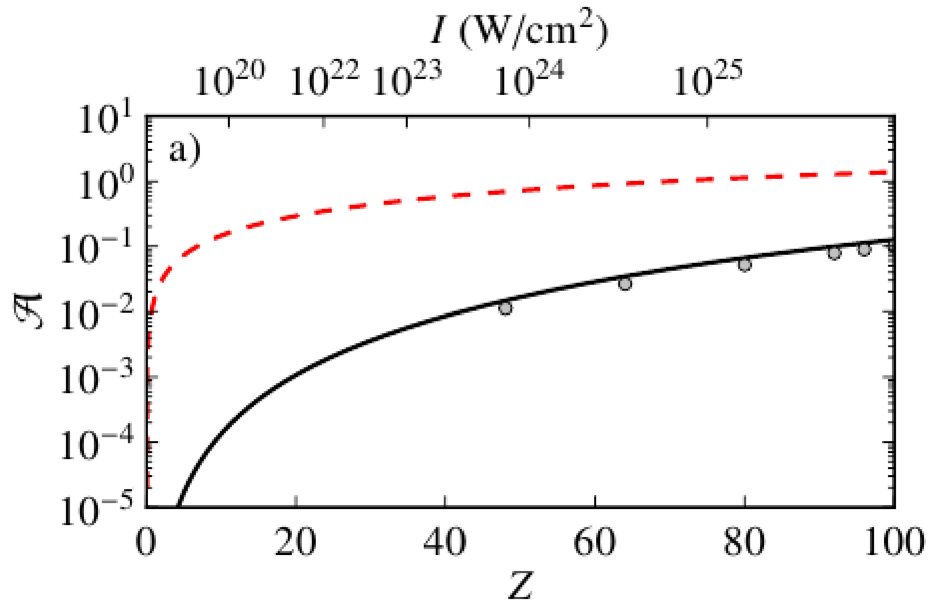


standard SFA

dressed SFA

$$E_0/E_a = 1/25$$

Spin asymmetry



----- standard SFA
—— dressed SFA

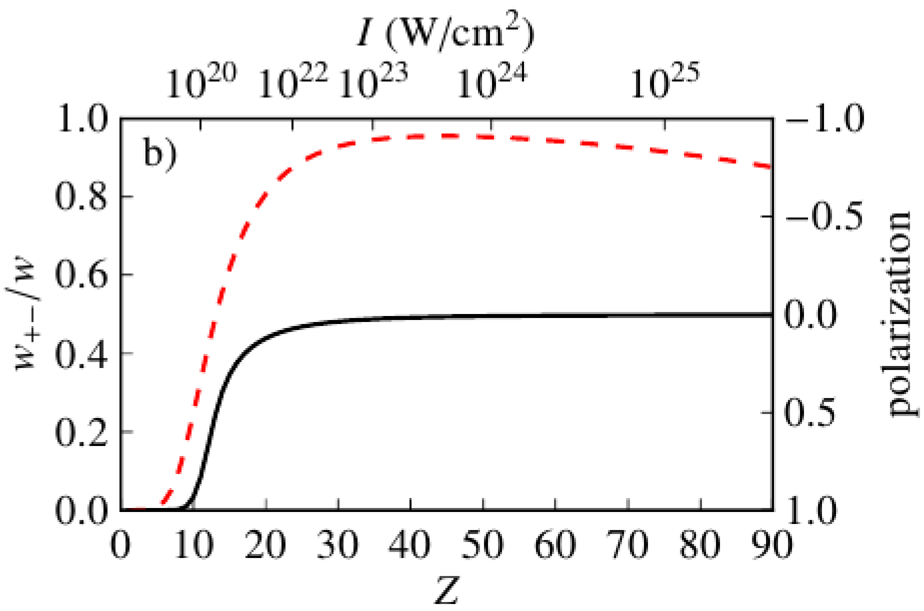
$$E_0/E_a = 1/25$$

$$\mathcal{A} \equiv (w_{--} - w_{++})/w \approx \begin{cases} 2\left(2I_p/c^2\right)^{1/2} & \text{(standard SFA)} \\ 2\left(2I_p/c^2\right)^{3/2} & \text{(dressed SFA)} \end{cases}$$

spin orientation w.r.t. laser **B** field

Inclusion of spin dynamics in the bound state strongly suppresses spin asymmetry!

Spin flip probability



----- standard SFA
—— dressed SFA

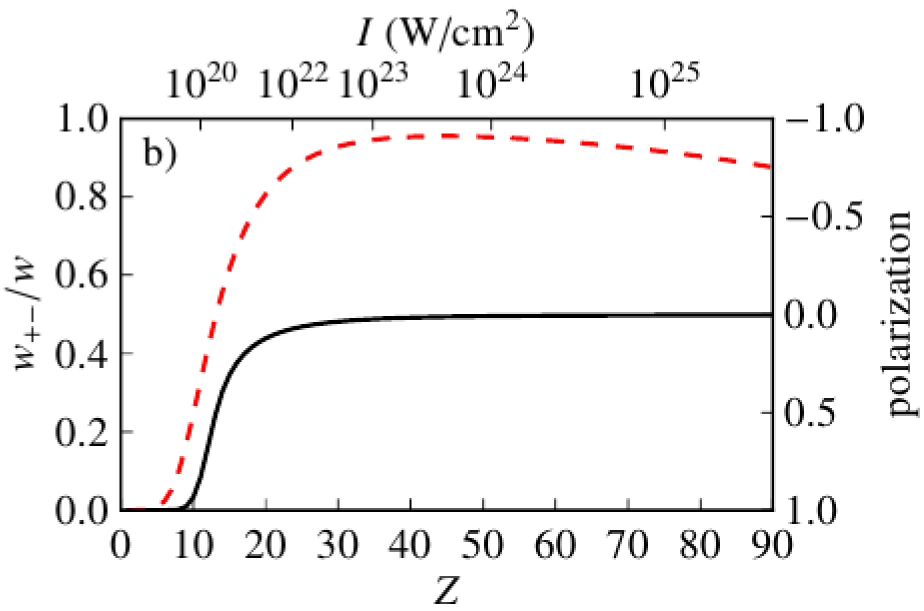
$$E_0/E_a = 1/25$$

(spin orientation w.r.t. laser **k**)

relevant parameter is: $\mu \equiv (E_0/E_a)\xi_0^2,\ \xi_0 \equiv E_0/(c\omega)$

Klaiber, Yakaboylu, Müller, Bauke, Paulus & Hatsagortsyan, JPB **47**, 065603 (2014)

Spin flip probability



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(spin orientation w.r.t. laser **k**)

relevant parameter is: $\mu \equiv (E_0/E_a)\xi_0^2,\ \xi_0 \equiv E_0/(c\omega)$

for $\mu \gg 1$:

$$\frac{w_{+-}}{w} \approx \begin{cases} 1 - \dfrac{I_p}{2c^2} & \text{(standard SFA)} \\ 1/2 + O(1/\mu) & \text{(dressed SFA)} \end{cases}$$

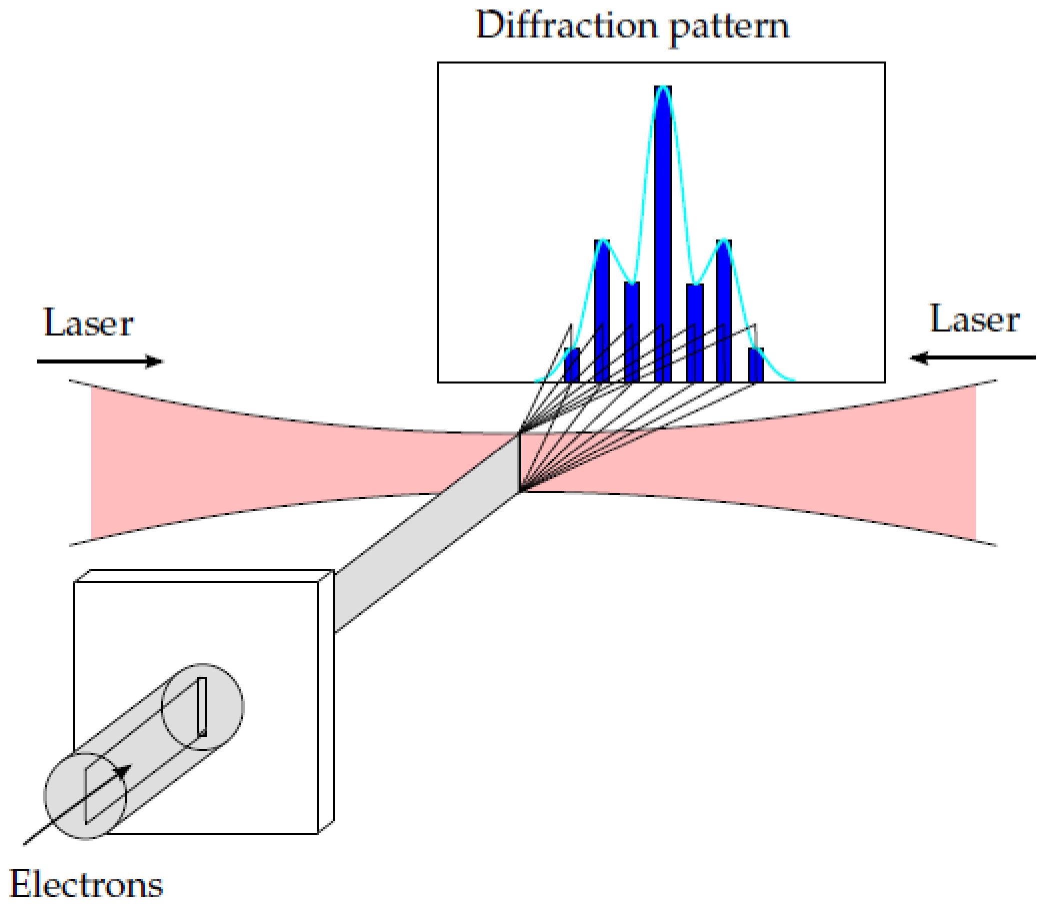
Klaiber, Yakaboylu, Müller, Bauke, Paulus & Hatsagortsyan, JPB **47**, 065603 (2014)

Spin effects in

Kapitza-Dirac scattering

Kapitza-Dirac scattering

(Kapitza & Dirac, 1933;
Fedorov 1967)

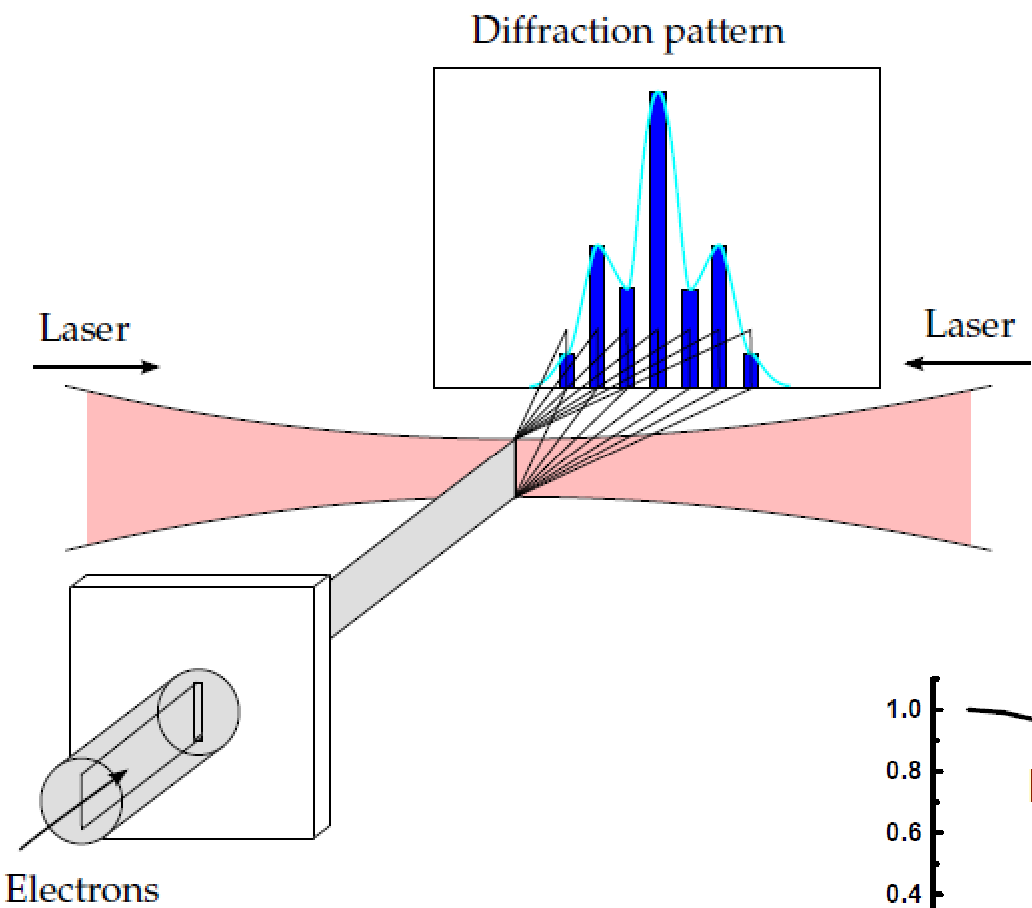


Bragg angle: $\sin\theta = \lambda_{dB}/\lambda$

Conclusively observed: Freimund, Aflatooni & Batelaan, Nature (2001)

Kapitza-Dirac scattering

(Kapitza & Dirac, 1933; Fedorov 1967)

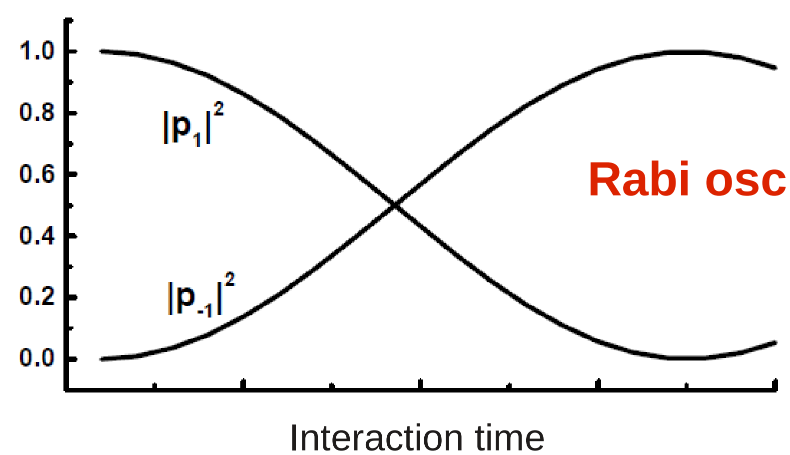


Bragg angle: $\sin\theta = \lambda_{dB} / \lambda$

standard case:

1 photon of $\hbar k$ absorbed,
1 photon of $-\hbar k$ emitted

→ momentum transfer $2\hbar k$



Rabi oscillations

Conclusively observed: Freimund, Aflatooni & Batelaan, Nature (2001)

Theory of relativistic Kapitza-Dirac effect

Dirac equation:

$$\mathrm{i}\frac{\partial}{\partial t}\psi(\boldsymbol{x},t) = \left[c\left(-\mathrm{i}\nabla + \frac{e}{c}\boldsymbol{A}(\boldsymbol{x},t)\right)\cdot\alpha + \beta mc^2\right]\psi(\boldsymbol{x},t)$$

in presence of standing laser wave $\boldsymbol{A}(\boldsymbol{x},t) = -\frac{\boldsymbol{E}}{k}\cos(\boldsymbol{k}\cdot\boldsymbol{x})\sin(ckt)$

Expansion of electron state into plane waves:

$$\psi(\boldsymbol{x},t) = \sum_{n,\zeta} c_n^\zeta(t)\psi_{n,\boldsymbol{p}}^\zeta(\boldsymbol{x})$$ with $$\psi_{n,\boldsymbol{p}}^\zeta(\boldsymbol{x}) = u_{n,\boldsymbol{p}}^\zeta \mathrm{e}^{\mathrm{i}(\boldsymbol{p}+n\boldsymbol{k})\cdot\boldsymbol{x}}$$

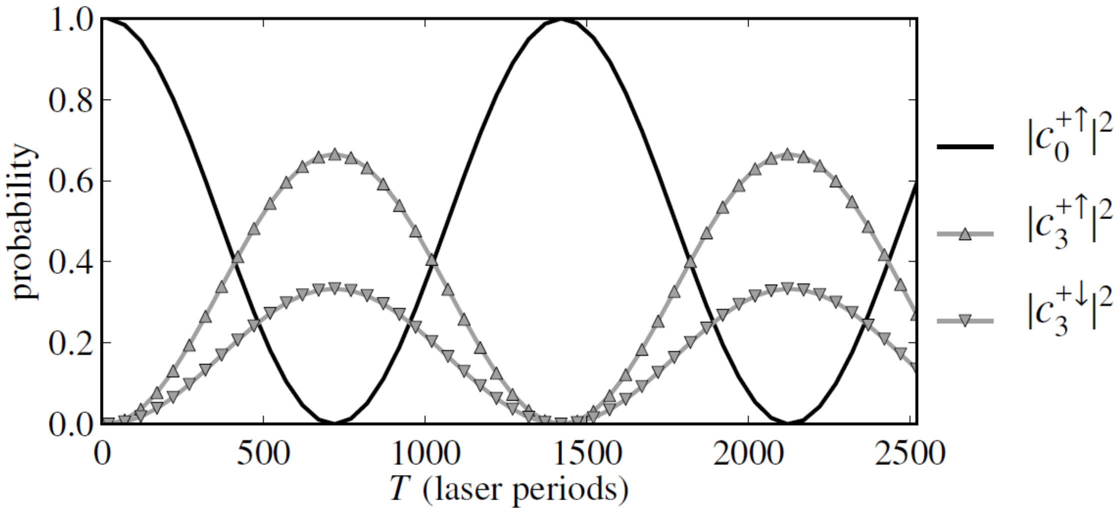
⇒ Obtain coupled system of ODEs for expansion coefficients

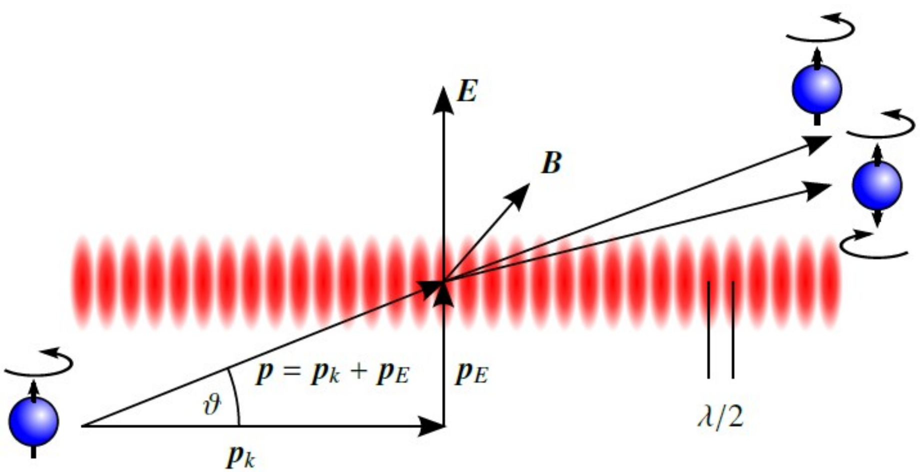
Relativistic Kapitza-Dirac effect of Klein-Gordon particles studied in:
Haroutunian & Avetissian, Phys. Lett. A (1975); Fedorov & McIver, Opt. Commun. (1980)

Relativistic Kapitza-Dirac effect in XFEL beams

relativistic electron beam (180 keV/c), XFEL of $\hbar\omega = 3$ keV, $I = 10^{23}$ W/cm^2,

scattering by <u>three-photon</u> exchange may occur with or without spin flip:





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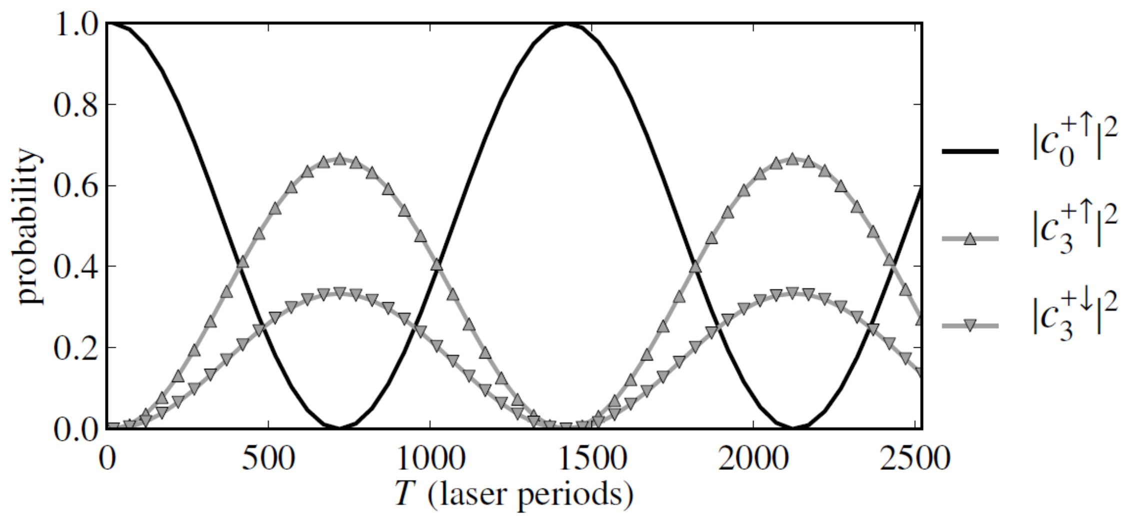
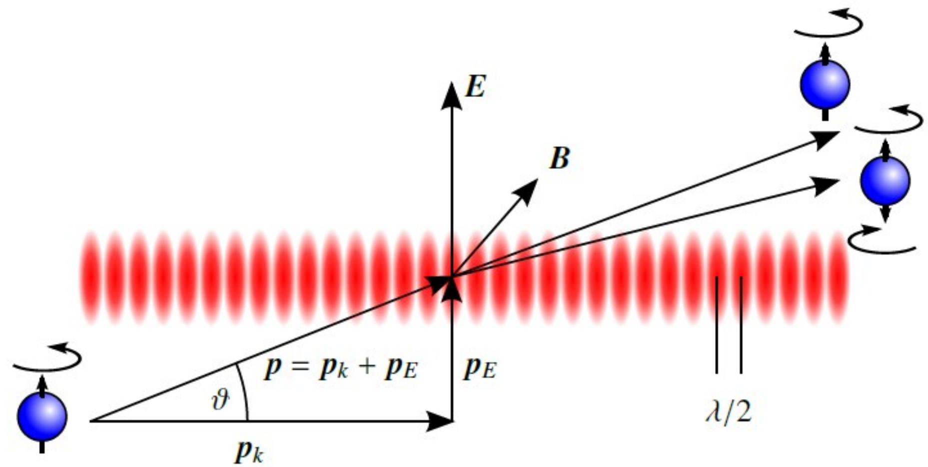


$$e\boldsymbol{A} \cdot \boldsymbol{p}/(mc)$$

$$e\boldsymbol{\sigma} \cdot \boldsymbol{B}/(2mc)$$



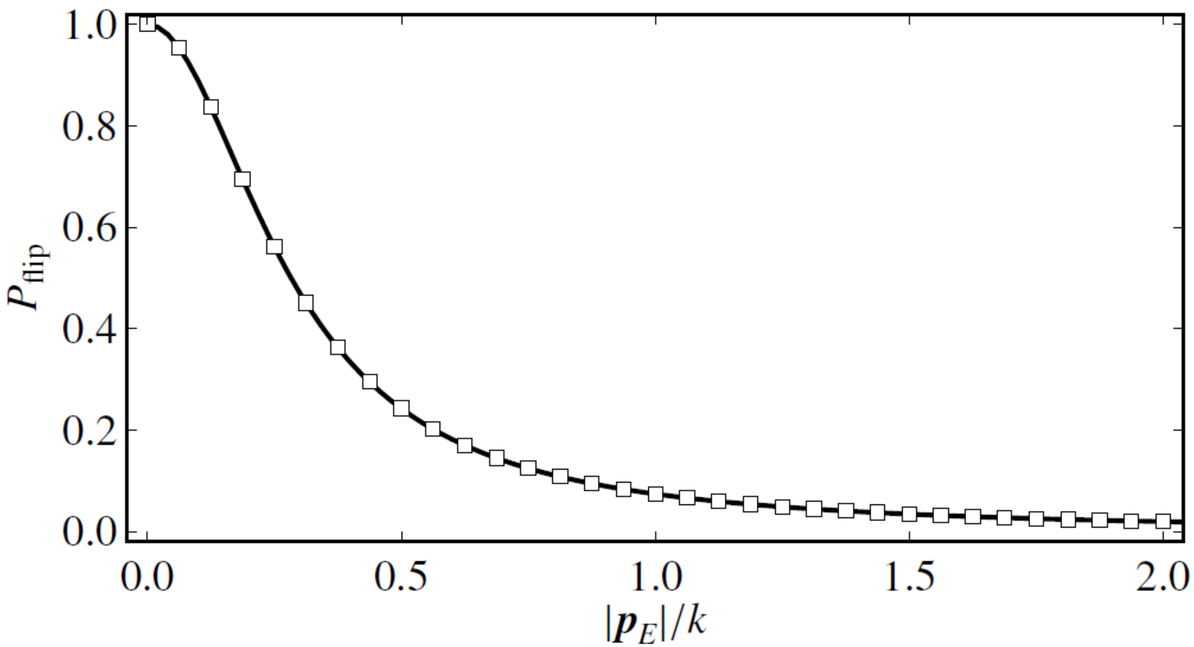
Spin-flip probability and Rabi frequency

For short times ($t \ll \Omega_R$):

$$|c_3^{+\uparrow}(T)|^2 = \left(\frac{1}{2}\Omega_0 T\right)^2 \left(\frac{5}{\sqrt{2}}\frac{|\boldsymbol{p}_E|}{k}\right)^2$$

$$|c_3^{+\downarrow}(T)|^2 = \left(\frac{1}{2}\Omega_0 T\right)^2 ,$$

$$\Rightarrow \quad P_{\text{flip}} \equiv \frac{|c_3^{+\downarrow}(T)|^2}{|c_3^{+\uparrow}(T)|^2 + |c_3^{+\downarrow}(T)|^2} = \frac{1}{\frac{25}{2}\left(\frac{|\boldsymbol{p}_E|}{k}\right)^2 + 1}$$



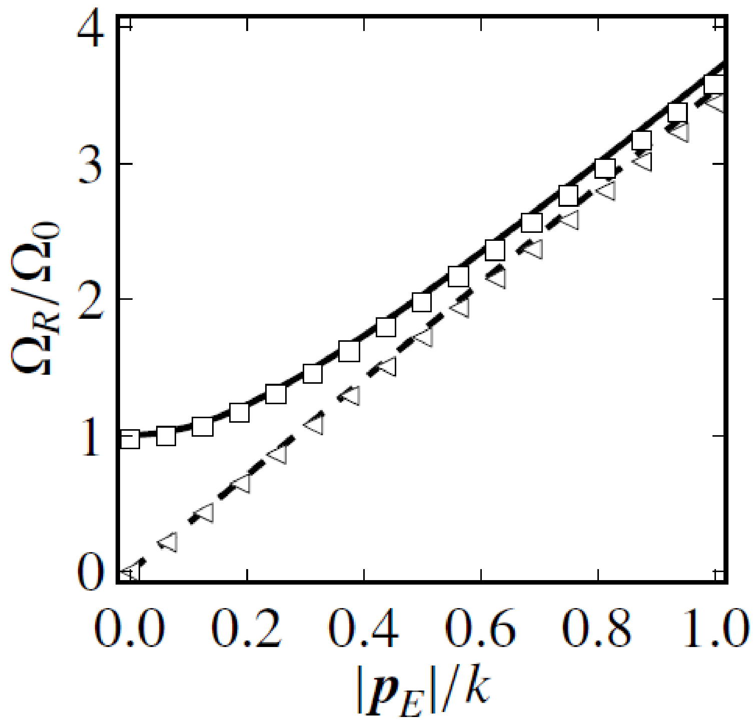
Rabi frequency:

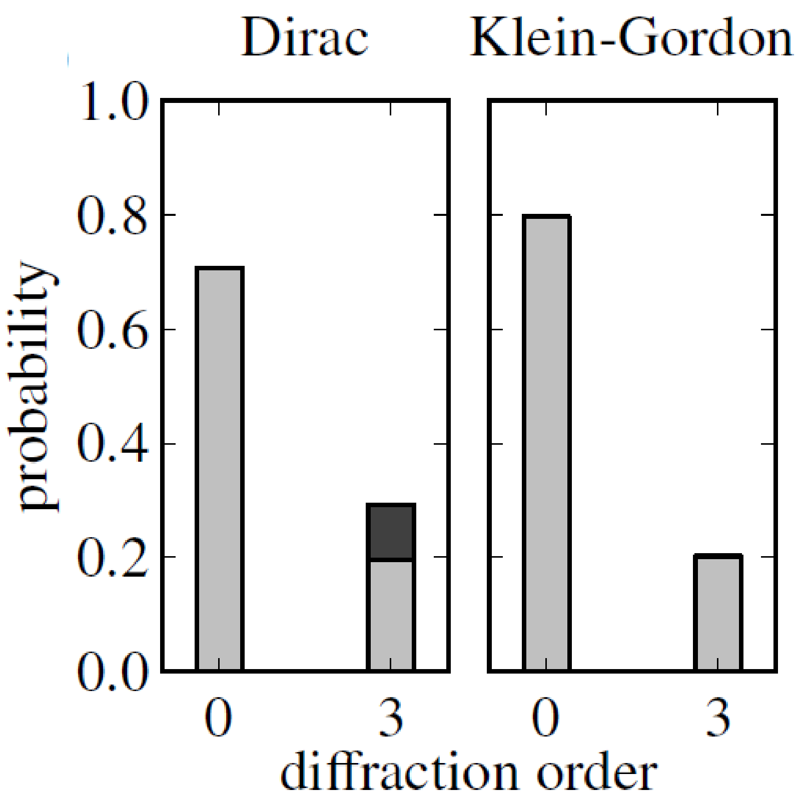
$$\Omega_R = \Omega_0 \sqrt{\frac{25}{2}\left(\frac{|\boldsymbol{p}_E|}{k}\right)^2 + 1}$$

Dirac versus Klein-Gordon particles

$$\Omega_R = \Omega_0 \sqrt{\frac{25}{2}\left(\frac{|\boldsymbol{p}_E|}{k}\right)^2 + 1}$$

$$\Omega_{R,\,\text{spinless}} = \Omega_0 \frac{5}{\sqrt{2}} \frac{|\boldsymbol{p}_E|}{k}$$



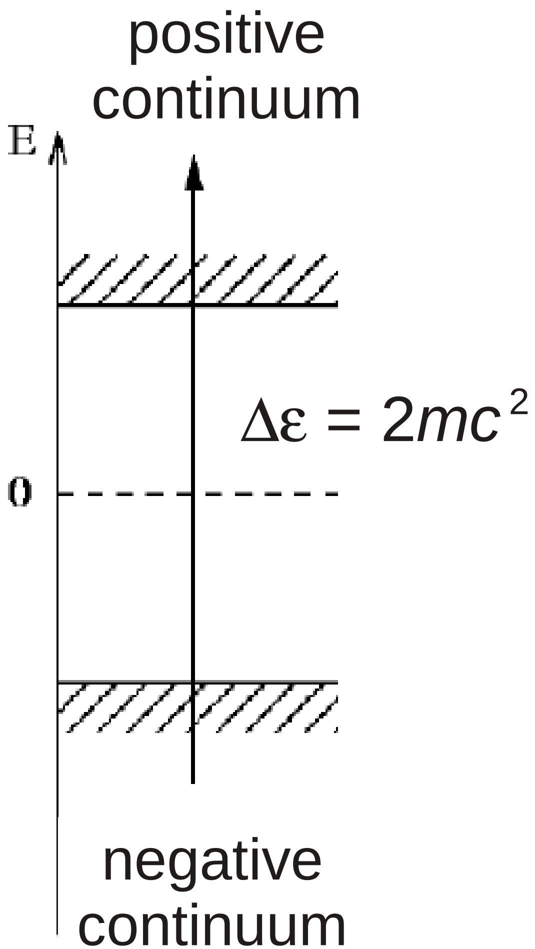


In the limit $\boldsymbol{p}_E \rightarrow 0$, scalar particles are not scattered at all!

Ahrens, Bauke, Keitel & Müller, PRL **109**, 043601 (2012)

Spin effects in strong-field pair production

e^+e^- pair creation in proton-laser collisions



Pair creation requires

$\hbar\omega \sim 2mc^2 \approx 1$ MeV

(multiphoton regime)

or

$E \sim E_{cr} = mc^2/e\lambda_C \approx 10^{16}$ V/cm

(tunneling regime)

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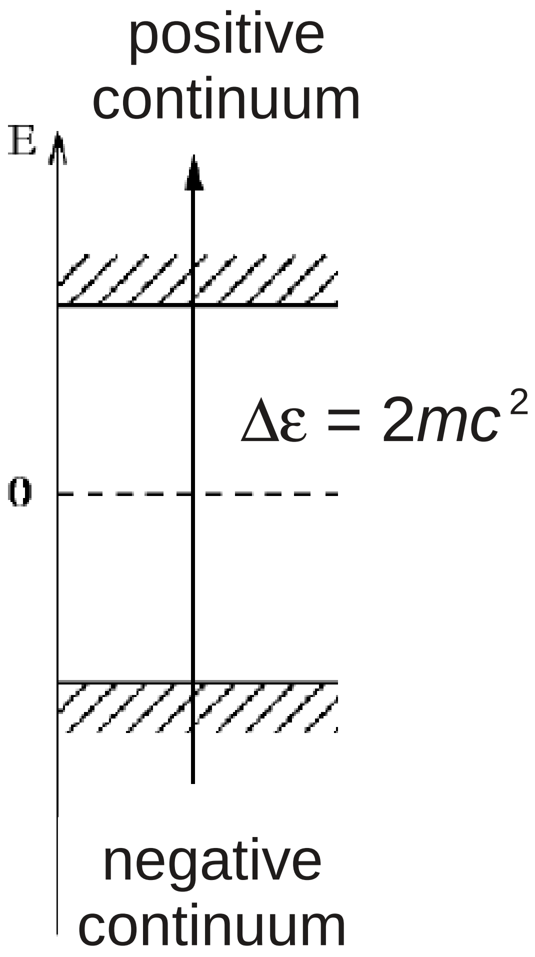

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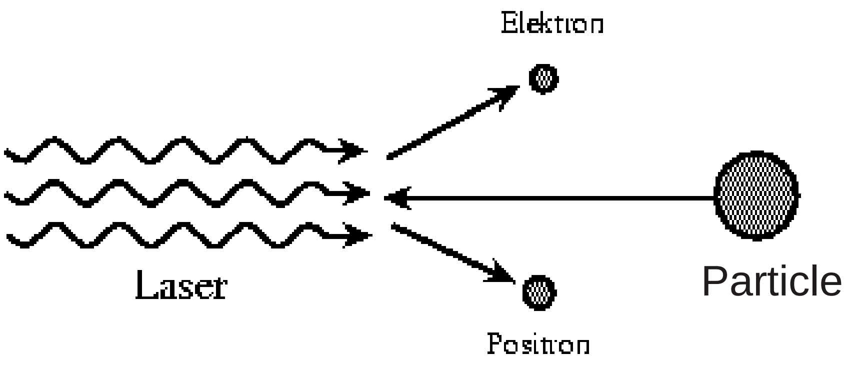


Exploit relativistic Doppler shift:

rest frame: $\hbar\omega'$ and E' enhanced by 2γ

Theory of spin-resolved Bethe-Heitler pair creation

Motivation: Future particle physics experiments planned with polarized $e^{\pm}$ beams; produced via linear Bethe-Heitler effect with ~20 MeV photons

Amplitude for transition from negative- to positive-energy Volkov state:

$$S_{p_+ s_+, p_- s_-} = \frac{i}{c} \int \overline{\Psi}^{(-)}_{p_-, s_-} \not{A}_N \Psi^{(+)}_{p_+, s_+} \; d^4x$$

$$= \frac{iZ}{\sqrt{q^0_+ q^0_-}} \sum_{n \geq n_0} \mathfrak{M}^{(n)}_{p_+ s_+, p_- s_-} \frac{4\pi}{Q^2_n} 2\pi\delta(Q^0_n)$$

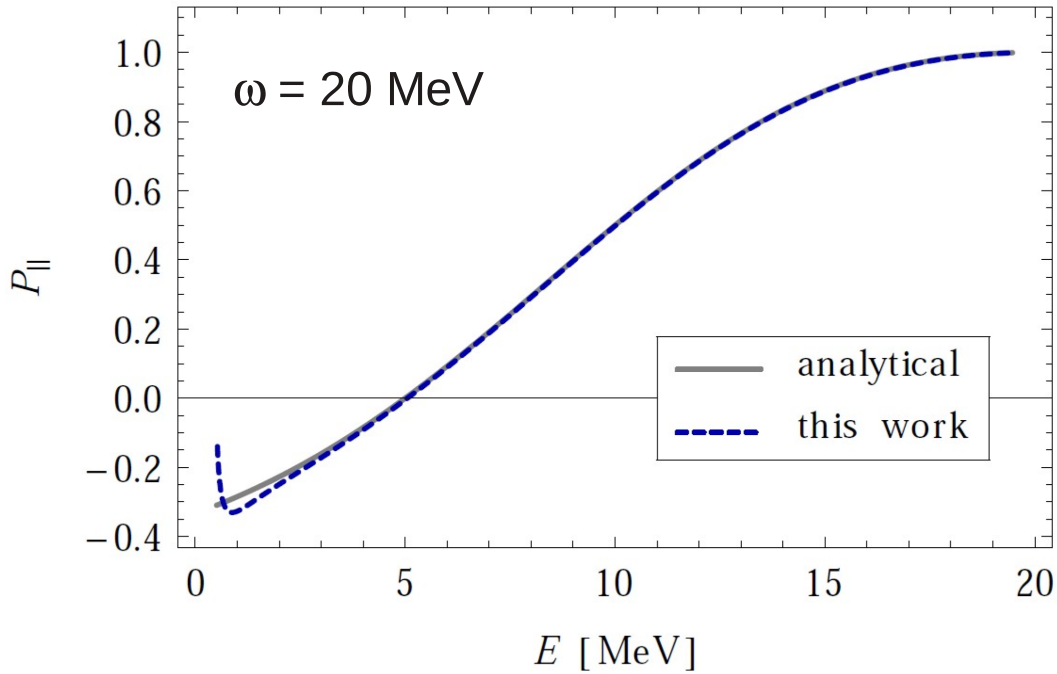
Spin projectors:

$$\left|\mathfrak{M}^{(n)}_{p_+ s_+, p_- s_-}\right|^2 = \mathrm{Tr}\left\{\Gamma^{(n)} \left[\left(\frac{\not{p}_+ - c}{2c}\right)\left(\frac{1 + \gamma^5 \not{s}_+}{2}\right)\right] \overline{\Gamma^{(n)}} \left[\left(\frac{\not{p}_- + c}{2c}\right)\left(\frac{1 + \gamma^5 \not{s}_-}{2}\right)\right]\right\}$$

Laser field is circularly polarized (right-handed photons); use **helicity states** for leptons.

Longitudinal spin polarization: High-energy limit

1-photon case: McVoy, Phys. Rev. (1958); Olsen & Maximon, Phys. Rev. (1959)



$$P_{\|}(E) = \frac{d\sigma_{\mathrm{R}} - d\sigma_{\mathrm{L}}}{d\sigma_{\mathrm{R}} + d\sigma_{\mathrm{L}}}$$

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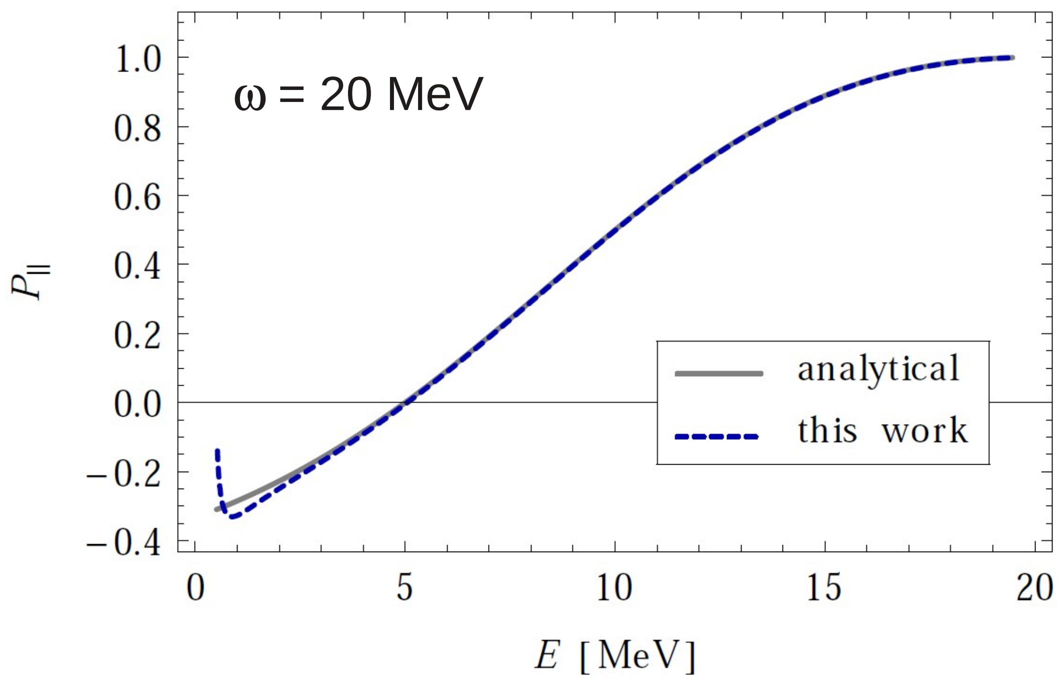
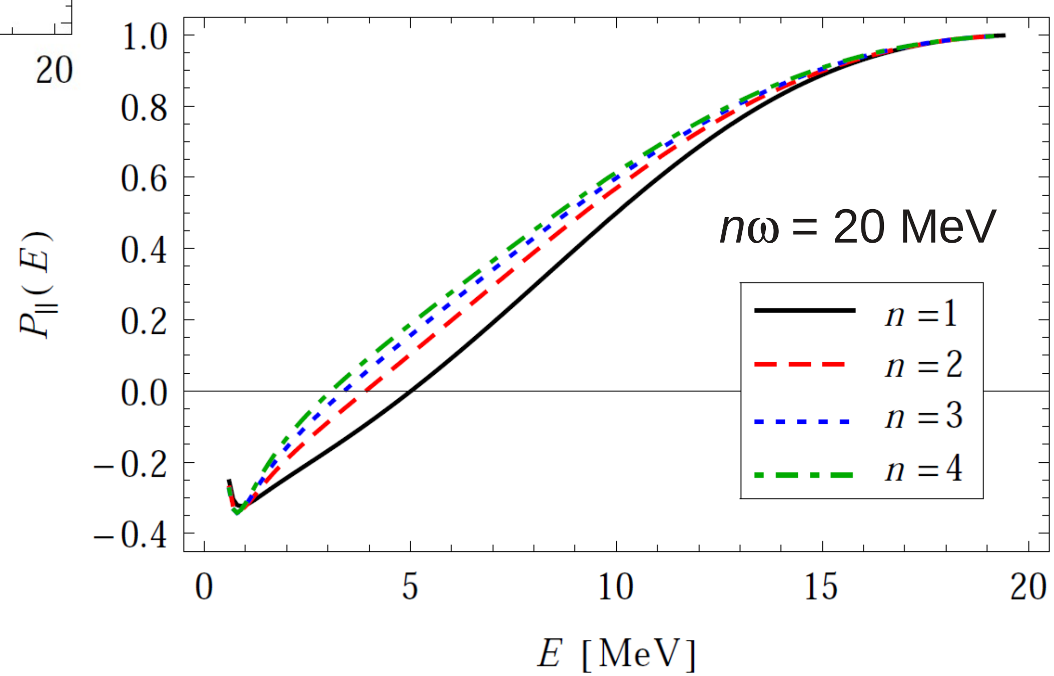


$$P_\parallel(E) = \frac{d\sigma_\mathrm{R} - d\sigma_\mathrm{L}}{d\sigma_\mathrm{R} + d\sigma_\mathrm{L}}$$

Polarization degree increases when few photons with **$n\omega \gg mc^2$** are absorbed:



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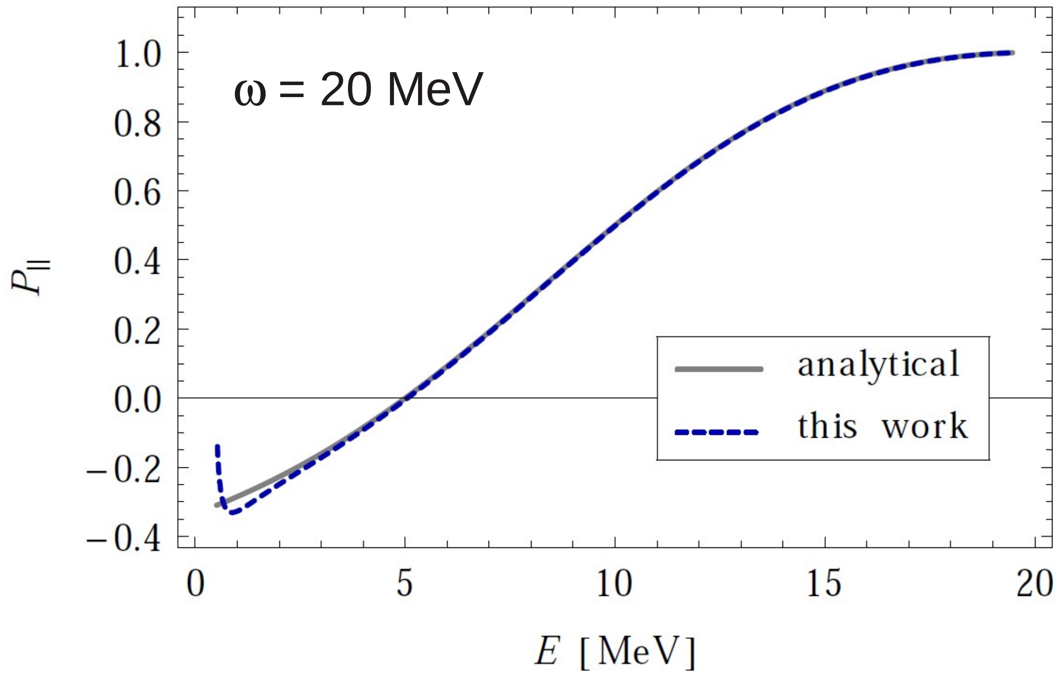
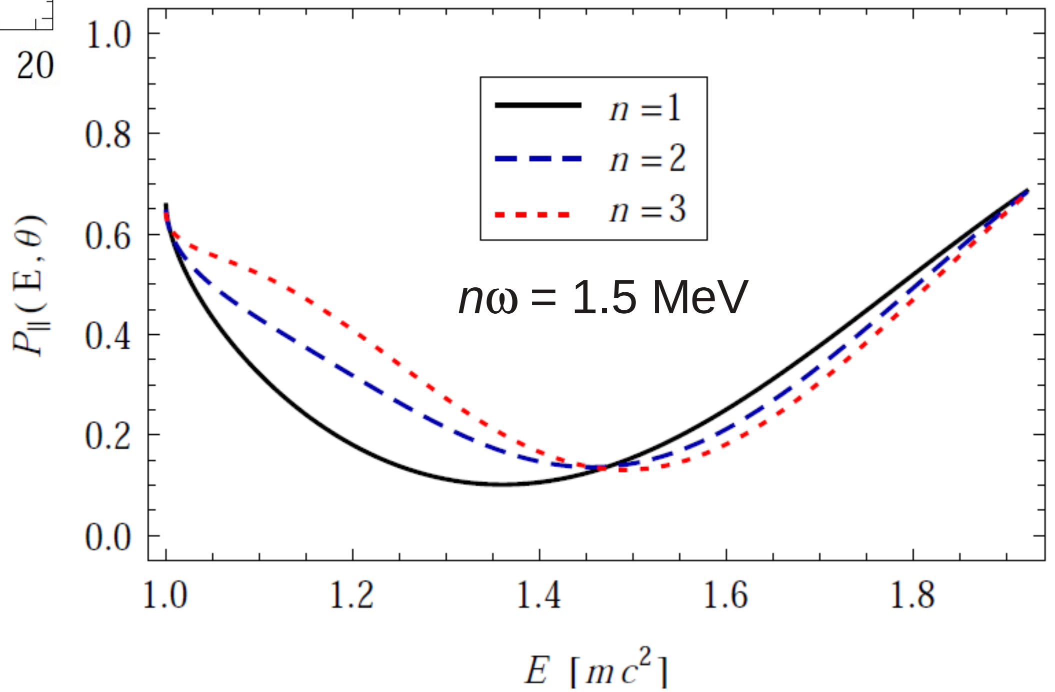


$$P_{\|}(E) = \frac{d\sigma_{\mathrm{R}} - d\sigma_{\mathrm{L}}}{d\sigma_{\mathrm{R}} + d\sigma_{\mathrm{L}}}$$

For a genuinly multiphoton process
this is not longer true...

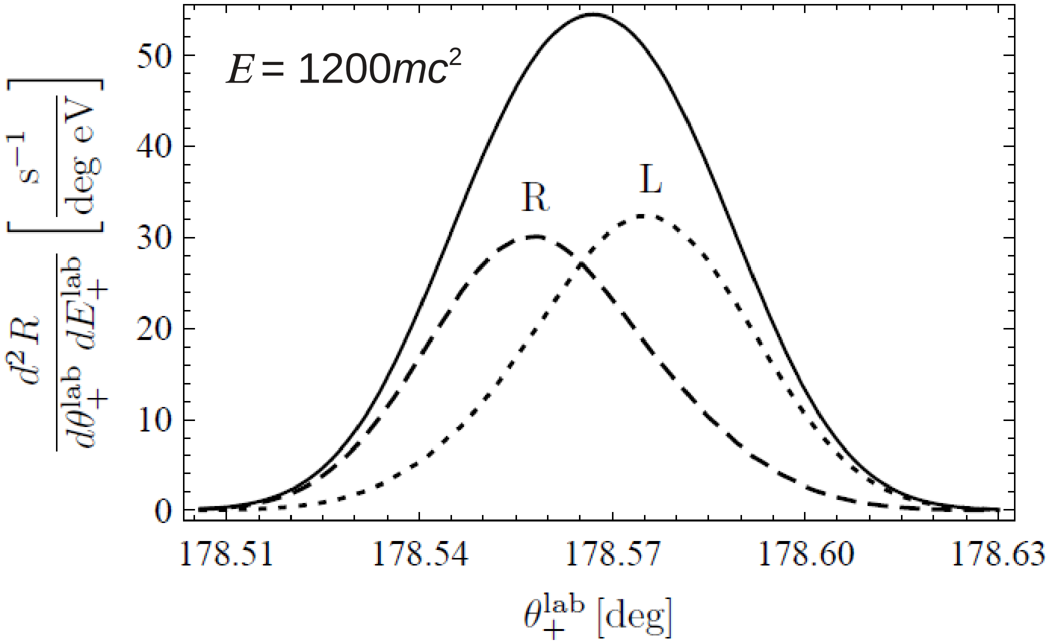


T.-O. Müller & C. Müller, PRA **86**, 022109 (2012)

Pair creation in the quasistatic regime

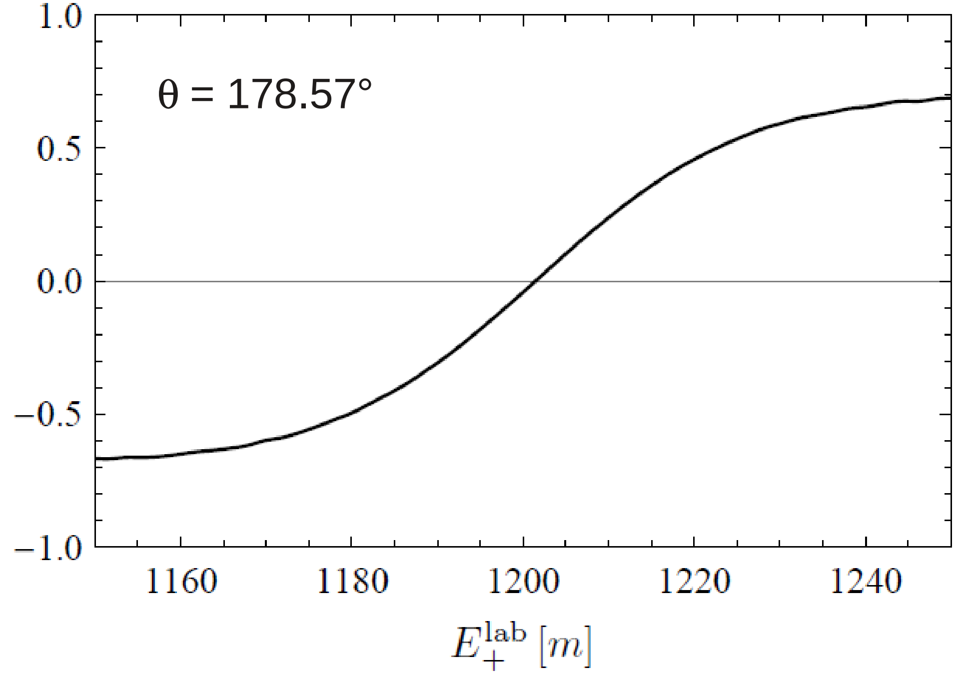
Interesting question: Is the helicity transfer by thousands of circularly polarized photons more efficient than by a single one of the same total energy?

ω_L = 2.5 eV, $I_L = 10^{22}$ W/cm^2 (ξ = 30), γ_p = 3000

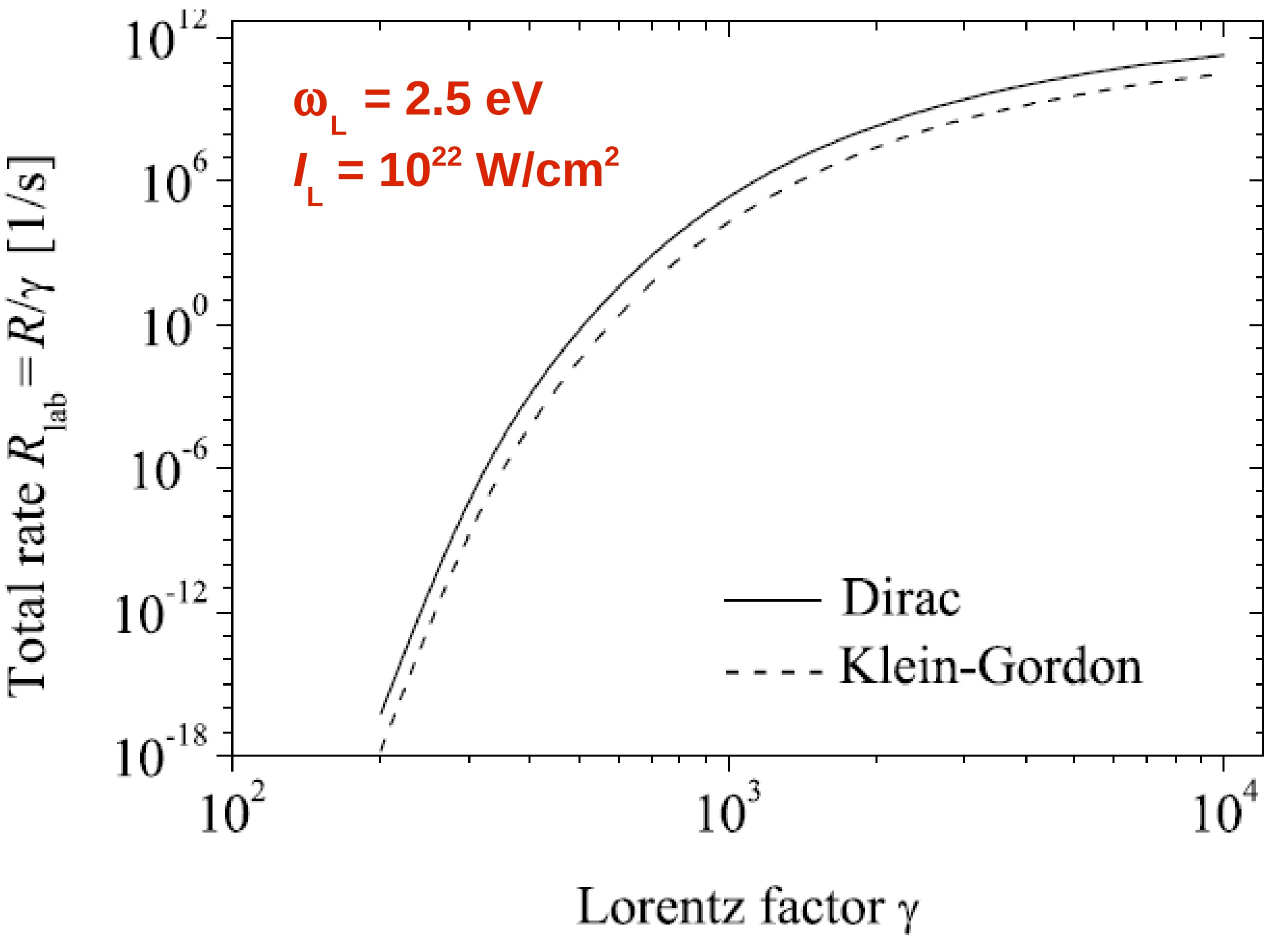


Right-handed positrons emitted under slightly smaller angles than left-handed ones

Attainable degrees of longitudinal polarization smaller than in high-energy limit of Bethe-Heitler pair creation by single γ-photon



Dirac versus Klein-Gordon particles



Electron spin **enhances** rate as compared to spinless particle by factor ~ 7

T.-O. Müller & C. Müller, Phys. Lett. B **696**, 201 (2011)

Same factor found in single-photon case by Pauli & Weiskopf (1934)

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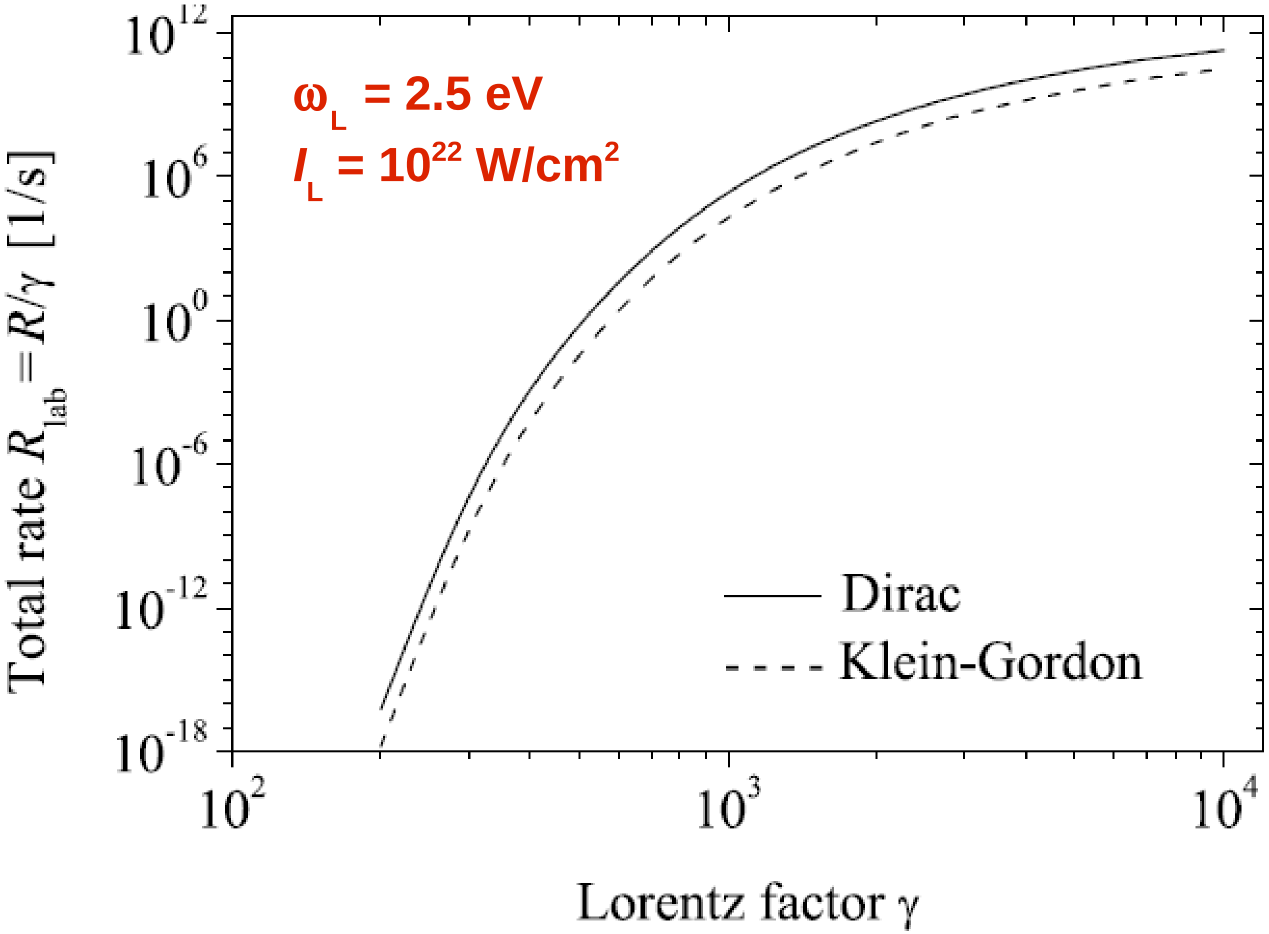

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Same factor found in single-photon case by Pauli & Weiskopf (1934)

Similar result for strong-field Breit-Wheeler process: S. Villalba-Chavez & CM, Phys. Lett. B **718**, 992 (2013)

Internal polarization vector of the positron

Differential rate for multiphoton Bethe-Heitler pair creation can be written as

$$dR/d\boldsymbol{p} = A + \boldsymbol{\zeta} \cdot \boldsymbol{B} \propto (1 + \boldsymbol{\zeta} \cdot \boldsymbol{\zeta}_f)$$

where $\boldsymbol{\zeta}$ is the polarization vector in the positron rest frame corresponding to the spinor $v_{\boldsymbol{p},\lambda}$. The actual **polarization** is then given by

$$\boldsymbol{\zeta}_f = \boldsymbol{B}/A$$

In the quasistatic limit (ξ >> 1) one finds

$$\boldsymbol{\zeta}_f = \left[\frac{\chi}{\sqrt{2}} + \sqrt{6}\frac{(p_\perp - m\xi)^2}{m^2}\right]\frac{\boldsymbol{p}_\perp}{p_\perp}$$

Di Piazza, Milstein & Müller, Phys. Rev. A 82, 062110 (2010)

Summary

- Sizeable spin-flip probability in relativistic strong-field ionization of highly-charged ions
- Kapitza-Dirac scattering involving three laser photons is sensitive to the electron spin
- Helicity transfer in multiphoton Bethe-Heitler pair creation depends on the number of absorbed photons
- For practical purposes of producing polarized positron beams, a single high-energy photon is more suitable
- Both strong-field QED processes are less probable for Klein-Gordon particles

Thank you for your attention!