QUANTUM VACUUM OPTICS

TOM HEINZL

FRONTIERS OF INTENSE LASER PHYSICS
KITP, SANTA BARBARA

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EXTREME FIELDS WITH PLYMOUTH UNIVERSITY

With V. Dinu, A. Ilderton, M. Marklund, O. Schroeder, H. Schwoerer, R Sauerbrey, G. Torgrimsson, A. Wipf

Outline

1. Introduction
2. Light-by-light scattering
3. Vacuum birefringence
4. Vacuum emission
5. Discussion and conclusion

1. Introduction

High intensity lasers provide…

- **Large photon density**
  - Laser = external background field
    - But: Radiation (back) reaction
  - Laser = classical field
    - Nonlinear classical physics
  - Quantum regime: strong-field QED
    - difficult to access

- **Strong fields**
  - Alternating and pulsed
  - Near null (plane waves)
  - Effects can be elusive

- Goal: probe via
  - Matter
  - Light

  in SF QED regime

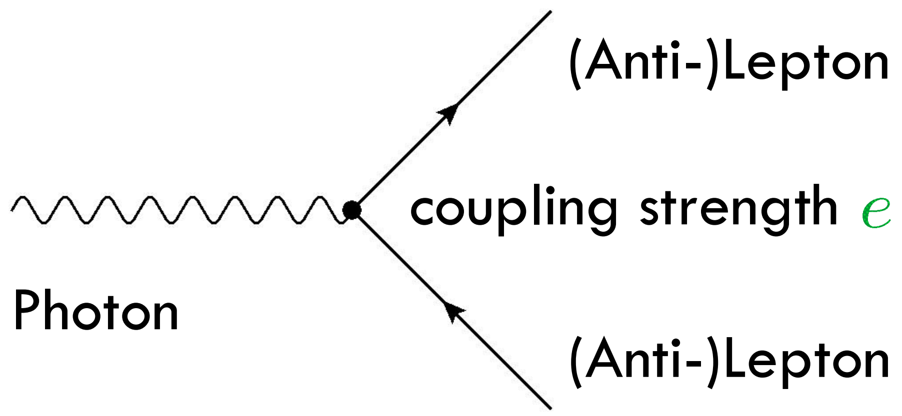
QED I

- Microscopic theory of light & matter: QED
- Parameters:
  - c and $\hbar$: relativistic quantum field theory
  - e and m : electron charge and mass
- Combinations:
  - fine structure constant $\alpha = e^2/4\pi\hbar c = 1/137$
  - Compton wavelength $\lambda\!\!\!^{-}_e = \hbar/mc \simeq 400$ fm
  - QED electric field (Sauter 1931, Schwinger 1951)

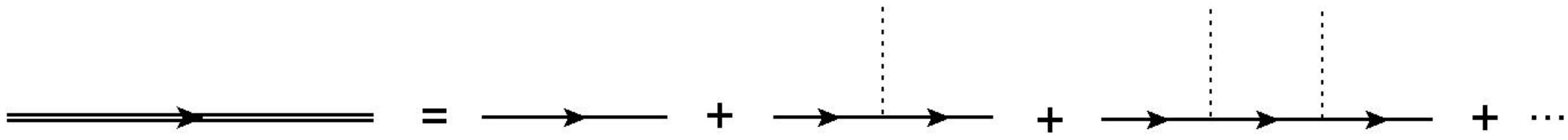
$$E_S = m^2c^3/e\hbar = 1.3 \times 10^{18}\,\mathrm{V/m}$$

QED II

- Elementary interaction (vertex):

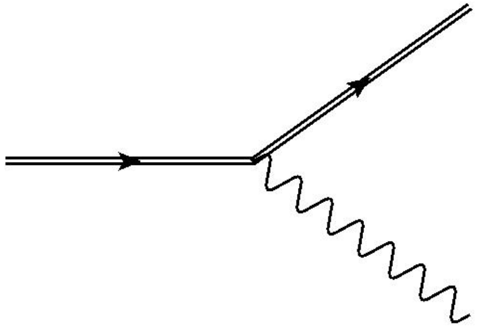


- Coupling to external laser field (··········) yields dressed (Volkov) electron



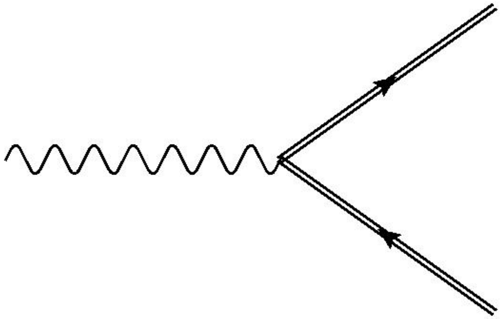
Elementary processes

- Nonlinear Thomson/ Compton scattering



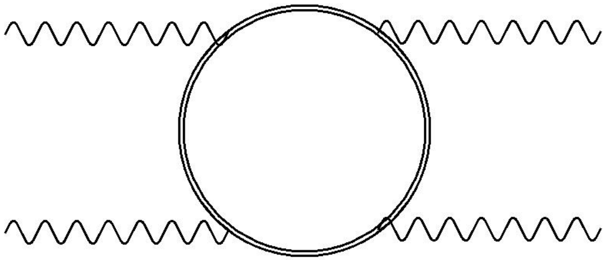
$$e + n\gamma_L \to e' + \gamma$$

- Pair production



$$\gamma + n\gamma_L \to e^+e^-$$

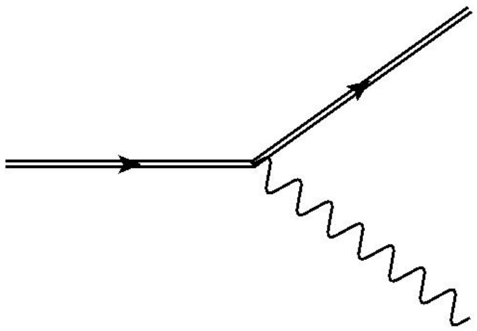
- Light-by-light scattering, e.g.



$$\gamma_1 + \gamma_2 + n\gamma_L \to \gamma_1' + \gamma_2'$$

NL Thomson & Compton

- **Charge laser scattering**
  - Nonlinear when



$$a_0 \simeq \frac{eE\lambdabar}{mc^2} \gtrsim 1$$

  - Classical (Thomson) unless high energy (Compton)

$$\gamma_e \hbar\omega / mc^2 \equiv \gamma_e \nu_0 \gtrsim 1$$ (cf. SLAC E-144)

  - Radiation reaction effects important when

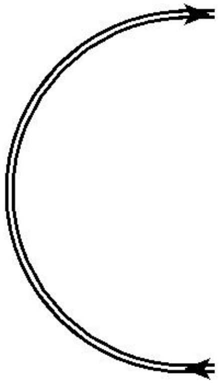
$$N\alpha\, \gamma_e \nu_0\, a_0^2 \equiv N \epsilon_{\text{rad}}\, a_0^2 \gtrsim 1$$

with N = no. of cycles per pulse

(S. Bulanov, TH, M. Marklund et al., arxiv:1310.0152)

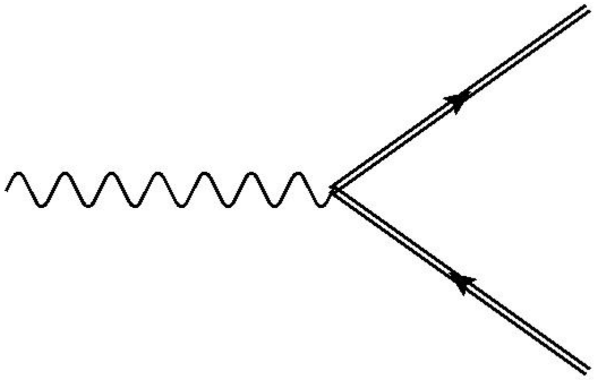
Pair production

- *Vacuum* (Sauter-Schwinger) PP:
  - Zero for PW
  - Nonperturbative (all orders in α)
  - Exp. suppressed, so conservative estimate:



$$I \gtrsim 10^{27}\ \text{W/cm}^2$$

- *Stimulated* PP:
  - Threshold suppressed



$$E_{\text{CM}} \geq 2mc^2(1 + a_0^2)$$

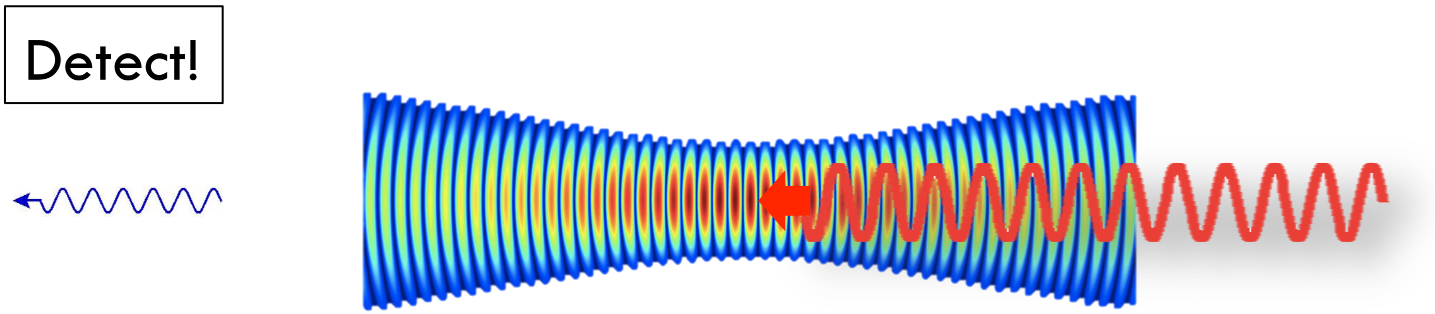
  - Lab:

$$E_\gamma \gtrsim 30\ \text{GeV}$$ (cf. SLAC E-144)

2. Light-by-light (LBL) scattering

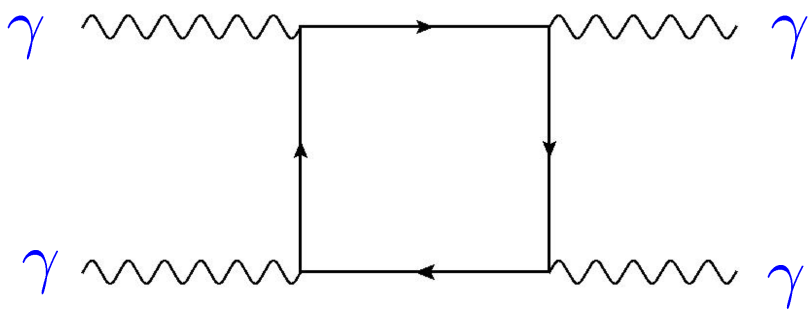
Idea

- Idea: use **purely photonic** probes and targets
  - Target: ultra-intense optical laser focus
    - “infinitely many” photons
  - Probe: suitable photons (e.g. X-ray)
    - High flux, so still “many” photons
  - Result: very “few” scattered photons



QED III

- $\gamma\gamma$ scattering in QED



Interaction of "light with light" mediated through **virtual** matter

- Features:
  - 1-loop: purely quantum
  - UV finite!
  - Small: amplitude $O(\alpha^2)$

Some history

- Halpern 1934:

  … effects as consequences of hitherto unknown properties of corrected electromagnetic equations. We are seeking, then, scattering properties of the "vacuum."

- Euler, Kockel 1935:

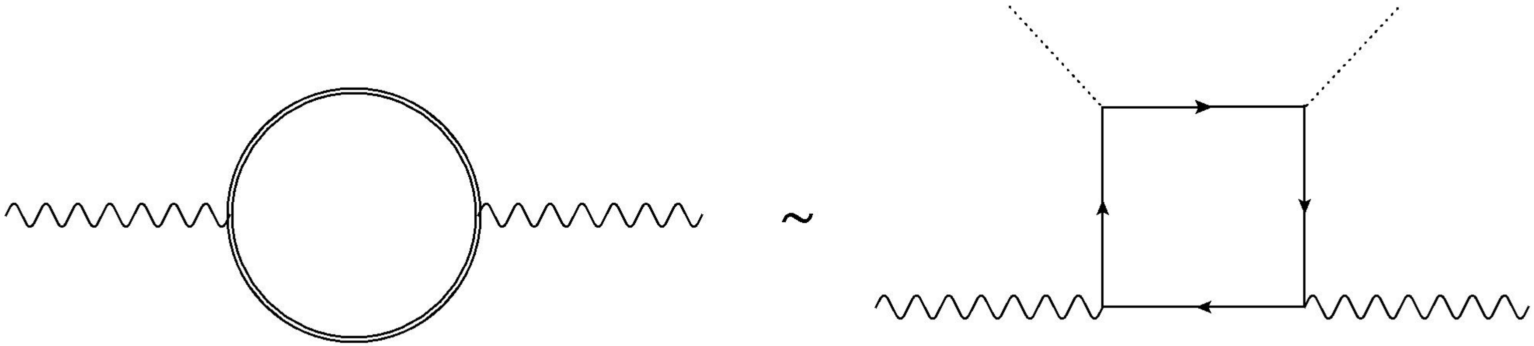
  "Halpern (1934) and Debye (in a discussion with Prof. Heisenberg) have pointed out that according to Dirac's theory there must be scattering of visible light by light. Namely, there are processes in which two light quanta virtually produce a pair (electron and positron) which annihilates immediately afterwards. These processes … can happen even if there is not enough energy to produce a real pair."

“Quantum vacuum optics”

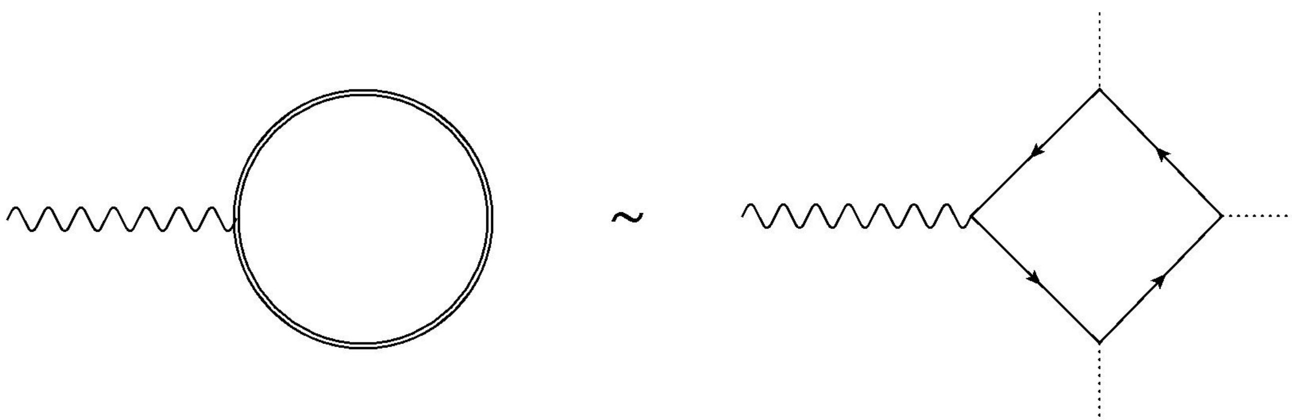
- $\gamma\gamma$ scattering: effects as in light-*matter* interactions
  - Reflection (Gies, Karbstein, Seegert, arxiv:1305.2320)
  - Refraction and birefringence (Toll, 1952)
  - Diffraction (di Piazza, Hatsagortsyan, Keitel, PRL 2006)
  - Light bending or aberration (De Lorenci, Klippert, PRD 2002)
  - Absorptive effects due to PP (Toll, 1952)
  - Nonlinear (optics) effects
    - photon splitting (Adler, Ann. Phys. 1971)
    - wave mixing (Moulin, Bernard, Opt. Commun. 1999, Lundström et al., PRL 2006)
  - …

Examples

- Photon laser scattering: refraction, deflection etc.



- Vacuum emission: 4-wave mixing, etc.



Observation?

- LBL scattering with **real** photons **never observed**
- only for **virtual** photons – Delbrück scattering (off nuclei, $\hbar\omega \gtrsim mc^2$) (Jarlskog et al., PRD 1973; Schumacher et al., PLB 1975)
- **Difficulty:**
  - Low energy: flux large, but cross section too small

$$\sigma_{\gamma\gamma} \simeq 10^{-66}\ \text{cm}^2\ , \quad \hbar\omega \ll mc^2$$

  - High energy: cross section larger, but flux too small

$$\sigma_{\gamma\gamma} \simeq 10^{-31}\ \text{cm}^2\ , \quad \hbar\omega \simeq mc^2$$

  - Current bound (laser regime) (Bernard et al., Eur. Phys. J, 2000)

$$\sigma_{\gamma\gamma} \leq 1.5 \times 10^{-48}\ \text{cm}^2$$

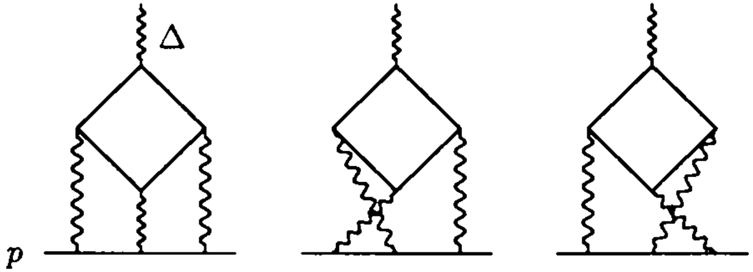
Remark

- Indirect evidence for LBL scattering from anomalous magnetic moments ($g-2$, **virtual** sub-diagrams)
- Deviation from Dirac value 2 known to NNNLO (including 891 4-loop diagrams)

Jegerlehner, Nyffeler, arxiv: 0902.3360

$$\left.\frac{g-2}{2}\right|_{\text{th}} = \frac{1}{2}\frac{\alpha}{\pi} - 0.32848\ldots\left(\frac{\alpha}{\pi}\right)^2 + 1.18124\ldots\left(\frac{\alpha}{\pi}\right)^3 - 1.9144\ldots\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$

- At NNLO: LBL diagrams contribute 30% of $1.18124\ldots!$



$$c_{3,\gamma\gamma} = 0.37100529\ldots$$

Laporta, Remiddi, PLB 1991

3. Vacuum birefringence

(from LBL scattering)

3. Vacuum birefringence

3.1 Generalities

(Some) references

- Theory

J. Toll, PhD thesis, Princeton, 1952

R. Baier and P. Breitenlohner, Nuovo Cim., 1967

N. Narozhny, JETP, 1968

E. Brezin, C. Itzykson, PRD, 1970

TH, B. Liesfeld, K.-U. Amthor, H. Schwoerer, R. Sauerbrey and A. Wipf, Opt. Commun., 2006

G. Shore, NPB, 2007

V. Dinu, TH, A. Ilderton, M. Marklund and G. Torgrimsson, PRD, 2014 (I & II)

…

- Experiment

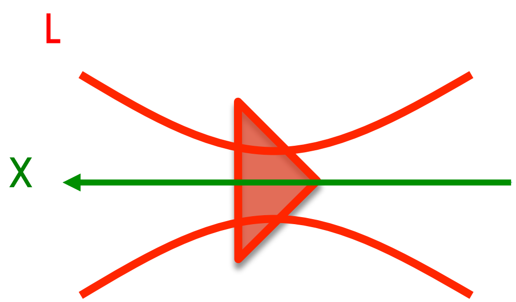
Zavattini et al., PVLAS, arxiv:1201.2309

Rizzo et al., BMV, arxiv:1302.5389

HIBEF @ DESY: in preparation

Scenario

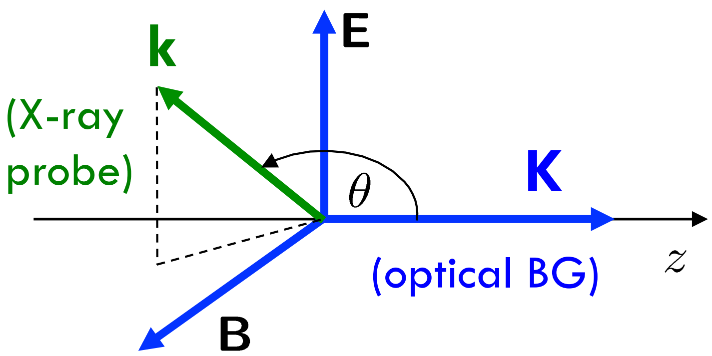
- Probe intense laser focus with X-ray probe



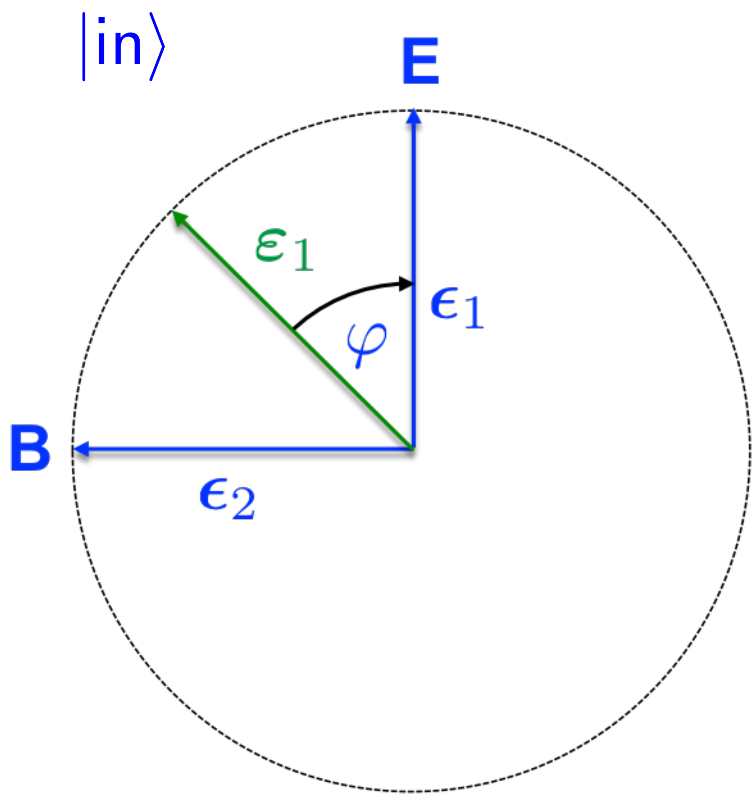
(HIBEF @ DESY – 'flagship experiment')

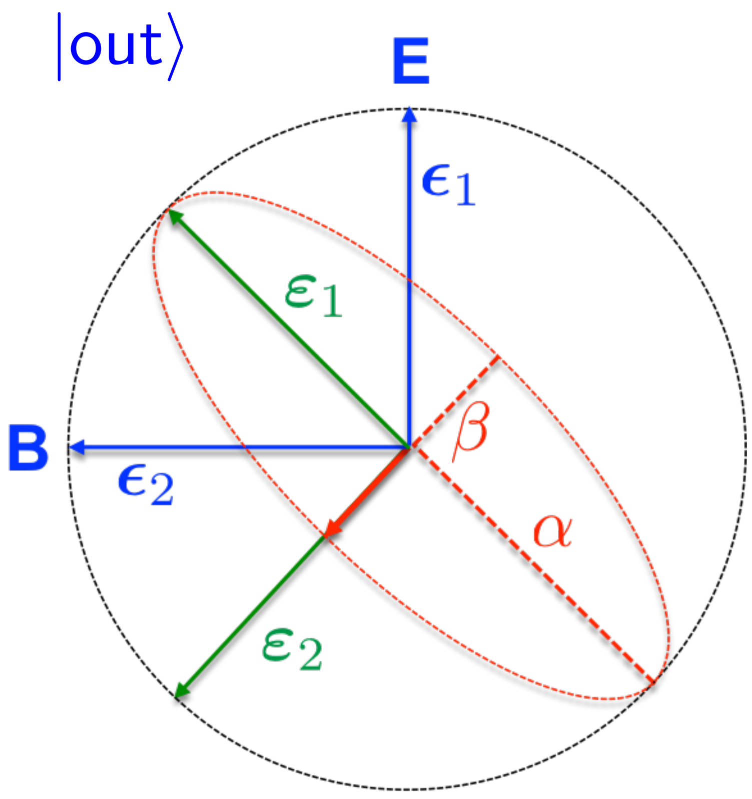
- Optimal: counter-propagation ($\theta = \pi$)

- X-rays:
  - XFEL, e.g. HIBEF @ DESY
  - Via Compton back-scattering from multi-GeV electron beams (e.g. ELI-NP)



Experimental signature: Ellipticity

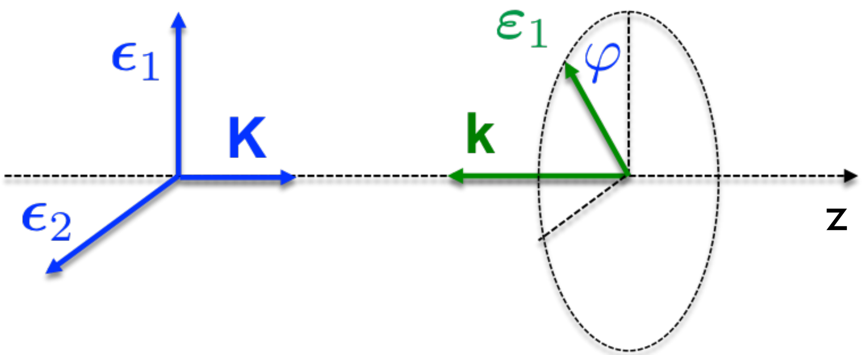




Optimal: $\varphi = \pi/4$

$\delta \equiv \beta/\alpha$

Polarisation transport I



- Transformation of BG basis (cf. G. Baym's text)

$$|\epsilon_i\rangle \to |\epsilon_i'\rangle \equiv U_{ij}(z)|\epsilon_j\rangle$$

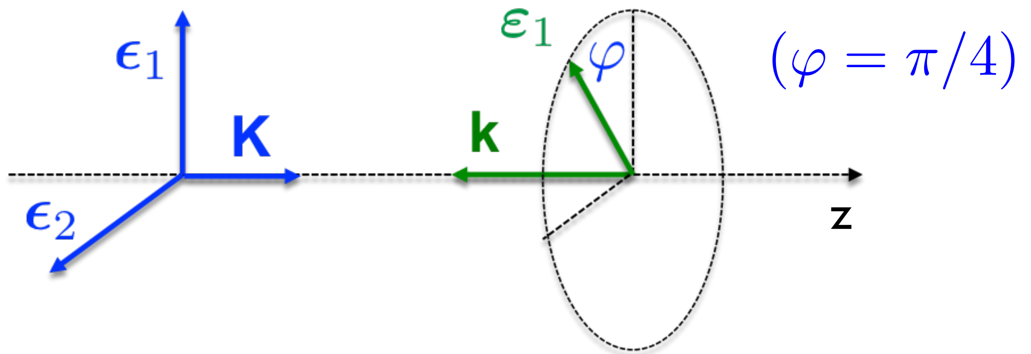
- "Transfer" matrix ($k_0 \equiv \omega_k/c$)

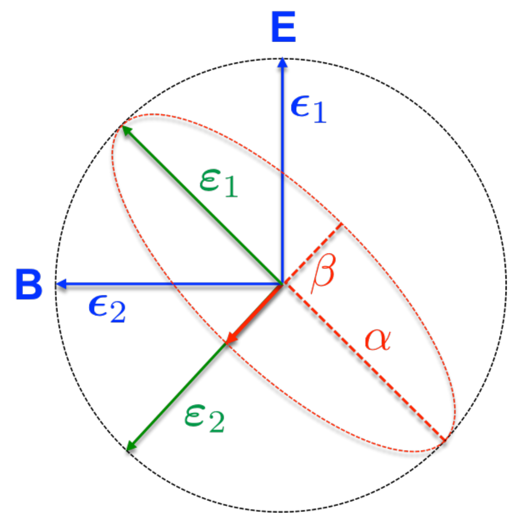
$$U(z) = \exp(ik_0 N z)$$

- Matrix of refractive indices

$$N \equiv \text{diag}(n_1, n_2)$$

Polarisation transport II





- Transformation of **probe** polarisation

$$|\varepsilon_1'\rangle = \alpha|\varepsilon_1\rangle + \beta|\varepsilon_2\rangle$$

- Non-flip amplitude

$$\alpha = \langle\varepsilon_1|\varepsilon_1'\rangle \simeq \exp(ik_0 z)$$

- Flip amplitude

$$\beta = \langle\varepsilon_2|\varepsilon_1'\rangle \simeq \frac{i}{2}\exp(ik_0 z)\,\overbrace{k_0 z\,(n_2 - n_1)}^{\Delta\phi}$$

$\Delta\phi$ ← Phase shift

Ellipticity again

- Ellipticity
  - field strength ratio
  - = amp ratio:

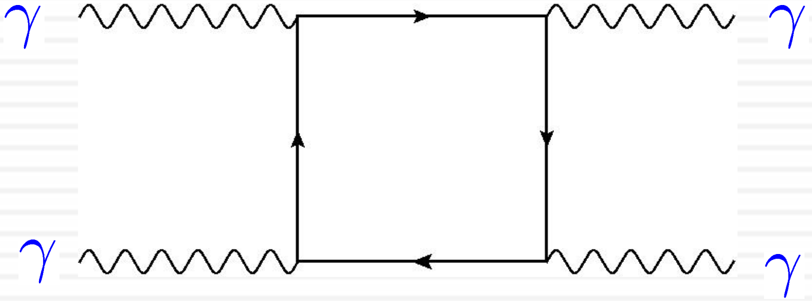
$$\delta = \beta/\alpha = (i/2)\Delta\phi$$

- Observable
  - **intensity** ratio
  - fraction of photons with flipped polarisation:

$$|\delta|^2 = |\beta/\alpha|^2 = (\Delta\phi/2)^2 = N_{\parallel}/N_{\perp}$$

3. Vacuum birefringence

3.2 QED Calculation



Textbook theory I (Akhiezer and Berestetskii)

- **Ingredients:** 4 wave and 4 polarisation vectors, so 4th rank tensors…

$$d\sigma = \frac{1}{(2\pi)^2}\left(\frac{e^2}{4\pi}\right)^4 \frac{1}{16\omega^2}\,|e_{1\mu}e_{2\nu}e_{3\lambda}e_{4\sigma}I_{\mu\nu\lambda\sigma}(-k_1,\,-k_2,\,k_3,\,k_4)|^2\,do \tag{54.9}$$

$$I_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) = \frac{4}{\left(\frac{e^2}{4\pi}\right)^2}\{aS_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) + bR_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4)\}.$$

$$\begin{aligned} S_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) = 4\{&k_{2\mu}k_{1\nu}k_{4\lambda}k_{3\sigma} + k_{3\mu}k_{4\nu}k_{1\lambda}k_{2\sigma} + k_{4\mu}k_{3\nu}k_{2\lambda}k_{1\sigma} \\ &- \delta_{\mu\nu}k_{4\lambda}k_{3\sigma}(k_1k_2) - \delta_{\lambda\sigma}k_{2\mu}k_{1\nu}(k_3k_4) - \delta_{\mu\lambda}k_{4\nu}k_{2\sigma}(k_1k_3) \\ &- \delta_{\nu\sigma}k_{3\mu}k_{1\lambda}(k_2k_4) - \delta_{\mu\sigma}k_{3\nu}k_{2\lambda}(k_1k_4) - \delta_{\nu\lambda}k_{1\sigma}k_{4\mu}(k_2k_3) \\ &+ \delta_{\mu\nu}\delta_{\lambda\sigma}(k_1k_2)(k_3k_4) + \delta_{\mu\lambda}\delta_{\nu\sigma}(k_1k_3)(k_2k_4) + \delta_{\mu\sigma}\delta_{\nu\lambda}(k_1k_4)(k_2k_3)\}, \end{aligned}$$

Textbook theory II (Akhiezer and Berestetskii)

$$\begin{aligned}
R_{\mu\nu\lambda\sigma}(k_1, k_2, k_3, k_4) = {} & k_{4\mu}k_{1\nu}k_{2\lambda}k_{3\sigma} + k_{2\mu}k_{3\nu}k_{4\lambda}k_{1\sigma} + k_{3\mu}k_{1\nu}k_{4\lambda}k_{2\sigma} \\
& + k_{2\mu}k_{4\nu}k_{1\lambda}k_{3\sigma} + k_{4\mu}k_{3\nu}k_{1\lambda}k_{2\sigma} + k_{3\mu}k_{4\nu}k_{2\lambda}k_{1\sigma} + \delta_{\mu\nu}[k_{2\lambda}k_{1\sigma}(k_3k_4) \\
& + k_{1\lambda}k_{2\sigma}(k_3k_4) - k_{2\lambda}k_{3\sigma}(k_1k_4) - k_{4\lambda}k_{1\sigma}(k_2k_3) - k_{4\lambda}k_{2\sigma}(k_1k_3) \\
& - k_{1\lambda}k_{3\sigma}(k_2k_4)] + \delta_{\lambda\sigma}[k_{4\mu}k_{3\nu}(k_1k_2) + k_{3\mu}k_{4\nu}(k_1k_2) - k_{4\mu}k_{1\nu}(k_2k_3) \\
& - k_{2\mu}k_{3\nu}(k_1k_4) - k_{3\mu}k_{1\nu}(k_2k_4) - k_{2\mu}k_{4\nu}(k_1k_3)] + \delta_{\mu\lambda}[k_{1\nu}k_{3\sigma}(k_2k_4) \\
& - k_{3\nu}k_{2\sigma}(k_1k_4)] + \delta_{\nu\lambda}[k_{2\mu}k_{3\sigma}(k_1k_4) + k_{3\mu}k_{2\sigma}(k_1k_4) - k_{2\mu}k_{1\sigma}(k_3k_4) \\
& - k_{4\mu}k_{3\sigma}(k_1k_2) - k_{3\mu}k_{1\sigma}(k_2k_4) - k_{4\mu}k_{2\sigma}(k_1k_3)] + \delta_{\nu\sigma}[k_{2\mu}k_{4\lambda}(k_1k_3) \\
& + k_{4\mu}k_{2\lambda}(k_1k_3) - k_{3\mu}k_{4\lambda}(k_1k_2) - k_{2\mu}k_{1\lambda}(k_3k_4) - k_{4\mu}k_{1\lambda}(k_2k_3) \\
& - k_{3\mu}k_{2\lambda}(k_1k_4)] + \delta_{\mu\sigma}[k_{1\nu}k_{4\lambda}(k_2k_3) + k_{4\nu}k_{1\lambda}(k_2k_3) - k_{1\nu}k_{2\lambda}(k_3k_4) \\
& - k_{3\nu}k_{4\lambda}(k_1k_2) - k_{3\nu}k_{1\lambda}(k_2k_4) - k_{4\nu}k_{2\lambda}(k_1k_3)] + \delta_{\mu\nu}\,\delta_{\lambda\sigma}[(k_1k_4)(k_2k_3) \\
& + (k_1k_3)(k_2k_4)] + \delta_{\mu\lambda}\,\delta_{\nu\sigma}[(k_1k_2)(k_3k_4) + (k_1k_4)(k_2k_3)] \\
& \qquad + \delta_{\mu\sigma}\,\delta_{\nu\lambda}[(k_1k_2)(k_3k_4) + (k_1k_3)(k_2k_4)], \qquad (54.18)
\end{aligned}$$

Effective Lagrangian

- **Simplification: Low energy** $(\ll mc^2)$ + BG
- Use LO Heisenberg-Euler Lagrangian (1936)

$$\mathcal{L}_{\text{HE}} = \mathcal{S} + c_1 \mathcal{S}^2 + c_2 \mathcal{P}^2 \,, \qquad \left\{ \begin{array}{c} c_1 \\ c_2 \end{array} \right\} = \frac{4\alpha^2}{45\, m^4} \left\{ \begin{array}{c} 4 \\ 7 \end{array} \right\}$$

- with basic invariants

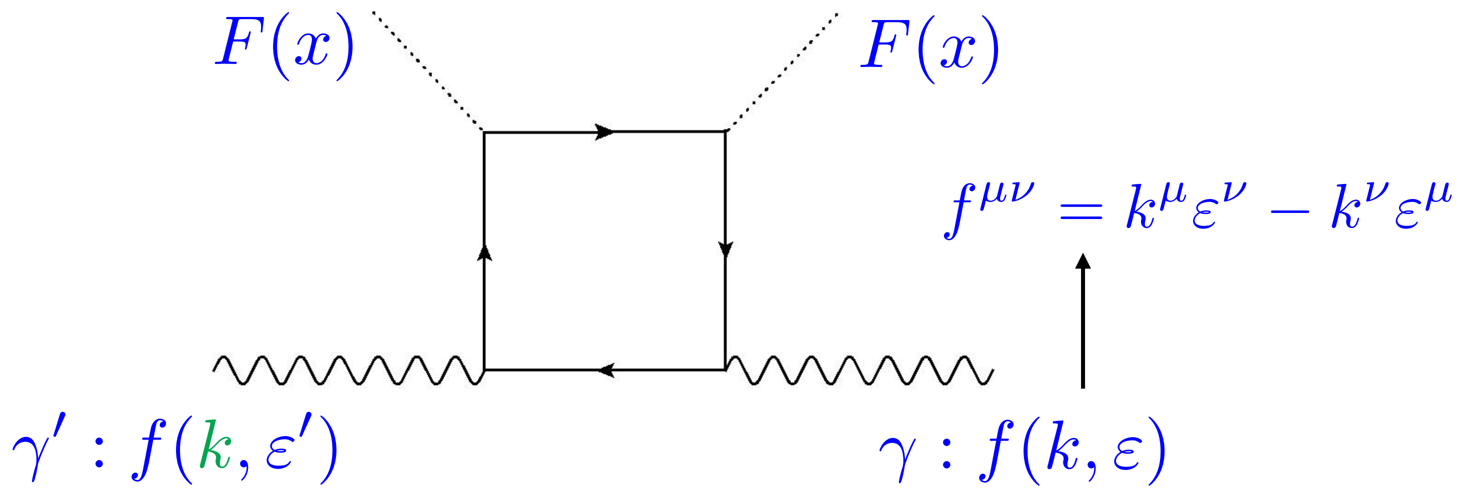
$$\mathcal{S} \equiv \frac{1}{4} \text{tr}\, F^2 = (E^2 - B^2)/2$$

$$\mathcal{P} \equiv \frac{1}{4} \text{tr}\, F\tilde{F} = \mathbf{E} \cdot \mathbf{B}$$

- F, $\tilde{F}$ field strength tensor and its dual

Scattering amplitude I

- Lowest order LBL scattering (**off BG**)



- Need **forward** scattering amplitude $(k' = k)$:

$$S_{\text{fi}} = \langle k, \varepsilon' | S | k, \varepsilon \rangle$$

Scattering amplitude II

- Standard Feynman rules yield equivalent rep.s:

$$S_{\mathrm{fi}} \sim c_1 \,\mathrm{tr}\,(Ff)\,\mathrm{tr}\,(Ff') + c_2\,\mathrm{tr}\,(\tilde{F}f)\,\mathrm{tr}\,(\tilde{F}f')$$

$$\sim c_1(\varepsilon, Fk)(Fk, \varepsilon') + c_2(\varepsilon, \tilde{F}k)(\tilde{F}k, \varepsilon')$$

$$\equiv c_1(\varepsilon, b_k)(b_k, \varepsilon') + c_2(\varepsilon, \tilde{b}_k)(\tilde{b}_k, \varepsilon')$$

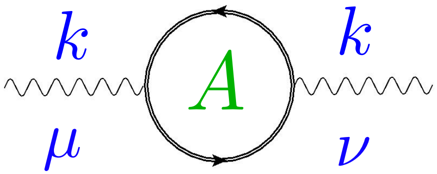
$$\equiv (\varepsilon, \Pi(k)\varepsilon')$$

with **polarisation tensor** $\Pi^{\mu\nu}(k)$

‘Traditional’ analysis (Toll 1952)

- VacPol tensor

$$\Pi^{\mu\nu}(k;A) =$$



  - **Two** nontrivial eigenvalues

$$\Pi_{1,2} = -c_{1,2}\, b_k^2 \equiv c_{1,2}(k, Tk)$$

  with $T =$ **energy momentum tensor**

  - **Two** dispersion relations (‘deformed LC’)

$$k^2 + \Pi_{1,2} = (g^{\mu\nu} + c_{1,2}T^{\mu\nu})k_\mu k_\nu = 0$$

  - **Two** indices of refraction (Toll 1952; Narozhny 1968; Brezin, Itzykson 1970)

$$n_{1,2} = 1 + \Pi_{1,2}/2\omega_k^2$$

Scattering analysis

- Non-flip amplitude:

$$A_{11} = 1 + \langle \varepsilon_1 | S | \varepsilon_1 \rangle = 1 + 2F^2 k_0 z \,(c_1 + c_2) = 1 + O(\alpha^2)$$

- Flip amplitude:

$$A_{12} = \langle \varepsilon_2 | S | \varepsilon_1 \rangle = 2F^2 k_0 z \,(c_1 - c_2)$$

0 for BI !

- Ellipticity:

$$\delta = A_{12} = (1/2) k_0 z \; (n_1 - n_2)$$

Exp. feasibility (TH et al., Opt. Commun., 2006)

- Rewrite ellipticity

$$\delta = \frac{4\alpha}{15}\frac{z}{\lambda}\epsilon_L^2 \, , \qquad \epsilon_L^2 \equiv \frac{E^2}{E_S^2}$$

- Optimal scenario: XFEL ($\lambda \simeq 10^2 \lambda_e$) & HP laser
  - HIBEF ($\epsilon_L \simeq 10^{-4}$): $\delta^2 \simeq 10^{-11}$
  - ELI ($\epsilon_L \simeq 10^{-2}$): $\delta^2 \simeq 10^{-7}$
  - X-ray polarimetry:
    - Current record in polarisation purity: 2.4×10^{-10} @ 6.5 keV
      (Marx, Uschmann, Paulus et al., PRL **110**, 2013)

3. Vacuum birefringence

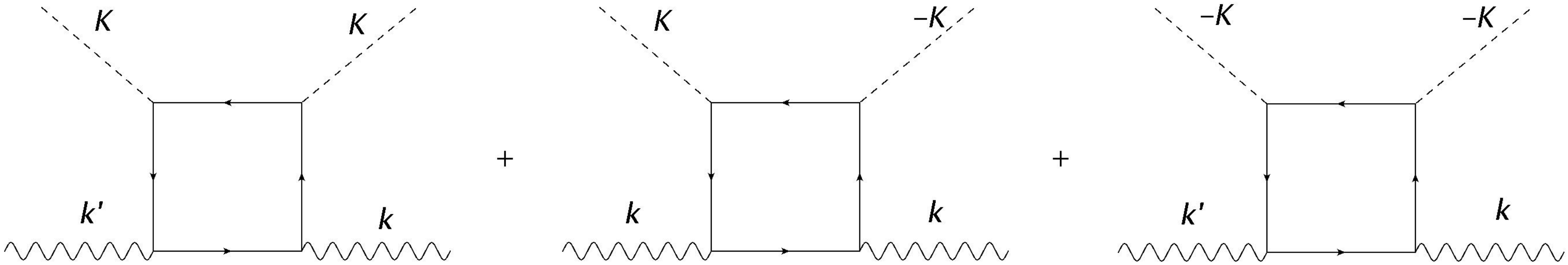
3.3 Generalisations

Directional and BG effects

- Non-forward (e.g. reflection, deflection,…)
- Non-constant BG, e.g. **pulsed** PW: finite duration

$$S_{\mathrm{fi}} = \langle k', \varepsilon' | S | k, \varepsilon \rangle \sim \varepsilon_\mu k_\alpha W^{\mu\alpha,\nu\beta}(q)\, \varepsilon'_\nu k'_\beta$$

  - $q = k' - k$ momentum transfer
  - $W^{\mu\alpha,\nu\beta}(q)$ F.T. of $c_1 FF + c_2 \tilde{F}\tilde{F}$: 'intensity **form factor**'



I. Affleck, J. Phys A21 (1988)

Finite size effects: Gaussian beams

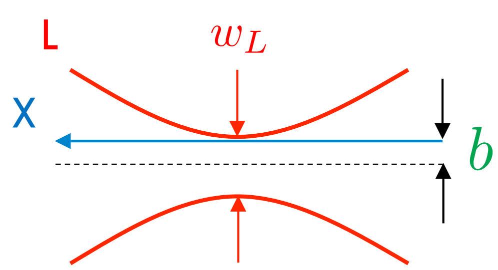
- finite longitudinal and transverse size (z_0 , w_0):
  - Rayleigh length and waist:

$$z_0 = w_0^2/\pi\lambda$$

  - Small parameter: beam divergence

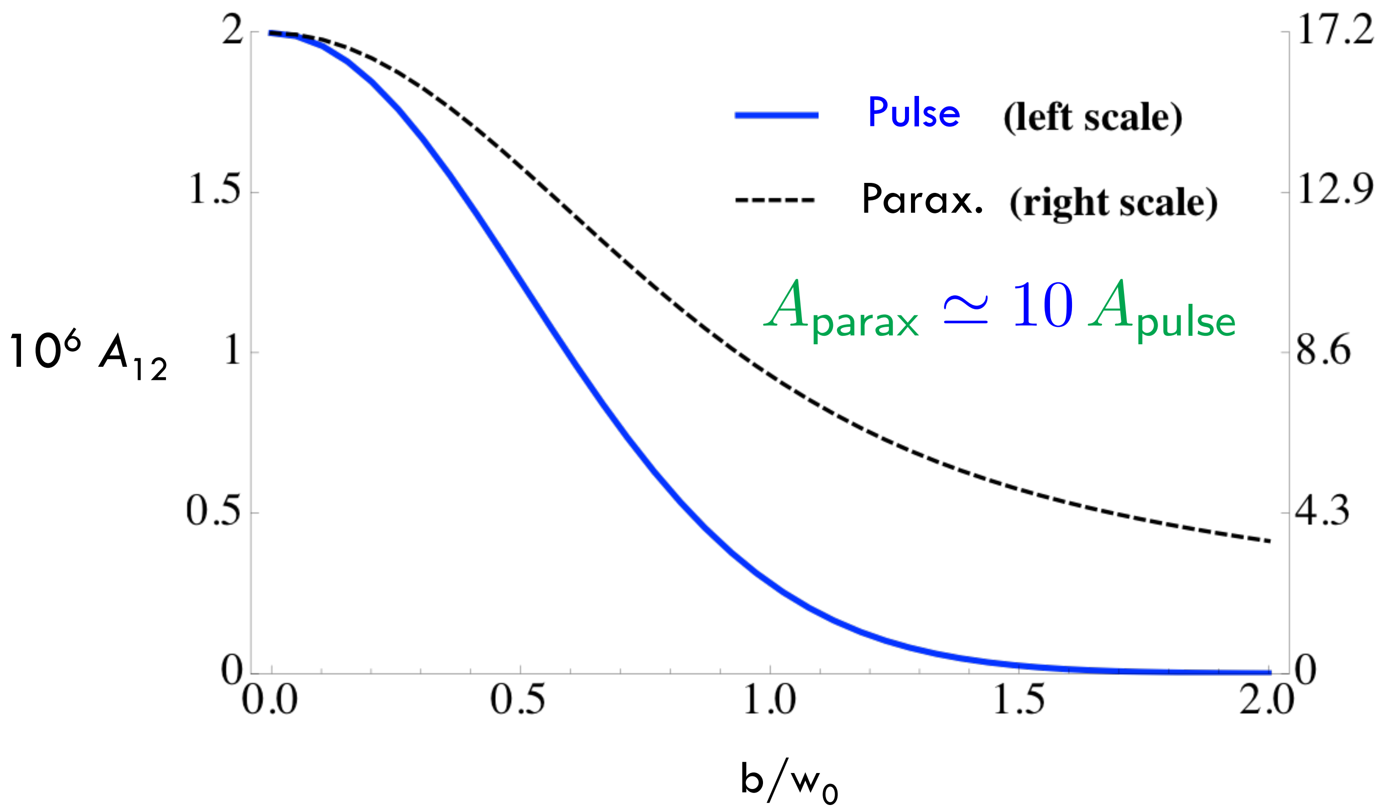
$$\sigma \equiv w_0/z_0 \lesssim 1/\pi$$

  - Paraxial approximation: $O(\sigma^0)$
  - New phase shift: $\Delta\phi = \Delta\phi(b)$
  - Dependence on impact parameter
  - Most realistic: finite **space-time** extent
  - Dependence on pulse duration



Finite pulse effects

- Graph (V. Dinu, TH, A. Ilderton, M. Marklund and G. Torgrimsson, PRD **89**, **90** (2014))
  - Flip amp. A_{12} – paraxial vs. pulsed Gaussian beam

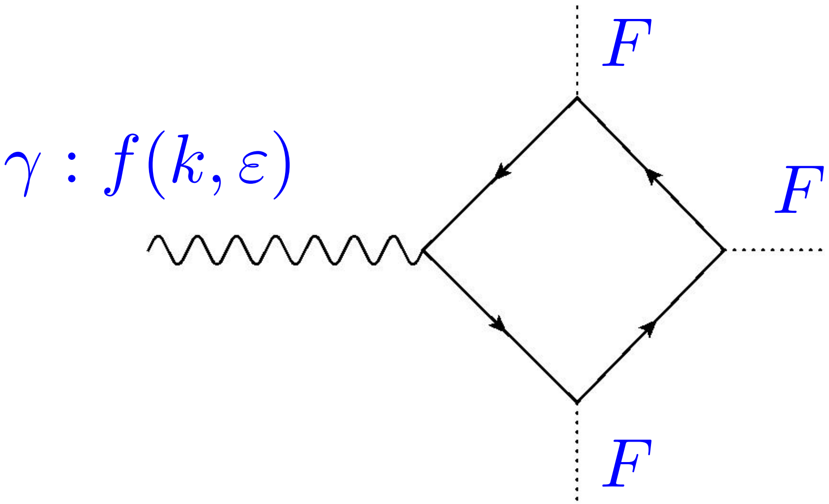


Vacuum Emission

(work in progress)

Scattering amplitude I

- LO Feynman diagram:



- Scattering amplitude:

$$S_{\mathsf{fi}} = \langle k, \varepsilon | S | \mathsf{vac} \rangle$$

Scattering amplitude II

- Feynman rules yield:

$$S_{\mathrm{fi}} \sim c_1\,\mathcal{S}\,\mathrm{tr}\,(F\,f) + c_2\,\mathcal{P}\,\mathrm{tr}\,(\tilde{F}\,f)$$

$$\sim (c_1 - 2c_2)\,\mathcal{S}\,\mathrm{tr}\,(F\,f) + 4c_2\,\mathrm{tr}\,(F^3 f)$$

- Zero for plane waves where invariants

$$\mathcal{S} = \mathcal{P} = 0$$

- Use e.g. Gaussian beams instead…

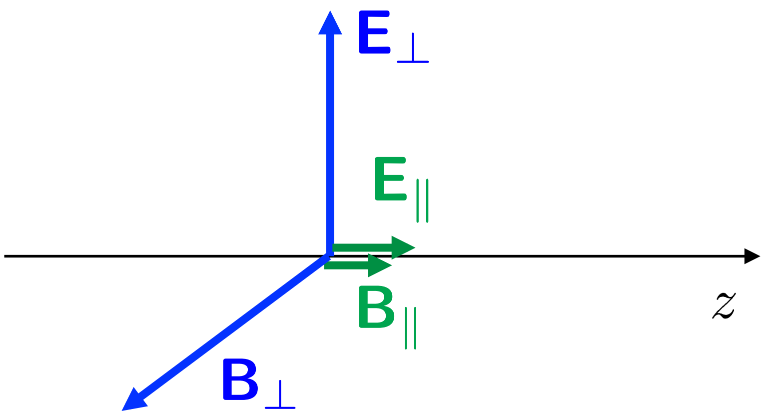
Gaussian beams

- recall parameter: beam divergence $\sigma \equiv w_0/z_0 \lesssim 1/\pi$
- Field strength tensor = "deformation" of PW

$$F^{\mu\nu} = F \begin{pmatrix} 0 & -1 & 0 & \frac{2i\sigma x}{1+2iz} \\ 1 & 0 & \frac{2i\sigma y}{1+2iz} & 1 \\ 0 & -\frac{2i\sigma y}{1+2iz} & 0 & 0 \\ -\frac{2i\sigma x}{1+2iz} & -1 & 0 & 0 \end{pmatrix} + c.c.$$

- Invariants nonzero:

$$\mathcal{S},\ \mathcal{P} = O(\sigma^2)$$



Outlook and Conclusion

Summary

- Nonlinear scattering and PP:
  - Quantum regime difficult to reach
  - Need high energy and/or extreme intensity

- Light-by-light scattering:
  - **Low-energy** quantum regime
  - still small cross sections: $O(\alpha^4)$
  - Wealth of effects: quantum vacuum optics

Conclusion

- **Theory**
  - Perform *systematic* study of vacuum optics effects
  - Identify most feasible/interesting of these

- **Experiment**
  - New strong-field QED experiment urgently needed!
  - Vacuum birefringence experiment feasible @ $10^{22}\ \mathrm{W/cm^2}$ (HIBEF)
    - Requires careful optimisation and fine tuning of parameters
    - but at current sensitivity limits

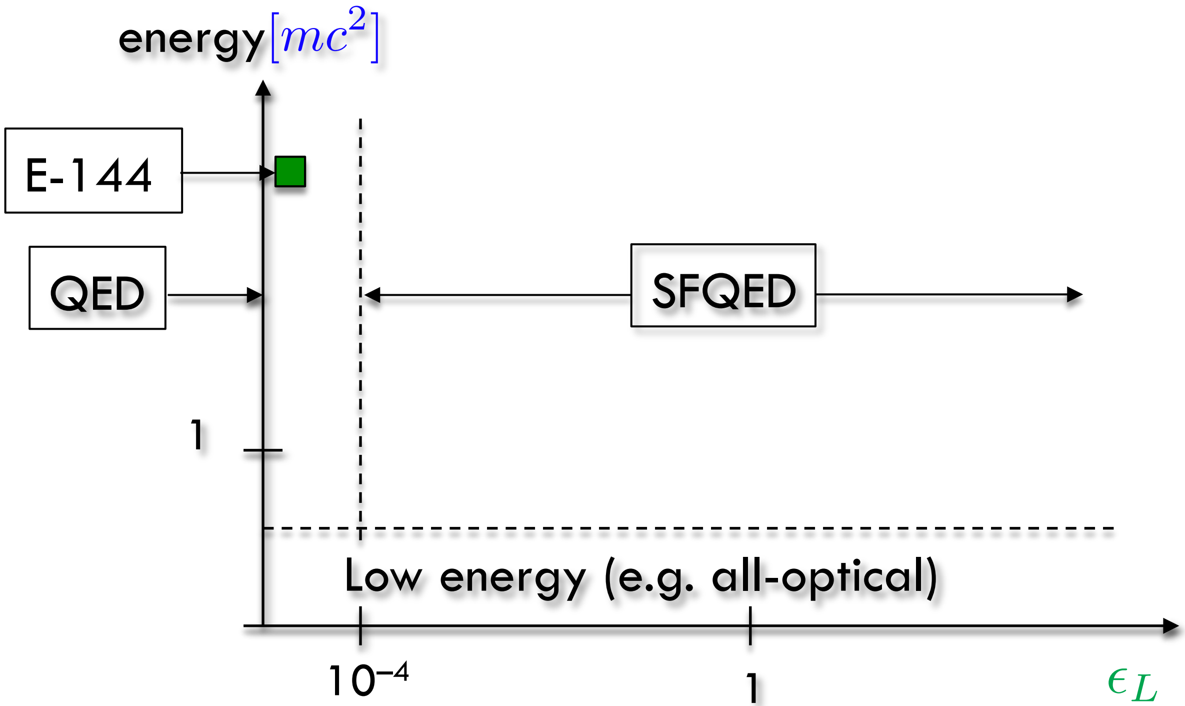
Thank you very much…

…for your attention!

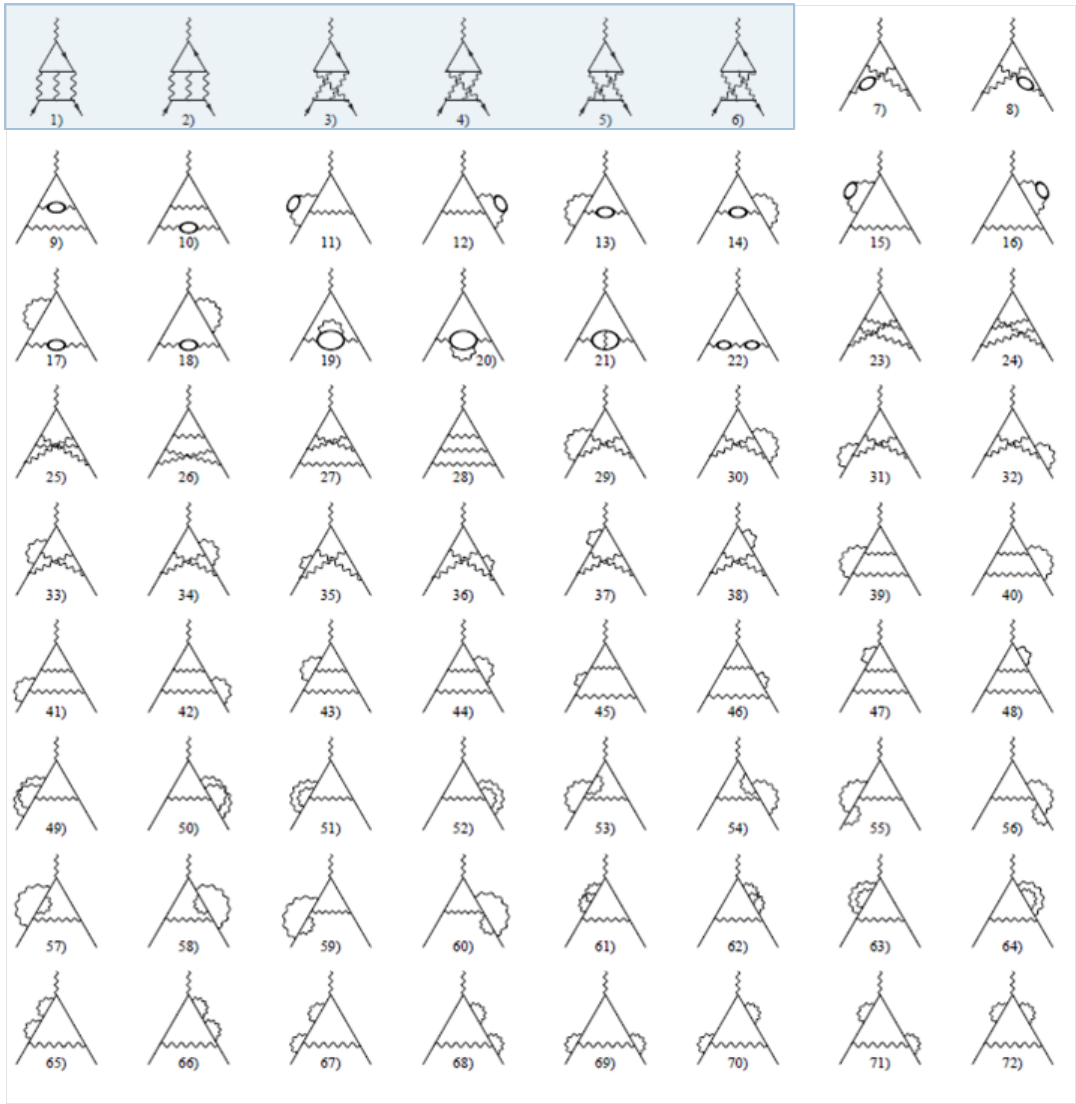
Appendix

Parameters

- Simultaneous expansion in α , ν_X and ϵ_L
  - Large external field parameter ϵ_L :
  - Incarnation of strong-field QED – unprobed region of SM!



g – 2: NNLO


1) 2) 3) 4) 5) 6) 7) 8)
9) 10) 11) 12) 13) 14) 15) 16)
17) 18) 19) 20) 21) 22) 23) 24)
25) 26) 27) 28) 29) 30) 31) 32)
33) 34) 35) 36) 37) 38) 39) 40)
41) 42) 43) 44) 45) 46) 47) 48)
49) 50) 51) 52) 53) 54) 55) 56)
57) 58) 59) 60) 61) 62) 63) 64)
65) 66) 67) 68) 69) 70) 71) 72)

LBL and g – 2

- Numerical values:

$$\left.\frac{g-2}{2}\right|_{\text{th}} = 0.00115965218279(771)$$

(Th: Kinoshita et al., 2008)

$$\left.\frac{g-2}{2}\right|_{\text{exp}} = 0.00115965218073(28)$$

(Exp: Gabrielse et al., 2008)

$$\left.\frac{g-2}{2}\right|_{\gamma\gamma=0} = 0.001159964207$$

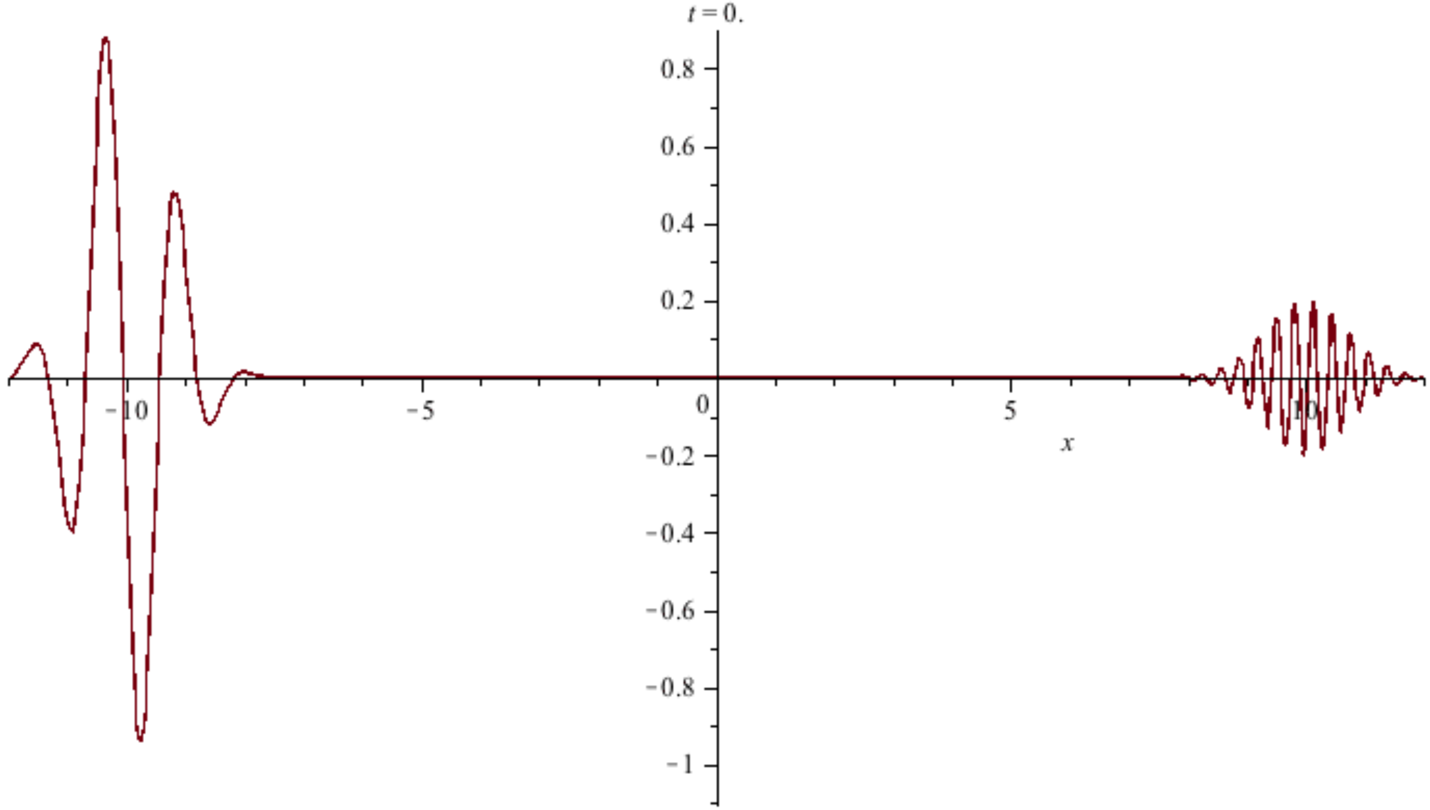
(LBL contribution = 0)

2 3 4

- **NB:** additional LBL terms at (numerically known) NNNLO

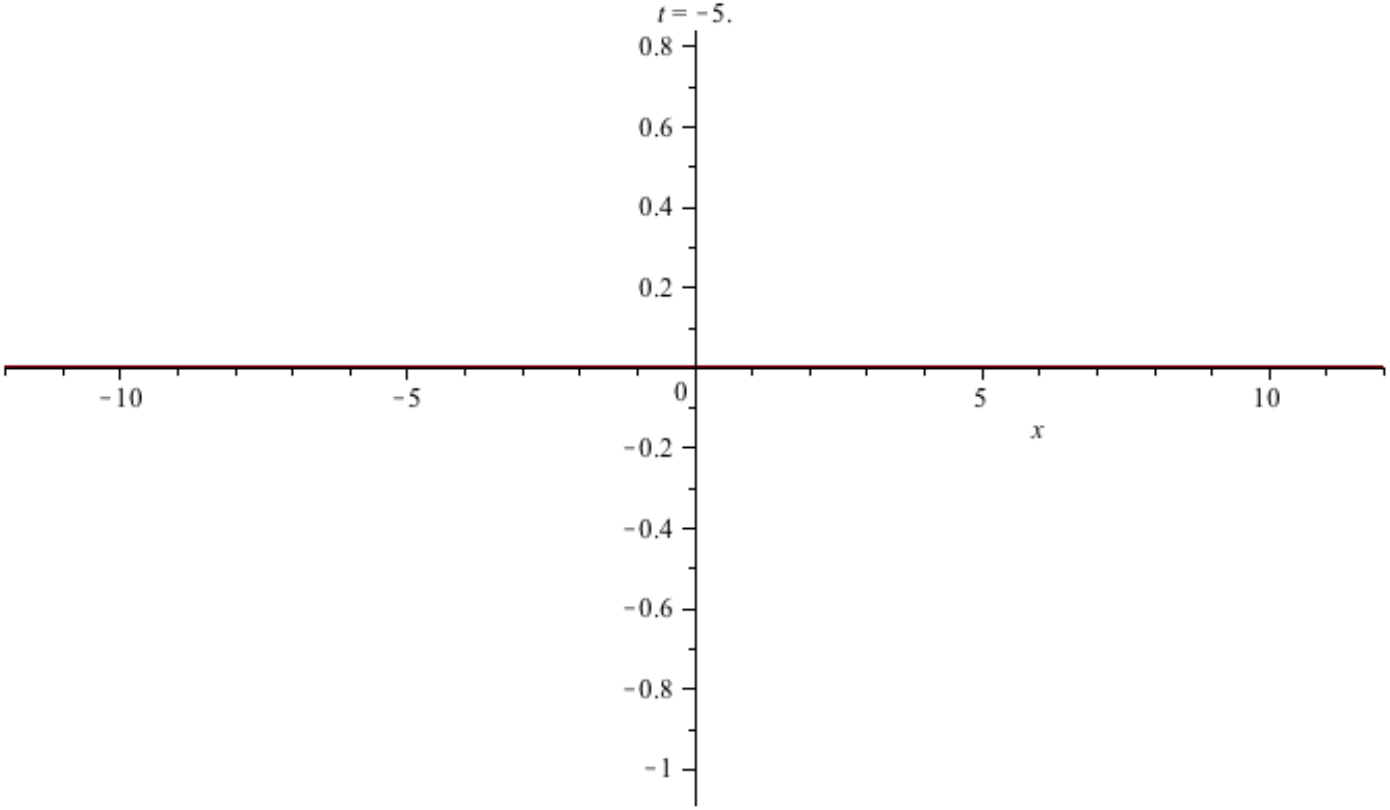
ϵ_L

Colliding pulsed plane waves



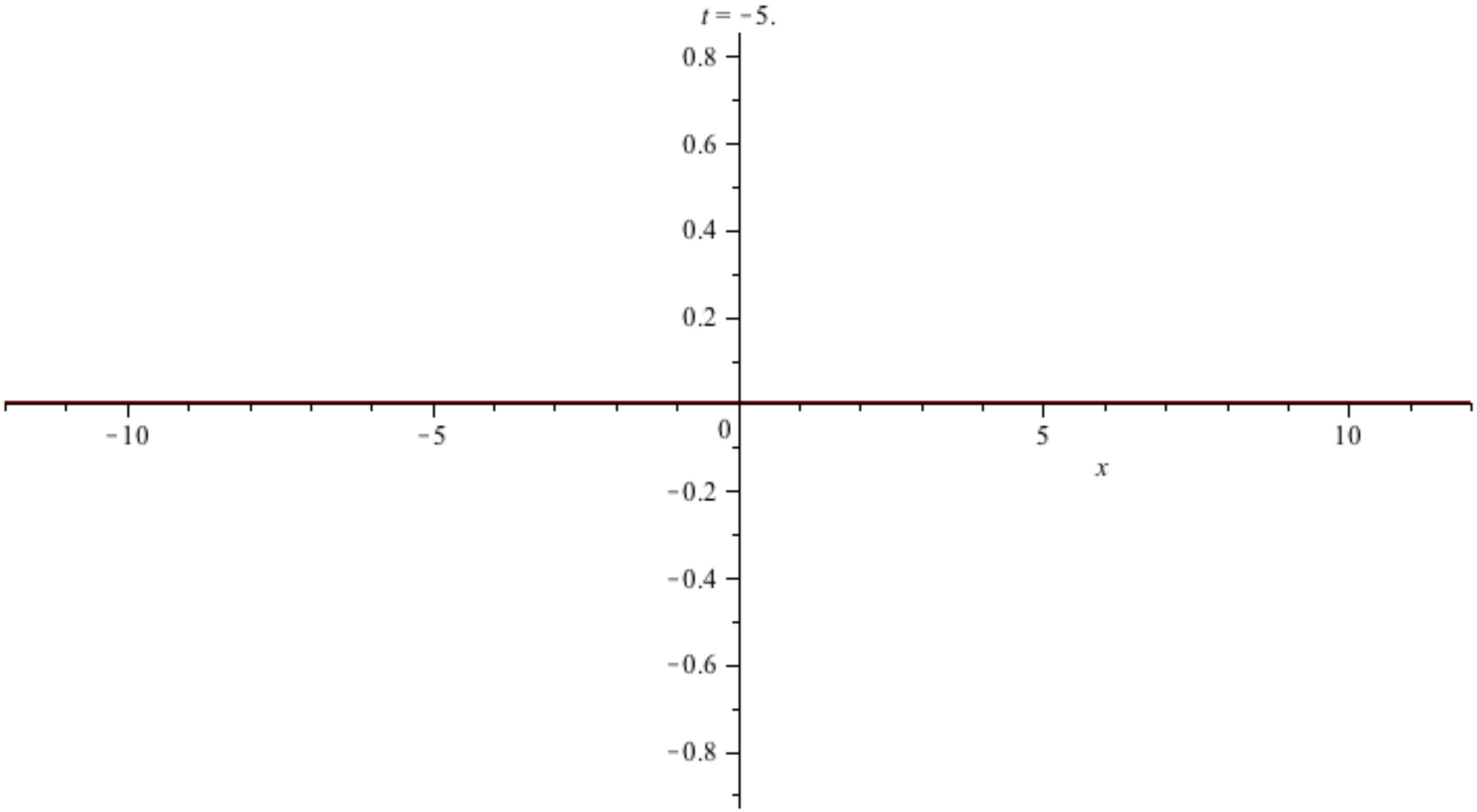
Eternal lifetime – unrealistic !

Colliding Gaussian beams I



Good space-time overlap

Colliding Gaussian beams II



Bad space-time overlap – jitter !