8/1/08

Rozansky, Saulina, A.K.

3d TFT

1. Chern - Simons
2. CS w/ finite group
3. Top gauge theories
4. Top σ-model $M_3 \to X$

Rozansky - Witten model

$$\phi : M_3 \longrightarrow (\underset{2n}{X}, \Omega)$$

holomorphic symplectic form

$$\eta \in \phi^* T_X$$
$$\rho \in \phi^* T_X \otimes \Omega^1_{M_3}$$

$$Q = \int_{M_3} \left(\eta \frac{\delta}{\delta \phi} + d\phi \frac{\delta}{\delta \rho} \right)$$

$$S = \int_{M_3} \| d\phi \|^2 + \ldots$$

$$= \int Q(\cdot) + \Omega(\underset{-1}{\rho}, \nabla\rho + \frac{1}{3} \underset{+1}{R} \eta\rho\rho)$$

Riemann tensor

$$x(A, dA) + x(A^3)$$

tangent vector fields analog of Lie algebra
have only $\mathbb{Z}/2\mathbb{Z}$ grading

Path integral localizes on $d\phi = 0$

Cate

RW model on $M_3 = S^1 \times \Sigma$

is equivalent to B-model on Σ

$S^2 \rightsquigarrow H^\bullet(\mathcal{O}_X)$ (space of local operators)

$\Sigma_1 \rightsquigarrow H^\bullet(\Omega^\bullet_X) \cong H^\bullet(\Lambda^\bullet TX)$

$\Sigma_g \rightsquigarrow H^\bullet(\Lambda^\bullet \Omega_X^{\oplus g})$

$S^1 \rightsquigarrow D_{2\text{-per}}(Coh(X))$

Physical version:

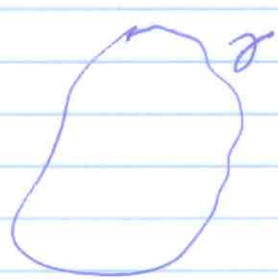
Ob: $\mathbb{Z}_2$-graded smooth VB $E = E_+ \oplus E_-$

$\bar{D} : \Gamma(E) \to \Gamma(E \otimes \Omega^{0,\text{odd}})$
$\bar{D}^2 = 0$

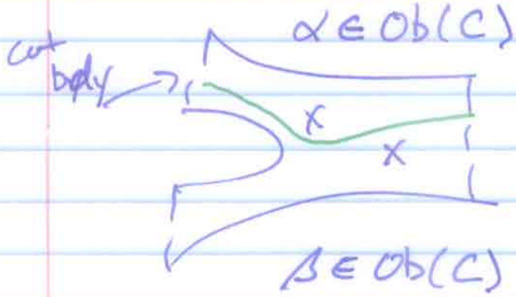
$E, \bar{\partial}$

$\Big\downarrow$

$\mathrm{Hol}_\gamma(\phi^* A) + \ldots$



$\subset M_3$



x local insertion

— line " "

Cut n-structure

$\overline{M}^n = (M_0 \hookrightarrow M_1 \hookrightarrow M_2 \hookrightarrow \dots \hookrightarrow M_n)$

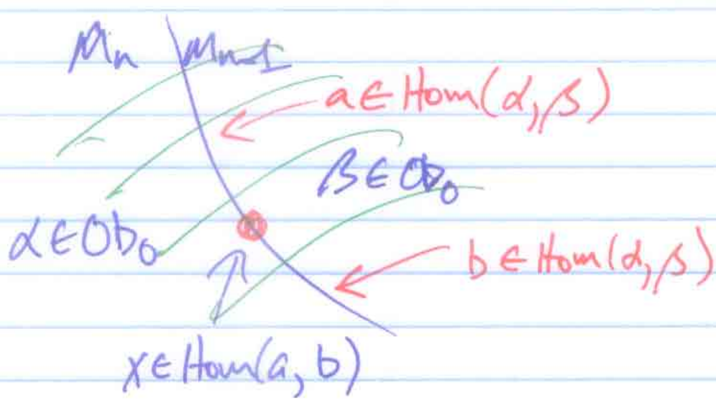
$\partial \overline{M}^n = \partial_b M_n \cup \partial_{cut} M_n$

$\partial_b M_n \subset M_{n-1}$

$\partial M_i \subset \partial_{cut} M_n$

$\partial_{cut} \partial_{cut} M_n = 0^{0<n}$

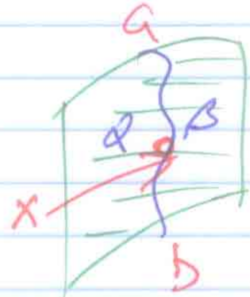
$C^N \quad Ob_0, Ob_1, \dots Ob$

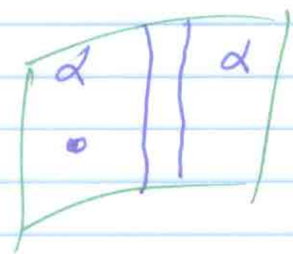


Ex

Ob_0 : all CY manifolds

$$\mathrm{Hom}(X, Y) = D^b(\mathrm{Coh}(X \times Y))$$





2-category of bdry conditions

0th approximation

Ob_1: complex Lagrangian submanifolds (and CY)

Let Y be such submanifold of X

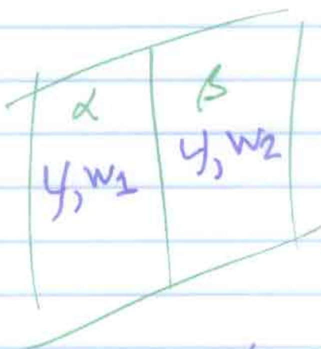
$$\mathrm{Hom}(Y, Y) = D_{2\text{-per}}(Y)$$

$$id = \mathcal{O}_Y$$

$$\mathrm{Hom}(y, y) = H^{\bullet}(\mathcal{O}_y)$$

1^{st} approximation

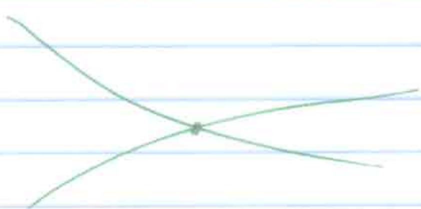
$$(Y,\ W \in H^{even}(\mathcal{O}_Y))$$

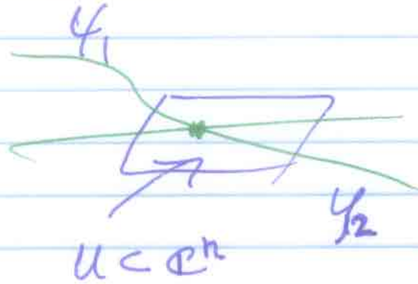


$\mathrm{Hom}((Y, w_1), (Y, w_2))$

$=$ B-branes on Y w/ superpotential $w_1 - w_2$

$$\left(\begin{array}{c} E = E_+ \oplus E_-,\ \bar{D} \\ \bar{D}^2 = 1\cdot(w_1 - w_2) \end{array} \right)$$





$\mathrm{Hom}(Y_1, Y_2)$

$=$ B-branes on U w/ superpotential W

$$\mathcal{O}_U / \partial W$$

monoidal structure on $D_{2\text{-per}}(Y)$

is deformed by an element $\beta \in \cancel{H}\ H^1(\mathrm{Sym}^2 TY)$

$$0 \to TY \to TX|_Y \to NY \to 0$$

$$\beta \in H^1(NY^* \otimes TY) = H^1(TY \otimes TY)$$

$$\mathcal{L}_1 \circledast_{\hbar} \mathcal{L}_2 = \left(\mathcal{L}_1 \otimes \mathcal{L}_2,\ \overline{D} = \bar{\partial}_{\mathcal{L}_1 \otimes \mathcal{L}_2} + \hbar \underbrace{s(F_1 F_2)}_{(0,3)\text{ form}} + \dots\ O(\hbar^2)\right)$$