Mirror Symmetry III

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Mirror Symmetry for hyperKähler manifolds

Compact: parameter space for metrics

$$\Gamma \backslash O(3,k-3)/O(3)\times O(19)$$

Real grassmannian

coordinates in triples $\quad 3(k-3)$

metrics + B-field:

$$\tilde{\Gamma} \backslash O(4,k-2)/O(4)\times O(k-2)$$

$$4(k-2)$$

Same phenomena as in Gukov's lecture

$\mathbb{C}^2/\mathbb{Z}_2$ and its blowups

$$\left.\begin{array}{l}\text{HK quotient by } U(1) \\ \quad 3 \text{ moment maps} \longrightarrow \mathbb{R}^3 \\ \\ \text{B-field}\end{array}\right\}\ \text{hyperKähler parameter space}$$

Comment S. Gukov

Manifold compact $\longrightarrow$ quaternion

non-compact $\longrightarrow$ sigma model

coupling constant in gravity

volume of hyperKähler space in question

$$\tilde{\Gamma} \backslash O(4,k-2)/O(4)\times O(k-2)$$

$$\xleftrightarrow[\text{Map}]{\text{Mirror}} \tilde{\Gamma} \backslash \widehat{O(4,k-2)}/O(4)\times O(k-2)$$

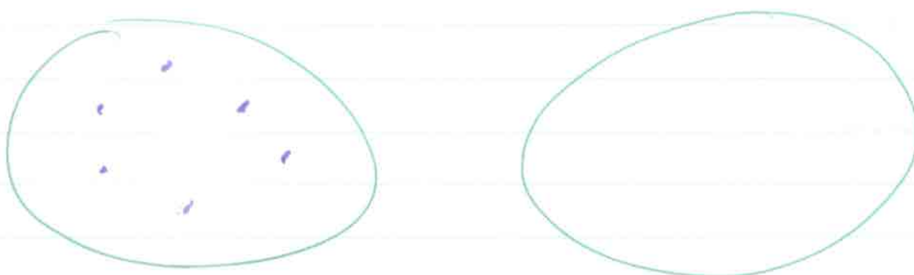
<u>S-Duality</u>

$$\mathcal{M}_G \longleftrightarrow \mathcal{M}_{^LG}$$

$$\tau \qquad\qquad -1/n\tau$$

Homological mirror symmetry
is about <u>twisted</u> 2D theories

Physical D-branes form a subcategory



$D^b(\mathrm{Coh}\, X)$ $\qquad\qquad$ $\mathrm{Fuk}(X)$

[G. Moore] Both larger and smaller
Doesn't encode stability (smaller)
Non-SUSY (larger)

Twisting $(4,4)$ - theories:

There is a choice of 2 complex structures (left and right)

$\left.\begin{array}{l} A\text{-twist} \\ B\text{-twist} \end{array}\right\}$

$\vdots$

<u>We must chose a complex structure</u>

A-branes

B-branes

$I, J, K \qquad IJ = K$

$\omega_I, \omega_J, \omega_K$

orthonormal basis of chosen positive 3-plane in $H^2(X, \mathbb{R})$

HK with a chosen cx structure + B

$$S^2 \longrightarrow \mathcal{M}_{HK \times cx} \longleftarrow H^2_{prim}(X, \mathbb{R}) = \omega^\perp \text{ in } H^2(X,\mathbb{R})$$

$$\mathcal{M}_{HK \times cx} \downarrow \mathcal{M}_{HK}$$

$$H^2_{prim}(X,\mathbb{R}) = \underbrace{H^{(2,0)} \oplus H^{(1,1)} \oplus H^{(0,2)}}_{\text{Hodge structure}}$$

$$\mathcal{M}_{Hodge} = \Gamma \backslash O(2, k-3) / O(2) \times O(k-3)$$

+ basis H^2 prim

$\omega \in K \subseteq H^2(X, \mathbb{R})$

$B \in H^2(X, \mathbb{R}/\mathbb{Z})$

$$\begin{array}{ccc} \text{cx Kähler cones} & \longrightarrow & \mathcal{M}_{HK + cx} \\ & & \downarrow \\ & & \mathcal{M}_{Hodge} \end{array}$$

Fix cx structure, A or B twist

Lagrangian cycles or holo cycles $\in (\mathrm{Re}\,\alpha)^{\perp} \cap (\mathrm{Im}\,\alpha)^{\perp}$

$= \omega_J^{\perp} \cap \omega_K^{\perp}$

$\int_\gamma \omega_I > 0 \Rightarrow \gamma = c_1(\text{alg cycle})$

Extent to $H^0 \oplus H^2 \oplus H^4$ Mukai vector

Special Lagrangian cycles $\lambda \subset \omega_I^{\perp} \cap \omega_J^{\perp}$

K3 surfaces of algebraic type

$H^2(X,\mathbb{Z})$ even unimodular lattice of signature $(3,19)$

$\cup$| primitive

$\Lambda^{1,\rho-1}$

$H \oplus H \oplus H \oplus E_8 \oplus E_8$

$(\Lambda^{1,\rho-1})^\perp$ has signature $(2, 20-\rho)$

$$\left\{ \overset{cx}{K3} \mid \Lambda^{1,\rho-1} \subseteq H^2(X,\mathbb{Z}) \text{ is type } (1,1) \right\} \xleftrightarrow{\text{mirror}} \left\{ cx\ K3 \;\middle|\; \begin{matrix} {}^M\Lambda^{1,19-\rho} \\ \cap \\ \omega_J^\perp \cap \omega_K^\perp \end{matrix} \right\}$$

(hyp twisted)

$$\Lambda^{1,\rho-1} \subseteq \omega_J^\perp \cap \omega_K^\perp \}$$

$$\begin{cases} \omega_I \in \Lambda^{1,\rho-1} \otimes \mathbb{R} \\ B \in \Lambda^{1,\rho-1} \otimes \mathbb{R}/\mathbb{Z} \end{cases}$$

cx structure is H.S. on $(\Lambda^{1,\rho-1})^\perp$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \oplus {}^M\Lambda^{1,19-1}$$

anticipated

Arnold "strange duality"

Elliptically fibered $\rightsquigarrow$ obscure Mukai paper

$$\begin{tikzcd} T^n \arrow[r] & X \arrow[d] \\ & B \end{tikzcd}$$

ω_I s.t. T^n holomorphic Lagrangian submanifold

ω_J

$$\begin{tikzcd} T^n \arrow[r] & X \arrow[d] \\ & B \end{tikzcd}$$

special Lagrangian submanifold

"Mirror symmetry is T-duality"

$$T^n \longrightarrow (T^n)^*$$

(metric + B-field)

$$\begin{tikzcd} X|_U \arrow[rrrr, leftrightarrow, bend left] \arrow[d] & \subseteq X \arrow[d] \arrow[r, dashed] & {}^M X \arrow[d] & \supseteq {}^M X|_U \arrow[d] \\ U & \subseteq B & {}^M B & \supseteq U \end{tikzcd}$$

Change complex structure

$T^n \subseteq X$ holo Lagrangian submanifold

SYZ aspect of Langlands duality:

HK manifolds $\mathcal{M}_{Hit}(C, G) \longrightarrow \mathcal{B}$

$$\mathcal{M}_{Hit}(C, {}^L G) \longrightarrow \mathcal{B}$$

fibers are holo Lagrangian tori in one of the complex structures

Hausel & Thaddeus

fibers are naturally dual tori
(~~once~~ discrete B-field is needed)