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Mirror Symmetry & Langlands Duality I

1. Review of mirror symmetry (today)
2. MS in hyperKähler case (tomorrow)
3. MS & Langlands Duality (end of week)

There are many forms of mirror symmetry, all closely related.

I. MS in $\mathcal{N}=(2,2)$ SCFT (+ Topological field theory)

Perturbative Superstring thy: (late 80s)
$\to$ SCFT on the "worldsheet" Σ

Worldsheet SUSY: $\mathcal{N}=(p,q)$

$\mathcal{N}=(2,2)$ | $\mathcal{N}=2$ in 2d is dim'l reduction of $\mathcal{N}=1$ in 4d

susy algebra: 9112056
$\longrightarrow$ representation thy
gauge multiplet
chiral, twisted chiral multiplets

Discrete series |
reps labeled by central charge $\in \mathbb{Q}$, <1

Gepner | $(\otimes D_{c_i})^G$

$c \in \mathbb{Z}$ eg $c=3, \ldots$

$\updownarrow$

σ-model on Fermat-type hypersurfaces in proj. space

$\varphi : \Sigma \longrightarrow X$ Kähler

geometric data: Riemannian metric, 2-form "B-field" ...

$\int \|d\varphi\|^2 + \int \varphi^* B + \cdots$

eg) $\left(\bigotimes^5 D_5\right)^{\mathbb{Z}_5} \longleftrightarrow \left(x_1^5+x_2^5+x_3^5+x_4^5+x_5^5=0\right) \subseteq \mathbb{CP}^4$

$\mathcal{N}=(2,2)$ SCA has an outer automorphism
$U(1)$ R-symmetry (from dim red.)
for L, R $\qquad (t_L, t_R): \varphi \to t_L^p t_R^q \varphi$

$(p,q) \longmapsto (-p,q)$
$\qquad \longmapsto (p,-q)$

σ-model action depends on choice of almost-cplx str.

$X \to \bar{X} \rightsquigarrow (p,q) \longmapsto (-p,-q)$
$X \to ??? \rightsquigarrow (p,q) \longmapsto (-p,q)$

(conformal $\Rightarrow c_1(X)=0$)

Greene-Plesser:

$$\text{outer}\left(\left(\otimes D_i\right)^G\right) = \left(\otimes D_i\right)^{\tilde{G}}$$

$$\updownarrow \qquad\qquad\qquad \updownarrow$$

$$\sum x_i^{a_i}=0 \qquad\qquad \widetilde{\left(\sum x_i^{a_i}=0\right)/\Gamma}$$

eg) $\sum_{i=1}^5 x_i^5 = 0 \qquad \left(\sum_{i=1}^5 x_i^5=0\right)/\mathbb{Z}_5^3$

$h^{1,1}=1,\ h^{2,1}=101 \qquad h^{1,1}=101,\ h^{2,1}=1$

$\mathcal{N} = (2,2)$ SCFT

$\Updownarrow$

topological (2,2) thys $\left\langle \begin{array}{l} \text{A twist} \\ \text{B twist} \end{array} \right.$ mirror symmetry

Candelas, dela Ossa, Green, Parkes :

A-twist on quintic

$$\sum_{\substack{\text{rational} \\ \text{curves}}} (\cdot\cdot) e^{dt}$$

Btwist on quintic mirror

Variation of Hodge str, <u>not</u> rational curves

Batyrev's construction:

$\underset{\text{C.Y.}}{X} \subseteq$ (toric variety) $\longleftrightarrow$ $X^{mir} \subseteq$ (mirror toric variety)

Witten's GLSM

$\varphi : \Sigma \longrightarrow \mathbb{C}^N$

geometric data:

include gauge field(s) & superpot compatible w/ $\mathcal{N}=(2,2)$

include argument why in IR, expect SCFT

How to do mirror constr for GLSMs (related to Batyrev's combinatorics) is well understood.

II. MS in (2,2) SQFT

work of Vafa & collaborators :

LG models, σ-models w/ non CY targets, ...

III. Kantsevich's 1994 proposal

"homological mirror symmetry"

MS should be "about":

$$D^b(\mathrm{Coh}\,X) \simeq \mathrm{Fuk}(X^{\text{mirror}})$$

alg. of open string ~~alg~~ amplitudes

IV. MS in type II string theory

II A or B on $X^6 \times \mathbb{M}^{1,3}$

even & odd coh exchanged, so A & B exchanged

$$\text{IIA on } X \times M^{1,3} \simeq \text{IIB on } X^{\text{mirror}} \times M^{1,3}$$

$$\Rightarrow \overset{\text{IIA}}{D}\text{-Branes on } X \times M^{1,3} \simeq \overset{\text{IIB}}{D}\text{-Branes on } X^{\text{mir}} \times M^{1,3}$$

$$(\text{brane}) \cap X = \begin{cases} \text{holo. submfld + stable ~~vector~~ bundle} \\ \text{special Lagrangian submfld} \end{cases}$$

SYZ (1996): $\quad \lfloor \dim_{\mathbb{R}} X = 6$, X is CY

$$\{D0\text{-branes on } X\} \cong X$$

$$\updownarrow$$

$$\left\{\begin{matrix} D3\text{-branes} \\ \text{in some class} \end{matrix} \text{ on } X^{\text{mir}}\right\} \cong X$$

Lagrangian 3-tori on X^{mir}

V. SYZ duality:

$X = (CY)$ of arb. dim.

$X \cong \left\{ \begin{array}{c} \text{Lagrangian } n\text{-tori} + \text{bundle} \\ \text{data on } Y \end{array} \right\}$

$$\begin{array}{c} X \\ \downarrow \pi \\ B \end{array}$$

for general b: $\pi^{-1}(b) = T^n$ — flat bundles on a T^n in Y

<u>T-duality</u>

$$\begin{array}{c} Y \\ \downarrow ? \\ B' \end{array}$$

might hope $B \cong B'$

if remove singular fibres,

$(B \setminus \text{singular}) \cong (B' \setminus \text{singular})$

but interesting physics in singular fibres