

Representation  
ThyGeometry  $\mathcal{M}$   
(categorification)  
eg  $D^b(\text{coh}(\mathcal{M}))$ (Topological) Gauge Thy $G$  cont. simple Lie grplocalizes to solns of PDEs on a 4-fold  $M$ eg  $F_A^+ = 0$ ,  $F_A = 0$ , etc

$\leadsto \mathcal{M} :=$  moduli spc of solns to PDEs  
will depend on Rep Thy data  
(eg ~~Thy~~  $G$ , reps, ...)

twisted  
 $\mathcal{N}=4$  SYMon  $M^4 = C \times \Sigma$ "where we do  
the GL""questions on  $\Sigma$ "  
ie. EFT

ramification  $\leadsto$  codim 2 objects in 4d gauge theory  
supported on  $D \subset M^4$

$$D = \prod_{x \in C} x \times \Sigma$$

## Operators

eg) Donaldson Thy

$$\langle \mathcal{O} \rangle = \int \mathcal{D}A e^{-S} \mathcal{O}$$

↳ oper. supported on a  $d$ -dim subfld of  $M$

• supported @ pt PEM

eg)  $\mathcal{O}(p) = \text{Tr } \varphi^k(p)$  (Donaldson Thy)

• supported on lines curves - lineopers  $\gamma \subset M$

eg) Wilson oper  
& Hooft oper.

$\langle \mathcal{O}_\gamma \rangle =$  knot invariant of  $\gamma$  ordered curve in 3mfld

Wilson Oper  $W_{\gamma, R} = \text{Tr}_R \text{Hol}_\gamma(A)$   
curve  $\gamma$ , rep of G  $R$

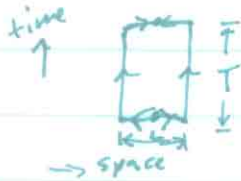
eg)  $G = U(1)$   
external electric charge  $q \in \mathbb{Z}$

$$W_{\gamma, q} = \mathcal{P} \exp \left( q \int_\gamma A \right)$$



$$= \exp \left( q \int_D F_A \right)$$

$\langle W_{r,g} \rangle$  detects "solitonic" strings  
confinement in 4d gauge thry



$$\langle W_{r,g} \rangle = \exp(-T V(L))$$

confinement:  $V(L) \sim (\text{const}) L$

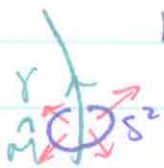


$\rightarrow$  area law

$$\langle W \rangle = \exp(-\text{Area}(\mathbb{D}))$$

### 't Hooft Oper

keep gauge connections on  $M \setminus \gamma$



$$\int_{S^2} \frac{F}{2\pi} = 1$$

for general  $G$ ,  $\rho: U(1) \rightarrow G$

cf  $\rho: SU(2) \rightarrow G$  for surf. ops

summarizing: line ops

- Wilson labeled by rep  $R$  of  $G$
- 't Hooft labeled by rep  $R$  of  $G$

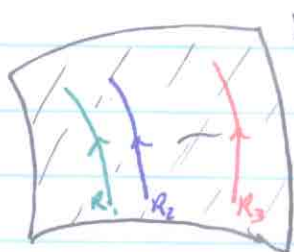
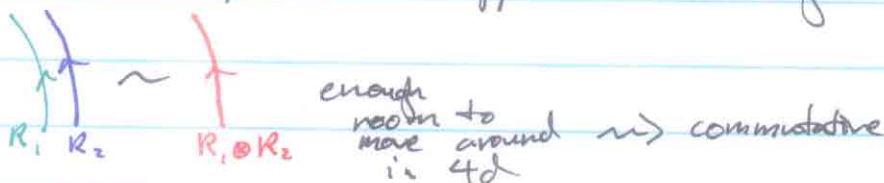
analog for DCM

- imitate constr. of Wilson/'t Hooft ops
- 4d gauge thry on  $M$  + 2d thry on  $D$   
( $\sigma$ -model)  $\varphi: D \rightarrow Q$

"good" ones = BRST symmetry  $Q_{BRST}$   
(SUSY)

- What classifies these surface opers? (discrete data + continuous params)

- What line opers are supported on a given surface oper?



DCM

algebra of line opers becomes non-commutative  
= affine Hecke algebra