

# Dim.-4 Proton Decay and Underlying Gauge Symmetry

KITP seminar on May 8, 2006

Taizan Watari (UC Berkeley)

In collaboration

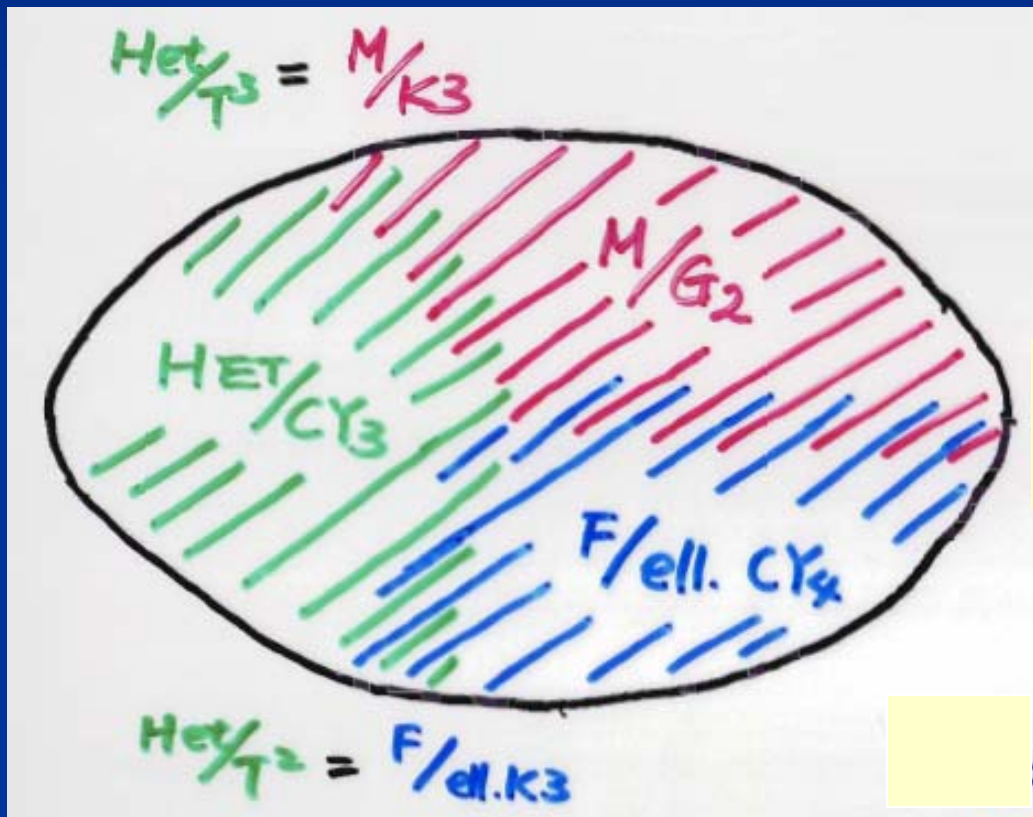
with Radu Tatar (Liverpool)

# Contents

- Introduction
- Underlying Gauge Symmetry
- Up-type Yukawa Matrix in M-theory Vacua
- B - L Symmetry Breaking  
in Heterotic String Vacua
- LSP (Lightest Supersymmetry Particle) Decay
- Summary

# Introduction

# $N = 1$ Supersymmetric String Landscape



String Duality

T-dual,  
Mirror

Het, M, F vacua can have  
qualitatively similar properties  
at the 1<sup>st</sup> order approximation.

Each corner of string moduli space (landscape) requires different mathematical techniques.

But, there is one “common” feature:

- 16 - SUSY YM multiplets of  $G$
- $G \rightarrow H$  broken by  $\langle \text{adj.} \rangle$ 
  - vector bundles (Het, Type I, Type IIA, IIB)
  - intersecting branes (Type IIA, IIB)
  - deformation of ADE singularities (M-, F-theory)
- matter chiral multiplets from  $\mathfrak{g}/\mathfrak{h}$

G/H '60s

$\pi^\pm, \pi^0$  : COORDINATES OF  $SU(2)_L \times SU(2)_R / SU(2)_V$

$H = SU(2)_V$  ACTS LINEARLY ON  $(\pi^+, \pi^0, \pi^-)$ .

FULL  $G = SU(2)_L \times SU(2)_R$  SYM. CONTROLS

$$\mathcal{L} = \partial_\mu \pi^a \partial^\mu \pi^a + \dots$$

$$+ \bar{N} i \gamma^\mu \left( \partial_\mu + \frac{(\partial_\mu \pi^a) \tau^b}{F_\pi} \tau^c \epsilon_{abc} + \gamma_5 \frac{\partial_\mu \pi^a}{F_\pi} \tau^a \right) N$$

+ .....

Quarks, leptons and Higgs multiplets instead of pions  
in supersymmetric G/H

Yukawa couplings of quarks and leptons  $\longrightarrow$  G

# Determining G

## DETERMINATION OF G

### ASSUMPTIONS

- ALL YUKAWA COUPLINGS FROM YM INT. OF G.
- Georgi - Glashow  $H = SU(5)$ .
- $W \ni \gamma_{d.e} \bar{5} \cdot 10 \cdot \bar{H}(5)$  BUT  $W \ni \bar{5} \cdot 10 \cdot \bar{5}$

$$\bar{5} = (\bar{D}, L) \text{ AND } \bar{H}(5) = (\bar{H}_c, H_d)$$

FROM DIFFERENT IRR. COMPONENTS  
OF  $G/H$ .



# POWER OF $G + 16$ SUSY.

Lykken Poppitz Trivedi '98

$G = SO(11)$  in Type IIB or Type I. /  $E^2/Z_3$  or  $T^6/Z_3$ .

$$H = SU(5)$$

$$\begin{bmatrix} \text{adj.} & 10 & 5 \\ \bar{10} & & \bar{5} \end{bmatrix}_{11 \times 11}$$

3 CHIRAL MULTIPLETS OF  $(10 + \bar{5})$

$$W \ni \epsilon_{abc} \text{tr}_{SO(11)\text{-adj.}} (\Sigma^a (\partial^b - \Sigma^b) \Sigma^c)$$

$$\rightarrow W \ni \int \Sigma^a 10^b \bar{5}^c \epsilon_{abc}. \quad \text{DIM.} - 4 \text{ p DECAY.}$$

CERTAIN  $G$  ARE PREDICTIVE, FALSIFIABLE.

# UP-TYPE YUKAWA COUPLINGS

SPECTRUM :  $SU(5)_{GUT} - 10$  FROM  $SU(10)/SU(5)$ .

$SU(5)_{GUT} - \bar{5}$  FROM  $SU(6)/SU(5)$ .

NOT ENOUGH INFO.

UP-TYPE YUKAWA  $W \Rightarrow 10^i 10^{kl} H(5)^m \epsilon_{ijklm}$ .

→ NOT JUST SIMPLE  $\rightsquigarrow + \cdot \rightsquigarrow \Rightarrow \rightsquigarrow$

[ NOT IA. IB ORIENTIFOLDS. TYPE I. ]

$G = E_6, 7, 8$  IN HET. M. F.

$$\text{Res}_{SU(5) \times SU(5)}^{E_6} e_6\text{-adj.} = (\text{adj. } 1) + (1, \text{adj.}) + \left[ (10, 2) + (\overline{1^2 5}, 1) \right] + \text{h.c.}$$

$$W \Rightarrow \text{tr}_{E_6} (\Sigma [\Sigma' \Sigma']) \rightarrow (10, 2) \otimes (10, 2) \otimes \overline{(1^2 5, 1)}$$

$$\downarrow \quad \cdot \quad \downarrow \quad \downarrow$$

$$10 \quad \cdot \quad 10 \quad H(5)$$

$e_6/SU(5)$  CONTAINS  $10 + H(5)$ , BUT NOT  $\bar{5} \cdot \bar{H}(\bar{5})$ .

Res  $E_7$   
 $SU(5)_{GUT} \times SU(2)$   
 $\times U(1) \times U(1)$

$e_7$ -adj. = (adj. . 1) + (1 . adj.)

$$\left[ \begin{array}{l} + (5, 1)^{0,6} \\ + (\Lambda^2 5, 2)^{1,2} + (\Lambda^2 5, 1)^{2,9} \\ + (5, 2)^{1,-9} + (\overline{10} \otimes 5, 1)^{2,-2} \\ + (1, 2)^{3,0} \end{array} \right] + h.c.$$

$$W \Rightarrow (\Lambda^2 5, 2)^{1,2} \otimes (\Lambda^2 5, 2)^{1,2} \otimes (\Lambda^2 5, 1)^{2,9} \longrightarrow 10 \cdot 10 \cdot H(5),$$

$$\overline{(5, 2)}^{1,-9} \otimes (\Lambda^2 5, 2)^{1,2} \otimes \overline{(5, 1)}^{0,6} \longrightarrow \bar{5} \cdot 10 \cdot \bar{H}(\bar{5}), \quad \bar{H}(\bar{5}) \cdot 10 \bar{5}$$

$$(\Lambda^2 5, 1)^{2,9} \otimes \overline{(5, 2)}^{1,-9} \otimes (1, 2)^{3,0} \longrightarrow H(5) \cdot \bar{5} \cdot \bar{N}, \quad H \cdot \bar{H} \cdot S$$

$$E_7 / SU(5)_{GUT} \times \langle SU(2) \times U(1) \times U(1) \rangle$$

IRR. COMPONENTS OF  $E_7/SU(5) + SU(2)$   
 CONTAIN 10,  $\bar{5}$ ,  $H(5)$ ,  $\bar{H}(5)$ .

SUPER YM INTERACTIONS GENERATE

u.d.e YUKAWA + [  $\psi$  DIRAC YUKAWA  
 $S\bar{H}\bar{H}$ .

PROTON DECAY OPERATORS

DIM. -4  $(\bar{5}_2)^{1-2} \otimes (1^2_5)_2 \otimes (\bar{5}_2)^{1-2}$   
 DIM. -5  $(\bar{5}_2)^{1-2} \otimes (1^2_5)_2^3$

} NOT INV.  
 UNDER  
 $SU(2) \times U(1)^2$ .

$G = E_7$  IS THE MINIMAL CHOICE.

$E_8$  IS NOT NECESSARY.

## THE OTHER POSSIBILITIES

$$E_8 / SU(5)_{GUT} \times \langle \underbrace{SU(2) \times SU(2) \times U(1)}_{\substack{\downarrow \\ \downarrow}} \times U(1) \rangle$$

$$E_8 / SU(5)_{GUT} \times \langle SU(2) \times SU(3) \times U(1) \rangle$$

$$E_8 / SU(5)_{GUT} \times \langle \downarrow SU(4) \times U(1) \rangle$$

STILL,  $\bar{5}$  AND  $\bar{H}(\bar{5})$  HAVE DIFFERENT ORIGINS

AND DIM-9 PROTON DECAY IS ABSENT.

# Up-type Yukawa Matrix in M-theory Vacua

# Up-type Yukawa coupling in M-theory vacua

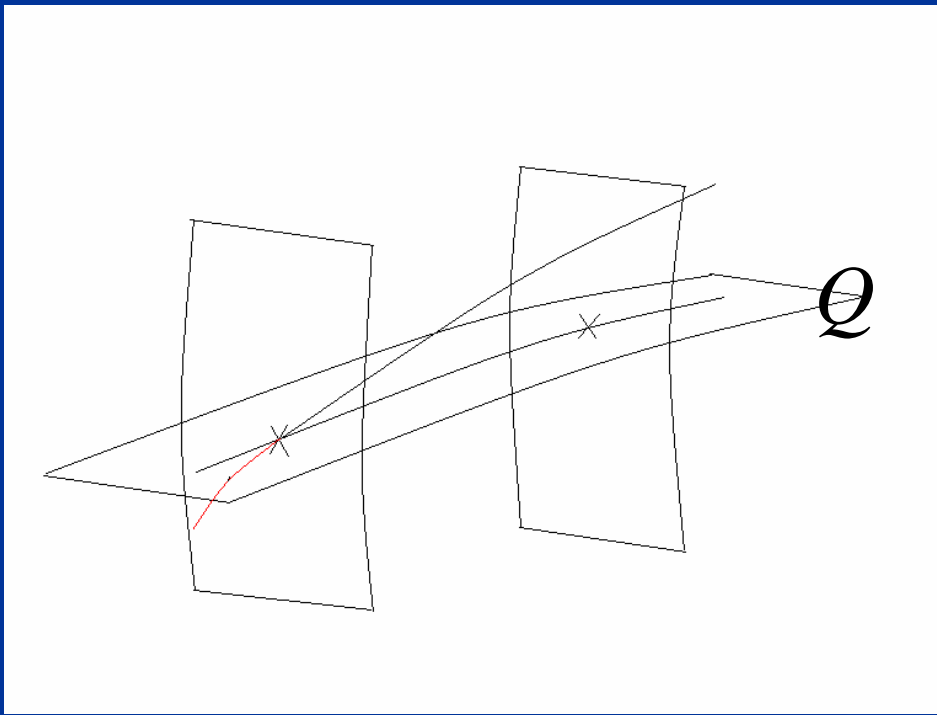
11 D SUGRA /  $G_2$  - holonomy manifold

locally ALE fibration on a compact 3-fold  $Q$

$H = SU(5)$

$A_4$  singular locus  $\simeq Q$

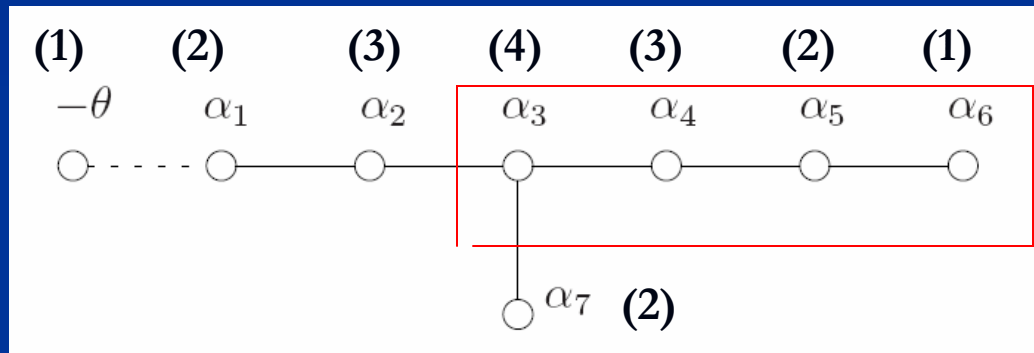
Matter chiral multiplets  
localized at  
enhanced singularities



Acharya '00  
Atiyah Witten '01,  
Acharya Witten '01, ...

# ALE fibre of $E_7$ or $E_8$ type

hyper-Kähler quotient of a quiver gauge theory associated with the extended Dynkin diagram



Kronheimer '89  
Douglas Moore '96

$$\sum n_i \alpha_i = 0.$$

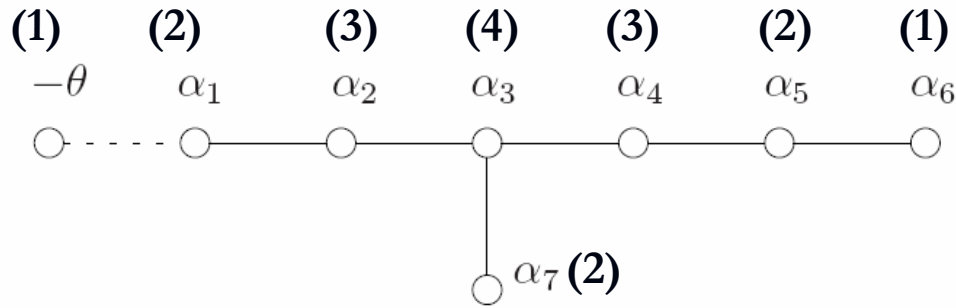
$$\sum_i n_i \zeta_i = 0.$$

$r = 7$  or  $8$  independent 2-cycles  $C_i$   $\sum_i n_i C_i = 0.$

$\zeta_i$ : size of a 2-cycle

enhanced  $A_4$  singularity when  $\zeta_{3,4,5,6} = 0.$   
[ SU(5) N=1 vector multiplet ]





Singularity is enhanced where some combination of  $\zeta_i$  (and  $C_i$ ) vanishes.

## Matter Identification

particles	10	$\bar{5}$	$H(5)$	$\bar{H}(\bar{5})$	$\bar{N}$
2-cycles	$-(C_2 + C_3 + C_7),$ $-(C_1 + C_2 + C_3 + C_7)$	$C_2,$ $(C_1 + C_2)$	$C_1 + 2C_2 + 2C_7$ $+3C_3 + 2C_4 + C_5$	$C_7$	$C_{-\theta},$ $C_{-\theta} + C_1$
singularity	$D_5$	$A_5$	$A_5$	$A_5$	$A_1 + A_4$

## Yukawa interactions from reconnection of M2-branes w/o p decay

$$10.10.H(5) \quad -(C_1 + C_2 + C_7) - (C_2 + C_7) + (C_1 + 2C_2 + C_7) \equiv 0,$$

$$\bar{5}.10.\bar{H}(\bar{5}) \quad C_2[+C_1] - (C_2 + C_7[+C_1]) + C_7 \equiv 0,$$

$$\bar{N}.\bar{5}.H(5) \quad C_{-\theta} + (C_1 + C_2) + (C_1 + 2C_2 + 2C_7) \equiv 0,$$

$$\text{dimension-4 : } \quad \bar{5}.10.\bar{5} \quad -(C_2 + C_7[+C_1]) + 2(C_2[+C_1]) \neq 0,$$

$$\text{dimension-5 : } \quad 10.10.10.\bar{5} \quad -3(C_2 + C_7[+C_1]) + (C_2[+C_1]) \neq 0.$$

$e_6/SU(5)$  CONTAINS  $10 + H(5)$ , BUT NOT  $\bar{5} \cdot \bar{H}(\bar{5})$ .

Res  $E_7$   
 $SU(5)_{GUT} \times SU(2)$   
 $\times U(1) \times U(1)$

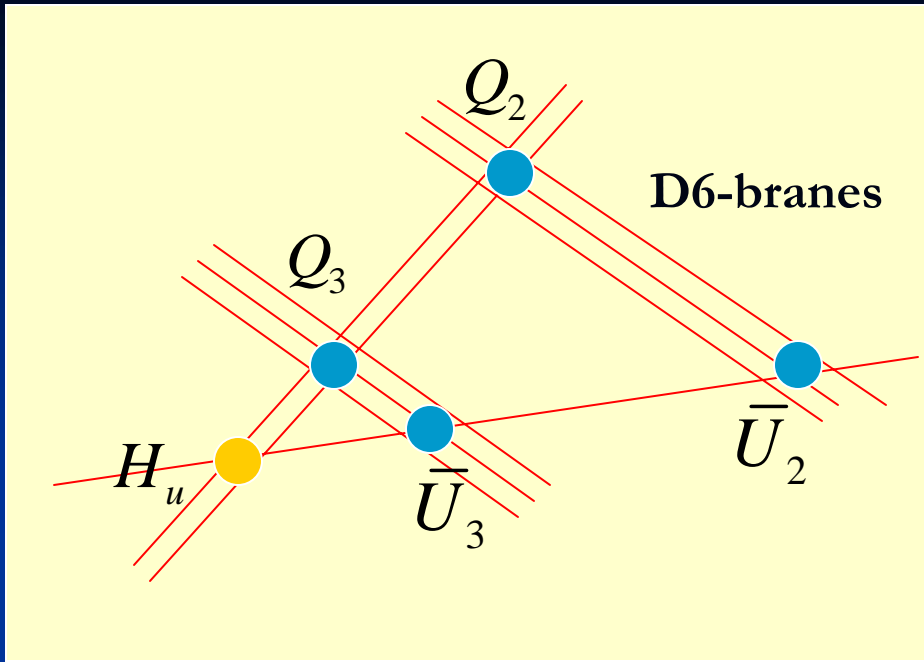
$e_7$ -adj. = (adj. . 1) + (1 . adj.)

$$\left[ \begin{array}{l} + (5, 1)^{0,6} \\ + (\Lambda^2 5, 2)^{1,2} + (\Lambda^2 5, 1)^{2,9} \\ + (5, 2)^{1,-9} + (\overline{10} \otimes 5, 1)^{2,-2} \\ + (1, 2)^{3,0} \end{array} \right] + h.c.$$

$$W \Rightarrow (\Lambda^2 5, 2)^{1,2} \otimes (\Lambda^2 5, 2)^{1,2} \otimes (\Lambda^2 5, 1)^{2,9} \longrightarrow 10 \cdot 10 \cdot H(5),$$

$$\overline{(5, 2)}^{1,-9} \otimes (\Lambda^2 5, 2)^{1,2} \otimes \overline{(5, 1)}^{0,6} \longrightarrow \bar{5} \cdot 10 \cdot \bar{H}(\bar{5}), \quad \bar{H}(\bar{5}) \cdot 10 \bar{5}$$

$$(\Lambda^2 5, 1)^{2,9} \otimes \overline{(5, 2)}^{1,-9} \otimes (1, 2)^{3,0} \longrightarrow H(5) \cdot \bar{5} \cdot \bar{N}, \quad H \cdot \bar{H} \cdot S$$



## Up-type Yukawa matrix of SUSY Standard Model

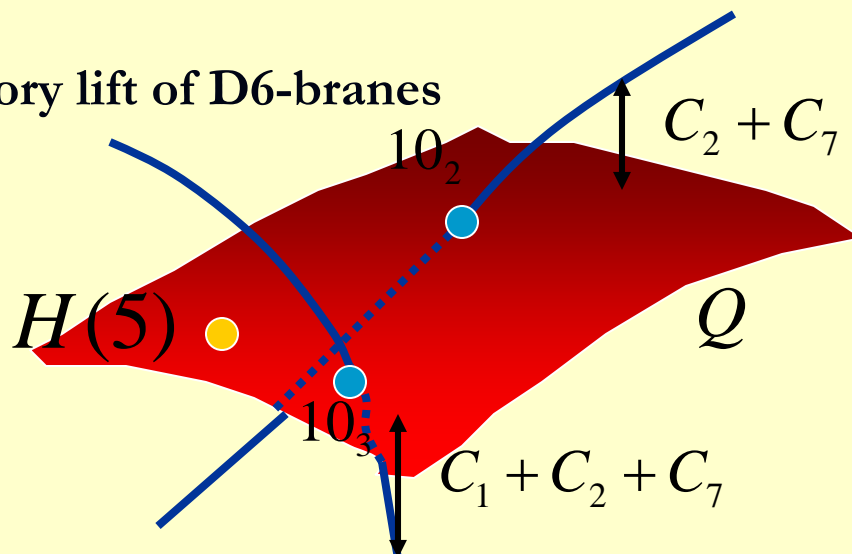
$$\propto e^{-\text{Area}} \quad \text{Cvetic Langacker Shiu '02}$$

$$\propto \begin{pmatrix} 1 & \delta \\ \delta' & \varepsilon \end{pmatrix}.$$

hierarchy:  $1 : \delta\delta'$

mixing :  $O(\delta), O(\delta')$ .

M-theory lift of D6-branes



## Up-type Yukawa matrix of SU(5) symmetric theories

$$10_2 10_3 H(5)$$

from minimal volume

$$10_3 10_3 H(5), \quad 10_2 10_2 H(5),$$

from M2-branes wrapping

Suppressed diagonal entries !

# B – L Symmetry Breaking in Heterotic String Vacua

## THE OTHER POSSIBILITIES

$$E_8 / SU(5)_{GUT} \times \langle \underbrace{SU(2) \times SU(2) \times U(1)}_{\text{red lines}} \times U(1) \rangle$$

$$E_8 / SU(5)_{GUT} \times \langle SU(2) \times SU(3) \times U(1) \rangle$$

$$E_8 / SU(5)_{GUT} \times \langle SU(4) \times U(1) \rangle$$

STILL,  $\bar{5}$  AND  $\bar{H}(\bar{5})$  HAVE DIFFERENT ORIGINS

AND DIM-9 PROTON DECAY IS ABSENT.

$G = E_8$ , SU(5) bdle  $V \longrightarrow$  unbroken SU(5).

$$248 = (\text{adj.}, 1) + (1, \text{adj.}) + [(V, 10) + (\wedge^2 V, \bar{5})] + \text{h.c.}$$

vector bdle moduli

gluons and photons

charged matter multiplets

Dimension-4 proton decay operators do not exist  
if the vector bundle  $V$  is reducible.

$$V_5 = L \oplus U_4, \quad \text{with } L \otimes \det U_4 \simeq \mathcal{O}_Z,$$

$$V_5 = U_3 \oplus U_2, \quad \text{with } \det U_3 \otimes \det U_2 \simeq \mathcal{O}_Z,$$

SU(5) symmetry may be broken down to the Standard Model gauge group  
via Wilson line or missing partner mechanism.

$$V_5 = L \oplus U_4, \quad \text{with} \quad L \otimes \det U_4 \simeq \mathcal{O}_Z,$$

$10 = (Q, \bar{U}, \bar{E})$  from  $H^1(Z; U_4)$

no massless modes from  $H^1(Z; L)$

$V_5$

$\bar{5} = (\bar{D}, L)$  from  $H^1(Z; U_4 \otimes L)$

$\bar{H}(\bar{5})$  from  $H^1(Z; \wedge^2 U_4)$

$\wedge^2 V_5$

$H(5)$  is from  $H^1(Z; \overline{\wedge^2 U_4})$

$$\begin{aligned} W \ni & (\mathbf{10}, U_4) \otimes (\mathbf{10}, U_4) \otimes \overline{(\bar{5}, \wedge^2 U_4)} \longrightarrow \mathbf{10} \cdot \mathbf{10} \cdot H(\mathbf{5}) \\ & + (\bar{5}, U_4 \otimes L) \otimes (\mathbf{10}, U_4) \otimes (\bar{5}, \wedge^2 U_4) \longrightarrow \bar{5} \cdot \mathbf{10} \cdot \bar{H}(\bar{5}) \\ & + (\bar{5}, U_4 \otimes L) \otimes (\mathbf{1}, U_4 \otimes L^{-1}) \otimes \overline{(\bar{5}, \wedge^2 U_4)} \longrightarrow \bar{5} \cdot \bar{N} \cdot H(\mathbf{5}). \end{aligned}$$

Some of vector bundle moduli  $\text{adj.}(V_5)$  are identified w RH-neutrinos.

$$V_5 = U_3 \oplus U_2, \quad \text{with} \quad \det U_3 \otimes \det U_2 \simeq \mathcal{O}_Z,$$

$$\begin{array}{ll}
 10 = (Q, \bar{U}, \bar{E}) \text{ from } H^1(Z; U_2) & \bar{5} = (\bar{D}, L) \text{ from } H^1(Z; U_3 \otimes U_2) \\
 \text{no massless modes from } H^1(Z; U_3) & \bar{H}(\bar{5}) \text{ from } H^1(Z; \wedge^2 U_3)
 \end{array}$$

$\underbrace{\hspace{15em}}_{V_5} \qquad \qquad \qquad \underbrace{\hspace{15em}}_{\wedge^2 V_5}$

$$H(5) \text{ is from } H^1(Z; \overline{\wedge^2 U_2})$$

### Proton decay operators

dim.-4	$\bar{5} \cdot 10 \cdot \bar{5}$	$(U_3 \otimes U_2)^2 \otimes U_2$	in $\wedge^5 V_5$
dim.-5	$10 \cdot 10 \cdot 10 \cdot \bar{5}$	$U_2^3 \otimes (U_3 \otimes U_2)$	vanish.



$$V_5 = L \oplus U_4, \quad \text{with} \quad L \otimes \det U_4 \simeq \mathcal{O}_Z,$$

The structure group is  $SU(4) \times U(1)$ , and the  $U(1)$  symmetry is not broken by the bdl configuration. This is the  $B - L$  symmetry; its gauge boson has mass terms through Green-Schwarz mechanism.

$$\mathcal{L} = \frac{1}{2g_{\text{YM}}^2} 2 \text{tr}_f(\mathbf{q}_X^2) D_X^2 + D_X \xi_X + D_X q_{X,i} \psi_i^\dagger \psi_i \rightarrow V = \frac{1}{2} \frac{g_{\text{YM}}^2}{2 \text{tr}_f(\mathbf{q}_X^2)} \left( \xi_X + q_{X,i} \psi_i^\dagger \psi_i \right)^2.$$

$$\xi_X = \frac{10M_G^2}{32\pi^2} \left[ \frac{2\pi l_s^2}{\text{vol}(Z)} \int c_1(L) \wedge J \wedge J - \frac{g_{\text{YM}}^2 e^{2\tilde{\phi}_4}}{2} \int c_1(L) \left( c_2(V_5) - \frac{1}{2} c_2(TZ) \right) \right].$$

Blumenhagen, Honecker, Weigand '05

Kahler moduli

dilaton

The Fayet—Iliopoulos parameter may not vanish.

If  $\xi < 0$ ,

the D-term condition (equation of motion of the gauge field)

$$\xi - 5|\bar{N}|^2 + 5|\bar{\bar{N}}|^2 - |10|^2 + 3|\bar{5}|^2 + 2|H|^2 - 2|\bar{H}|^2 = 0$$

can be satisfied by  $\langle \bar{\bar{N}} \rangle \neq 0$ .

This is equivalent to

$$0 \rightarrow L \rightarrow V_5 \rightarrow U_4 \rightarrow 0.$$

more general than the reducible limit.

The B – L gauge boson is Higgsed, and

the mass is typically of the order of Kaluza—Klein scale.

Broken B – L symmetry allows Majorana RH neutrino masses,  
and consequently the see-saw mechanism to work.

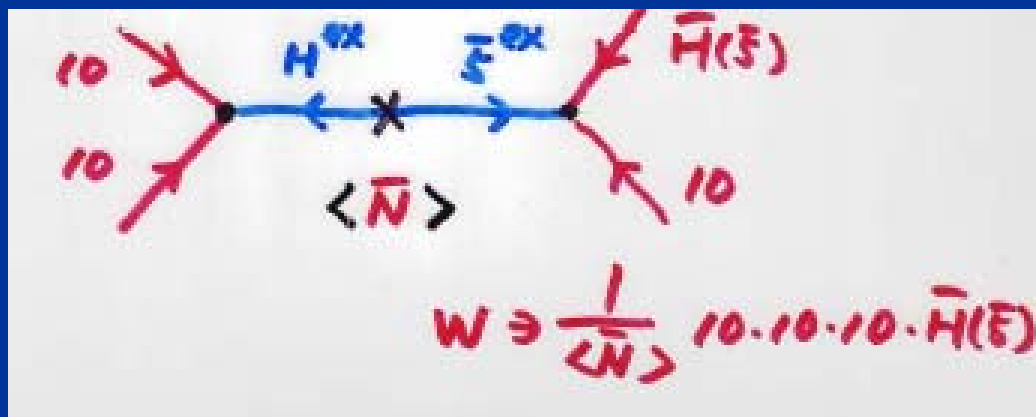
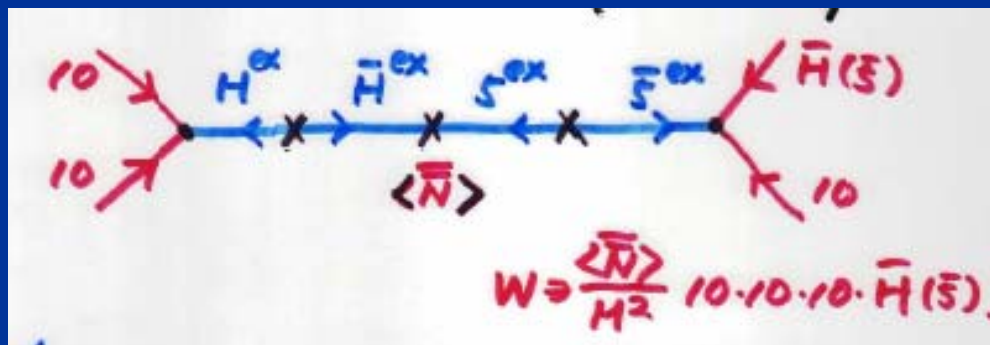
The original motivation, the absence of dim.-4 proton decay, is still maintained even if  $\langle \bar{\bar{N}} \rangle \neq 0$ .

This is because operators of the form  $W \neq \bar{5} \cdot 10 \cdot \bar{5} \cdot \bar{\bar{N}}^n$  is not allowed by the underlying gauge symmetry  $E_8$ .

# LSP Decay

$\langle \overline{\overline{N}} \rangle \neq 0$ . means that neither R-parity nor matter parity remains unbroken.

Indeed, after integrating out heavy states, an R-parity violating operator can be generated (Figure).



The lightest supersymmetry particle (LSP) is not stable. It decays through this operator, and is no longer a good candidate of dark matter.

LSP decay may be seen in future collider experiments.

Axion dark matter

# Summary

- Yukawa int. + absence of dim.-4 p decay determines the underlying gauge symmetry:  $E_7$  or  $E_8$ .
- In M-theory vacua, diagonal entries of the up-type Yukawa matrix tends to be suppressed.
- The  $B - L$  symmetry is spontaneously broken if the Fayet—Iliopoulos parameter is non-zero, allowing Majorana masses of right-handed neutrinos.
- If the sign of the FI parameter is correct, dim.-4 p decay is absent.
- The matter parity or R-parity is not conserved, and the LSP decay may be seen in future experiments.