High Energy Scattering in AdS/CFT

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Motivation

Describe high energy scattering in QCD

Important for DIS at small x (Bjorken variable)

If $Q^2 \gg \Lambda^2_{QCD}$ conformal symmetry is useful

Understand high energy scattering in AdS

String effects - Reggeization

Eikonal approximation

Unitarity

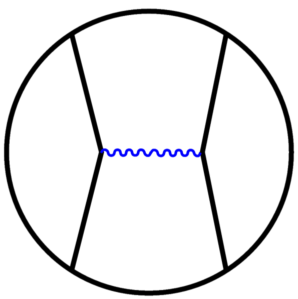
Black hole formation

Outline

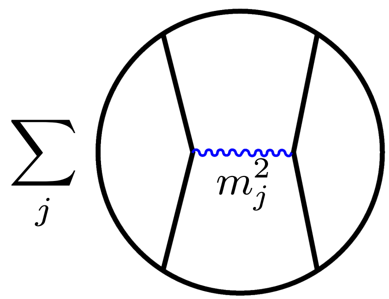
λ

Tree-level gravity

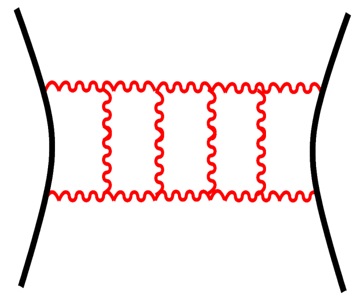
$N \gg \lambda \gg 1$



Reggeized graviton

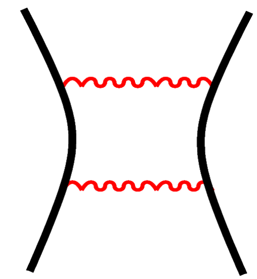


BFKL pomeron

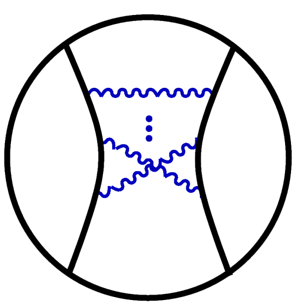


Perturbative CFT

$N \gg 1 \gg \lambda$



Eikonalized gravi-reggeon



$G \sim N^{-2}$

Multi pomeron

Saturation

AdS Eikonal?

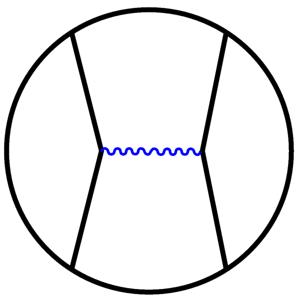
?

Outline

λ

Tree-level gravity

$N \gg \lambda \gg 1$



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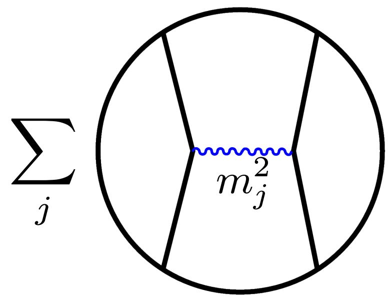

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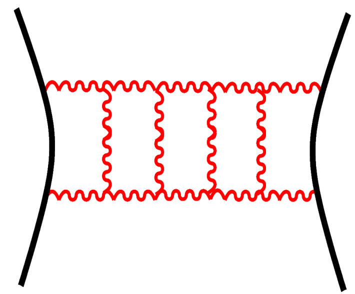

Perturbative CFT

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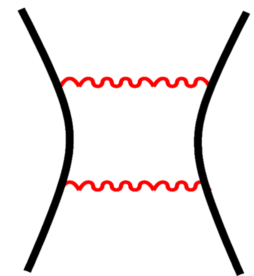
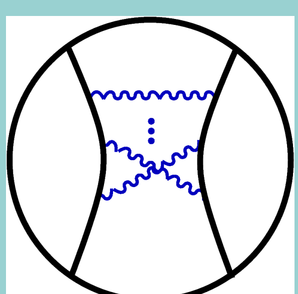


Eikonalized graviton



Multi pomeron
Saturation
AdS Eikonal?

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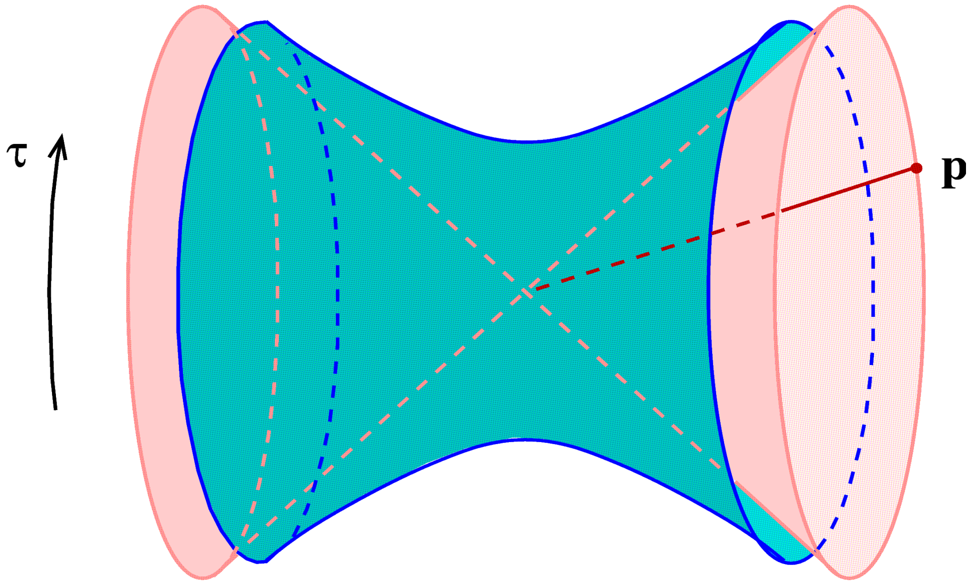
$G \sim N^{-2}$

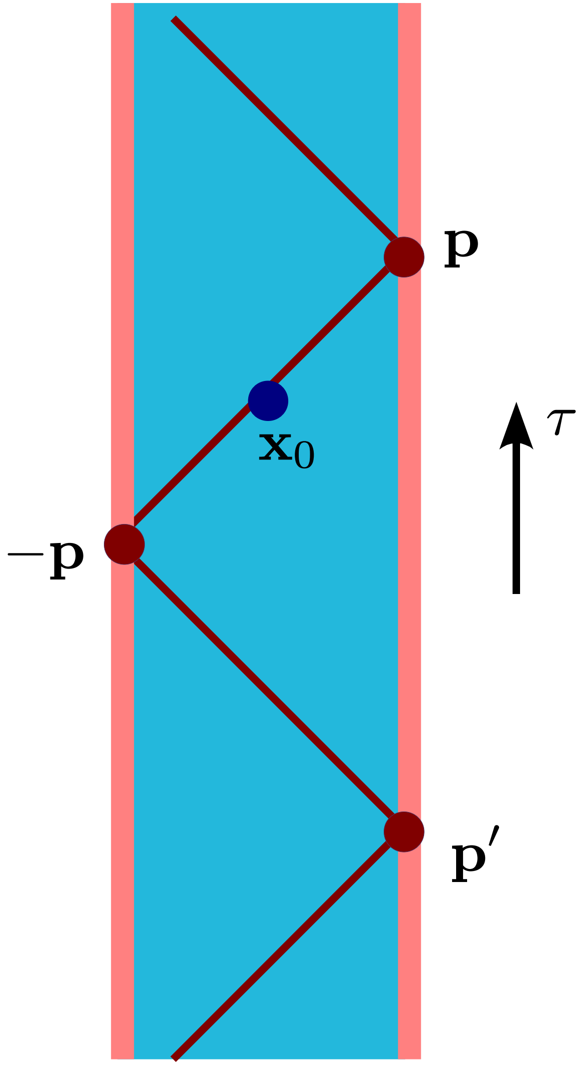
Anti-de Sitter Spacetime

$$\mathrm{AdS}_{d+1} = \left\{ \mathbf{x} \in \mathbb{R}^{2,d} : \mathbf{x}^2 = -1 \right\}$$

$$\partial \mathrm{AdS}_{d+1} = \left\{ \mathbf{p} \in \mathbb{R}^{2,d} : \mathbf{p}^2 = 0 \,, \; \mathbf{p} \sim \lambda \mathbf{p} \; (\lambda > 0) \right\}$$



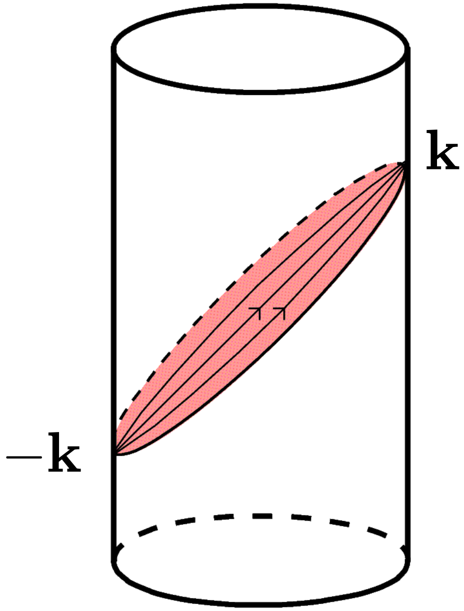
AdS_2



Null Geodesics in AdS are also null geodesics in the embedding space

$$\mathbf{x}(\lambda) = \mathbf{x}_0 + \lambda \mathbf{p} \;, \qquad \mathbf{x}_0^2 = -1 \;, \quad \mathbf{p}^2 = 0 \;, \quad \mathbf{x}_0 \cdot \mathbf{p} = 0$$

Wave Functions and Null Geodesic Congruences

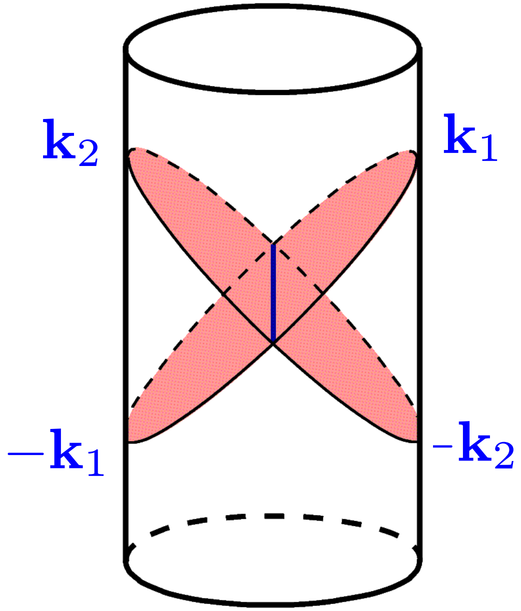


Null vector $\mathbf{k}$ defines surface of null geodesics

$$\mathbf{x} \cdot \mathbf{k} = 0$$

How to extend to full AdS?

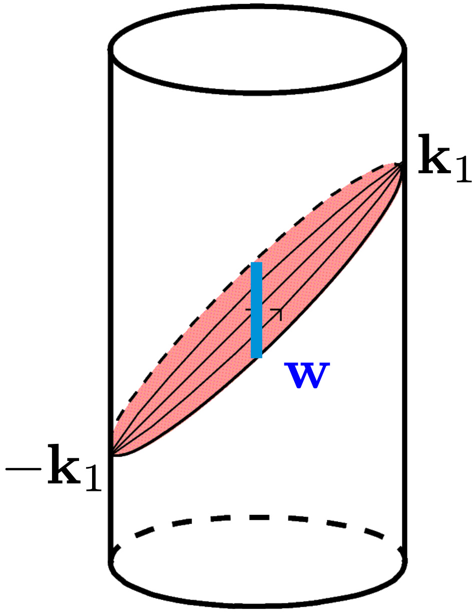
Start with two null vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ in $\mathbb{R}^{2,d}$



Transverse space is the hyperboloid H_{d-1}

$$\mathbf{w}^2 = -1 \qquad \mathbf{k}_1 \cdot \mathbf{w} = \mathbf{k}_2 \cdot \mathbf{w} = 0$$

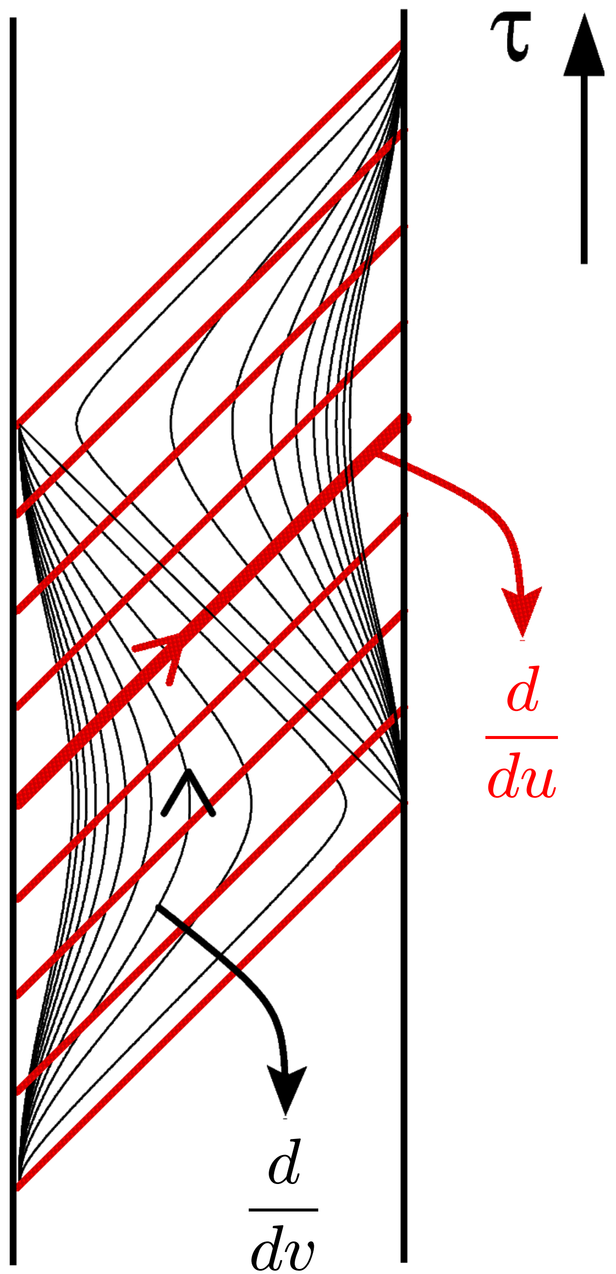
Wave Functions and Null Geodesic Congruences



Introduce $\{u, v, \mathbf{w}\}$ coordinates using AdS isometries

$$\mathbf{x} = \exp(v\mathbf{T}_2)\exp(u\mathbf{T}_1)\,\mathbf{w}$$

$\mathbf{T}_i$ are Lorentz generators

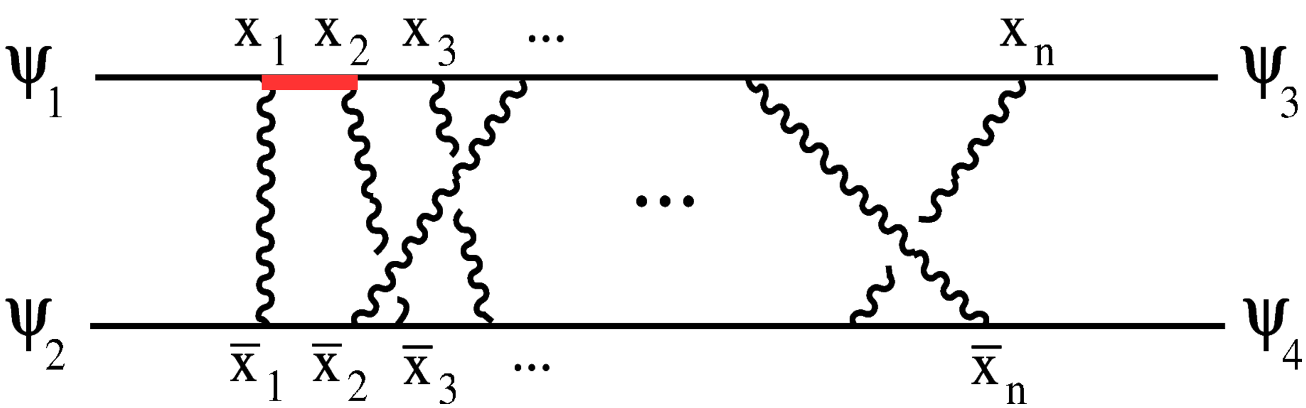


Eikonal wave functions for particle 1

$$\psi_1(\mathbf{x}) \simeq \exp(-i\omega v)\, F(v) F_1(\mathbf{w})$$

$$\psi_3(\mathbf{x}) \simeq \exp(i\omega v)\, F^\star(v) F_3(\mathbf{w})$$

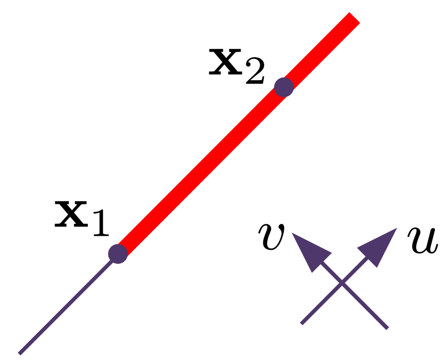
Eikonal Approximation



$$A = \sum_n (ig)^{2n} \int_{\mathrm{AdS}} d\mathbf{x}_1 \cdots d\mathbf{x}_n d\bar{\mathbf{x}}_1 \cdots d\bar{\mathbf{x}}_n \, \psi_3(\mathbf{x}_n)\Pi_{\Delta_1}(\mathbf{x}_n, \mathbf{x}_{n-1}) \cdots \Pi_{\Delta_1}(\mathbf{x}_2, \mathbf{x}_1)\psi_1(\mathbf{x}_1)$$

$$\psi_4(\bar{\mathbf{x}}_n)\Pi_{\Delta_2}(\bar{\mathbf{x}}_n, \bar{\mathbf{x}}_{n-1}) \cdots \Pi_{\Delta_2}(\bar{\mathbf{x}}_2, \bar{\mathbf{x}}_1)\psi_2(\bar{\mathbf{x}}_1) \sum_{\mathrm{perm}\ \sigma} \Pi_{\Delta}(\mathbf{x}_1, \bar{\mathbf{x}}_{\sigma_1}) \cdots \Pi_{\Delta}(\mathbf{x}_n, \bar{\mathbf{x}}_{\sigma_n})$$

$$\Pi_{\Delta_1}(\mathbf{x}_2, \mathbf{x}_1) \simeq \frac{1}{2\omega}\Theta(u_2 - u_1)\delta(v_2 - v_1)\delta_{H_{d-1}}(\mathbf{w}_2, \mathbf{w}_2)$$

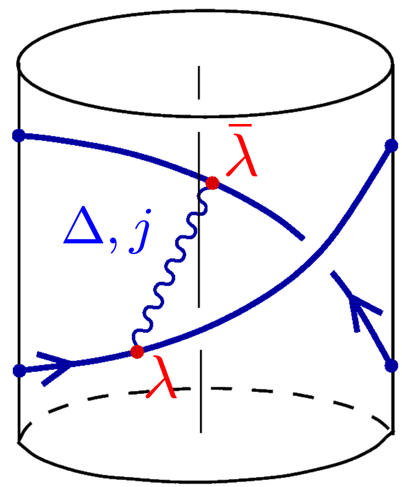


The propagator is non zero only along the classical trajectory of particle 1

Eikonal Amplitude

$$A_{eik} \simeq 8\omega^2 \int_{H_{d-1}} d\mathbf{w} d\bar{\mathbf{w}} F_1(\mathbf{w}) F_3(\mathbf{w}) F_2(\bar{\mathbf{w}}) F_4(\bar{\mathbf{w}}) \; e^{I/4}$$

$$I = \int d\lambda d\bar{\lambda}$$



$$= 2ig^2 s^{j-1} \, \Pi_\perp(\mathbf{w}, \bar{\mathbf{w}})$$

Scalar propagator of dimension $\Delta - 1$ on H_{d-1}

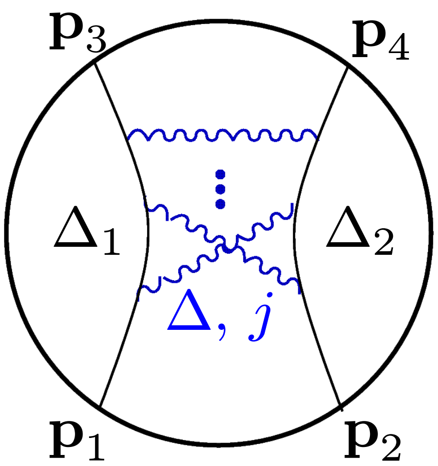
$s = -2\mathbf{k} \cdot \bar{\mathbf{k}}$

Graviton dominance

AdS/CFT Correspondence

Witten diagrams

$$A(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4) =$$



Bulk to boundary propagator
$\sim (-2\mathbf{p}_i \cdot \mathbf{x})^{-\Delta_i}$

Given ϕ_i such that $\psi_i(\mathbf{x}) = \int d\mathbf{p}_i\, \phi_i(\mathbf{p}_i)\, K_{\Delta_i}(\mathbf{p}_i,\mathbf{x})$ are of **eikonal type**

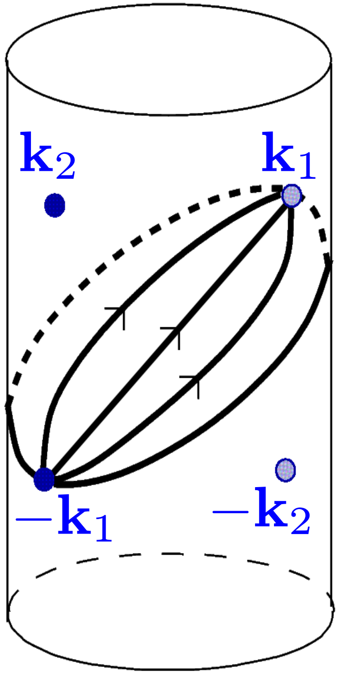
$$\int d\mathbf{p}_1 \cdots d\mathbf{p}_4\, \phi_1(\mathbf{p}_1)\cdots\phi_4(\mathbf{p}_4)\, A(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4) \simeq A_{eik}$$

CFT Eikonal Kinematics

$$A\left(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4\right) \sim \mathbf{p}_{13}^{-\Delta_1} \mathbf{p}_{24}^{-\Delta_2} \mathcal{A}(z, \bar{z})$$

$$\mathbf{p}_{ij} = -2\mathbf{p}_i \cdot \mathbf{p}_j$$

Cross ratios $\quad z\bar{z} = \dfrac{\mathbf{p}_{13}\mathbf{p}_{24}}{\mathbf{p}_{12}\mathbf{p}_{34}}\,, \qquad (1-z)\,(1-\bar{z}) = \dfrac{\mathbf{p}_{14}\mathbf{p}_{23}}{\mathbf{p}_{12}\mathbf{p}_{34}}$



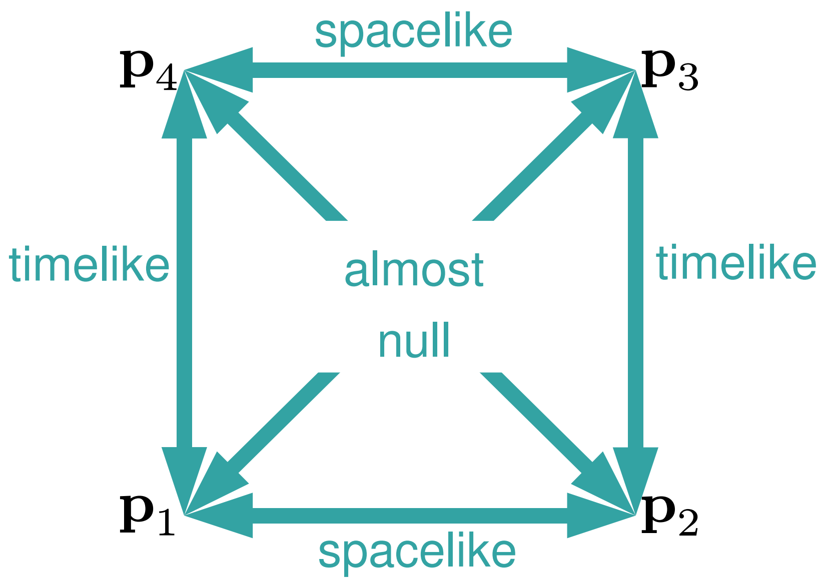
From bulk computation expect

$$\mathbf{p}_1 \sim -\mathbf{k}_1$$
$$\mathbf{p}_2 \sim -\mathbf{k}_2$$
$$\mathbf{p}_3 \sim \mathbf{k}_1$$
$$\mathbf{p}_4 \sim \mathbf{k}_2$$

→ Small cross ratios $z, \bar{z} \sim 0$

Lorentzian 4pt Function

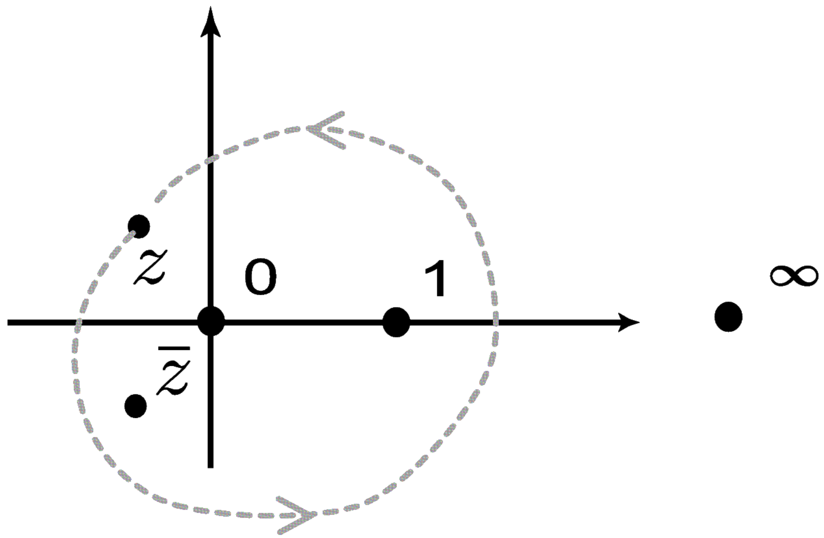
Causal relations between boundary points



$$\hat{\mathcal{A}}\,(z,\bar{z}) = \mathcal{A}^{\circlearrowleft}\,(z,\bar{z})$$

Lorentzian Euclidean

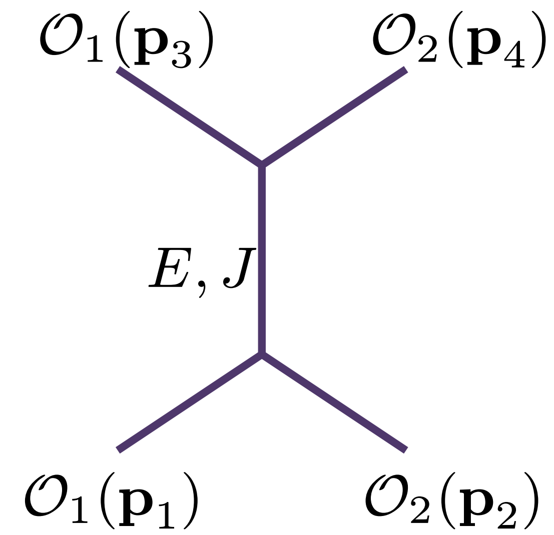
High energy kinematics is intrinsically Lorentzian



High energy scattering in AdS determines the behavior of $\hat{\mathcal{A}}(z,\bar{z})$ for small $z,\bar{z}$

Conformal Partial Waves

$$\mathcal{A}(z,\bar{z}) = \sum_{E,J} \sigma_{E,J} S_{E,J}(z,\bar{z})$$



For small cross ratios the sum is dominated by large dimension and spin and can be written in the **impact parameter representation**

$$z\bar{z} = \mathbf{p}^2\bar{\mathbf{p}}^2 \qquad z+\bar{z} = 2\mathbf{p}\cdot\bar{\mathbf{p}}$$

$$\hat{\mathcal{A}} \sim |\mathbf{p}|^{2\Delta_1}|\bar{\mathbf{p}}|^{2\Delta_2} \int_{\mathrm{M}} \frac{d\mathbf{y}}{|\mathbf{y}|^{d-2\Delta_1}} \frac{d\bar{\mathbf{y}}}{|\bar{\mathbf{y}}|^{d-2\Delta_2}} e^{-2i\mathbf{p}\cdot\mathbf{y}-2i\bar{\mathbf{p}}\cdot\bar{\mathbf{y}}} e^{-2\pi i\,\Gamma(\mathbf{y},\bar{\mathbf{y}})}$$

Phase Shift $$\Gamma(\mathbf{y},\bar{\mathbf{y}}) \simeq -\frac{g^2}{4\pi} s^{j-1}\Pi_\perp(r)$$

$$\cosh r = -2\frac{\mathbf{y}\cdot\bar{\mathbf{y}}}{|\mathbf{y}||\bar{\mathbf{y}}|}$$

$$s = 4|\mathbf{y}||\bar{\mathbf{y}}|$$

Anomalous dimensions of double trace operators

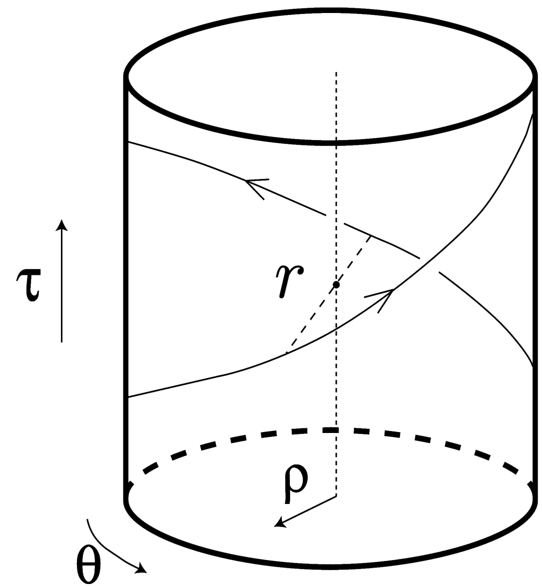
Double trace operator

$$\mathcal{O}_1 \partial_{\mu_1} \cdots \partial_{\mu_J} \partial^{2n} \mathcal{O}_2$$

Classical Dimension

$$E = \Delta_1 + \Delta_2 + J + 2n$$

Two string state



$$s = E^2 - J^2$$

$$\tanh\left(\frac{r}{2}\right) = \frac{J}{E}$$

Semiclassical description at large E and J

$$e^{-2\pi i \hat{H}} \simeq e^{-2\pi i E} \left(e^{-2\pi i \Gamma}\right)^2$$

Anomalous Dimension 2Γ

Anomalous dimensions of double trace operators

AdS coupling

$$2\Gamma(E,J) \simeq -\frac{g^2}{2\pi} s^{j-1} \Pi_\perp(r)$$

Scalar propagator of dimension $\Delta - 1$ on H_{d-1}

$$s = E^2 - J^2$$

$$\tanh\left(\frac{r}{2}\right) = \frac{J}{E}$$

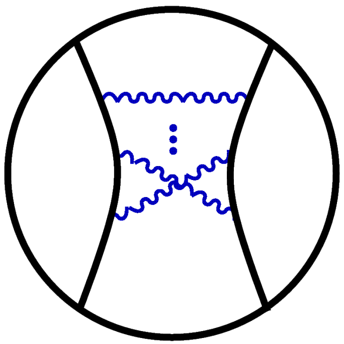
At high energies, **graviton exchange** is dominant

$$2\Gamma(E,J) \simeq -GJ^2 \frac{\Gamma\left(\frac{d-1}{2}\right)}{d\,2^{d-4}\pi^{\frac{d-1}{2}}} \left(\left(\frac{E}{J}\right)^2 - 1\right)^d F\left(d-1, \frac{d+1}{2}, d+1, 1-\left(\frac{E}{J}\right)^2\right)$$

- $d = 2$ → $2\Gamma(E,J) \simeq -8G(E-J)^2$
- $d = 4$ → $2\Gamma(E,J) \simeq -\frac{G}{2\pi}\frac{(E-J)^4}{EJ} = -\frac{1}{4N^2}\frac{(E-J)^4}{EJ}$ $\mathcal{N}=4$ SYM

Eikonal & Phase Shift

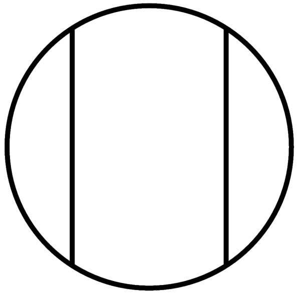
Eikonal graviton exchange

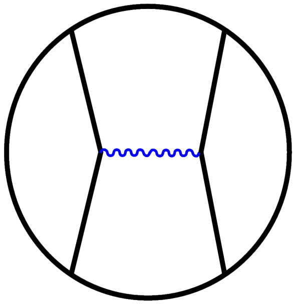


$$\hat{\mathcal{A}}(z,\bar{z}) \simeq \int_{\mathrm{IPR}} \left[e^{-2\pi i \Gamma(s,r)} \right]$$

$$\Gamma(s,r) \simeq -2Gs\Pi_{\perp}(r) \qquad G \sim \frac{1}{N^2}$$

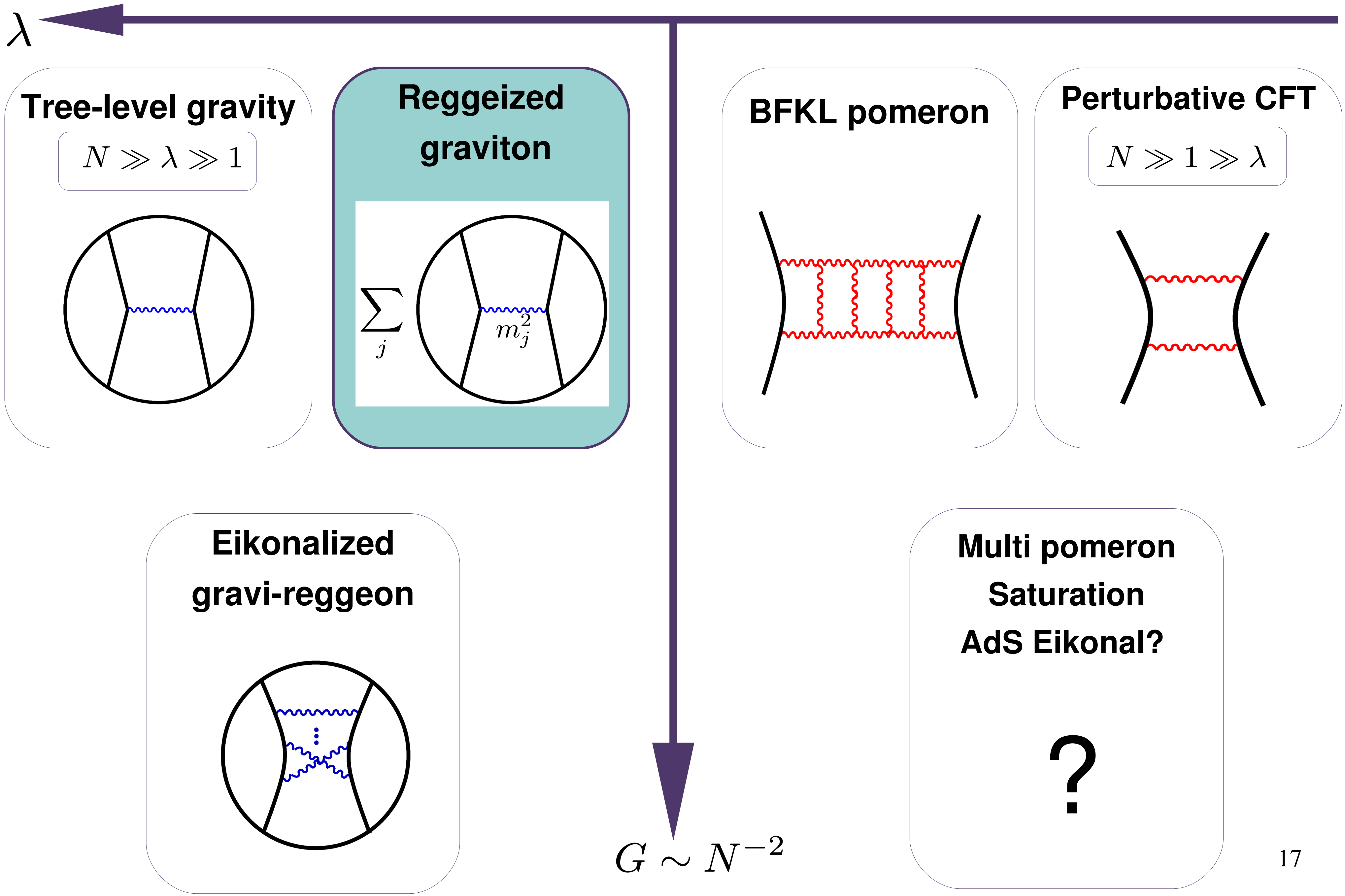
$$\hat{\mathcal{A}} \simeq \int_{\mathrm{IPR}} [1 - 2\pi i \Gamma + \ldots] \simeq \quad 1 \quad + \quad \frac{1}{N^2}\hat{\mathcal{A}}_{\mathrm{planar}} \quad + \ldots$$





Eikonal Approximation = Exponentiation of tree-level phase shift

Outline



λ
Tree-level gravity
$N \gg \lambda \gg 1$
Reggeized graviton
$\sum_j$
m_j^2
BFKL pomeron
Perturbative CFT
$N \gg 1 \gg \lambda$
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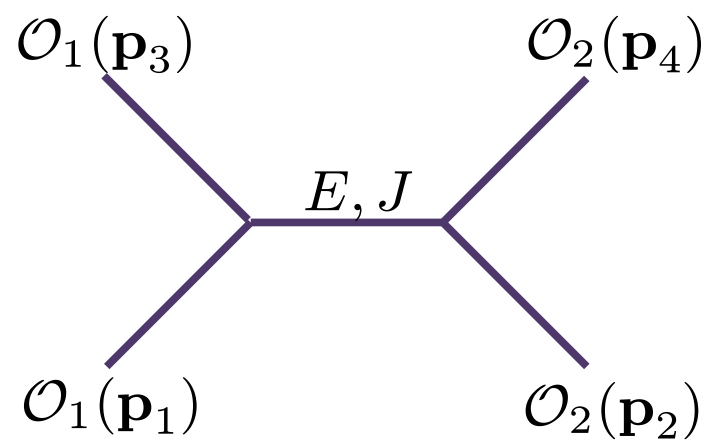
$G \sim N^{-2}$

Conformal Regge Theory

T-channel conformal partial wave decomposition

$$\mathcal{A}(z,\bar{z}) = \sum_{E,J} g_{E,J} \mathcal{T}_{E,J}(z,\bar{z})$$



Euclidean OPE: $\mathcal{T}_{E,J}(z,\bar{z}) \sim \sigma^{2+E}$

Lorentzian behavior: $\hat{\mathcal{T}}_{E,J}(z,\bar{z}) \sim \sigma^{1-J}$

$$4z = \sigma e^{\rho}$$
$$4\bar{z} = \sigma e^{-\rho}$$

Unbounded sum over spins requires Regge theory methods

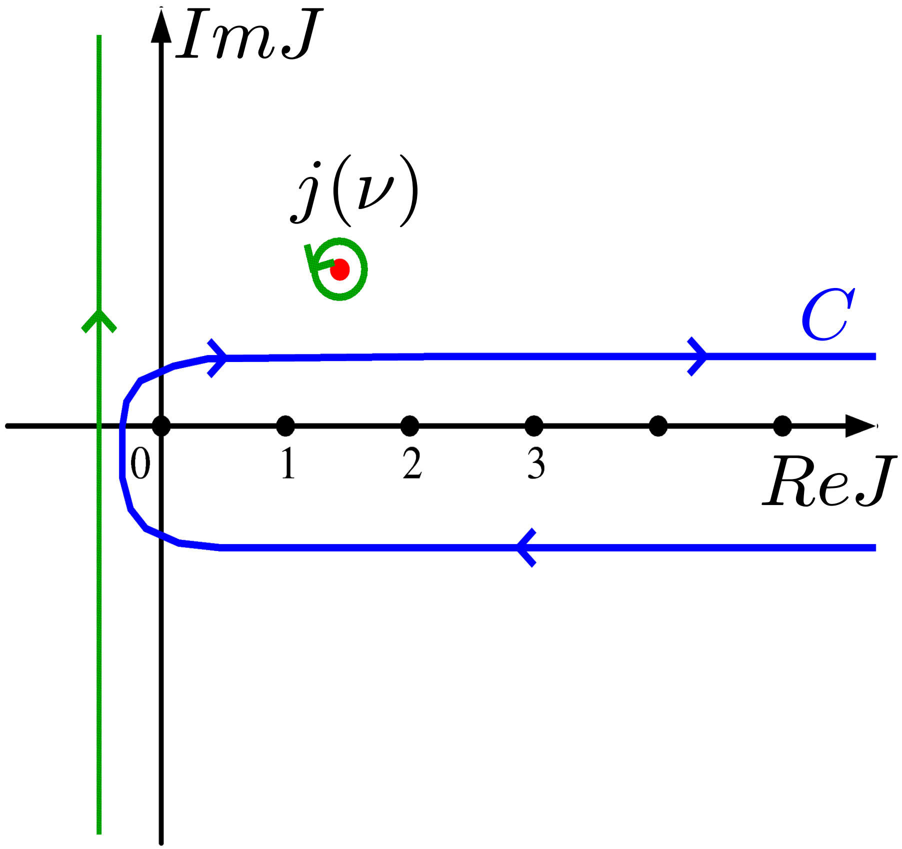
Integral representation of Lorentzian amplitude

$$\hat{\mathcal{A}} = \sum_J \int d\nu g_J(\nu)\, \hat{\mathcal{T}}_{i\nu,J}$$

Conformal Regge Theory

Sommerfeld-Watson representation

$$\hat{\mathcal{A}} = \int d\nu \int_C \frac{dJ}{\sin \pi J} (-)^J g(\nu, J)\, \hat{\mathcal{T}}_{i\nu,J}$$



Leading **Regge pole**

$$g(\nu, J) \sim \frac{\alpha(\nu)}{J - j(\nu)}$$

$$\hat{\mathcal{A}}(z, \bar{z}) \simeq 2\pi i \int d\nu (-)^{j(\nu)} \alpha(\nu) \sigma^{1-j(\nu)} \Omega_{i\nu}(\rho)$$

$$4z = \sigma e^{\rho}$$
$$4\bar{z} = \sigma e^{-\rho}$$

Harmonic functions on H_3

$$\left(\Box_{H_3} + \nu^2 + 1\right) \Omega_{i\nu}(\rho) = 0$$

Reggeon Phase Shift

$$\hat{\mathcal{A}}(z,\bar{z}) \simeq 2\pi i \int d\nu (-)^{j(\nu)} \alpha(\nu) \sigma^{1-j(\nu)} \Omega_{i\nu}(\rho) \simeq \int_{\mathrm{IPR}} [-2\pi i \Gamma]$$

$$\Gamma(s,r) \simeq \int d\nu \beta(\nu) s^{j(\nu)-1} \Omega_{i\nu}(r)$$

Graviton exchange gives

$$j(\nu) = 2 \qquad\qquad \beta(\nu) = -\frac{2G}{\nu^2+4}$$

Valid for $\alpha' \ll \ell^2 \quad (\lambda \to \infty)$

String Corrections

$$j(\nu) = 2 + \sum_{n=1} \lambda^{-n/2} j_n(\nu)$$

$$\sqrt{\lambda} = \ell^2/\alpha'$$
$$\nu = \ell\sqrt{-t}$$
$$\ell \to \infty$$

Require $j(\nu) \to 2 + \frac{\alpha'}{2} t$ in the **flat space limit**:

$$\frac{j_1(\ell\sqrt{-t})}{\ell^2} \to \frac{t}{2} \qquad \frac{j_n(\ell\sqrt{-t})}{\ell^{2n}} \to 0$$

Massless graviton for any value of λ: $j(\pm 2i) = 2$

Dimension
$2 + i\nu$

$$j(\nu) = 2 - \frac{\nu^2 + 4}{2\sqrt{\lambda}} + \dots$$

$$\beta(\nu) \sim G \frac{\Gamma(1 - j(\nu)/2)}{\sqrt{\lambda}\,\Gamma(j(\nu)/2)} + \dots$$

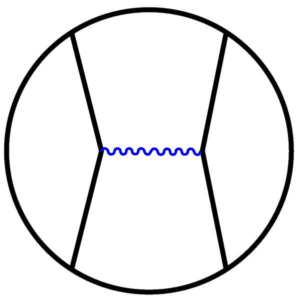
Decreasing intercept: $j(0) = 2 - \frac{2}{\sqrt{\lambda}} + \dots$

Outline

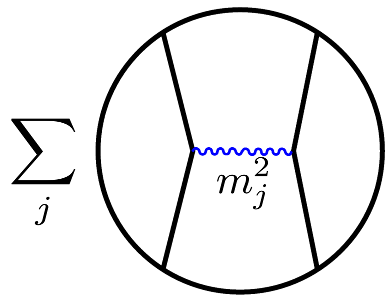
λ

Tree-level gravity

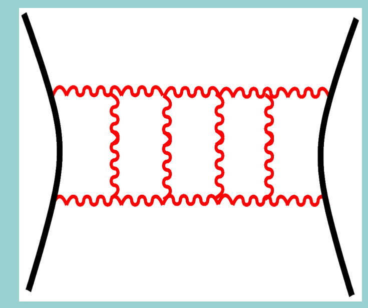
$N \gg \lambda \gg 1$



Reggeized graviton

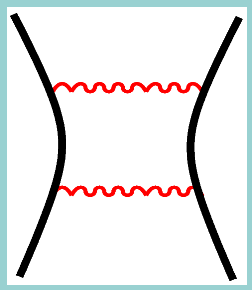


BFKL pomeron

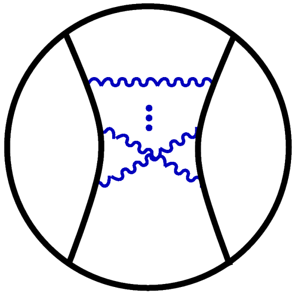


Perturbative CFT

$N \gg 1 \gg \lambda$



Eikonalized gravi-reggeon



$G \sim N^{-2}$

Multi pomeron

Saturation

AdS Eikonal?

?

Weak Coupling

$$\langle TrZ^2(\mathbf{p}_1)TrW^2(\mathbf{p}_2)Tr\bar{Z}^2(\mathbf{p}_3)Tr\bar{W}^2(\mathbf{p}_4)\rangle = \frac{\mathcal{A}(z,\bar{z})}{\mathbf{p}_{13}^2\mathbf{p}_{24}^2}$$

Known to order λ^2

Leading behavior $(\sigma \to 0)$ of **Lorentzian amplitude** is

$$\hat{\mathcal{A}} \simeq -\frac{\lambda^2}{8\pi^2}\frac{\rho^2}{\sinh^2\rho}$$

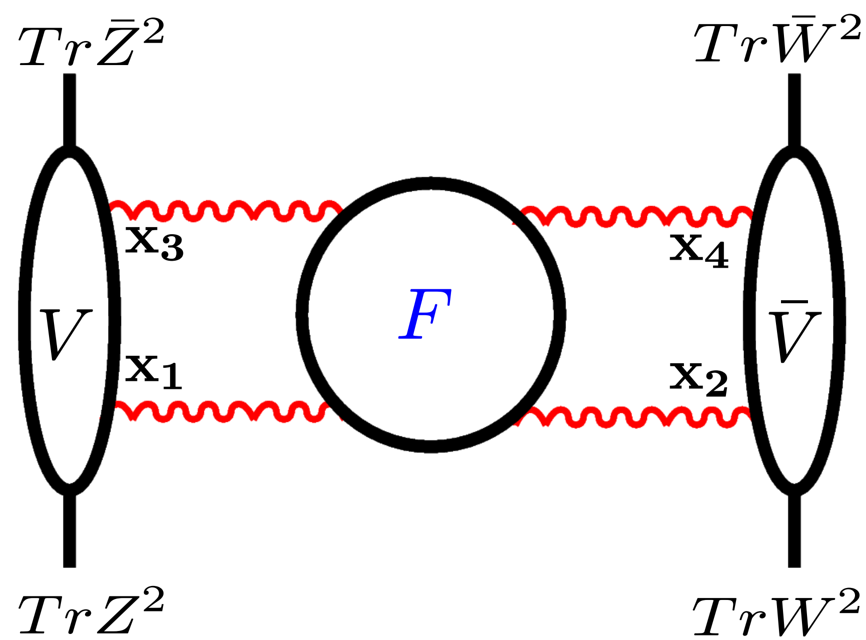
$$4z = \sigma e^{\rho}$$
$$4\bar{z} = \sigma e^{-\rho}$$

corresponding to

$$j(\nu) = 1 \qquad \alpha(\nu) = -i\frac{\pi\lambda^2}{16}\frac{\sinh\frac{\pi\nu}{2}}{\nu\cosh^3\frac{\pi\nu}{2}}$$

BFKL Pomeron

High energy hadron scattering (with $s \gg |t| \gg \Lambda_{QCD}$) is dominated by hard (BFKL) perturbative **Pomeron** exchange



The hard **Pomeron** is a colorless state of two gluons with ladder interactions (Reggeized gluons)

$$z\bar{z} = \mathbf{p}^2\bar{\mathbf{p}}^2 = \sigma^2/16$$

$$z + \bar{z} = 2\mathbf{p}\cdot\bar{\mathbf{p}}$$

Impact Factor **Impact Factor**

$$\hat{\mathcal{A}}(\mathbf{p},\bar{\mathbf{p}}) = \int_{\mathbb{R}^2 = \partial H_3} d\mathbf{x_1} d\mathbf{x_2} d\mathbf{x_3} d\mathbf{x_4} V(\mathbf{p},\mathbf{x_1},\mathbf{x_3}) F(\sigma,\mathbf{x_1},\mathbf{x_2},\mathbf{x_3},\mathbf{x_4}) \bar{V}(\bar{\mathbf{p}},\mathbf{x_2},\mathbf{x_4})$$

BFKL kernel has transverse **conformal invariance**

$$F = \int d\nu \frac{\nu^2}{(1+\nu^2)^2} \sigma^{1-j(\nu)} \mathcal{G}_\nu(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3},\mathbf{x_4})$$

BFKL Pomeron

Performing the transverse conformal integrals we recover the Regge form

$$\hat{\mathcal{A}}(z,\bar{z}) \simeq 2\pi i \int d\nu (-)^{j(\nu)} \alpha(\nu) \sigma^{1-j(\nu)} \Omega_{i\nu}(\rho)$$

$$4z = \sigma e^{\rho}$$
$$4\bar{z} = \sigma e^{-\rho}$$

with

$$\alpha(\nu) = -i \frac{\pi \lambda^2}{16} \frac{\sinh \frac{\pi\nu}{2}}{\nu \cosh^3 \frac{\pi\nu}{2}} + \dots$$

$$j(\nu) = 1 + \frac{\lambda}{4\pi^2} \left[2\Psi(1) - \Psi\left(\frac{1+i\nu}{2}\right) - \Psi\left(\frac{1-i\nu}{2}\right) \right] + \dots$$

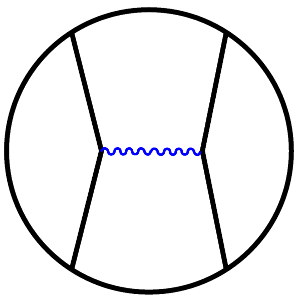
Raising intercept: $j(0) = 1 + \frac{\log 2}{\pi^2} \lambda + \dots$

Outline

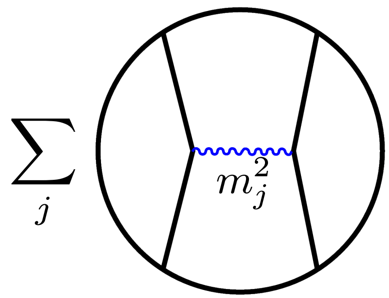
λ

Tree-level gravity

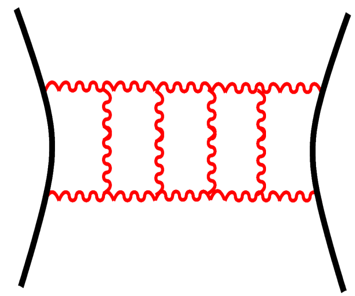
$N \gg \lambda \gg 1$



Reggeized graviton

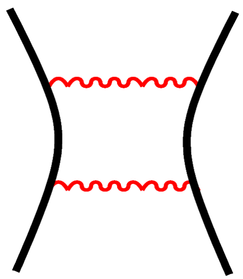


BFKL pomeron

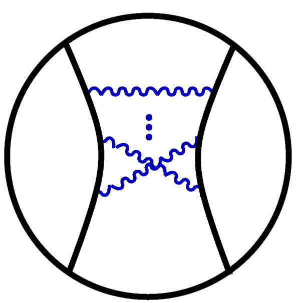


Perturbative CFT

$N \gg 1 \gg \lambda$



Eikonalized gravi-reggeon



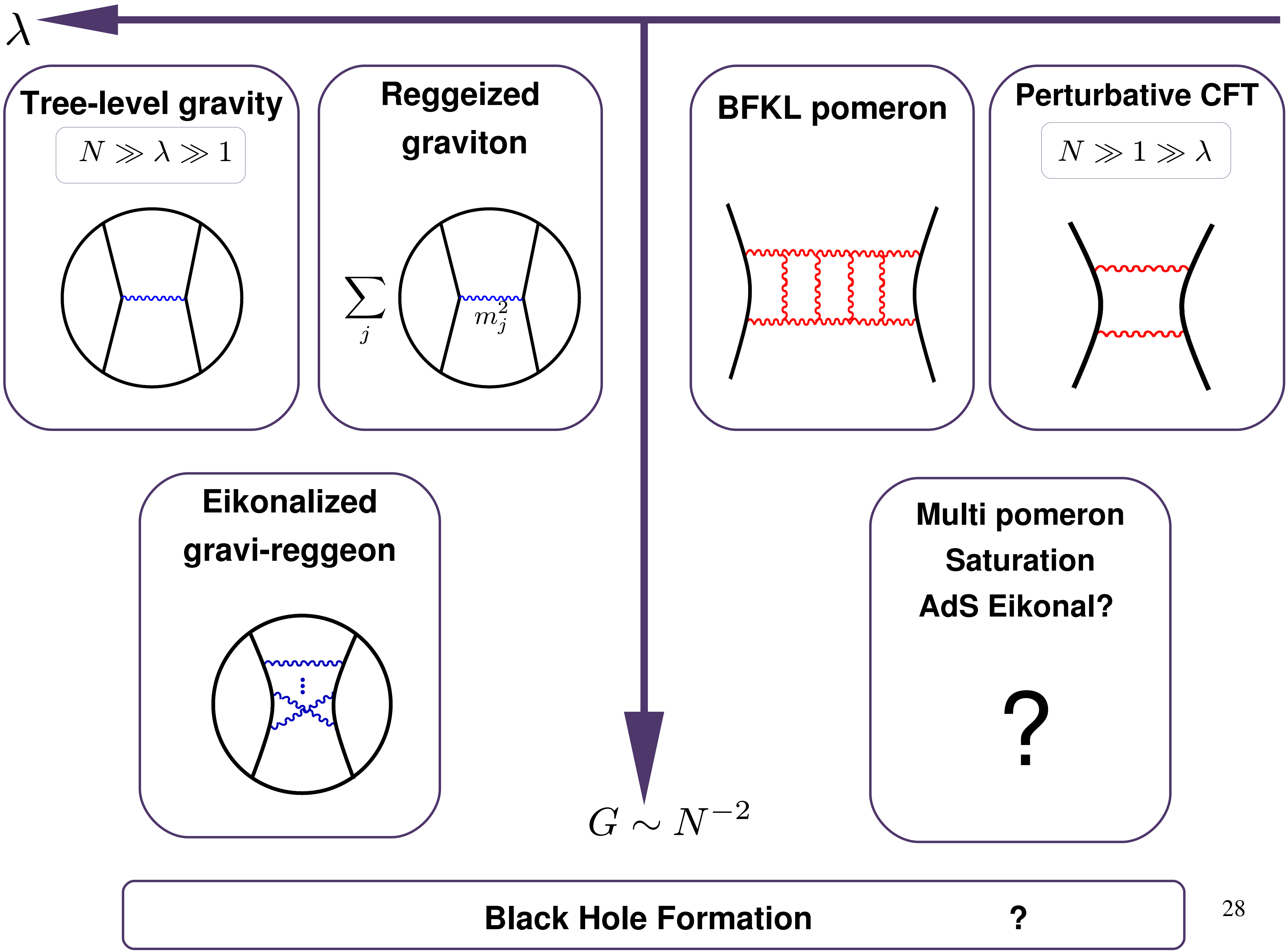
Multi pomeron
Saturation
AdS Eikonal?

?

$G \sim N^{-2}$

Multi pomeron & other open questions

- At ultrahigh energies, single pomeron exchange **breaks unitarity**. Non–linear effects of **pomeron self–interaction** are important (e.g. Balitsky–Kovchegov equation)
- At ultrahigh energies, the increase of the **gluon density** in the hadron wave–functions **saturates** due to non–linear effects
- **4–dimensional eikonal** resummation of BFKL pomeron **fails** to reproduce saturation physics
- Is there a kinematical regime where **AdS eikonal** resummation is dominant at weak coupling?
- Is there an analog to **black hole formation** at weak coupling?
- Is there any **unitarity bound** for the CPW expansion?


λ
Tree-level gravity
$N \gg \lambda \gg 1$
Reggeized graviton
$\sum_j$
m_j^2
BFKL pomeron
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