STRINGS AND MATRICES IN FLAT SPACE AND ON PP-WAVES FROM SUSY GAUGE THEORIES

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SUMMARY

- 't Hooft - large N Yang-Mills and string theory
- The AdS-CFT correspondence
  - questions
- Penrose limits
- String quantization on pp waves
- Strings from Super Yang Mills:
  - string spectrum
  - perturbative computation
  - discretized worldsheets
  - other fields and diagrams
- M theory on the pp wave
  - D0 brane lagrangian
  - supersymmetric solutions
- Future work: open strings, string interactions, Matrix model
- Conclusions

't Hooft: large N Yang-Mills and string theory

- 't Hooft limit: $N \to \infty$, $g^2 N$ = fixed [illegible]
- Yang Mills diagrams:

- gauge fields: adjoint → double lines
- planar diagrams → genus 0 string worldsheets
- quarks: fundamental → worldsheet boundaries
- nonplanar diagrams → worldsheets with holes
- coupling factor:

$$(g^2 N)^{\frac{1}{2}V_3 + V_4} (1/N)^{2H + 2L - 2}$$

H = holes
L = quark loops

→ $g_s = 1/N$ = closed string coupling
$g^2 N$ = vertex factor

The AdS-CFT correspondence

- large number N of D3-branes near horizon: $AdS_5 \times S_5$

$$ds^2 = R^2(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2) + R^2(d\psi^2 \cos^2\psi + d\theta^2 + \sin^2\theta\, d\Omega_3'^{(2)})$$

$$R^4 = 4\pi \alpha'^2 g_s N$$

- CFT:
  - no S matrices
  - observables – correlators on Euclidean space
- Correspondence:
  - $SU(N)$ Super Yang Mills, coupling g_{YM} = string theory on $AdS_5 \times S_5$, $2\pi g_s = g_{YM}^2$
  - Limit:
    - $N \to \infty$, $g^2 N$ = fixed and large
    - $\alpha' \to 0$ ($R/\sqrt{\alpha'}$ = fixed, large)
  - String theory ≃ sugra
- gauge inv. SYM operators → boundary values for sugra field

$$\langle e^{-\int \mathcal{O}\cdot\phi_0}\rangle_{CFT} = e^{-S_{sugra}[\phi[\phi_0]]} \quad (5)$$

$$\rightarrow \langle \mathcal{O}(x_1)\dots\mathcal{O}(x_n)\rangle_{CFT} = \frac{\delta^n}{\delta\phi_0(x_1)\dots\delta\phi_0(x_n)} S_{sugra}[\phi[\phi_0]]\Big|_{\phi_0=0}$$

- AdS : ϕ_0 = boundary source for ϕ
- CFT : ϕ_0 = source for gauge invariant operator $\mathcal{O}$

$\cdot \mathrm{Tr}\, X^{(a_1}\dots X^{a_n)} C^{I}_{a_1\dots a_n} \Leftrightarrow$ sugra modes $\phi^{(a_1\dots a_n)}$

Questions:

- derive supergravity from SYM, but string theory? ($\Delta \sim (g_s N)^{1/4}$ – string states $\sim N^{1/4} \Rightarrow$ Planck)
- know string spectrum in flat space, but not on $AdS_5\times S_5$.

Penrose limits (6)

Penrose:

- Look near a null geodesic $\Rightarrow$ space becomes a pp wave.
- formally: In the neighbourhood of a null geodesic I can put the metric in the form

$$ds^2 = dV(dU + \alpha\, dV + \sum_i \beta_i dY^i) + \sum_{ij} C_{ij}\, dY^i dY^j$$

Take limit: $U = u \quad Y^i = y^i/R \quad (1)$

$V = v/R^2 \quad R\to\infty$

- Interpretation:
  - boost along a direction, while taking the scale of the metric to infinity

$$\begin{cases} t' = \cosh\beta\, t + \sinh\beta\, X \\ x' = \sinh\beta\, t + \cosh\beta\, X \end{cases}$$

$$\Rightarrow \begin{array}{l} x'-t' = e^{-\beta}(x-t) \\ x'+t' = e^{\beta}(x+t) \end{array}$$

then scale all coordinates (t, x) by $1/R$, with $e^\beta = R\to\infty \Rightarrow (1)$

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· pp waves:

$$ds^2 = 2dx^+dx^- + (dx^+)^2 H(x^+, x^i) + \sum_i dx_i^2$$

$$R_{++} = -1/2\, \partial_i^2 H(x^+, x^i)$$

· pp wave solutions to sugra:

· 11d sugra:

$$ds^2 = 2dx^+dx^- + (dx^+)^2 H(x^+, x_i) + \sum_{i=1}^{9} dx_i^2$$

$$F_4 = dx^+ \wedge \varphi \qquad d\varphi = d * \varphi = 0$$

$$\partial_i^2 H = \frac{1}{12}|\varphi|^2$$

· In particular, $\varphi = 0$, $H = \frac{1}{|x - x_0|^7}$ → D0 brane

· Subclass: $H = \sum_{ij} A_{ij} x^i x^j$

$$2\,\mathrm{tr} A = \frac{1}{12}|\varphi|^2$$

- not flat at infinity!

• $\varphi = 0 \to \mathrm{tr} A = 0$ solution

· preserves 1/2 susy: $\Gamma_- \varepsilon = 0$

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· In particular, maximal susy solution: Kowalski-Glikman 1984 proved that the only susy backgrounds are M_{11}, $AdS_7 \times S_4$, $AdS_4 \times S_7$ and pp wave with

$$\begin{cases} \varphi = \mu\, dx^1 dx^2 dx^3 \\ A_{ij} x^i x^j = -\sum_{i=1,2,3} \frac{\mu^2}{9} x_i^2 - \sum_{i=4}^{9} \frac{\mu^2}{36} x_i^2 \end{cases}$$

· Obs: Aichelburg-Sexl metric: Shockwave - boosted black hole

· δ function source:

$$\delta^n(x^i, x^0) \to \delta^{n-1}(x^i)\delta(x^+)$$

$$\to H = \delta(x^+)\, h(x^i)$$

· 10d IIB solutions - similar Blau, Figueroa O'Farrill, Hull, Papadopoulos Oct. 2001

· pp wave solutions:

$$ds^2 = 2dx^+dx^- + H(x^i, x^+)(dx^+)^2 + \sum_{i=1}^{8}(dx^i)^2$$

$$F_5 = dx^+ \wedge (\omega + * \omega) \qquad \partial_i^2 H = -32|\omega|^2$$

$$d\omega = d * \omega = 0$$

1/2 susy.

$$ds^2 = 2dx^+dx^- - \mu^2 (dx^+)^2 \sum_{i=1}^{8} x_i^2 + \sum_{i=1}^{8} dx_i^2$$

$$F = \frac{\mu}{2} dx^+ \wedge (dx^1 dx^2 dx^3 dx^4 + dx^5 dx^6 dx^7 dx^8)$$

maximal susy

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- Maximal susy solutions are Penrose limits of $AdS \times S$:
  - $AdS_5 \times S_5$ - boost along equator of S_5; expand near $\rho=0, \theta=0$.
  - $AdS_4 \times S_7$ and $AdS_7 \times S_4 \to$ same Penrose limit.
  - Other cases:
    - $AdS_3 \times S_3$ with a combination of R-R and NS-NS fields:

$$ds^2 = 2dx^+ dx^- - \mu\left(\sum_{i=1}^{4} y_i^2\right)(dx^+)^2 + d\vec{y}^2$$
$$H^{NS} = c_1 \mu \cos\alpha\, dx^+ (dx^1 dx^2 + dx^3 dx^4)$$
$$H^{RR} = c_2 \mu \sin\alpha\, dx^+ (dx^1 dx^2 + dx^3 dx^4)$$

    - near horizon NS5-brane $\to$ Nappi-Witten geometry

$$ds^2 = 2dx^+ dx^- - (dx^+)^2 (x_1^2 + x_2^2) + \sum_{j=1}^{8} dx_j^2$$
$$H = \mu\, dx^+ dx^1 dx^2$$

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String quantization on pp-waves

- GS action in RR background
  - can be solved explicitly (Metsaev Dec 2001)
- Usually in RR background it's hard even to write down explicitly the action
- D branes propagating on supercoset manifolds - superfield formalism actions:

$$S = \int_M d^n\sigma \sqrt{g}\, g^{ij} L_i^A L_j^A + S_{WZ}$$

- String action in general background:

$$S = -\frac{1}{2}\int_{\partial M_3} d^2\sigma \sqrt{g}\, g^{ij} L_i^{\hat{a}} L_j^{\hat{a}} + i\int_{M_3} s^{IJ} L^{\hat{a}} \wedge \bar{L}^I \gamma^{\hat{a}} \wedge L^J$$

$L^{\hat{a}}$ = bos. supervielbein (sugra vielbeins pulled back)
L^I = ferm. —"—

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· On supercosets, supervielbeins;

$$L^A = L_0^A + 2\theta^\alpha f^A_{\alpha\beta}\left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\right)^\beta{}_\gamma (D\theta)^\gamma$$

$$(\mathcal{M}^2)^\alpha{}_\beta = -\theta^\gamma f_\gamma{}^\alpha{}_A \theta^\delta f_\delta{}^\beta{}_A$$

$f^A_{\alpha\beta}$ = structure constants of Fermi-Fermi algebra,

$$\{F_\alpha, F_\beta\} = f^A_{\alpha\beta} B_A$$

$$(D\theta)^\alpha = d\theta^\alpha + (L_0^A B_A)^\alpha$$

· gauge fix · κ symmetry $\Gamma^+\theta = 0$

· bosonic light cone gauge

$\sqrt{g}g^{ij} = \eta^{ij}$

$x^+(\sigma,\tau) = \tau$

$\Rightarrow$ String action on the 10d ppwave:

$$S = \frac{1}{2\pi\alpha'}\int dt \int_0^{2\pi\alpha' p^+} d\sigma \Big[\frac{1}{2}\dot{z}^2 - \frac{1}{2}z'^2 - \frac{1}{2}\mu^2 z^2 + i\bar{S}(\not\partial + \mu I)S\Big]$$

$I = \Gamma^{123}$

$S = \begin{cases}\text{Majorana 2d Spinor} \\ SO(8) \text{ pos. chirality spinor}\end{cases}$

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$\Rightarrow$ massive scalars and spinors

Fourier expand on $\sigma \Rightarrow$

$$H = -p_+ = \sum_{n\in\mathbb{Z}} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}}$$

$n>0$ left movers

$n<0$ right movers

$P = \sum_{n\in\mathbb{Z}} n N_n = 0$ - no momentum on string.

Limits:

· flat space: $\mu \to 0$ (metric $\to$ flat, $F_5 \to 0$)

Indeed $\mu\alpha' p^+ \ll 1 \Rightarrow \omega_n = \frac{n}{\alpha' p^+}$

· large RR background: $\mu\alpha' p^+ \gg 1 \Rightarrow$

$\omega_n = 1 + \ldots$

- we will explore this

· Strings on 6d ppwave ($AdS_3 \times S_3$)

$$S = \frac{1}{2\pi\alpha'}\int dt \int^{2\pi\alpha' p^+} d\sigma \, \frac{1}{2}\Big[|\dot{z}_i|^2 - |z_i' + i\cos\alpha\,\mu z_i|^2 - \sin^2\alpha\,\mu^2|z_i|^2\Big] + \bar{S}\big(\sigma^0\partial_0 + \sigma^1(\partial_1 + \cos\alpha\, I) + \sin\alpha\,\mu I\big)S$$

$$H = p^- = \sum_{n\in\mathbb{Z}} N_n \sqrt{\sin^2\alpha\,\mu^2 + \left(\cos\alpha\,\mu + \frac{n}{\alpha' p^+}\right)^2}$$

AdS-CFT correspondence in Penrose limit. (13)

· $AdS_5 \times S_5 \to 10d$ ppwave

$$ds^2 = 4dx^+dx^- - \mu^2 \vec{x}^2 (dx^+)^2 + d\vec{x}^2$$

Boost along equator ψ on S_5.

$$\begin{cases} \text{energy } E = i\partial_t \\ \text{ang. mom. } J = -i\partial_\psi \end{cases}$$ rotates dir. 5-6 ⊥ D3 brane

Map: $E \to$ conformal dimension Δ of operator

$J \to$ R charge rotating 5-6 of operator

momenta on ppwave $\begin{cases} 2p^- = -p_+ \rightsquigarrow \Delta - J \\ 2p^+ = -p_- \rightsquigarrow \frac{\Delta + J}{R^2} \end{cases}$ parameters of SYM operator

So: string theory: p^-, p^+ finite

$\Rightarrow$ in SYM, $\Delta + J \sim R^2$, $\Delta - J \sim 1$

$J \sim O(R^2) = O(\sqrt{N})$

$\Rightarrow$ Operators with large number of fields!

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So: Limit: $R \to \infty \Rightarrow g^2 N \to \infty$ in AdS limit

or:

$$\begin{cases} g^2_{YM} = \text{fixed (and} \ll 1) \\ N \to \infty \end{cases}$$: keep string interactions

· Look only at operators with $\Delta \sim J \sim \sqrt{N}$ - near BPS! $\Delta - J$ small.

· Extra condition:

giant gravitons (D3 branes wrapped on S^3) must be unimportant: $\frac{r}{\sqrt{\alpha'}} = \sqrt{2\pi g p^+ \mu \alpha'} \ll 1$

$$\Rightarrow J \ll \frac{\sqrt{N}}{g_{YM}}$$

· Next, find operator for which $(\Delta - J)_n = W_n|_{string} = \sqrt{1 + 4\pi g N \frac{n^2}{J^2}}$

· Obs: $n=0$: sugra modes (same as in flat space)

Metsaev-Tseytlin - sugra modes in pp wave

Strings from Super Yang Mills

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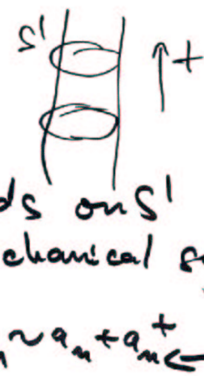
$\mathcal{N}=4$ SYM $\to$ CFT

- CFT: operator-state correspondence

2d: plane $\longleftrightarrow$ cylinder

(radial quantization)



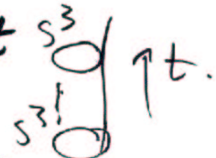
dim. reduce fields on S^1

$\Rightarrow$ quantum mechanical system

$\partial^n X \to X_n \sim a_n + a_n^\dagger \longleftrightarrow a_n^\dagger |0\rangle$

4d : $\mathbb{R}^4 \longleftrightarrow S^3 \times \mathbb{R}_t$

(radial quantization)



dim. reduce fields on S^3

$\Rightarrow$ quantum mechanical system

$\partial^n X \to X_n$ ... we will see

String spectrum

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- SYM operators:
  - $Z = \phi^5 + i\phi^6$ has $J=1 \Rightarrow \Delta - J = 0$
    $\hookrightarrow$ unique!

$\Rightarrow |0, p^+\rangle = \frac{1}{\sqrt{J} N^{J/2}} Tr[Z^J]$

- $\Delta - J = 1$: 8b. $\begin{cases} \phi^i, & i=1,2,3,4 \text{ transverse} \\ D_i Z = \partial_i Z + [A_i, Z] & i=1,2,3,4 \text{ in } \mathbb{R}^4. \end{cases}$

- 8f : $\chi^a_{J=1/2}$

- $\Delta - J = 2$: $\bar{Z}$, $F_{\mu\nu}$, $\chi^a_{J=-1/2}$, $D_i D_j Z$, $D_j \phi_i$, $D_j \chi^a_{J=1/2}$

:

- $Tr[Z^J]$ = sugra mode in a part. wavefct.
- string theory : act with zero modes on vacuum $|0, p^+\rangle$.
  $a_0^i, i=1,...,8$, $S_0^b, b=1...8$ to generate all the multiplet
- Sugra : act with all the symmetries:
  J^{+I} doesn't commute with $P^- \sim \Delta - J$

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· SYM : $Tr[Z^J] \to \sim \sum_l Tr(Z^l \phi^r Z^{J-l}) \sim Tr[\phi^r Z^J]$

- change $\Delta - J$: ϕ^r has $\Delta - J = 1$.

· So, operators : · vacuum : BPS : $p^- = \Delta - J = 0$

· small nr. of insertions → near BPS $p^- = \Delta - J$ small

· 2 fields: e.g. $\sim \sum_{l=1}^{J} Tr[\phi^r Z^l \psi^b_{J=1/2} Z^{J-l}]$

- neglect 2 symmetry op. on same state
- "dilute gas approx." → low nr. of insertions

$$\left\{ \begin{array}{l} a_0^i = \{\phi^i, D_i Z\} \\ b_0^b : \{\psi^b_{J=1/2}\} \end{array} \right\}$$ sum over positions of insertions.

· Non-zero modes (string states):

add momentum on the string $\sim e^{in\sigma}$

e.g. $a^{+4}_{n}|0,p^+\rangle \sim \sum_{l=1}^{J} Tr[Z^l \phi^4 Z^{J-l}] e^{\frac{2\pi i n l}{J}}$

- is actually $= 0$ by Tr cyclicity !

momentum constraint (total mom. $= 0$)

→ need at least 2 osc.

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· 10d Symmetry algebra: contraction of $AdS_5 \times S_5$ algebra

$p^+, p^I, J^{+I}, J^{ij}, J^{i'j'}, Q^+, \bar{Q}^+$

const., fixed

$p^-, Q^-, \bar{Q}^-$ → commute with H.

Hamilt. H

$$[p^-, Q^+_\alpha] = \mu Q^+_\beta \Pi^\beta{}_\alpha$$

$$[p^-, J^{+I}] = P^I ; \ [p^-, p^I] = \mu^2 J^{+I}$$

$$[P^I, J^{+J}] = -\delta^{IJ} p^+$$

$$\{Q^+_\alpha, \bar{Q}^+_\beta\} = 2\gamma^-_{\alpha\beta} P^+$$

$$\{Q^-_\alpha, Q^-_\beta\} = 2\gamma^+_{\alpha\beta} P^- + \mu(\gamma^+ \gamma^{ij} \Pi)_{\alpha\beta} J^{ij} + \mu(\gamma^+ \gamma^{i'j'} \Pi')_{\alpha\beta} J^{i'j'}$$

String th:

$$\left\{ \begin{array}{l} P^I = p_0^I \\ J^{+I} = -i x_0^I p^+ \end{array} \right.$$

→ $a_0^I = \frac{1}{\sqrt{2}}(P^I - J^{+I}/p^+)$, $a_0^{+I} = \frac{1}{\sqrt{2}}(P^I + J^{+I}/p^+)$ → bos. osc.

$$\left\{ \begin{array}{l} Q^+ = 2\sqrt{p^+}\gamma^- \theta_0 \\ \bar{Q}^+ = 2\sqrt{p^+}\gamma^- \bar{\theta}_0 \end{array} \right.$$

$\theta_0 = \frac{\theta_0^1 + i\theta_0^2}{\sqrt{2}}$ → ferm. osc

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Similarly 11d pp-wave:

$p^I_+ J^{+I} \sim a^I$ $[a^I, a^{+J}] = p^+ \delta^{IJ}$

$Q^+_\alpha \sim b_\alpha + (\Gamma^-)_{\alpha\beta} b^+_\beta$ $\{b_{\alpha\dot\beta}, b^{+\gamma\dot\delta}\} = p^+ \delta^\gamma_\alpha \delta^{\dot\delta}_{\dot\beta}$

$P^- = H$ $[H, Q_-] = -\frac{1}{12} I Q_-$ → split Q_- in Q, S

$[H, a^i] = -\frac{\mu}{3} a^i$; $[H, a^{i'}] = -\frac{\mu}{6} a^{i'}$

$[H, b_{\alpha\dot\beta}] = -\frac{\mu}{4} b_{\alpha\dot\beta}$

$[H, Q_{\alpha\dot\beta}] = \frac{\mu}{12} Q_{\alpha\dot\beta}$ $S^+_{\alpha\dot\beta} = Q_{\alpha\dot\beta}$

$[H, S^{\gamma\dot\delta}] = -\frac{\mu}{12} S^{\gamma\dot\delta}$

$\{Q_{\alpha\dot\beta}, S^{\gamma\dot\delta}\} = \delta_\alpha^\gamma \delta_{\dot\beta}^{\dot\delta} h + i\frac{\mu}{6} \delta_{\dot\beta}^{\dot\delta} (\gamma_{ij})_\alpha^\gamma J_{ij} + i\frac{\mu}{12} \delta_\alpha^\gamma (\gamma_{i'j'})_{\dot\beta}^{\dot\delta} J_{i'j'}$

⋮

Similarly to 10d, a^I, $b_{\alpha\dot\beta}$ give center of mass osc. of the system ($U(1)$ piece of D0 action)

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Picture:

Discretized string with J sites ⟷ SYM operators (of dim. J)

string bits

$|0, p_+\rangle \longleftrightarrow Tr[Z^J]$

$1 Z = 1$ bit : carries $J = 1$ → 1 unit of $P^+ = \frac{\Delta + J}{R^2}$

Insertion of a_n^+ (momentum $e^{in\sigma}$) ⟷ insertion of $\phi e^{in\sigma} = \phi e^{in\frac{2\pi l}{J}}$

string states ⟷ sum over "words"

Planar diagrams dominate ($J \sim g_{YM}\sqrt{N}$)

locality ⟷ planarity

We will see that,

string hamiltonian acting on states ⟷ Feynman diagrams 'acting' on operators

- We want to derive that a state with momentum n has:

$$\Delta - J \to \omega_n = \sqrt{1 + \frac{4\pi g N}{J^2} n^2}$$

Perturbative calculation (21)

$\mathcal{O}^*(0)$

$\mathcal{O}(x)$ free

Planar diagrams only $\Rightarrow$ locality: interaction can only hop 1 site: $l \to l \pm 1$

interaction term: $Tr([Z,\phi][\bar{Z},\bar{\phi}])$

"1-loop" $(\mathcal{O}(g^2_{YM}))$ + compute: $\langle \mathcal{O}(x)\mathcal{O}^*(0)\rangle = \frac{\mathcal{N}}{|x|^{2J+2}}$

$$\cdot\left[1 - \frac{4\pi g N n^2}{J^2}\cdot \log|x|\Lambda\right]$$

$$\Rightarrow (\Delta - J)_n = 1 + 2\pi g N \frac{n^2}{J^2}$$

. "Dilute gas" approximation: insertions are not near each other $\Rightarrow$ treat them independently ($\mathcal{O}(1/J)$ error)

$$\Rightarrow \Delta - J = \sum_n N_n (\Delta - J)_n .$$

- Obs: Near BPS: gives n^2/J^2 suppression

$\to$ otherwise, corr. $\sim \mathcal{O}(g^2_{YM} N) \to \infty$

Discretized worldsheets (22)

$\Rightarrow$ Hamiltonian description

. operator - state correspondence: look at SYM states on $S^3 \times \mathbb{R}$ ($\hookrightarrow$ radius 1.)

. KK expand fields: tower $\phi_n \leftrightarrow \partial^n \phi$

ϕ_n: N^2 harmonic oscillators

. Z: lowest mode of fields $Z \leftrightarrow a^{+i}{}_j$

$Tr[Z^J] \longleftrightarrow Tr((a^+)^J)|0\rangle \equiv |0\rangle'$

insertion of $\phi \leftrightarrow$ insertion of $b^{+i}{}_j$

$\Rightarrow$ States = words, up to cyclicity of Tr.

. But: Gopakumar, Gross '94 (Voiculescu, Dijkgraaf, N...)

$N \to \infty$ matrix structure encoded in "free random variables", associated with Cuntz oscillators.

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So: $\frac{a_\alpha{}^i{}_j}{\sqrt{N}} \to a_\alpha$ $\begin{cases} a_\alpha a_\beta^+ = \delta_{\alpha\beta} \\ \sum_\alpha a_\alpha^+ a_\alpha = 1 - |0\rangle\langle 0| \end{cases}$

and no other relations.

- describes the "word" structure (up to cyclicity of trace) - no commutation possible.

$\Rightarrow$ states: $a^+ \dots a^+ b^+ a^+ \dots a^+ b^+ a^+ \dots |0\rangle$

In $a_\alpha{}^i{}_j$ picture: Hamiltonian acting on states $\Longleftrightarrow$ all possible contractions of H with operators

· Few b^+'s, many a^+'s $\Rightarrow$ forget a^+'s.

· Define $b_j = 1$ (independent) (Cuntz oscillator at each site J along the string.

$\Rightarrow b_j b_j^+ = 1$; $b_j^+ b_j = 1 - |0\rangle\langle 0|_j$

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Then, calculate

$$H_{int} = g_{YM}^2 \mathrm{Tr}([Z,\phi][\bar{Z},\bar{\phi}]) \to$$

$$\to g\frac{N}{2\pi}(b_j + b_j^+ - b_{j+1} - b_{j+1}^+)^2$$

$H = \sum_j b_j^+ b_j + H_{int}$.

Then, string state:

$$\sum_{l=1}^{J} \mathrm{Tr}[Z^l \phi Z^{J-l}] e^{2\pi i l \frac{n}{J}} \to \sum_{l=1} b_l^+ |0\rangle' e^{2\pi i l \frac{n}{J}}$$

Define: $b_n^+ = \frac{1}{\sqrt{J}} \sum_{l=1}^{J} e^{2\pi i l \frac{n}{J}} b_l^+$

$\Rightarrow$ In the "dilute gas" approximation

$[b_m, b_m^+] = \delta_{mm} + O(1/J)$ - usual oscillator

Then

$$H \simeq \sum_i A_i^+ A_i + \frac{\beta^2}{2} \frac{(A_i + A_i^+)^2}{2}$$

$A_i \sim b_n \pm b_{-n}$

- perturbed harmonic osc.

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After a Bogoliubov transformation,

$$\omega_n \simeq \sqrt{1 + 4\pi g N n^2/J^2}$$

Approx: - perturbative in n/J ✓
- dilute gas approx. ✓

Interpretation:

$$H = \sum_j b_j^\dagger b_j + \frac{gN}{2\pi}\left(b_j + b_j^\dagger - b_{j+1} - b_{j+1}^\dagger\right)^2$$

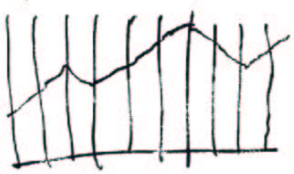
is discretized hamiltonian of string:

$$H = \int_0^L d\sigma \frac{1}{2}\left[\dot\phi^2 + \phi'^2 + \phi^2\right] \qquad L = J\sqrt{\frac{\pi}{gN}}$$

ϕ insertion at site j $\to b_j^\dagger \to$ quantum fluctuation at site j of string field ϕ.

. What do we sum?

We iterate H_{int}:



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Other fields and diagrams

What do we neglect?

. genuinely higher loop expressions
- nonrenormalization theorems?

. other fields: $\Delta - J = 2, 3, \ldots$

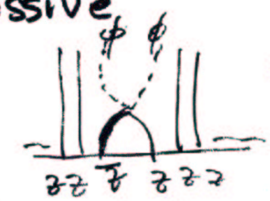
argument: e.g. $\bar Z$ $(\Delta - J = 2)$

compute perturbatively,

$$(\Delta - J)_n = 2 + \frac{gN}{4\pi}\left(4 + \frac{4\pi^2 n^2}{J^2}\right) + \ldots$$

$\Rightarrow$ in $gN \to \infty$ limit becomes very massive

- decay: $\sim gN$



. $\Delta - J = 1$ ~~$\Delta - J = 1$~~ don't get mass because they are Goldstone bosons and fermions of broken symmetries
$\to$ masses protected by susy algebra

M theory on the pp wave (27)

· D0 brane lagrangian:

· U(1): Supercoset supervielbein formalism:

$$S = \int dt \left(2 L^+_t L^-_t + L^I_t L^I_t \right)$$

$i = 1,2,3$; $i' = 4, \dots, 9$; $I = (i, i')$

$$\begin{cases} L^+ = dx^+ \quad ; \quad L^I = dx^I \\ L^- = dx^- - \frac{\mu^2}{2}\frac{x^{i2}}{9}dx^+ - \frac{\mu^2}{2}\frac{x^{i'2}}{36}dx^+ + \frac{1}{2}\bar{\theta}\Gamma^- D\theta \end{cases}$$

· U(N): generalize by usual $[X,X]^2$ and $\psi\gamma[X,\psi]$ terms and Myers term $F_{-ijk}\mathrm{Tr}X^iX^jX^k$

$\to S = S_0 + S_{mass}$

$$S_0 = \int dt\, \mathrm{Tr}\Big[\frac{1}{2(2R)}(D_0\phi^I)^2 + \psi^T D_0\psi + \frac{(2R)}{4}[\phi^J,\phi^K]^2 + i(2R)\psi^T\gamma^J[\psi,\phi^J]\Big]$$

$$S_{mass} = \int dt\, \mathrm{Tr}\Big[-\frac{\mu^2}{2(2R)}\Big(\frac{\phi^{i2}}{9} + \frac{\phi^{i'2}}{36}\Big) - \frac{\mu}{4}\psi^T\gamma_{123}\psi - i\frac{\mu}{3}\mathrm{Tr}\,\phi^i\phi^j\phi^k\epsilon_{ijk}\Big]$$

· Interpretation:
DLCQ quantization: look in the sector of the theory with $2p^+ = p_- = N/R$.

Supersymmetric solutions: (28)

- Supersymmetry is time dependent (no Q commutes with H)

$\epsilon(t) = e^{-\frac{\mu}{12}\gamma_{123}t}\epsilon_0$ → linearly realized

$\epsilon(t) = e^{\frac{\mu}{4}\gamma_{123}t}\epsilon_0$ → non linearly realized

- Ground states:
  - fully susy: $[\phi^i,\phi^j] = i\mu\,\epsilon_{ijk}\phi^k$, $\dot{\phi}^i = 0$, $\phi^{i'} = 0$

→ fuzzy S^2 of radius $r \sim \pi\frac{\mu}{3}\frac{N}{(2R)} \sim \frac{1}{(\text{coupling})}$

N = size of $SU(2)$ rep.
= giant graviton = spherical M2.

Obs. Giant gravitons on S^5: spherical M5

$r^4 \sim \frac{1}{(\text{coupling})}$ → not a classical solution?
→ probably is $\phi^i = 0$ vacuum

  - 1/2 susy: $(\phi^4 + i\phi^5)(t) = e^{i\frac{\mu}{6}t}(\phi^4 + i\phi^5)$

$\gamma_{12345}\epsilon_0 = \epsilon_0$

= momentum waves.

Future work

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- Open strings: $\mathcal{N}=2$ SCFT. $AdS_5 \times S_5/\mathbb{Z}_2$
  - has quarks in the fundamental
  - open string has quarks at the end
- String interactions
  - understand nonplanar diagrams.
- Other pp-waves.
  - study $AdS_3 \times S_3$ wave.
  - near horizon NS5 ppwave Nappi-Witten model
  - other quotients
  - nonconformal theories?
- Study the new Matrix model
- Understand holography
  - how does the correspondence work in this case?

Conclusions

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- String theory on pp waves can be solved explicitly
- We have derived string theory from $\mathcal{N}=4$ SYM in a special limit, realizing 't Hooft's idea in spirit.
- Flat space arises as a limit of the pp-wave, and it corresponds to a well defined limit in SYM theory.
- We can write down a new Matrix model which has a discrete set of ground states.
- There are many open questions for further study.