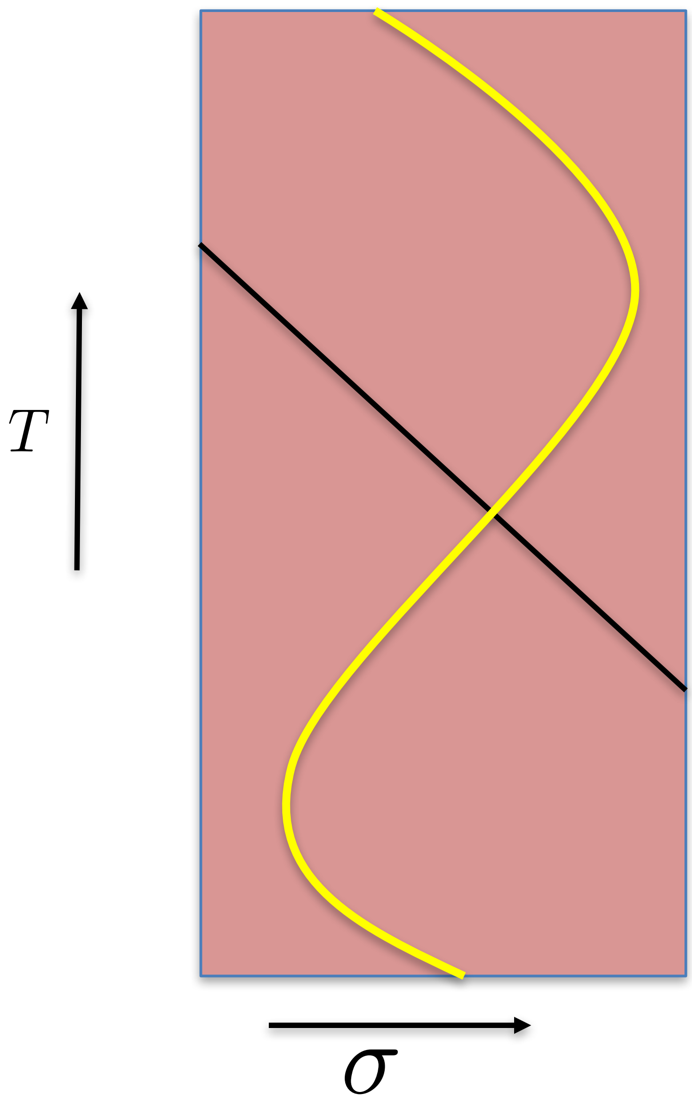
How to make a wormhole
Understanding global AdS_2

Juan Maldacena

Based on ongoing work with Xiaoliang Qi .

Also on previous work:
- Gao, Jafferis, Wall
- JM, Douglas Stanford and Zhenbin Yang.
- Ioanna Kourkoulou and JM

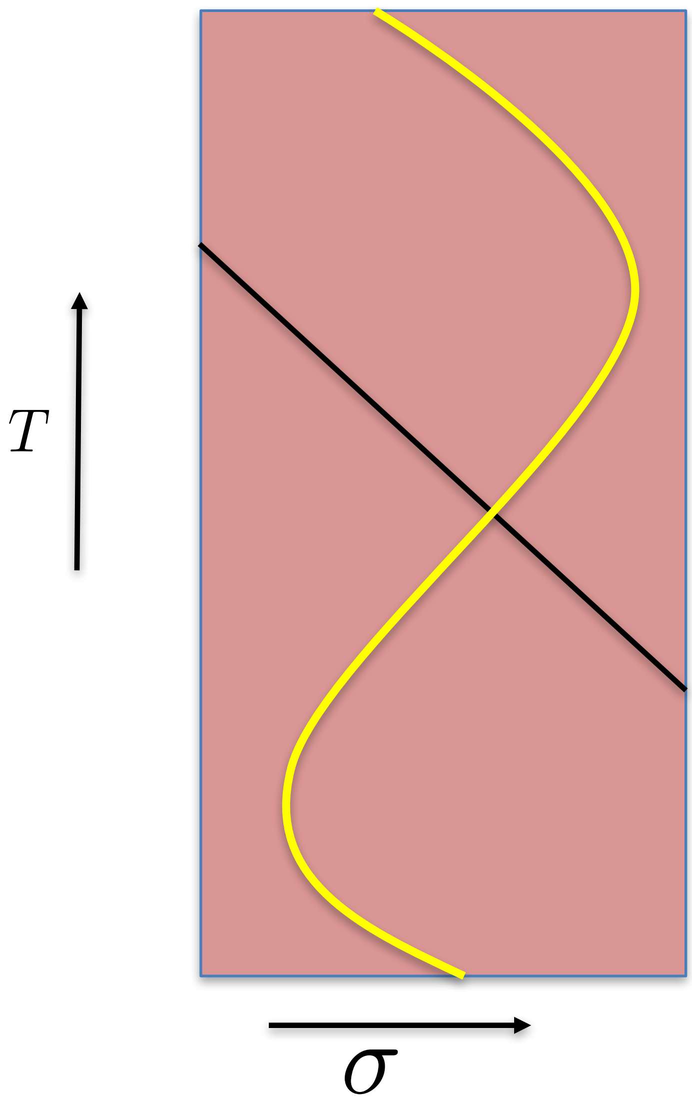
AdS_2 - Global coordinates



$$ds^2 = \frac{-dT^2 + d\sigma^2}{(\sin\sigma)^2}$$

- SL(2,R) isometries
- Two boundaries
- Causally connected
- Particle dynamics → oscillatory behavior → gapped spectrum
- Global coordinates

AdS_2 : A traversable wormhole



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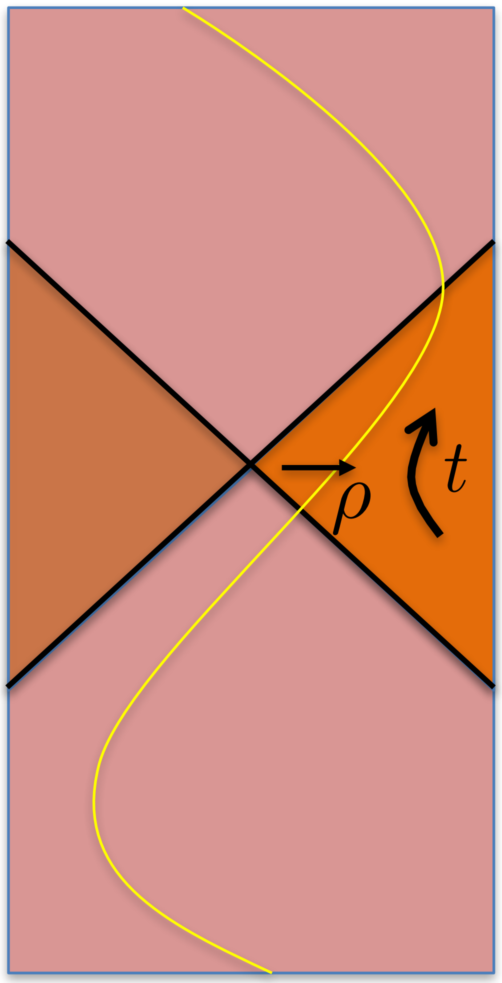
AdS_2 - thermal (Rindler) coordinates



$$ds^2 = -dt^2 \sinh^2 \rho + d\rho^2$$

- Two boundaries
- Cover only a portion of AdS_2
- <u>Causally disconnected</u>

T, t are conjugate to two different elements of the SL(2,R) isometries of AdS_2

Non-travesable wormhole

AdS_2 vs $NAdS_2$ asymptotic boundary conditions

- Exact AdS_2 boundary conditions do not make sense

JM, Michelson, Strominger

- Need to break some of the AdS_2 isometries slightly
- We should think about nearly-AdS_2

- Nearly AdS_2 with t-isometry → TFD of Nearly CFT_1
- Nearly AdS_2 with T-isometry → ?

First recall some facts about nearly AdS_2 boundary conditions…

Nearly AdS_2 gravity

Keep the leading effects that perturb away from AdS_2

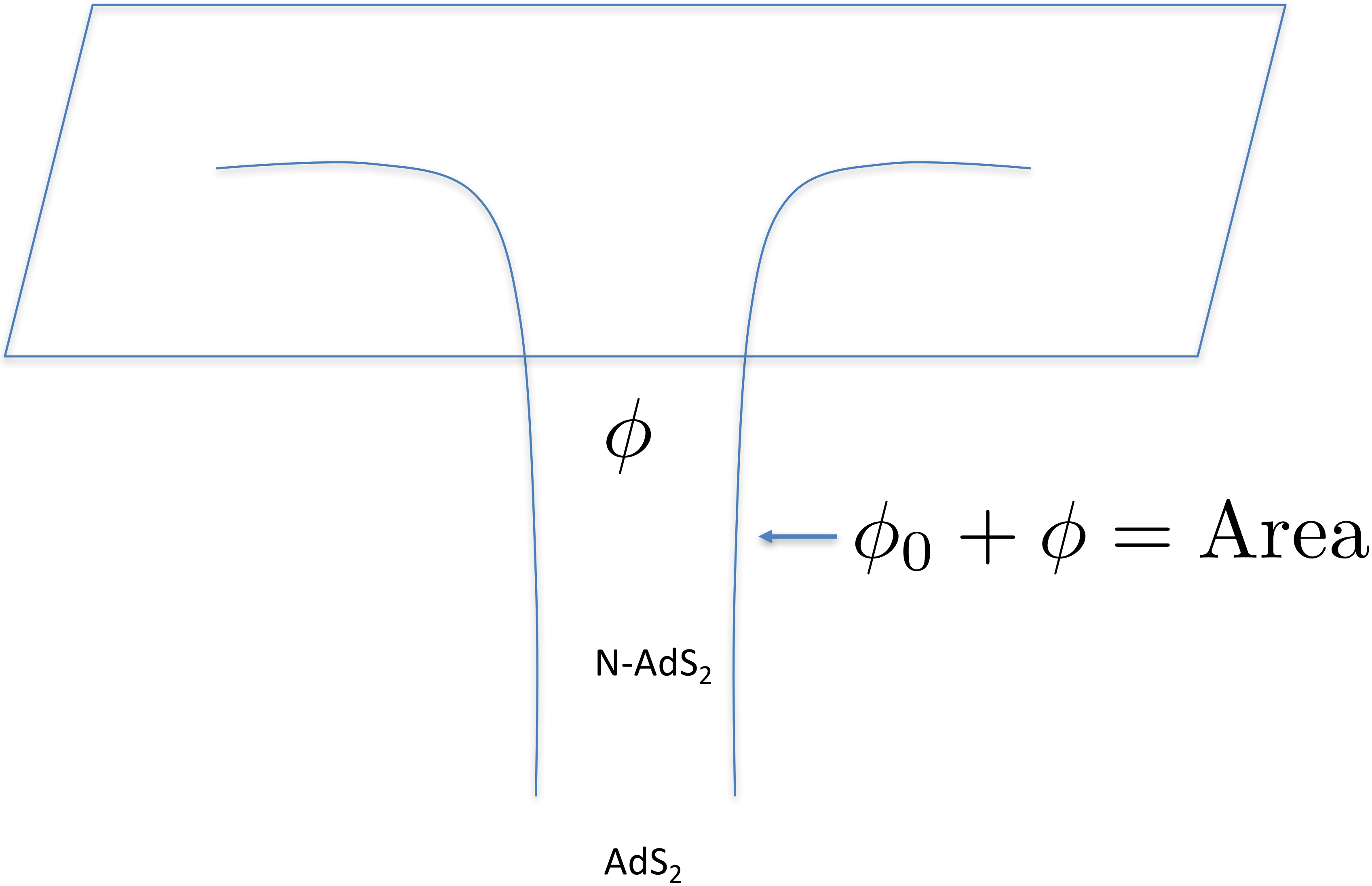
Teitelboim Jackiw
Almheiri Polchinski

$$\int d^2x\sqrt{g}\phi(R+2) + \phi_0 \int d^2x\sqrt{g}R$$

Topological term.
Ground state entropy

Gives leading gravitational dynamics.

Universal description for near extremal black holes.



ϕ
$\phi_0 + \phi = \text{Area}$
N-AdS$_2$
AdS$_2$

Nearly AdS_2 gravity

Keep the leading effects that perturb away from AdS_2

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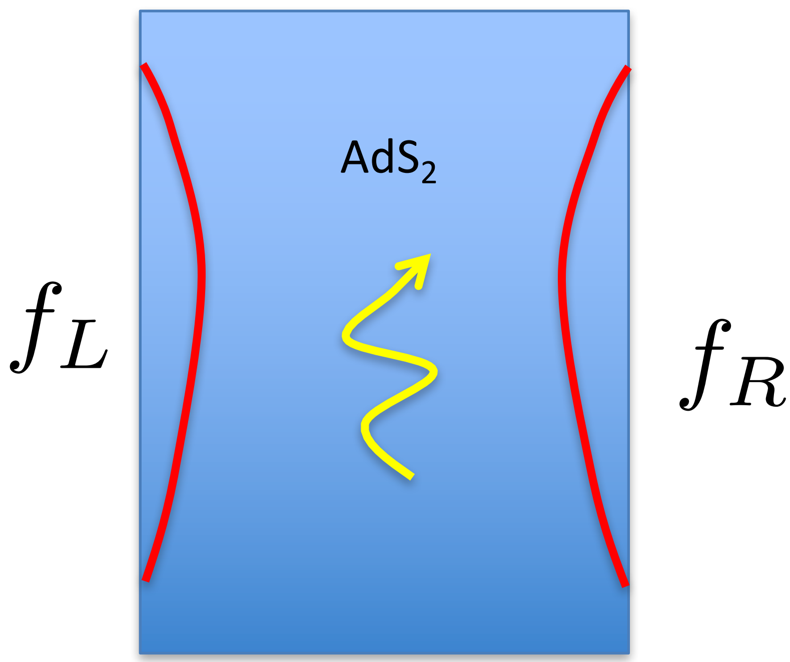
$$\int d^2x\sqrt{g}\phi(R+2) + \phi_0 \int d^2x\sqrt{g}R$$

Comes from the area of the additional dimensions.

Ground state entropy

No bulk excitations → only "boundary gravitons" → location of the physical boundary in AdS_2

Gravitational dynamics



$$\int \phi(R+2)$$

Rigid AdS$_2$

Physical boundary given by dilaton

Dynamics is in the position of the boundary.

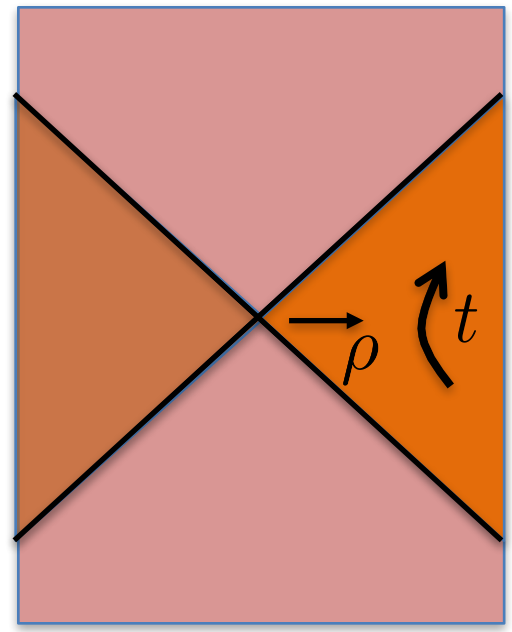
Boundary graviton: encodes the motion of the boundary.

$$(H_{f_L} \times H_{\rm bulk} \times H_{f_R})/SL(2,R)$$

- With pure gravity, the only solution with ϕ growing towards both boundaries is the thermal AdS_2 , with t- isometry.

- We need some sort of matter.

- No ordinary matter →



$$-e^{-2\omega}\partial_+\phi\Big|_{-\infty}^{+\infty} = \int_{-\infty}^{\infty} dx^+ T_{++} < 0$$

Integrate

$$-\partial_+(e^{-2\omega}\partial_+\phi) = e^{2\omega}T_{++} \ , \qquad ds^2 = -e^{2\omega}dx^+dx^-$$

(Raychaudhuri eqn)

A small digression..

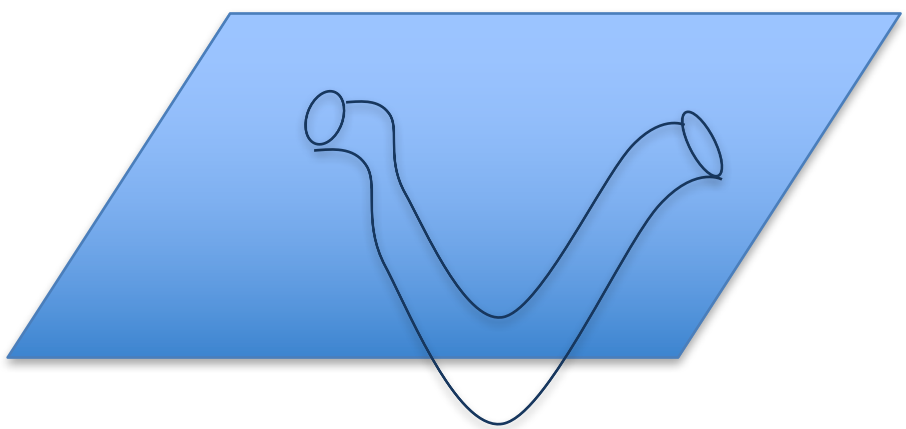
Topological Censorship in GR

- We cannot have disconnected boundaries that are causally connected through the bulk.
- We cannot have non-trivial topology in asymptotically flat space (traversable wormhole).

Even if the length of the wormhole is larger than the distance between its two mouths.

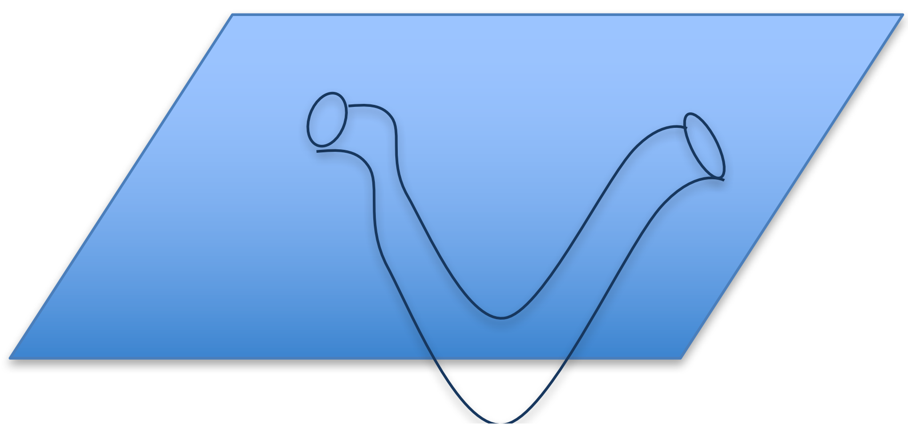
Friedman, Schleich, Witt
Galloway, Schleich, Witt, Woolgar

- If we obey the positive null energy condition.



Not true when we include quantum effects

- Quantum effects should connect the boundaries.
- Do not obey the positive null energy condition.
- In principle, we could have non-trivial topology in asymptotically flat space (traversable wormhole) (mouths should be closer than length of the wormhole)



Back to AdS_2 …

We will look at a simple example

- Nearly-AdS_2 gravity
- Plus matter
- Plus boundary conditions connecting the two sides (as in Gao-Jafferis-Wall)

$$S_{int} = \mu \int du \chi_L(u) \chi_R(u)$$

u is proper length along the boundary, or boundary time.

- This generates negative null energy and allows for a solution with the global time isometry, where ϕ grows towards both boundaries

In parallel we will look at a similar problem in the SYK model.

Sachdev, Ye, Kitaev model (SYK)

Quantum mechanical model, only time.

$$\{\psi_i, \psi_j\} = \delta_{ij}$$

N Majorana fermions

$$H = \sum_{i_1,\cdots,i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

$$(H_q = J_{i_1 \cdots i_q} \psi^{i_1} \cdots \psi^{i_q})$$

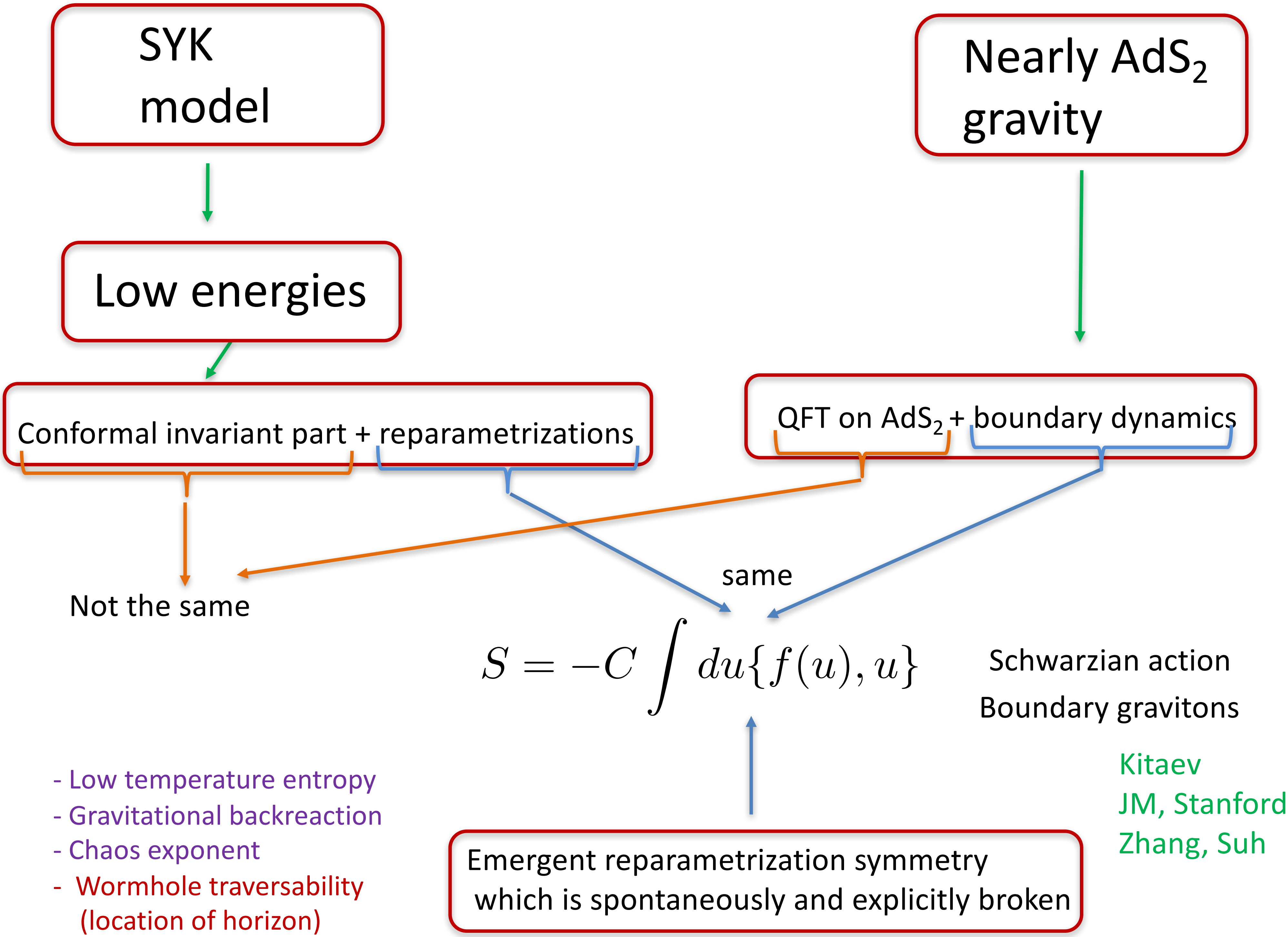
random couplings

$$\langle J^2_{i_1 i_2 i_3 i_4} \rangle = J^2 / N^3$$

J = single dimension one coupling.

N large, strong coupling 1 << (time) J << N (still exponentially many energy levels)

- The SYK model has some properties in common with nearly AdS_2 gravity.
- It has the same gravitational dynamics.
- This dynamics is expected to be universal for any system with an almost conformal symmetry in the IR (which is not integrable).



SYK model
Nearly AdS_2 gravity
Low energies
Conformal invariant part + reparametrizations
QFT on AdS_2 + boundary dynamics
Not the same
same
$S = -C \int du \{ f(u), u \}$
Schwarzian action
Boundary gravitons
Kitaev
JM, Stanford
Zhang, Suh
- Low temperature entropy
- Gravitational backreaction
- Chaos exponent
- Wormhole traversability (location of horizon)
Emergent reparametrization symmetry which is spontaneously and explicitly broken

Two copies of SYK + Interaction

$$H = H_L + H_R + \mu \int du \sum_i \psi_L^i(u) \psi_R^i(u)$$

Two copies of SYK + Interaction

$$H = H_L + H_R + \mu \int du \sum_i \psi_L^i(u)\psi_R^i(u)$$

↓ Low energies

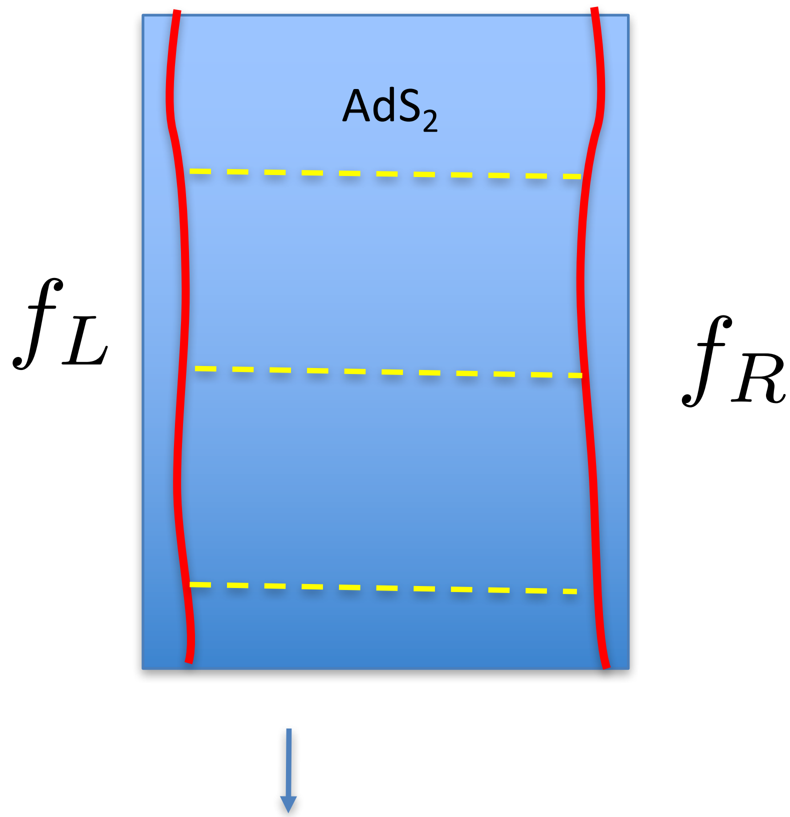
$$S = \frac{N\alpha_S}{J} \int du \{f_L(u), u\} + \{f_R(u), u\} + N\mu \int du \left[\frac{f_L'(u) f_R'(u)}{|f_L(u) - f_R(u)|^2} \right]^\Delta$$

+ Global SL(2,R) gauge symmetry → set total SL(2,R) charge to zero.

$$f(u) = \tan(T(u)/2)$$

AdS$_2$ gravity +

Interaction



$$S = \frac{N\alpha_S}{J}\int du \{f_L(u), u\} + \{f_R(u), u\} + N\mu \int du \left[\frac{f_L'(u) f_R'(u)}{|f_L(u) - f_R(u)|^2}\right]^\Delta$$

+ Global SL(2,R) gauge symmetry → set total SL(2,R) charge to zero.

$$f(u) = \tan(T(u)/2)$$

$$T_L(u) = T_R(u) = (\text{constant})u = T'u$$

is a solution of the equations of motion.

zero SL(2,R) charges

$$N \frac{1}{J}(T')^2 \propto N\mu \left(\frac{T'}{J}\right)^{2\Delta}$$

$$\left(\frac{T'}{J}\right)^{2(1-\Delta)} \propto \frac{\mu}{J}$$

A solution always exists for small $\frac{\mu}{J} \ll 1$

$$\text{for} \quad 0 < \Delta < \frac{1}{2}, \qquad \mu \ll T' \ll J$$

$$T_L(u) = T_R(u) = (\text{constant})u = T'u$$

$$\text{for} \quad 0 < \Delta < \frac{1}{2} \ , \qquad \mu \ll T' \ll J$$

↑

sets the scale of the energy gap. Relation between AdS_2 time and boundary time, u.

Field in AdS_2 corresponding to a boundary operator of dimension Δ →

$$E = E_u = T'(\Delta + n)$$

Spectrum governed by conformal symmetry
(like in higher dimensional global AdS)

Schwarzian, or dynamical boundary degree of freedom (boundary graviton) →
For small perturbations: one harmonic oscillator with energy

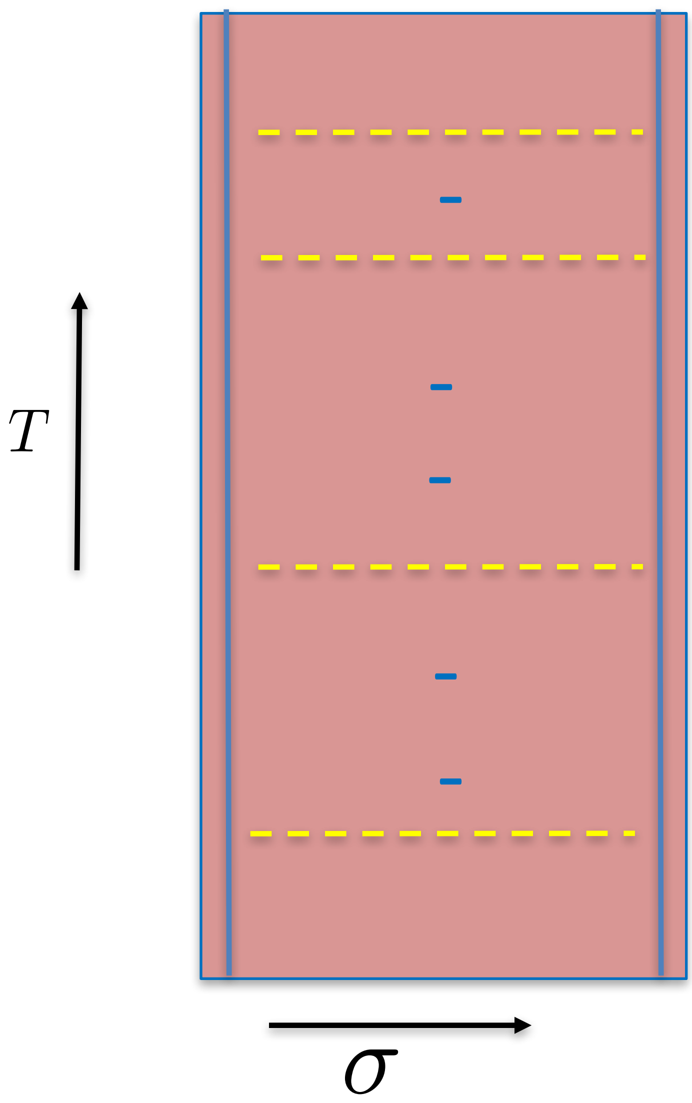
$$E = T'\sqrt{2(1-\Delta)}(n + \frac{1}{2})$$

Stable equilibrium

Same energy scale as the particles inside

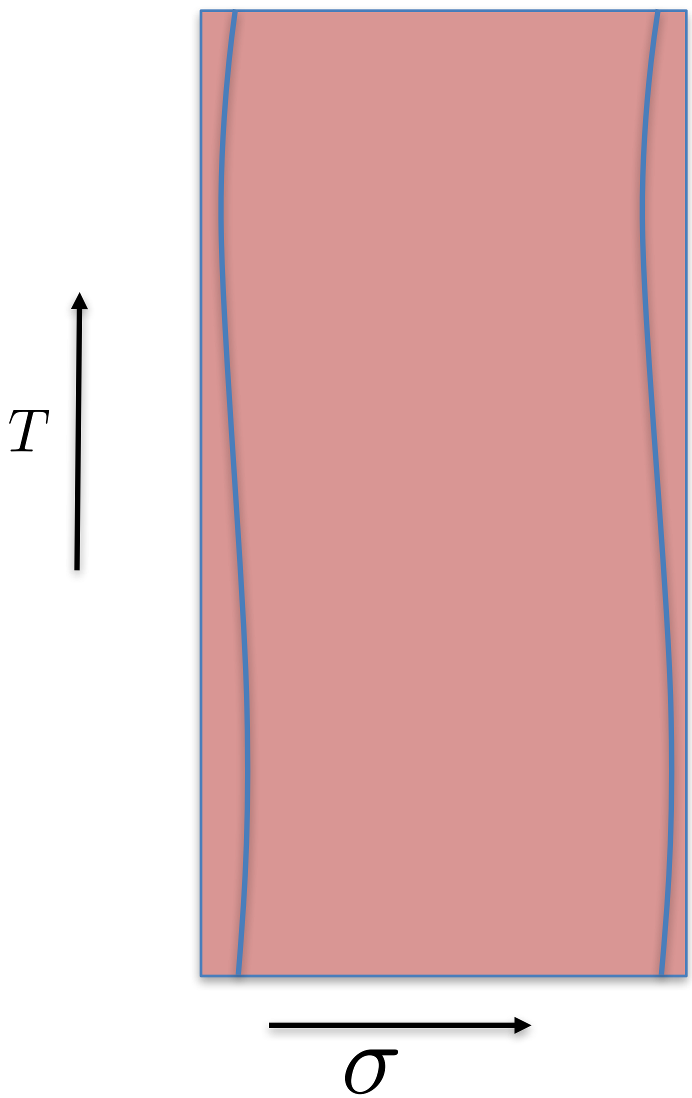
↑ This part is not conformal invariant.

Nearly AdS_2



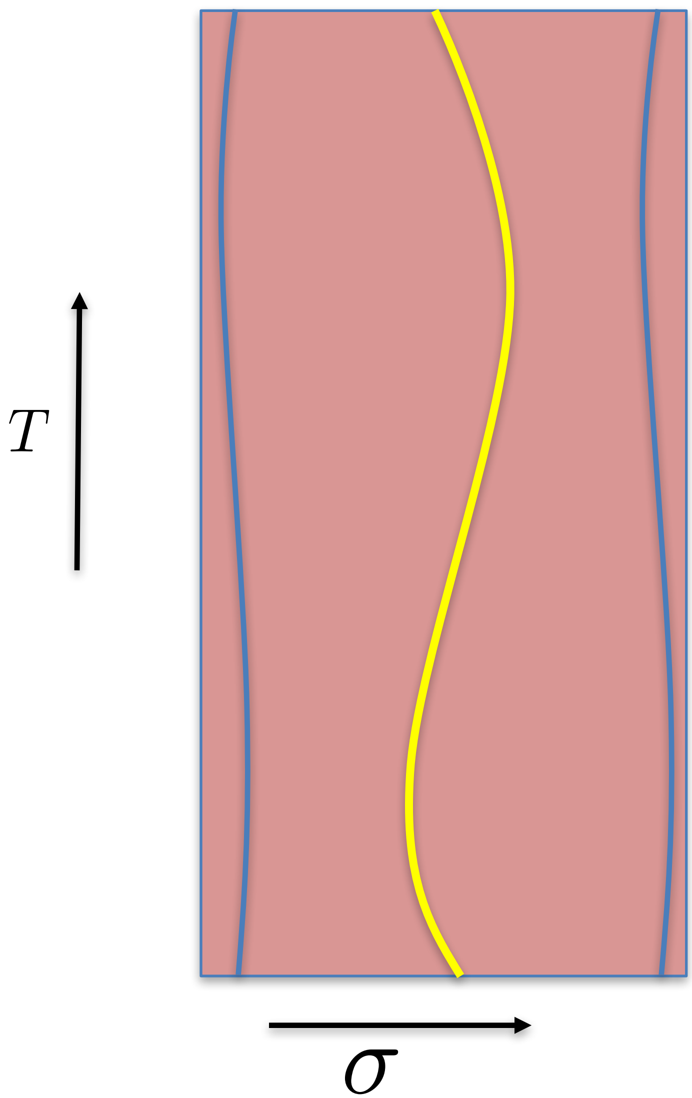
Casimir force due to the boundary conditions connecting the left and right sides → attractive force between the two boundaries.

Nearly AdS_2



Small oscillations around equilibrium position.

Nearly AdS_2



Adding matter.

Matter leads to a conformal spectrum to leading order.

In SYK we can also solve the theory beyond the low energy limit.

SYK analysis

- Large N → special set of diagrams
- Give rise to a closed set of equations for the fermion propagator

$$G(t_1, t_2) = \frac{1}{N} \sum_i \langle \psi^i(t_1) \psi^i(t_2) \rangle$$

$$\partial_{t_1} G - \Sigma * G = \delta(t_{12})$$

$$\Sigma(t_1, t_2) = J^2 [G(t_1, t_2)]^{q-1}$$

For usual SYK

Two coupled SYK systems

- Similar equations.
- We now have both left and right systems

$$G_{LL} \ , \quad G_{LR} \ , \qquad G_{RL} \ , \qquad G_{RR}$$

$$\partial_{t_1} G - \Sigma * G = \delta(t_{12})$$
$$\Sigma(t_1, t_2) = J^2 [G(t_1, t_2)]^{q-1}$$

For usual SYK

↓

$$\partial_{t_1} G_{LL} - \Sigma_{LL} * G_{LL} - \Sigma_{LR} * G_{RL} = \delta(t_{12})$$
$$\partial_{t_1} G_{LR} - \Sigma_{LL} * G_{LR} - \Sigma_{LR} * G_{RR} = 0$$
$$\Sigma_{LL}(t_1, t_2) = J^2 [G_{LL}(t_1, t_2)]^{q-1} \ ,$$
$$\Sigma_{LR}(t_1, t_2) = \mu \delta(t_{12}) + J^2 [G_{LR}(t_1, t_2)]^{q-1}$$

Two coupled
SYK systems

Simplest ansatz

Valid for $\frac{\mu}{J} \ll 1$

$$G(f_1, f_2) \propto \frac{\text{sign}(f_1 - f_2)}{|f_1 - f_2|^{2\Delta}}$$

Solves the low energy limit of the equations. The equations have a reparametrization symmetry in that limit.

$$f(u) = \tan(T(u)/2)$$

Include the leading reparametrization symmetry breaking effects.

→ Previous discussion.

Methods to analyze the equations

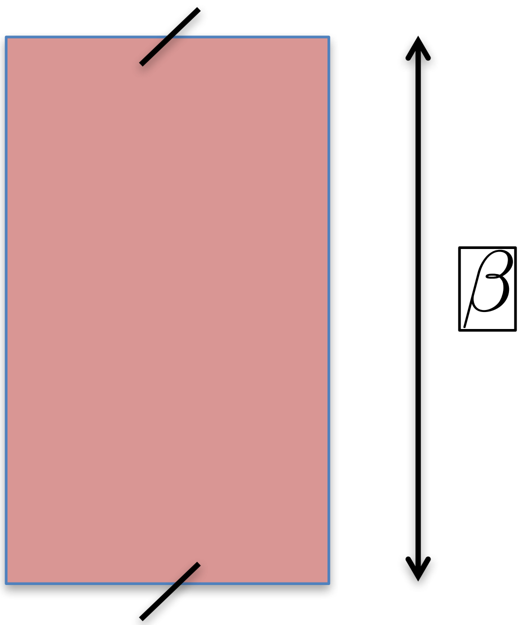
- Numerical
- Large q approximation → analytic solution.

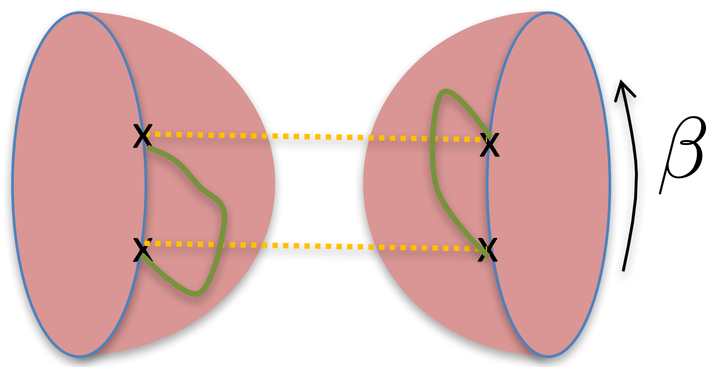
- We can now solve the equations not limited to the small $\frac{\mu}{J}$ approximation.

- It is also interesting to study the finite temperature situation.

Finite temperature and Hawking Page phase transition.

- Two possible finite temperature configurations in Euclidean space



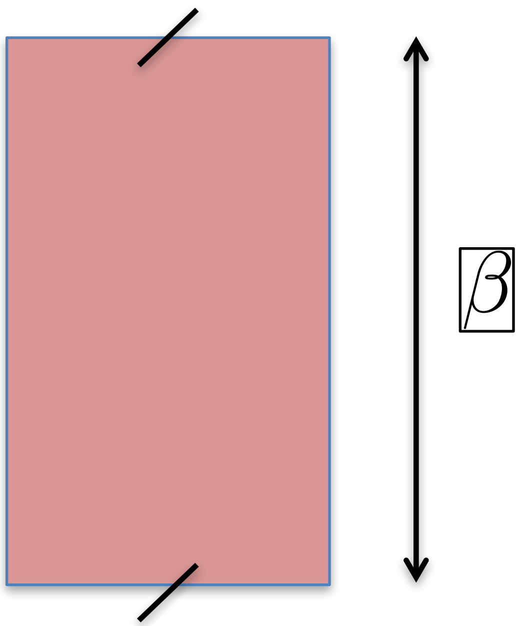


$$\log Z/N \sim 2S_0 + \frac{1}{\beta J} + \mu^2(\beta J)^{2\Delta}$$

"ground state" entropy contribution

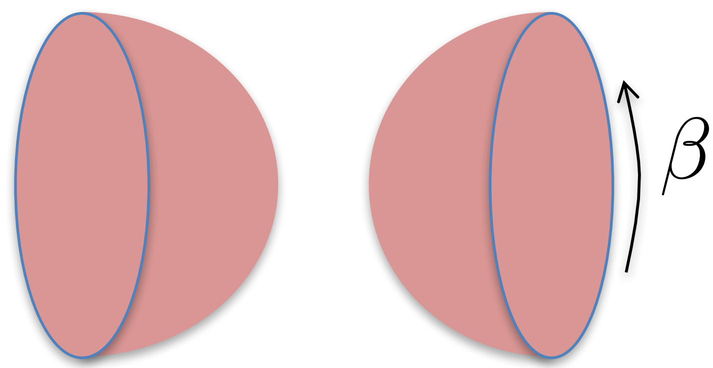
$$\log Z/N \sim -\beta(T')^2 + e^{-\beta T'\Delta} \sim -\beta\left(\frac{\mu}{J}\right)^{\frac{1}{(1-\Delta)}} + e^{-\beta T'\Delta}$$

Finite temperature and Hawking Page phase transition.



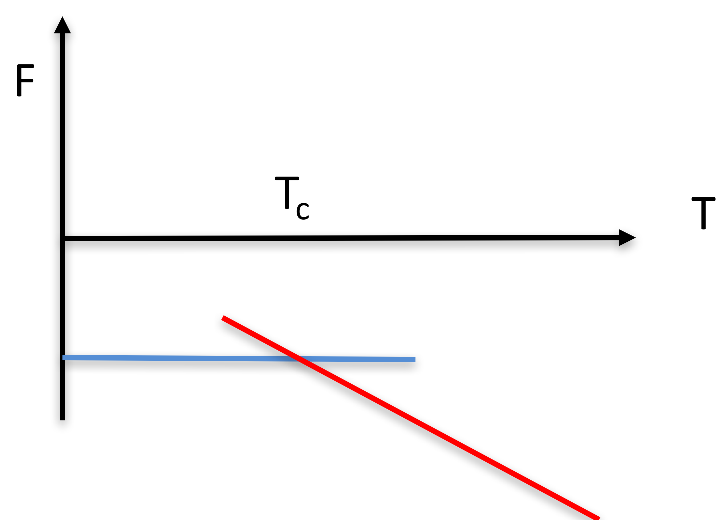
$$\log Z/N \sim \beta \left(\frac{\mu}{J}\right)^{\frac{1}{(1-\Delta)}}$$

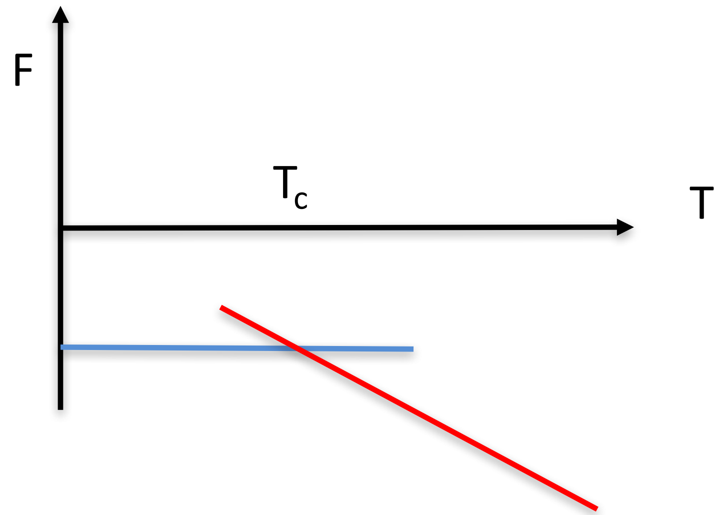
Low temperature



$$\log Z/N \sim 2S_0$$

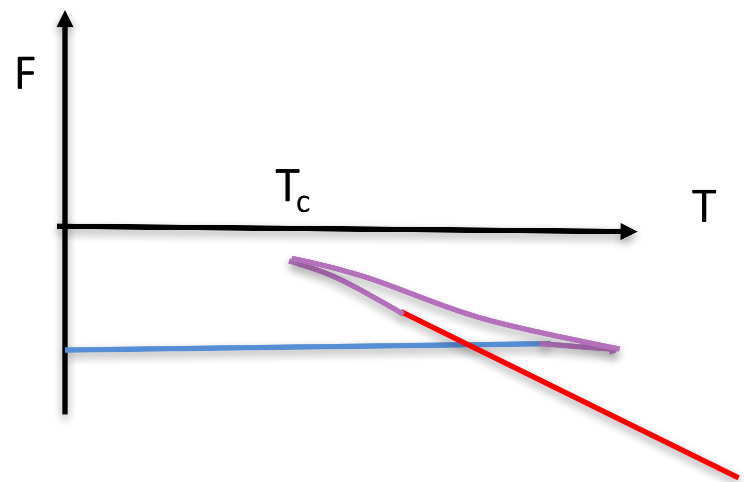
High temperature





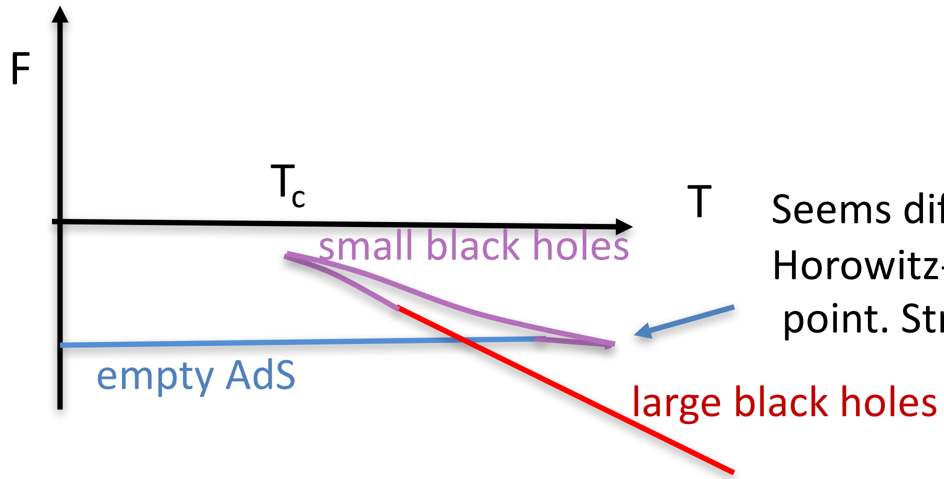
- Both phases can be described using the conformal approximation + Schwarzian correction, or as nearly AdS_2 gravity configurations plus further small corrections.
- Phase transition happens when both approximations are valid.
- But the underlying conformal solutions really different configurations. The SL(2,R) gauge symmetries act differently.

- At large q, analytically one can find



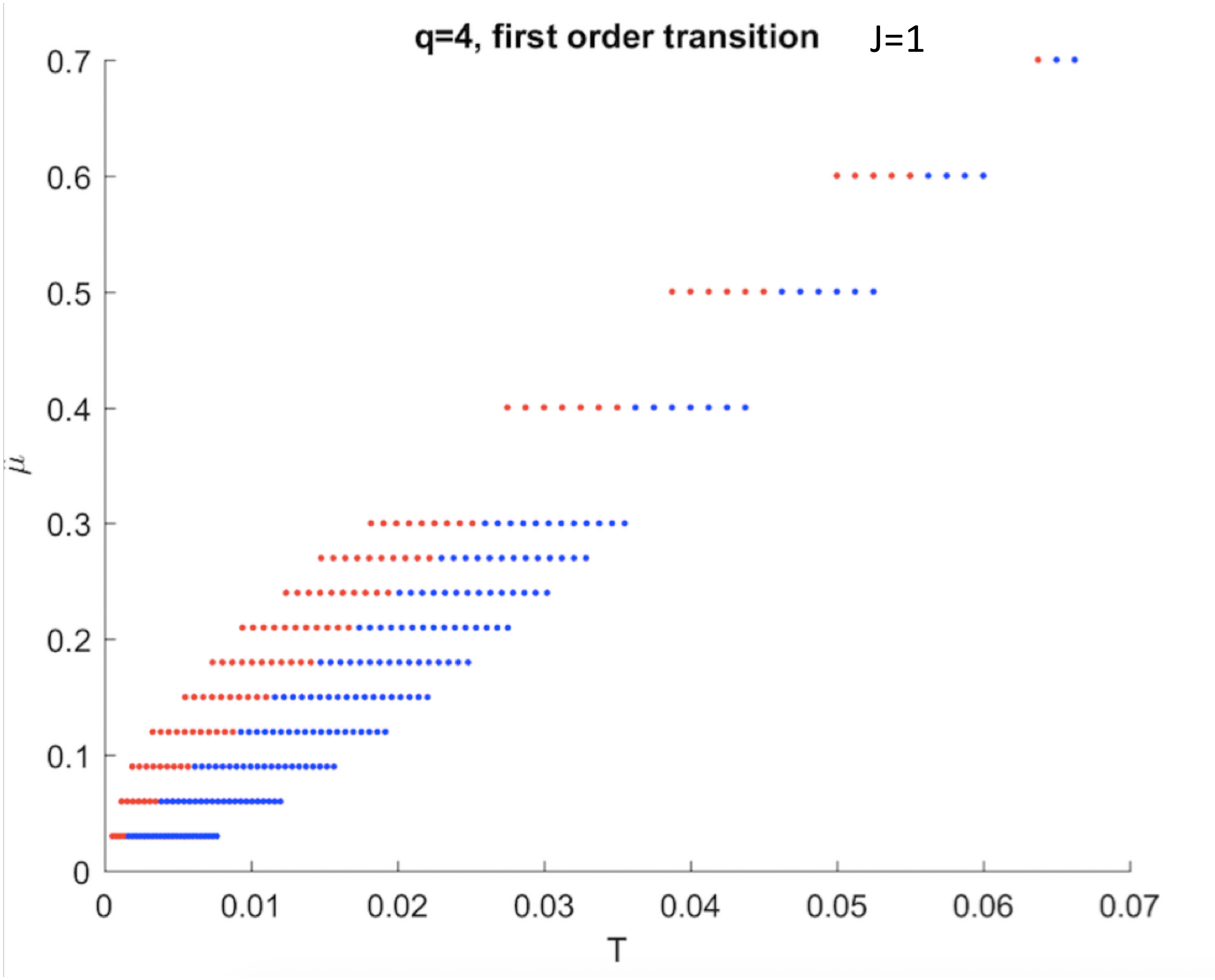
- One can go continuously between the two phases.
- But it is necessary to go beyond the conformal approximation. Beyond the Schwarzian approximation.
- We have to go beyond the Nearly AdS_2 approximation due to the backreaction of the matter inside AdS_2 .
- Once we put the boundary interaction, no clear way to distinguish the two phases.

- At large q, analytically one can find



- In the cannonical ensemble: 1st order phase transition (like Hawking-Page).
- In the microcannonical ensemble → continuous behavior.
- Continuous connection between the phase with no black holes and the one with a ``small black hole''

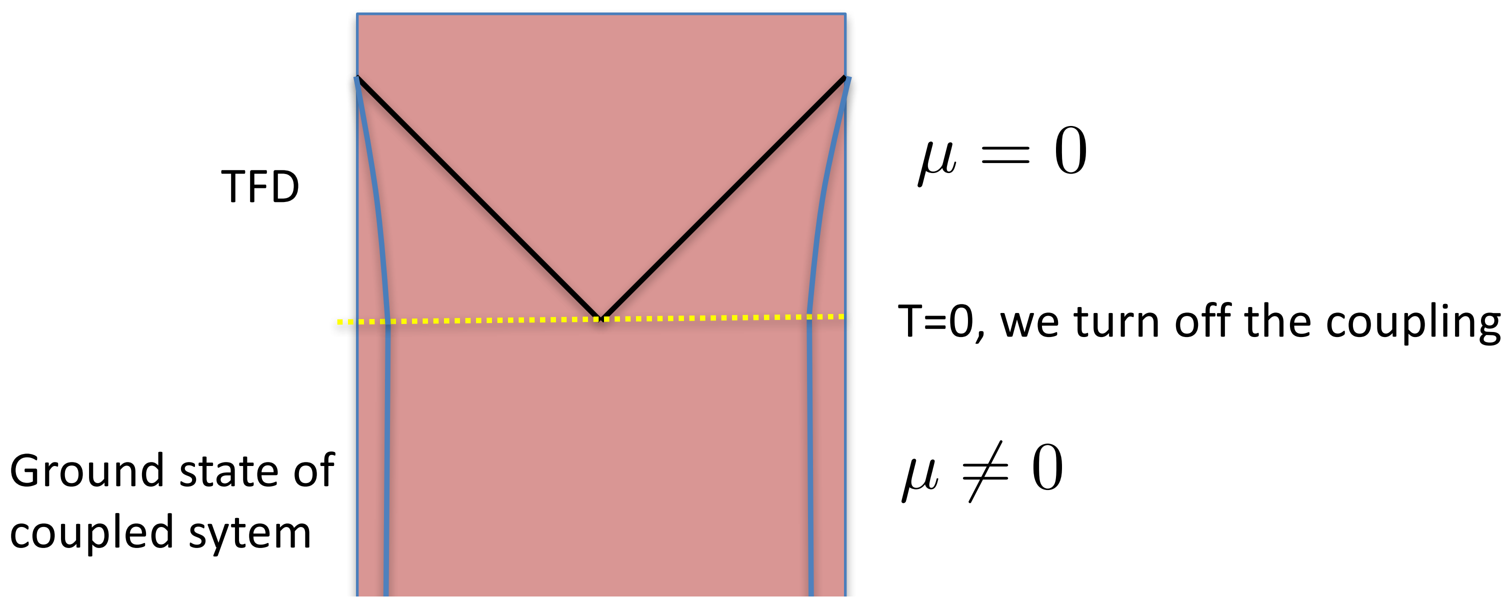
Numerical Analysis



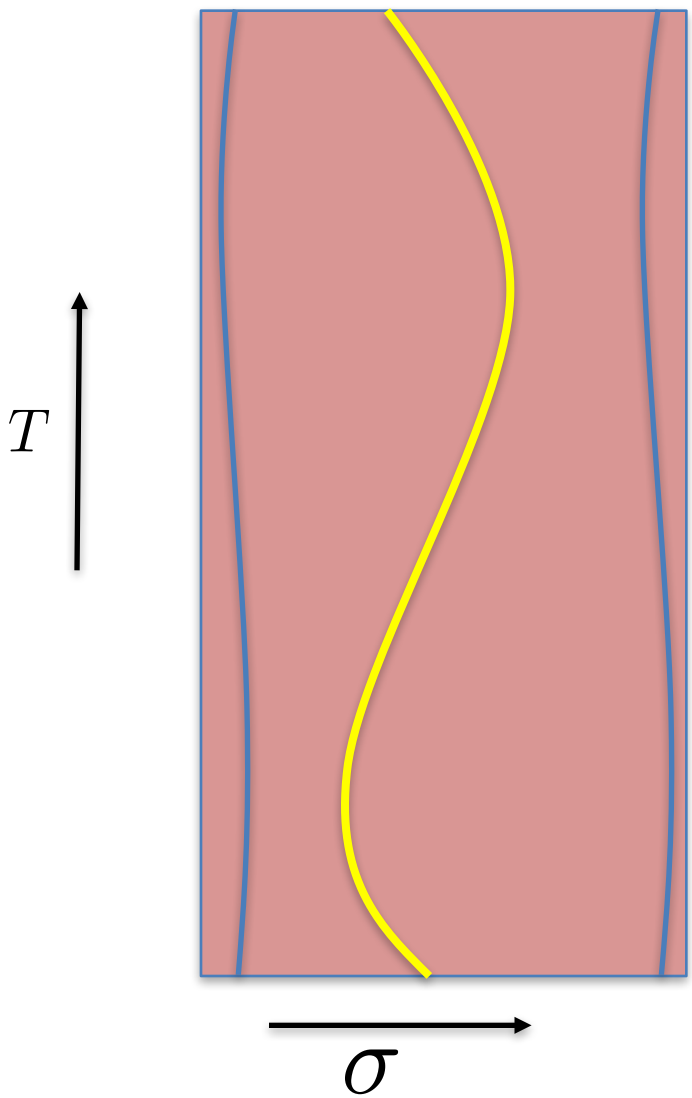
Making the TFD

- Create two SYK systems.
- Couple term. $\mu \neq 0$
- Couple them further to a heat sink and let them cool down to find its ground state.
- At t=0, turn off the left-right coupling. $\mu = 0$

- → Get a state that is close to the TFD.



Matter oscillations



Bulk matter particle → Oscillations → excitation goes from mostly from the left SYK to mostly on the right SYK.

Governed by conformal symmetry.

Additional comment

- Even if the couplings are different, we still get a solution which looks connected.
- The energy gap becomes smaller if the couplings are different, it decreases as the couplings get less correlated.
- We do not need perfect matchings of energies to build a state that is close to the TFD double, or that behaves as if the gravity dual was connected.

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Conclusions

- As a variant of the Gao-Jafferis-Wall teleportation idea, we can generate states that lead to traversable wormholes, similar to AdS_2 in global coordinates.
- Can be analyzed in the SYK context.
- Discussed thermal aspects and the phase transition.
- Realized a state close to the TFD as the ground state of the coupled system.

Future

- In this case we had N fields in the bulk.
- To fully find an "ordinary gravity" example, we need to think about cases with a small number of bulk fields.
- It would also be nice to find examples of near extremal black holes in asymptotically flat or $AdS_{d>2}$ spacetimes which are connected by these wormholes. (No causality violation).
- With rotating black holes ?