Supertubes and the 4D black hole

Per Kraus, UCLA

with I. Bena: hep-th/0402144, hep-th/0408186, hep-th/0502xxx

Introduction

Goal

- Find smooth sugra solutions for ground states of 4D black hole, i.e. D1-D5-KK system

Motivations

- Interesting to find smooth geometries for $\frac{1}{8}$ BPS 3-charge systems
- Will provide geometries dual to chiral primaries of $(4, 0)$ CFT.
- For zero entropy, BPS black rings are singular. Near ring geometry is that of D1-D5-KK. Want to resolve this singularity to find smooth microstate geometries.
- Relation to black hole entropy Mathur

Results

- We'll find all $U(1) \times U(1)$ invariant geometries (chiral primaries with equal length cycles)
- Solutions are asymptotically flat in 4D
- Solutions carry electric and magnetic charges

$$N_e = \frac{N_1 N_5}{n}, \quad N_m = N_K$$

and angular momentum

$$J = \frac{1}{2} \frac{N_1 N_5 N_K}{n} = \frac{1}{2} N_m N_e$$

- Same J as for widely separated electric and magnetic charges. Marginal stability.

Naive metric for D1-D5-KK

- Recall metric of KK-monopole (Gross, Perry; Sorkin)

$$ds^2 = -dt^2 + d\vec{x}_\perp^2 + ds_{TN}^2$$

$$ds_{TN}^2 = Z_K(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) + \frac{1}{Z_K}(R d\psi + Q_K(1 - \cos\theta)d\phi)^2$$

$$Z_K = 1 + \frac{Q_K}{r}$$

- Absence of Dirac string $\Rightarrow Q_K = \frac{1}{2} N_K R$.
- Small r behavior:

$$ds_{TN}^2 \approx d\tilde{r}^2 + \tilde{r}^2(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\psi}^2 + \cos^2\tilde{\theta} d\tilde{\phi}^2),$$

$$(\tilde{\psi}, \tilde{\phi}) \cong (\tilde{\psi} + \frac{2\pi}{N_K}, \tilde{\phi} + \frac{2\pi}{N_K})$$

So have R^4/Z_{N_K} singularity at origin.

Naive metric for D1-D5-KK cont.

- $D1 - D5 - KK$ metric given by harmonic function rule

$$ds^2 = \frac{1}{\sqrt{Z_1 Z_5}}(-dt^2 + dx_5^2) + \sqrt{Z_1 Z_5}\, ds^2_{TN} + \sqrt{\frac{Z_1}{Z_5}}\, ds^2_{T^4}$$

$$Z_{1,5} = 1 + \frac{Q_{1,5}}{r}$$

- Near horizon geometry is

$$AdS_3 \times S^3/Z_{N_K} \times T^4, \quad \ell^2_{AdS} \sim \sqrt{Q_1 Q_5}\, Q_K$$

- With compact x_5, geometry is singular

$$ds^2_{AdS} = \frac{\ell^2}{z^2}(-dt^2 + dz^2 + dx_5^2), \quad x_5 \cong x_5 + 2\pi R_5, \quad \text{singular}$$

D1-D5-KK CFT

- Recall that $D1 - D5$ CFT has $(4,4)$ susy with R-symmetry

$$SU(2)_L \times SU(2)_R \approx SO(4) \quad \text{from } S^3$$

Replacing $S^3 \to S^3/Z_{N_K}$, R-symmetry is reduced as

$$SU(2)_L \times SU(2)_R \to SU(2)_L$$

corresponding to $(4,0)$ susy.

- central charge is $c = 6N_1 N_5 N_K$.
- Ramond ground states carry angular momentum

$$-\frac{c}{12} \le J \le \frac{c}{12} \quad \Rightarrow \quad |J| \le \frac{1}{2} N_1 N_5 N_K$$

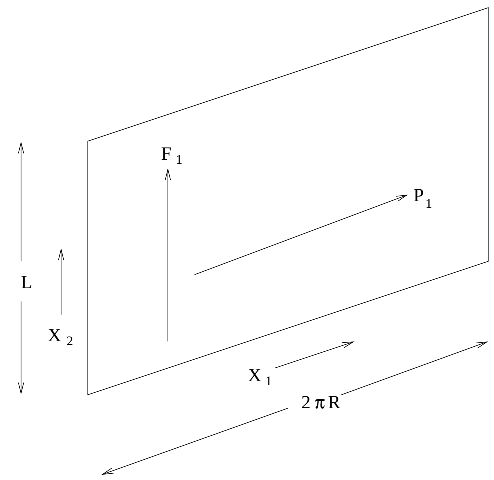
- Chiral primaries related to Ramond ground states by spectral flow (bulk diffeomorphism).
- Bulk geometries will be nonsingular due to expansion into KK-supertube: $D1 - D5 \to kk$.

Review of supertubes *(Mateos, Townsend)*

Start with a flat Dp-brane in $x^{0,1,\dots p}$, and turn on worldvolume electric and magnetic fields

$$2\pi F_{02} = 1, \qquad 2\pi F_{12} = B$$

Induces F1-strings, D(p-2)-branes, and P_1:



$$\begin{aligned} N_{p-2} &\approx BRL \\ N_{F1} &\approx RT_p/B \\ P_1 &\approx RLT_p \end{aligned}$$

- $N_{p-2}N_{F1} - J = 0, \quad J \equiv P_1 R$

Born-Infeld action gives

$$\mathcal{L}_{BI} = -(-\det[\eta_{\mu\nu} + 2\pi F_{\mu\nu}])^{1/2} \approx -B$$

and so the energy is

$$\mathcal{H} = \pi_E F_{02} - \mathcal{L}_{BI} = Q_{F1} + Q_{p-2}$$

- BPS, and no contribution from Dp-brane

Open string quantization

Fluxes described by open string metric:

$$\langle X^\mu(\tau_1)X^\nu(\tau_2)\rangle = -G^{\mu\nu}\ln|\tau_1-\tau_2|^2 + \frac{i}{2}\theta^{\mu\nu}\epsilon(\tau-\tau')$$

$$G^{\mu\nu} = \begin{pmatrix} -1+B^{-2} & -B^{-1} & 0 \\ -B^{-1} & 0 & 0 \\ 0 & 0 & B^{-2} \end{pmatrix}$$

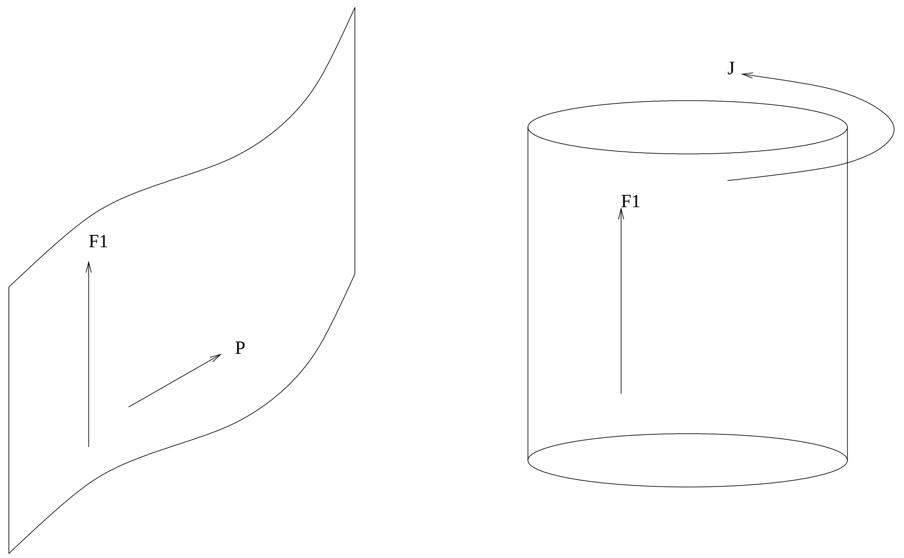
- $G^{11} = 0! \quad \Rightarrow \quad \langle X^1(z_1)X^1(z_2)\rangle = 0$

So we can start with a zero momentum vertex operator $\epsilon_\mu \partial_{n,t} X^\mu$ and attach a factor $e^{ip_1 X^1}$ to get a dimension 1 primary

$$V = \epsilon_\mu \partial_{n,t} X^\mu e^{ip_1 X^1}, \quad G^{\mu\nu}\epsilon_\mu p_\nu = 0$$

- Adds momentum P_1 but no energy or other charge.
- Multiple such operators can be added, and exponentiated

The Dp-brane can change its shape and local flux density at no cost in energy



- In the tubular case J is angular momentum. For a circular tube

$$J = N_{p-2} N_{F1}$$

- Supertube radius is $R^2 \sim g_s$, so at weak coupling the tube structure is lost. Makes counting at weak coupling more subtle.

- But since tubes become large at strong coupling, they are more directly related to finite size gravitational description.

3-charge supertubes *(Bena, P.K.)*

To compare with black hole physics would like a tube carrying D1-D5-P charges. But more convenient to dualize and take D0-D4-F1 since F1 appears in supertube construction.

Starting from

$$D0 + F1 \quad \to \quad d2$$

and dualizing, we have

$$\begin{aligned} D4 + F1 &\quad \to \quad d6 \\ D0 + D4 &\quad \to \quad ns5 \end{aligned}$$

- So we expect a tube with 3 independent dipole charges: d2, d6, and ns5.
- In absence of ns_5 can find Born-Infeld description in terms of fluxes on $d6$-brane. Can include ns_5 by T-dualizing ns5 $\to$ kk $\approx A_N$ singularity. Or, work in M-theory (Elvang et. al.)

Supertube with ns5 dipole charge

- Including NS5 in the flat case yields a brane carrying charges D2-D6-NS5-P. These yield a 4d black hole after compactification on T^6.
- Entropy given by quartic $E_{7(7)}$ invariant:

$$S = 2\pi\sqrt{J_4}$$

$$\begin{aligned} -J_4 &= x^{ij}y_{jk}x^{kl}y_{li} - x^{ij}y_{ij}x^{kl}y_{kl}/4 \\ &+ \epsilon^{ijklmnop}(x^{ij}x^{kl}x^{mn}x^{op} + y^{ij}y^{kl}y^{mn}y^{op}) \end{aligned}$$

with the charges identified as

$$\begin{aligned} x_{12} &= N_{D0}, \quad x_{34} = N_{D4}, \quad x_{56} = N_{F1}, \quad x_{78} = 0 \\ y^{12} &= n_{d6}, \quad y^{34} = n_{d2}, \quad y^{56} = n_{ns5}, \quad y^{78} = J \end{aligned}$$

- System now has finite size $S^2 \times T^6$ horizon. As before, we can instead curl up one direction into a circle and compactify on T^5. Result should be a horizon of topology $S^1 \times S^2$ in $D = 5$ — a black ring. Entropy should agree with above. Related approach (Cyrier, Guica, Mateos, Strominger).

Black rings in D1-D5 CFT

Supergravity solution for 3-charge supertube was found by (Elvang, Emparan, Mateos, Reall) and generalized further by (Bena, Warner; EEMR; Gauntlett, Gutowski)

In IIB frame solutions carries charges

$$N_1 \quad D1(5), \quad N_2 \quad D5(56789), \quad N_3 \quad P(5)$$

and dipole charges

$$n_1 \quad d5(x6789), \quad n_2 \quad d1(x), \quad n3 \quad kk(x56789)$$

- N_i are conserved charges measured at infinity. Ring itself is best thought of as made up of charges $\overline{N}_i$:

$$\overline{N}_1 = N_1 - n_2 n_3, \quad \text{and permutations}$$

- “Harmonic" functions Z_i are no longer harmonic

$$Z_1 = 1 + \frac{\overline{Q}_1}{\Sigma} + \frac{q_2 q_3 \rho^2}{\Sigma^2}$$

with $\Sigma = \sqrt{(\rho^2 - R^2)^2 + 4R^2\rho^2\cos^2\theta}$.

- $1/\Sigma$ is a harmonic function sourced on the ring: $\rho = R, \ \cos\theta = 0$. $R = 0$ gives BMPV.

- Solution carries angular momenta

$$J_\phi = J_{BMPV}, \quad J_\psi = -J_{BMPV} + J_{tube}$$

$$J_{BMPV} = -\frac{1}{2}\Sigma\, n_i \overline{N}_i - n_1 n_2 n_3, \quad J_{tube} \sim (q_1 + q_2 + q_3) R^2$$

- Entropy is

$$\begin{aligned} S &= 2\pi[-\frac{1}{4}(n_1^2\overline{N}_1^2 + n_2^2\overline{N}_2^2 + n_3^2\overline{N}_3^2) \\ &+ \frac{1}{2}(n_1 n_2 \overline{N}_1 \overline{N}_2 + n_1 n_3 \overline{N}_1 \overline{N}_3 + n_2 n_3 \overline{N}_2 \overline{N}_3) - n_1 n_2 n_3 (J_\psi + J_\phi)]^{1/2} \\ &= 2\pi\sqrt{J_4} \end{aligned}$$

- Solutions have 7 free parameters, but only 5 conserved charges. So these black objects have “hair". Makes it especially interesting to understand them on gauge theory side.

Decoupling limit

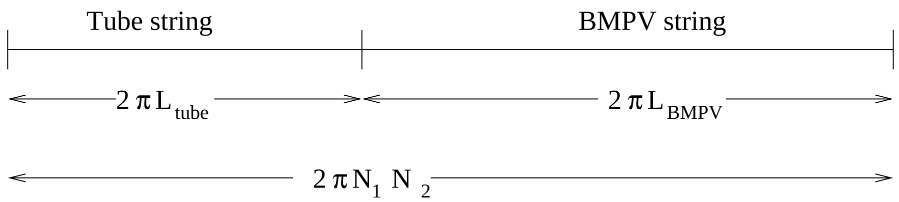
- As with usual D1-D5-P system, we drop the 1 from the D1 and D5 harmonic functions, but keep it in the P harmonic function.
- Solution is then asymptotic to the same $\mathrm{AdS}_3 \times S^3 \times T^4$ as for usual D1-D5-P, so we should be able to understand the black rings as states in the usual CFT.
- Work at orbifold point. Have an effective string of length $N_1 N_2$ which can be broken up into any number of integer length components. Each component has 4 bosons and 4 fermions. Fermions are doublets under $SO(4) \approx SU(2)_L \times SU(2)_R$ R-symmetry (rotation) group.
- Diagonal generators are

$$J_L = J_\psi - J_\phi, \quad J_R = J_\psi + J_\phi$$

- Black rings combine properties of BMPV and 2-charge supertubes, and we know how to describe these at orbifold point, so can hope for same with rings.

Black ring entropy in D1-D5 CFT

- Natural to divide effective string into a tube part and a BMPV part:



- Tube string further breaks up into components of length ℓ_c, and carries J_{tube} but no entropy. BMPV string carries J_{BMPV} and all entropy.
- L_{tube} fixed by $\frac{L_{tube}}{\ell_c} = J_{tube}$.
- $\ell_c = n_3$ for large class of states, but in general need to make phenomenological assumption for ℓ_c. Testable via time delay experiments.
- With this assumption, black ring entropy then takes BMPV form in terms of J_{BMPV}, L_{BMPV}, N_3, and angular momenta are correctly reproduced.

Near ring geometry

- In the UV (AdS boundary) we have the usual $(4,4)$ CFT with $c_{UV} = 6N_1N_2$.
- In the IR (near the ring) the dipole charges dominate, and we see the CFT of the D1-D5-KK system with $(4,0)$ susy and $c_{IR} = 6n_1n_2n_3$.
- In between have a highly nontrivial RG flow. Note $c_{IR} < c_{UV}$.
- In simplest zero entropy case (microstate?) define

$$\tilde{\psi} = \psi - \frac{1}{q_3}x^+, \quad \tilde{\phi} = \phi + \frac{1}{q_3}x^+, \quad \tilde{x}^+ = q_3\psi$$

to yield near ring

$$AdS_3 \times S^3/Z_{n_3} \times T^4$$

with

$$\ell_{AdS}^2 = \ell_{S^3}^2 = q_1q_2q_3^2, \quad V_{T^4} \sim (\frac{q_1}{q_2})^{1/2}$$

- Old angular coordinate becomes new coordinate parallel to AdS
- $\tilde{x}^+$ compact and cycle shrinks to zero size: singular.

Geometries for D1-D5-KK

- Look for solutions with Taub-NUT base metric (T^4 suppressed)

$$ds^2 = \frac{1}{\sqrt{Z_1 Z_5}}[-(dt+k)^2 + (dx_5 - k - s)^2] + \sqrt{Z_1 Z_5} ds^2_{TN}$$

$$ds^2_{TN} = Z_K(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) + \frac{1}{Z_K}(R d\psi + Q_K \cos\theta d\phi)^2$$

- Demand $U(1) \times U(1)$ symmetry:

$$k = k_\psi(r,\theta) d\psi + k_\phi(r,\theta) d\phi, \quad \text{same for s}$$

and $Z_{1,5} = Z_{1,5}(r,\theta)$.

- BPS equations reduce to (e.g. Gauntlett, Gutowski, Hull, Pakis, Reall)

$$ds = \star_4 ds, \quad da = -\star_4 da, \quad \nabla^2 Z_{1,5} = 0$$

(away from sources) with $a = k + \frac{1}{2}s$.

- We take

$$Z_{1,5} = 1 + \frac{Q_{1,5}}{\Sigma}, \quad \Sigma = (r^2 + \tilde{R}^2 + 2\tilde{R} r \cos\theta)^{1/2}$$

corresponding to ring of branes around KK circle.

- Need to find closed (anti) self-dual 2 forms $\Theta^+ = ds,\ \Theta^- = da$. All such $U(1) \times U(1)$ invariant 2-forms can be related to harmonic functions on R^3 base of Taub-NUT.

- Write Taub-NUT as

$$ds^2 = Z_K d\vec{x}^2 + \frac{1}{Z_K}(R d\psi + \vec{A} \cdot \vec{x})^2, \quad Z_K = 1 + \frac{Q_K}{|\vec{x}|}$$

Let P^- and $Z_K P^+$ be harmonic functions (with sources). Then 2-forms are

$$\Theta^{\pm}_{\psi i} = R \partial_i P^{\pm}, \quad \Theta^{\pm}_{ij} = A_i \partial_j P^{\pm} - \partial_i P^{\pm} A_j + Z_K \epsilon_{ij}{}^{k} \partial_k P^{\pm}$$

- k and s obtained by integration.

- Take general form

$$P^- = c_1 + \frac{c_2}{r} + \frac{c_3}{\Sigma}$$

$$Z_K P^+ = d_1 + \frac{d_2}{r} + \frac{d_3}{\Sigma}$$

  Fix coefficients by demanding smoothness and asymptotic flatness. Potential singularities at $r = 0$, $\Sigma = 0$, and Dirac-Misner strings at $\sin\theta = 0$.

- All free coefficients, as well as ring radius $\tilde{R}$ are uniquely fixed.

Properties of solutions

- Ring radius $\tilde{R}$ determined by

$$1 + \frac{Q_K}{\tilde{R}} = \frac{R_5^2}{4Q_1Q_5}$$

- Get 4D metric after KK reduction on x_5 and ψ.
- Mass given by

$$M = Q_1 + Q_5 + Q_K$$

- Gauge field $A^{(\psi)}$ carries both electric and magnetic charge

$$N_e = N_1 N_5, \quad N_m = N_K$$

- Angular momentum is

$$J = \frac{1}{2} N_K N_1 N_5 = \frac{1}{2} N_e N_m$$

- Easy to generalize solutions to allow for Z_n singularity at $\Sigma = 0$, corresponding to n coincident KK monopole rings. Only effect on charges is that J and N_e are reduced by n. Relation $J = \frac{1}{2} N_e N_m$ maintained.

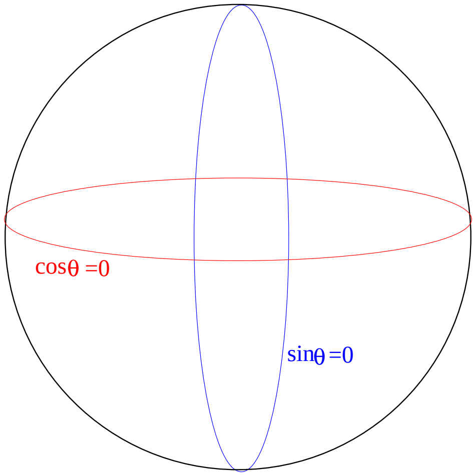
Near horizon decoupling limit

- Check identifications by taking near horizon limit. More complicated than just omitting 1s from harmonic functions.
- After series of coordinate transformations, find near horizon $AdS_3 \times S^3/Z_{N_K} \times T^4$, with Z_n conical defect

$$ds^2 = -(1+\frac{\tilde{r}^2}{\ell^2})dt^2 + \frac{dr^2}{(1+\frac{\tilde{r}^2}{\ell^2})} + \tilde{r}^2 d\chi^2 + \ell^2(d\tilde{\theta}^2 + \sin^2\theta d\tilde{\psi}^2 + \cos^2\tilde{\theta} d\tilde{\phi}^2)$$

$$(\tilde{\psi},\tilde{\phi}) \cong (\tilde{\psi}+\frac{2\pi}{N_K}, \tilde{\phi}+\frac{2\pi}{N_K}), \quad (\chi,\tilde{\phi}) \cong (\chi+\frac{2\pi}{n}, \tilde{\phi}+\frac{2\pi}{n})$$

- Sources for harmonic functions are mapped to nonintersecting circles on S^3:



$$\begin{aligned} r &= 0 \Rightarrow (\tilde{r}=0, \sin\tilde{\theta}=0) \\ \Sigma &= 0 \Rightarrow (\tilde{r}=0, \cos\tilde{\theta}=0) \end{aligned}$$

Comments and questions

- Found nonsingular 3-charge solutions representing the ground states of the D1-D5-KK system (a.k.a. 4D black hole).
- Can dualize to smeared D1-D5-P system.
- Solutions may resolve singularity of zero entropy black rings.
- What is generalization to non $U(1) \times U(1)$ invariant geometries? Probably need to deform Taub-NUT base.