Can AdS/CFT be useful for heavy-ion physics?

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A review of many people’s work

seminar at KITP, UCSB

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What is heavy-ion physics?

Heavy-ion collisions — study QCD at high energy density

Experiment: RHIC (Brookhaven, NY)
- Started in year 2000
- Collides Au nuclei
- CM energy $\sqrt{s} = 200 \text{ GeV per nucleon}$

Quest: find and study QGP [deconfined state of QCD] → field theory at finite temperature and density. Not obvious a priori that a thermal state will be produced.

Evidence for thermalization [lots of data and non-trivial calculations]:
- Particle abundances and ratios — reproduced by statistical models
- Elliptic flow — reproduced by hydrodynamic models
- Jet quenching — indication of short mean free path

Optimistically, QGP is hidden in the collision
What is AdS/CFT

(J. Maldacena hep-th/9711200, review: hep-th/9905111)

large $N_c$, $d=4$, $\mathcal{N}=4$ SYM $\mapsto$ IIB strings on $AdS_5 \times S^5$

$\lambda \leftrightarrow \left(\frac{R^2}{\alpha'}\right)^2$ string corrections to SUGRA

$\frac{\lambda}{4\pi N_c} \leftrightarrow g_s$ string loops

AdS/CFT is a tool to define/perform calculations in field theory

Applies to field theories beyond $\mathcal{N}=4$ SYM
RHIC + AdS/CFT = ♥ ?

Modest: Can AdS/CFT be useful to understand finite-temperature QCD?

Bold: Can AdS/CFT be useful to understand the dynamics of the collision?
**RHIC + AdS/CFT = ♥ ?**

**Modest:** Can AdS/CFT be useful to understand finite-temperature QCD?

**Bold:** Can AdS/CFT be useful to understand the dynamics of the collision?

**Comments**

- $\mathcal{N}=4$ SYM is the simplest example. Theories which are more similar to QCD can be treated by AdS/CFT methods.

- Application of AdS/CFT to thermal QCD is not exhausted. How far can we push this program?

- It is not a waste of time to do these calculations. Results are relatively easy to derive compared to the conventional methods.

- If there were an effective tool to do real-time computations in strongly coupled QCD at finite $T$ and $\mu$ — no need to invoke AdS/CFT. In the absence of such a tool, AdS/CFT is the best we have (for some questions).

- Understanding finite-temperature field theory is an interesting question by itself. AdS/CFT can be useful in searching for universal properties (shear viscosity example is encouraging).
**Why \( N=4 \) SYM may have something to do with thermal QCD**

**Q:** \( N=4 \) SYM is supersymmetric, while QCD is not  
**A:** At finite temperature, supersymmetry is broken anyway

**Q:** \( N=4 \) SYM is conformal, while QCD is asymptotically free  
**A:** Let’s look at the thermodynamics of QCD (e.g. F.Karsch, [hep-lat/0106019](https://arxiv.org/abs/hep-lat/0106019))

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**Graph:**

- **Left graph:**
  - \( \varepsilon/T^4 \) and \( \varepsilon_{SB}/T^4 \) as functions of \( T/T_c \)
  - Curves for 3 flavour, 2+1 flavour, and 2 flavour

- **Right graph:**
  - \( \varepsilon/\varepsilon_{SB} \) as a function of \( T/T_c \)
  - Curves showing convergence to a value
Why $\mathcal{N}=4$ SYM may have something to do with thermal QCD

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$T_c \approx 170$ MeV :: QCD perturbation theory is not to be trusted for $T \gtrsim T_c$

When depart from thermodynamics — real-time on the lattice is hard

Use strongly coupled $\mathcal{N}=4$ SYM as a model for QCD at $T \gtrsim T_c$
Will discuss application of AdS/CFT to:

- Momentum transport
- Electromagnetic response
- Energy loss by a heavy probe
- Thermalization

- AdS/CFT has more to say!
Momentum transport

- Conservation laws: \( \partial_{\mu}T^{\mu\nu} = 0 \) \( \Rightarrow \) \[
\begin{align*}
\partial_t \epsilon &= -\nabla \cdot \pi \\
\partial_t \pi^i &= -\nabla_j T^{ij}
\end{align*}
\]

- Constitutive relations:

\[
\begin{cases}
T^{ij} = \delta^{ij} \left[ \langle P \rangle + v_s^2 \delta \epsilon - \gamma_\zeta \nabla \pi \right] - \gamma_\eta \left( \nabla^i \pi^j + \nabla^j \pi^i - \frac{2}{3} \delta^{ij} \nabla \pi \right) + \ldots \\
\gamma_\eta \equiv \frac{\eta}{\langle \epsilon + P \rangle}, \quad \gamma_\zeta \equiv \frac{\zeta}{\langle \epsilon + P \rangle}, \quad v_s^2 = \partial P / \partial \epsilon
\end{cases}
\]

- Viscosities \( \eta, \zeta \) — input from microscopic physics

Two eigenmodes:

Shear mode: \( \pi_{\perp}(t, \mathbf{k}) = e^{-\gamma_\eta k^2 t} \pi_{\perp}(0, \mathbf{k}) \)

Sound mode: \( \pi_{\parallel}(t, \mathbf{k}) = e^{-\frac{1}{2}(\gamma_\zeta + \frac{4}{3} \gamma_\eta) k^2 t} \times \left[ \pi_{\parallel}(0, \mathbf{k}) \cos(kv_s t) - ikv_s \sin(kv_s t) \delta \epsilon(0, \mathbf{k}) \right] \)

Long-wavelength response is controlled by a small number of kinetic coefficients
Correlation functions in the hydrodynamic limit

Hydrodynamic modes ⇒ hydrodynamic singularities at small \( \omega, k \).

Example: \( S_{tx,tx}(\omega, k) = \frac{2\gamma \eta k^2}{\omega^2 + (\gamma \eta k^2)^2} (\epsilon + P) T \) relaxation of transverse momentum

Kubo formulas

\[
\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt \, e^{i\omega t} \int d^3 x \, \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0)] \rangle
\]

\[
\frac{4}{3} \eta + \zeta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt \, e^{i\omega t} \int d^3 x \, \langle [T_{xx}(t, \mathbf{x}), T_{xx}(0)] \rangle
\]

Connection to microscopic physics: Viscosities can be extracted from (small-frequency limits of) real-time correlation functions
Spectral function for stress

\[ \frac{1}{\omega} (\chi(\tilde{\omega}) - \chi^{T=0}(\tilde{\omega})) \left[ \frac{1}{\pi^2 N_c^2 T^4} \right] \]

\[ \chi(\omega, k) = -2 \text{Im} G_{xy,xy}^{\text{ret}}(\omega, k) \]

\[ \chi(\omega) \sim \omega, \quad \omega \ll 2\pi T \]

\[ \chi(\omega) - \chi^{T=0}(\omega) \sim e^{-\gamma \omega}, \quad \omega \gg 2\pi T \]

\[ \eta = \frac{\pi}{8} N_c^2 T^3 \]

\( T^3 \) by conformal invariance, \( N_c^2 \) counts d.o.f.

Spectral function for conserved energy-momentum

\[ \chi_{tx,tx}(\tilde{\omega}) \]

\[ \chi_{tt,tt}(\tilde{\omega}) \]

Hydrodynamic peaks clearly visible in dual classical gravity
Singularities of $G^{\text{ret}}(\omega, k)$

(A.Nunez, A.Starinets hep-th/0302026, PK, A.Starinets hep-th/0506184)

- Infinite series of poles
- $\omega_n = 2\pi n T (\pm 1 - i)$ as $n \to \infty$
- For conserved densities, $\omega_0 \to 0$ as $k \to 0$
- Hydro poles agree with Kubo formula

Singularities of $G^{\text{ret}}(\omega, k)$ are (quasi)normal modes of the dual gravity background
Real-time correlators are very different at strong and weak coupling.

\[ \chi(\tilde{\omega})/\tilde{\omega} \left[ \frac{1}{\pi^2 N_c^2 T^4} \right] \]

Red — strongly coupled SYM, \( \eta = O(1) \)
Dashed — free SYM, \( \eta = \infty \)
Purple — weakly coupled SYM, \( \eta \sim \frac{1}{\lambda^2} \)
Blue — SYM, \( T = 0 \)

Euclidean correlators are almost the same!

(picture from D. Teaney, hep-ph/0602044)

\[ G_E(\tau) = \int_0^\infty d\omega \chi(\omega) \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh(\beta \omega/2)} \]

Lattice \( G_E(\tau) \) has errorbars \( \sim 500\% \) (hep-lat/0406009)

\[ \therefore \text{Lattice determination of shear viscosity has no chance} \]
Universality: Lower bound on shear viscosity?

(PK, D. Son, A. Starinets hep-th/0405231)

η/s ∝ 1 at small coupling

η/s = 1/4π is finite at large coupling

η/s ≥ 1/4π in SYM

Is \[ \frac{\eta}{s} \geq \frac{1}{4\pi} \] universal?

We know \( \frac{\eta}{s} = \frac{1}{4\pi} \) is universal — proven for a large class of (but not all) field theories with gravity duals. Prove from first principles?
What is the viscosity measured at RHIC?

Of course, RHIC does not “measure” viscosity. Rather, measured angular distributions of particles are confronted with hydrodynamic models of the “fireball” evolution. Quantitatively: lentil-shaped reaction region ⇒ azimuthal anisotropy of particle distribution

$$\frac{d^2 N}{dp_T d\phi} = N_0 [1 + 2 v_2(p_T) \cos(2\phi) + \ldots]$$

“elliptic flow”

Elliptic flow from PHENIX and STAR (figure from nucl-ex/0501009). Yellow band — hydro calculations. To reproduce elliptic flow from hydro, $\eta/s$ must be small, within a factor of 4 of the conjectured bound. Perturbative $\eta/s$ is too high.

Hydro works well at low $p_T$, but not at high $p_T$

Optimistically: $\mathcal{N}=4$ SYM does very well with $\eta/s$
Will discuss application of AdS/CFT to:

Momentum transport

→ Electromagnetic response

Energy loss by a heavy probe

Thermalization

AdS/CFT has more to say!
How brightly does a plasma glow?

$\Gamma$ — number of photons per unit time per unit volume

Photon interaction: $e J^\text{EM}_\mu A^\mu$

e small $\Rightarrow$ photons do not thermalize

$$d\Gamma = \frac{d^3k}{(2\pi)^3} \frac{e^2}{2|k|} \eta^{\mu\nu} C^<_{\mu\nu}(k) \bigg|_{\omega=|k|} \text{ where } C^<_{\mu\nu}(x) = \langle J^\text{EM}_\mu(0) J^\text{EM}_\nu(x) \rangle$$

Wightman function: $C^<_{\mu\nu}(k) = -2n_B(\omega) \text{Im}C^\text{ret}_{\mu\nu}(k)$

\[ \therefore \text{Photon spectrum is determined by EM current-current spectral function} \]

true to leading order in $e$, but to all orders in $g$

Perturbative evaluation of $\frac{d\Gamma}{d^3k}$ is not easy, see e.g. P.Arnold, G.Moore, L.Yaffe, hep-ph/0111107
How brightly does $\mathcal{N}=4$ plasma glow?
(PK, A.Starinets, to appear)

But wait... $\mathcal{N}=4$ SYM does not have a photon

[\(U(1)\) gauge field coupled to a conserved current]

Let's introduce one! To do so:

Gauge a $U(1)$ subgroup of $SU(4)$ R-symmetry with coupling $e$

Add spectators $\psi_s$ to cancel the anomaly

Take $e$ to be small

$$J^\text{EM}_\mu = J^R_\mu + \sum \bar{\psi}_s \gamma_\mu \gamma_5 \psi_s$$

conserved non-anomalous couples to a $U(1)$ gauge field

To lowest order in $e$:

$$\langle J^\text{EM}_\mu J^\text{EM}_\nu \rangle = \langle J^R_\mu J^R_\nu \rangle + \text{spectator loop does not contribute to photon production (one-photon emission kinematically forbidden in a free theory)}$$

Photon emission rate is given by R-current spectral function
Photon emission spectrum of $\text{SYM}_{\text{EM}}$

Thick line: \[ \frac{x^3}{e^x-1} \]
Thin line: \[ \frac{x^3}{e^x-1} f_{\text{SYM}}(x) \]

At small $x$: $f_{\text{SYM}}(x) \to \text{const}$, $\text{Im} \Pi_\mu^\mu(\omega=k) \sim k$, consistent with hydrodynamics

At large $x$: decays as $f_{\text{SYM}}(x) \sim \ln(1/x)$

Emission rate is finite and $\lambda$-independent at strong coupling
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PHENIX data for photon production (nucl-ex/0410003)

\[ \therefore \] Not much room left for thermal QGP photons — but more precise measurements are planned
Will discuss application of AdS/CFT to:

- Momentum transport
- Electromagnetic response
- Energy loss by a heavy probe
- Thermalization

AdS/CFT has more to say!
Heavy probe energy loss

(C.Herzog, A.Karch, PK, C.Kozcaz, L.Yaffe, to appear)

Probe of mass $M$, moving through a thermal medium with $T \ll M$
(think about charm quark $M \approx 1.3\, \text{GeV}$, moving in a plasma at $T = 200\, \text{MeV}$)

Model: $\mathcal{N}=4\, \text{SYM} + \text{fundamental matter } N_f \ll N_c$ (A.Karch, A.Katz, [hep-th/0205236])

<table>
<thead>
<tr>
<th>D7</th>
<th>boundary</th>
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<tbody>
<tr>
<td>$u = u_m$</td>
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<td>AdS$_5$-Schw.</td>
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<tr>
<td>$u = u_h$</td>
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$\overset{\text{boundary}}{\text{D7}}$

Static string: $E = \frac{\sqrt{\lambda}}{2\pi} u_m, \quad T = 0$
$E = \frac{\sqrt{\lambda}}{2\pi} (u_m - u_h), \quad T \neq 0$

| $u = u_m$ |
| thermal mass |
| $u = u_h$ |

Heavy quark $E \gg T$ — string is classical

To analyze energy loss, solve classical equations of motion for a moving string
Stationary analytic solution

Take infinitely heavy “quark” \((u_m \to \infty)\), move it at constant speed

String profile: \(x(u, t) = vt\) solves e.o.m.

\(-g\) flips sign at \(u^4 = \frac{u_h^4}{1-v^2} > u_h^4\)

\(E, P\) become complex — solution unphysical

\[x(u, t) = X(u) + vt\]

Can find \(X(u)\) s.t. \(-g\) is positive everywhere

Source moves at constant speed

Momentum pumped in at the boundary

Momentum leaks off at the horizon

\[
\frac{dP}{dt} = -\pi \frac{1}{2\pi} \bigg|_{u=u_h} = -\sqrt{\lambda} \frac{v}{2\pi \sqrt{1-v^2}} (\pi T)^2 = -\left(\frac{\sqrt{\lambda T^2 \pi}}{2M}\right) \left(\frac{Mv}{\sqrt{1-v^2}}\right) = -\mu P
\]
Numerical solution

\[ \frac{dP}{dt} = -\mu P \]

- Both non-relativistic and relativistic quarks
- Drag coefficient independent of momentum
- Does not have a finite \( \lambda \to \infty \) limit

Rough estimate: \( M/T \approx 7 \), \( \alpha_s \approx 0.5 \), \( \lambda \approx 20 \) gives drag coefficient \( \mu \approx T \)

Perturbative \( \mu \), see e.g. G. Moore, D. Teaney, hep-ph/0412346, \( \mu \approx T/7 \)
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PHENIX data for electrons from heavy quark decay (nucl-ex/0510047)

Perturbative gluon radiation under-estimates energy loss for heavy quarks (but does well for light quarks)

Currently active area of research
Will discuss application of AdS/CFT to:

Momentum transport
Electromagnetic response
Energy loss by a heavy probe

→ Thermalization

AdS/CFT has more to say!
Thermalization

Thermal equilibrium in a collision established? If so:
On the partonic level at early stage, or on the hadronic level at a later stage?
At what temperature? How does the state evolve?

From AdS/CFT perspective [both solid results and speculations] :


however:

Any attempt to understand thermalization in heavy-ion collisions from AdS/CFT must:

– be able to distinguish between hadron-hadron and Au+Au collisions
– quantitatively understand the role of finite $N_c$

AdS/CFT is not (currently) useful for understanding thermalization
Will discuss application of AdS/CFT to:

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- Electromagnetic response
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- Thermalization

→ AdS/CFT has more to say!
A sample of challenges for AdS/CFT
(both easy and hard)

• Deformations of thermal $\mathcal{N}=4$ SYM [more realistic models for QCD]
• Finite ’t Hooft coupling corrections to $\mathcal{N}=4$ SYM [quasiparticle dynamics]
• Real-time dynamics with flavor [heavy-quark resonances]
• Finite $N_c$ corrections to $\mathcal{N}=4$ SYM [relaxation timescales]
• Non-linear hydrodynamics [classical gravity not enough]
• Gauge-field dynamics [effective theories]
• Partonic structure [initial state for the collision]
• Reason for universality of $\eta/s$ [or any other strong-coupling universality]
• Membrane paradigm and AdS/CFT [project for a GR raduate student]
• Black hole singularity [power vs exp decay of correlators at large $\omega$]

Can AdS/CFT be useful for heavy-ion physics?